Outline

1. Fundamentals of Polarized Light
2. Descriptions of Polarized Light
3. Scattering Polarization
4. Zeeman Effect
5. Hanle Effect

Electromagnetic Waves in Matter

- electromagnetic waves are a direct consequence of *Maxwell's equations*
- optics: interaction of electromagnetic waves with matter as described by *material equations*
- polarization properties of electromagnetic waves are integral part of optics

Plane-Wave Solutions

Plane Vector Wave ansatz

$$\vec{E} = \vec{E}_0 e^{i\left(\vec{k}\cdot\vec{x}-\omega t\right)}$$

- $\vec{k}$ spatially and temporally constant *wave vector*
- $\vec{k}$ normal to surfaces of constant phase
- $|\vec{k}|$ *wave number*
- $\vec{x}$ spatial location
- ω *angular frequency* ($2\pi\times$ frequency)
- t time
- $\vec{E}_0$ a (generally complex) vector independent of time and space
  - damping if $\vec{k}$ is complex
  - real electric field vector given by real part of $\vec{E}$

Polarization

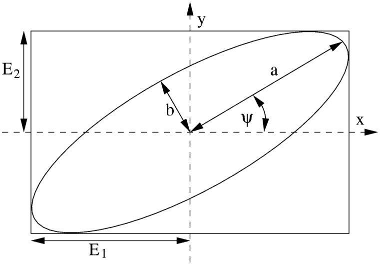
- spatially, temporally constant vector $\vec{E}_0$ lays in plane perpendicular to propagation direction $\vec{k}$
- represent $\vec{E}_0$ in 2-D basis, unit vectors $\vec{e}_1$ and $\vec{e}_2$, both perpendicular to $\vec{k}$

  $$\vec{E}_0 = E_1\vec{e}_1 + E_2\vec{e}_2.$$

  E_1, E_2: arbitrary complex scalars
- damped plane-wave solution with given ω, $\vec{k}$ has 4 degrees of freedom (two complex scalars)
- additional property is called *polarization*
- many ways to represent these four quantities
- if E_1 and E_2 have identical phases, $\vec{E}$ oscillates in fixed plane

Description of Polarized Light

Polarization Ellipse



Polarization

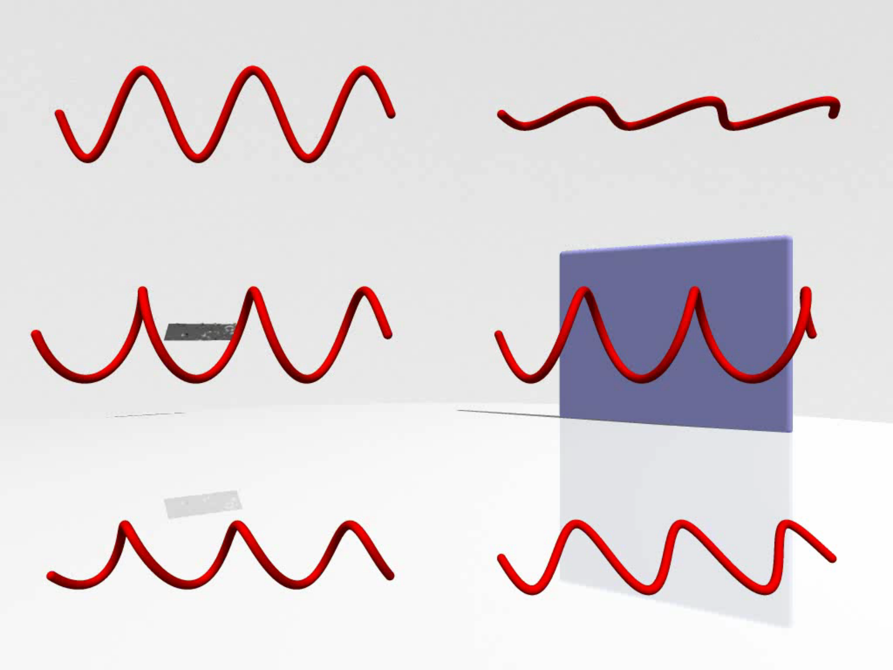
$$\vec{E}(t) = \vec{E}_0 e^{i\left(\vec{k}\cdot\vec{x}-\omega t\right)}$$

$$\vec{E}_0 = E_1 e^{i\delta_1}\vec{e}_x + E_2 e^{i\delta_2}\vec{e}_y$$

- wave vector in z-direction
- $\vec{e}_x$, $\vec{e}_y$: unit vectors in x, y directions
- E_1, E_2: (real) amplitudes
- $\delta_{1,2}$: (real) phases

