Computer Exercises OAF2

Computational accuracy

Compile and run the FORTRAN program machine.f.

Create, compile and run small FORTRAN programs for the examples given in BASIC and test the accuracy issues of the computer venus.

Sampling

Exercise

Computer problem: Reconstruction of a Gaussian profile through optimum sampling.

We investigate the sampling and reconstruction of a function. M(x) is a spectral line, broadened by the observation with a Gaussian profile. For simplicity the central wavelength x_0 is set to 0. We now sample this function at x_n with intervals τ where n=-N,-N+1,....+N. According to the optimal sampling theorem the reconstructed function $M_R(x)$ can be written as

$$M_R(x) = \sum_{n=-N}^{N} \mathrm{sinc}(\frac{x - x_n}{\tau}) M(x_n) \tag{1}$$

a: Write a computer code that computes this reconstruction. In addition make the code compute the difference between the reconstructed and the corrected value, $d(x_g) \equiv M_R(x_g) - M(x_g)$ at $x_g = (-3 + [i-1]0.1)\sigma)$ for i=1,61 (hence, $-3\sigma \leq x_g \leq 3\sigma$ and set $\sigma = 1$).

Investigate the effect of the number of samples. We sample at the Nyquist frequency if $\tau = \sigma$. Compute the root mean square difference between the reconstructed and the calculated (exact) function $e = \sqrt{\sum d(x_g)^2 / N_g}$ where N_g is the number of grid points used, for $N_g = 1, 2, 3, 5, 7$. *Note:* the reconstruction only makes sense between sampled points. Therefore compute e only for grid points $|x_g| < N\sigma$. At what value does the improvement of the reconstruction stop?

b: Now alter the program to allow for an offset with respect to zero of the linecenter. Repeat the test for N=2,3,5 for $x_0 = 0.1\sigma$ and $x_0 = 0.3\sigma$.

Finally investigate the effect of the sampling width. Repeat b: for sampling at τ/σ=0.25, 0.5, 1.3, 1.5, 2.0.

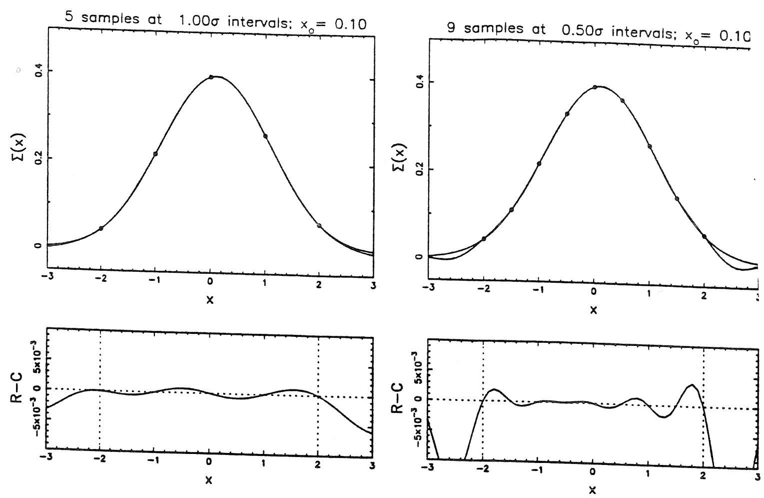

Figure 1: Example sinusoids and the Reconstructed - Computed panels.