Polarization Description

- 2 complex scalars not the most useful description
- at given $\vec{x}$, time evolution of $\vec{E}$ described by *polarization ellipse*
- ellipse described by axes a, b, orientation ψ



Jones Formalism

Jones Vectors

$$\vec{E}_0 = E_x \vec{e}_x + E_y \vec{e}_y$$

- beam in z-direction
- $\vec{e}_x$, $\vec{e}_y$ unit vectors in x, y-direction
- complex scalars $E_{x,y}$
- Jones vector

$$\vec{e} = \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

- phase difference between E_x, E_y multiple of π, electric field vector oscillates in a fixed plane $\Rightarrow$ *linear polarization*
- phase difference $\pm\frac{\pi}{2}$ $\Rightarrow$ *circular polarization*

Summing and Measuring Jones Vectors

$$\vec{E}_0 = E_x \vec{e}_x + E_y \vec{e}_y$$

$$\vec{e} = \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

- Maxwell's equations linear $\Rightarrow$ sum of two solutions again a solution
- Jones vector of sum of two waves $=$ sum of Jones vectors of individual waves if wave vectors $\vec{k}$ the same
- addition of Jones vectors: *coherent* superposition of waves
- elements of Jones vectors are not observed directly
- observables always depend on products of elements of Jones vectors, i.e. intensity

$$I = \vec{e} \cdot \vec{e}^{*} = e_x e_x^{*} + e_y e_y^{*},$$

Jones matrices

- influence of medium on polarization described by 2×2 complex *Jones matrix* J

$$\vec{e}' = \mathrm{J}\vec{e} = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix} \vec{e}\,.$$

- assumes that medium not affected by polarization state
- different media 1 to N in order of wave direction, combined influence described

$$\mathrm{J} = \mathrm{J}_N \mathrm{J}_{N-1} \cdots \mathrm{J}_2 \mathrm{J}_1$$

- order of matrices in product is crucial
- Jones calculus describes coherent superposition of polarized light

Quasi-Monochromatic Light

- monochromatic light: purely theoretical concept
- monochromatic light wave always fully polarized
- real life: light includes range of wavelengths $\Rightarrow$ *quasi-monochromatic light*
- quasi-monochromatic: superposition of mutually incoherent monochromatic light beams whose wavelengths vary in narrow range $\delta\lambda$ around central wavelength λ_0

$$\frac{\delta\lambda}{\lambda} \ll 1$$

- measurement of quasi-monochromatic light: integral over measurement time t_m
- amplitude, phase (slow) functions of time for given spatial location
- *slow*: variations occur on time scales much longer than the mean period of the wave

Polarization of Quasi-Monochromatic Light

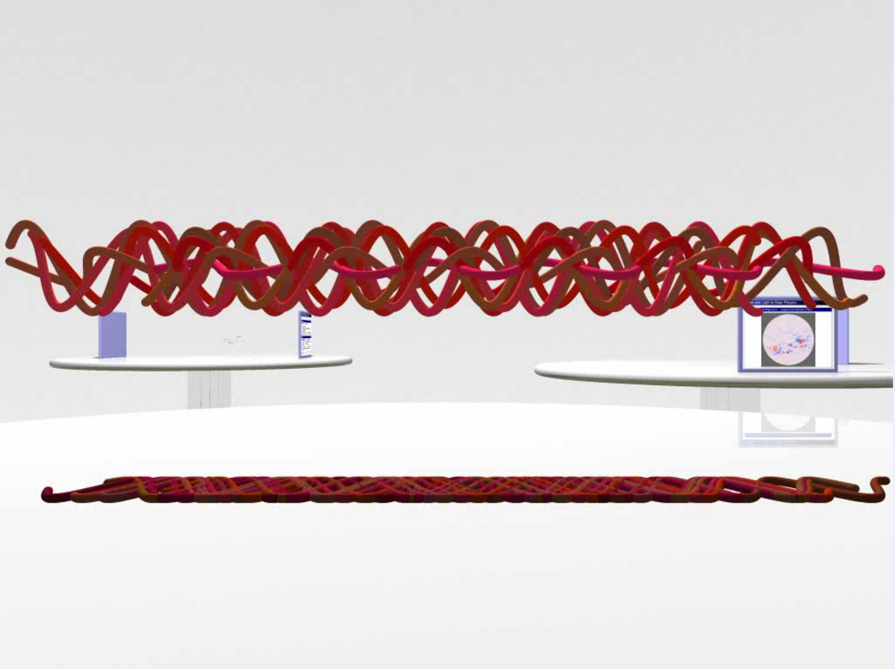
- electric field vector for quasi-monochromatic plane wave is sum of electric field vectors of all monochromatic beams

$$\vec{E}(t) = \vec{E}_0(t)\, e^{i\left(\vec{k}\cdot\vec{x}-\omega t\right)} .$$

- can write this way because $\delta\lambda \ll \lambda_0$
- measured intensity of quasi-monochromatic beam

$$\left\langle \vec{E}_x \vec{E}_x^* \right\rangle + \left\langle \vec{E}_y \vec{E}_y^* \right\rangle = \lim_{t_m->\infty} \frac{1}{t_m} \int_{-t_m/2}^{t_m/2} \vec{E}_x(t)\vec{E}_x^*(t) + \vec{E}_y(t)\vec{E}_y^*(t) dt ,$$

  $\langle\cdots\rangle$: averaging over measurement time t_m.
- measured intensity independent of time
- Quasi-monochromatic: frequency-dependent material properties (e.g. index of refraction) are constant within $\Delta\lambda$



Stokes Vector

- need formalism to describe polarization of quasi-monochromatic light
- directly related to measurable intensities
- Stokes vector fulfills these requirements

$$\vec{I} = \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} E_x E_x^* + E_y E_y^* \\ E_x E_x^* - E_y E_y^* \\ E_x E_y^* + E_y E_x^* \\ i\left(E_x E_y^* - E_y E_x^*\right) \end{pmatrix} = \begin{pmatrix} E_1^2 + E_2^2 \\ E_1^2 - E_2^2 \\ 2E_1 E_2 \cos\delta \\ 2E_1 E_2 \sin\delta \end{pmatrix}$$

  Jones vector elements $E_{x,y}$, real amplitudes $E_{1,2}$, phase difference $\delta = \delta_2 - \delta_1$

- $$I^2 \geq Q^2 + U^2 + V^2 \,.$$

Stokes Vector Interpretation

$$\vec{I} = \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} \text{intensity} \\ \text{linear } 0^\circ - \text{linear } 90^\circ \\ \text{linear } 45^\circ - \text{linear } 135^\circ \\ \text{circular left} - \text{right} \end{pmatrix}$$

- *degree of polarization*

$$P = \frac{\sqrt{Q^2 + U^2 + V^2}}{I}$$

  1 for fully polarized light, 0 for unpolarized light
- summing of Stokes vectors = *incoherent* adding of quasi-monochromatic light waves

Mueller Matrices

- 4×4 real Mueller matrices describe (linear) transformation between Stokes vectors when passing through or reflecting from media

$$\vec{I}' = \mathrm{M}\vec{I}\,,$$

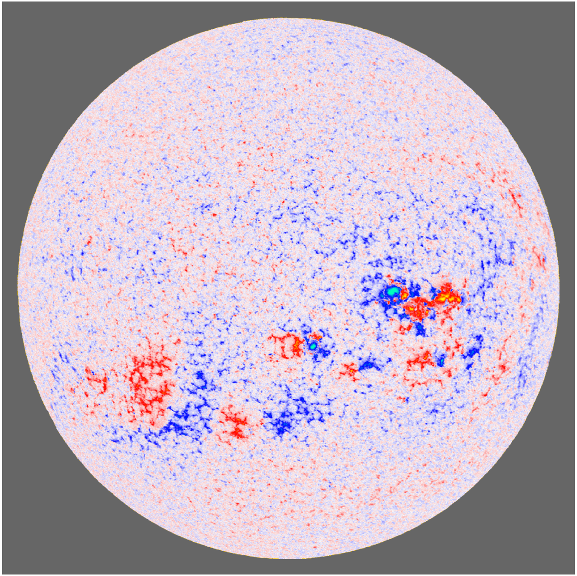
$$\mathrm{M} = \begin{pmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{pmatrix}$$

- N optical elements, combined Mueller matrix is

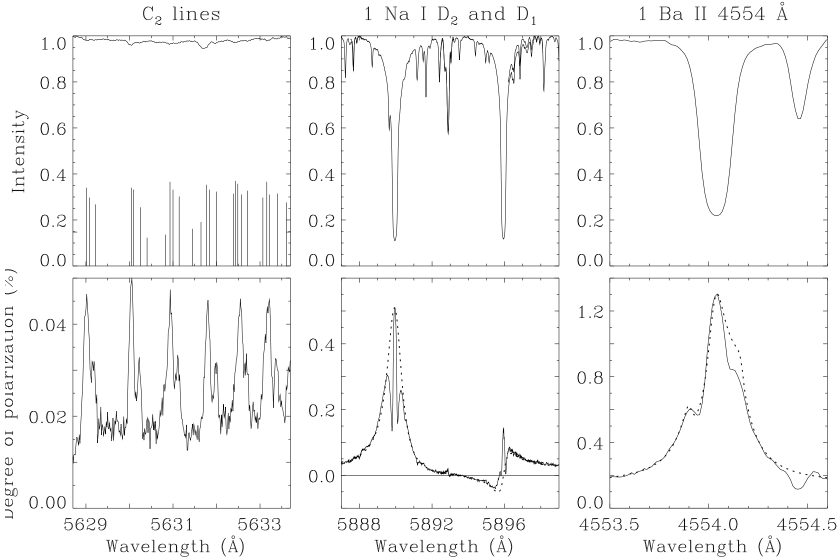
$$\mathrm{M}' = \mathrm{M}_N \mathrm{M}_{N-1} \cdots \mathrm{M}_2 \mathrm{M}_1$$

Polarized Light in Solar Physics

Magnetic Field Maps from Longitudinal Zeeman Effect



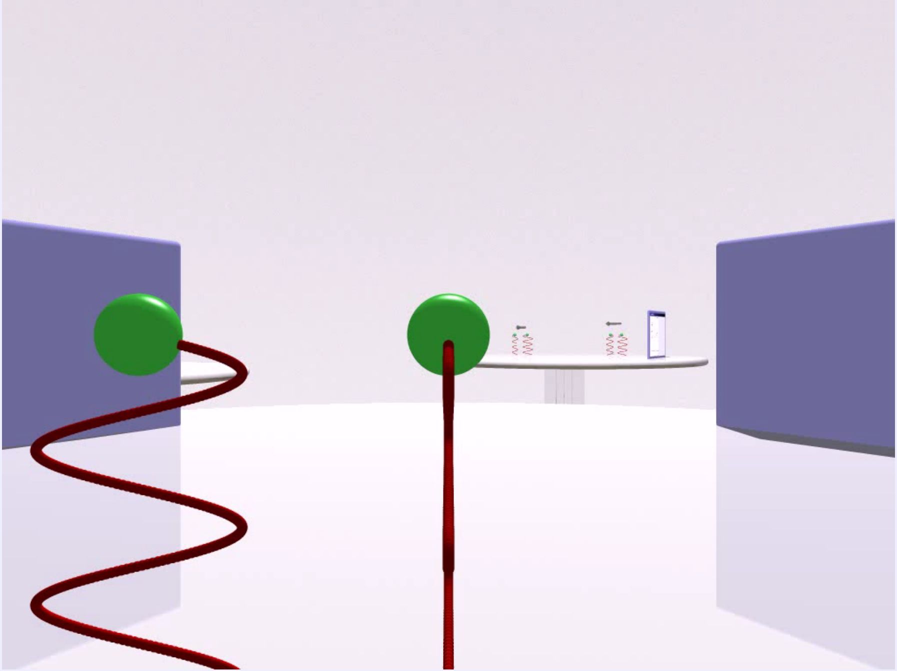
Second Solar Spectrum from Scattering Polarization

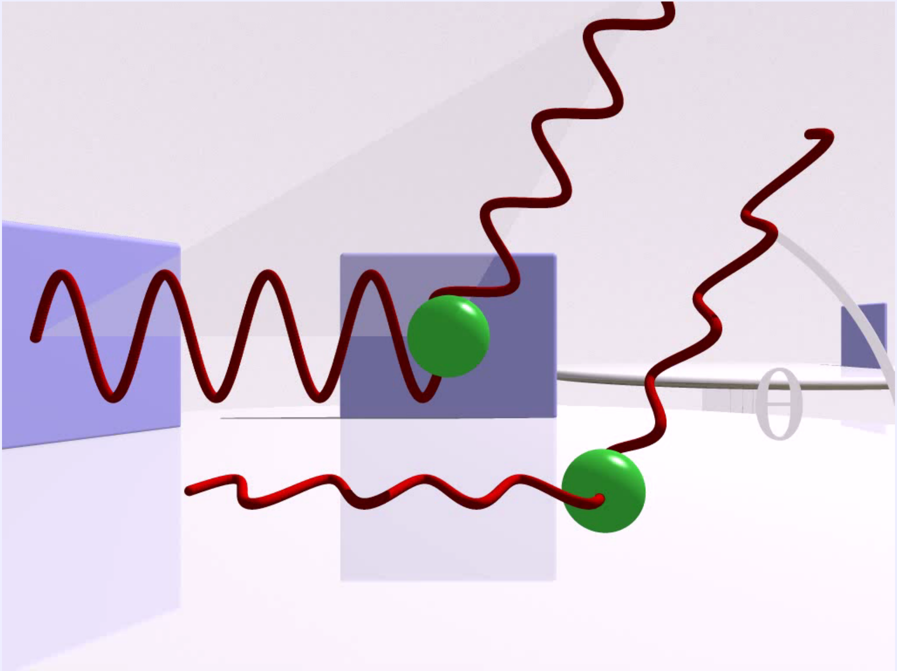


Scattering Polarization

Single Particle Scattering

- light is absorbed and re-emitted
- if light has low enough energy, no energy transfered to electron, but photon changes direction $\Rightarrow$ elastic scattering
- for high enough energy, photon transfers energy onto electron $\Rightarrow$ inelastic (Compton) scattering
- Thomson scattering on free electrons
- Rayleigh scattering on bound electrons
- based on very basic physics, scattered light is linearly polarized

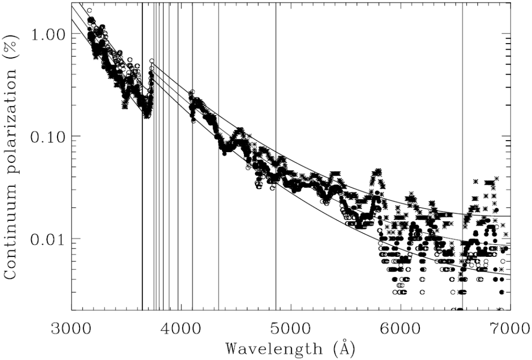



θ

Polarization as a Function of Scattering Angle

- same variation of polarization with scattering angle applies to Thomson and Rayleigh scattering
- scattering angle θ
- projection of amplitudes:
  - 1 for polarization direction perpendicular to scattering plane
  - $\cos\theta$ for linear polarization in scattering plane
- intensities = amplitudes squared
- ratio of $+Q$ to $-Q$ is $\cos^2\theta$ (to 1)
- total scattered intensity (unpolarized = averaged over all polarization states) proportional to $\frac{1}{2}\left(1+\cos^2\theta\right)$

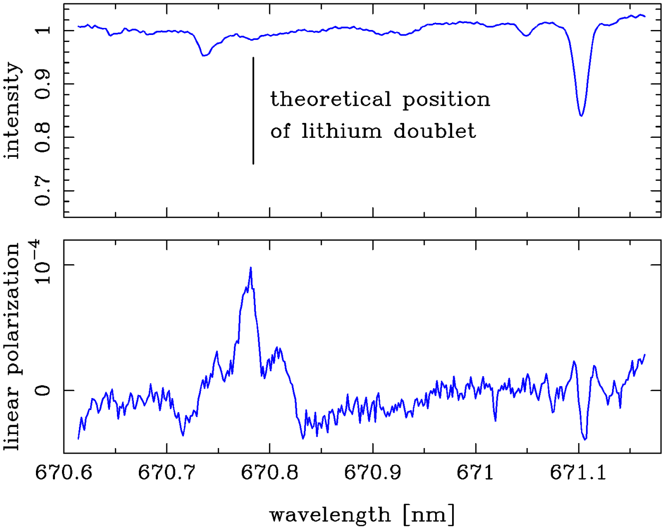
Solar Continuum Scattering Polarization



(from Stenflo 2005)

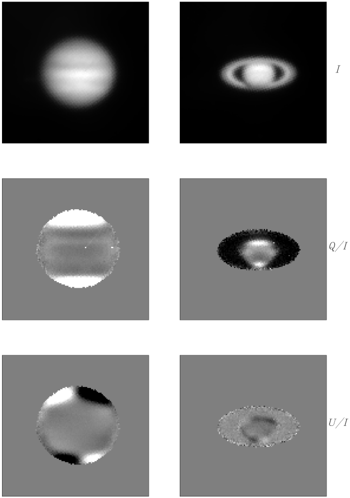
- due to anisotropy of the radiation field
- anisotropy due to limb darkening
- limb darkening due to decreasing temperature with height
- last scattering approximation without radiative transfer

Solar Spectral Line Scattering Polarization



resonance lines exhibit “large” scattering polarization signals

Jupiter and Saturn



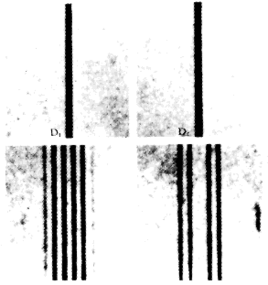
(courtesy H.M.Schmid and D.Gisler)

Planetary Scattered Light

- Jupiter, Saturn show scattering polarization
- multiple scattering changes polarization as compared to single scattering
- much depends on cloud height
- equivalent effect to study extrasolar planetary systems
- ExPo (Extreme Polarimeter) development here in Utrecht

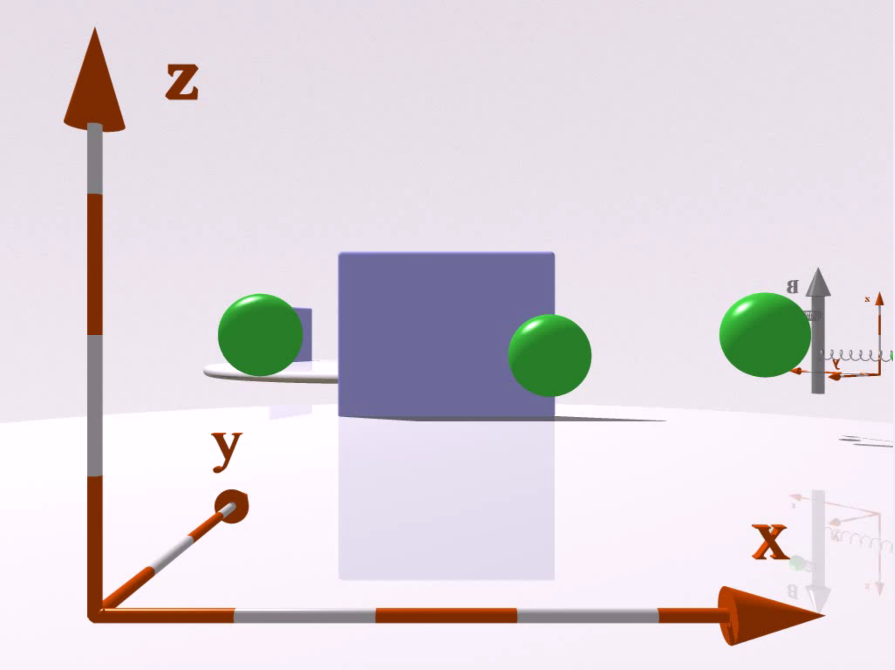
Zeeman Effect

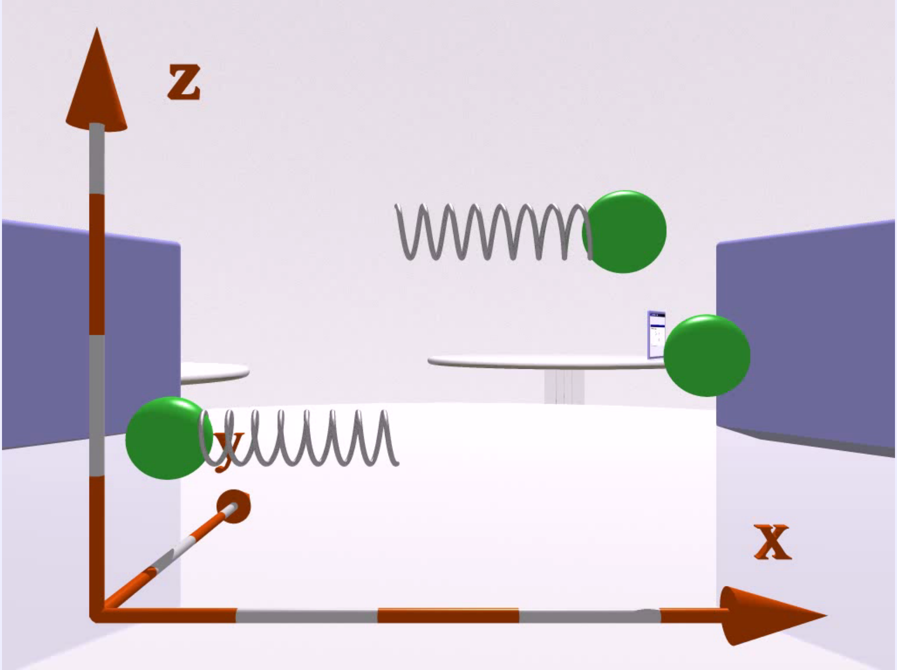
photos.aip.org/



Splitting/Polarization of Spectral Lines

- discovered in 1896 by Dutch physicist Pieter Zeeman
- different spectral lines show different splitting patterns
- splitting proportional to magnetic field
- split components are polarized
- *normal Zeeman effect* with 3 components explained by H.A.Lorentz using classical physics
- splitting of sodium D doublet could not be explained by classical physics (*anomalous Zeeman effect*)
- quantum theory and electron's intrinsic spin led to satisfactory explanation


z
y
x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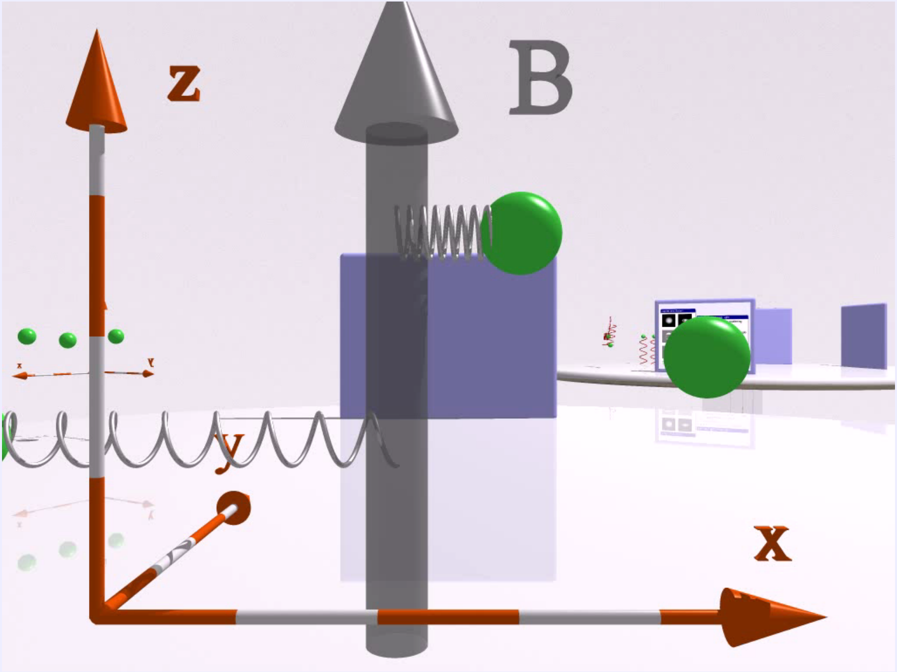
z
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Quantum-Mechanical Hamiltionian

- classical interaction of magnetic dipol moment $\vec{\mu}$ and magnetic field given by magnetic potential energy

$$U = -\vec{\mu} \cdot \vec{B}$$

  $\vec{\mu}$ the magnetic moment and $\vec{B}$ the magnetic field vector
- magnetic moment of electron due to orbit and spin
- Hamiltonian for quantum mechanics

$$H = H_0 + H_1 = H_0 + \frac{e}{2mc}\left(\vec{L} + 2\vec{S}\right)\vec{B}$$

H_0 Hamiltonian of atom without magnetic field
H_1 Hamiltonian component due to magnetic field
e charge of electron
m electron rest mass
$\vec{L}$ the orbital angular momentum operator
$\vec{S}$ the spin operator

Energy States in a Magnetic Field

- energy state $\langle E_{NLSJ}|$ characterized by
  - main quantum number N of energy state
  - $L(L+1)$, the eigenvalue of $\vec{L}^2$
  - $S(S+1)$, the eigenvalue of $\vec{S}^2$
  - $J(J+1)$, the eigenvalue of $\vec{J}^2$,
    $\vec{J} = \vec{L} + \vec{S}$ being the total angular momentum
  - M, the eigenvalue of J_z in the state $\langle NLSJM|$
- for the magnetic field in the z-direction, the change in energy is given by

$$\Delta E_{NLSJ}(M) = \langle NLSJM|H_1|NLSJM\rangle$$

The Landé g Factor

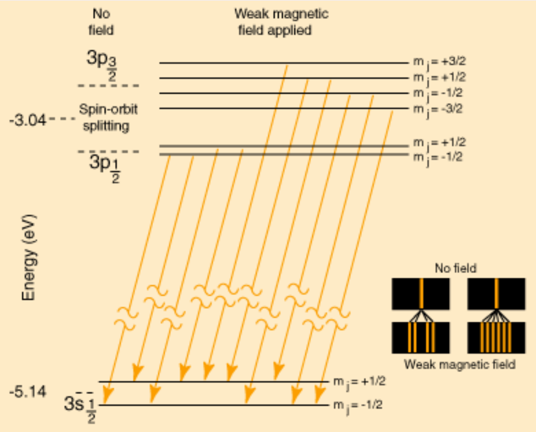
- based on pure mathematics (group theory, Wiegner Eckart theorem), one obtains

$$\Delta E_{NLSJ}(M) = \mu_0 g_L B M$$

  with $\mu_0 = \frac{e\hbar}{2m}$ the Bohr magneton, and g_L the Landé g-factor

- in LS coupling where B sufficiently small compared to spin-orbit splitting field

$$g_L = 1 + \frac{J(J+1) + L(L+1) - S(S+1)}{2J(J+1)}$$



hyperphysics.phy-astr.gsu.edu/hbase/quantum/sodzee.html

Spectral Lines - Transitions between Energy States

- spectral lines are due to transitions between energy states:

  lower level with $2J_l + 1$ sublevels M_l

  upper level with $2J_u + 1$ sublevels M_u

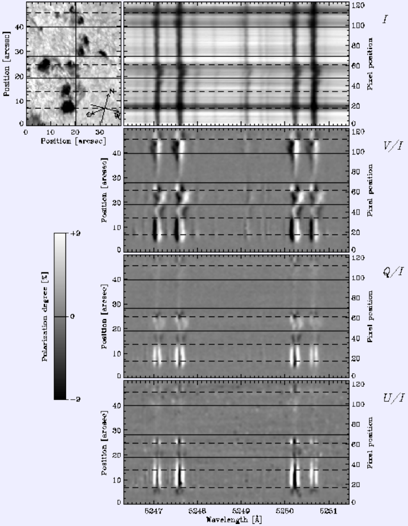
- not all transitions occur

Selection rule

- not all transitions between two levels are allowed
- assuming dipole radiation, quantum mechanics gives us the *selection rules*:
  - $L_u - L_l = \Delta L = \pm 1$
  - $M_u - M_l = \Delta M = 0, \pm 1$
  - $M_u = 0$ to $M_l = 0$ is forbidden for $J_u - J_l = 0$
- total angular momentum conservation: photon always carries $J_{\mathrm{photon}} = 1$
- *normal Zeeman effect*: line splits into three components because
  - Landé g-factors of upper and lower levels are identical
  - $J_u = 1$ to $J_l = 0$ transition
- *anomalous Zeeman effect* in all other cases

Effective Landé Factor and Polarized Components

- each component can be assigned an effective Landé g-factor, corresponding to how much the component shifts in wavelength for a given field strength
- components are also grouped according to the linear polarization direction for a magnetic field perpendicular to the line of sight
  - π components are polarized parallel to the magnetic field (**p**i for *parallel*)
  - σ components are polarized perpendicular to the magnetic field (**s**igma for German *senkrecht*)
- for a field parallel to the line of sight, the π-components are not visible, and the σ components are circularly polarized

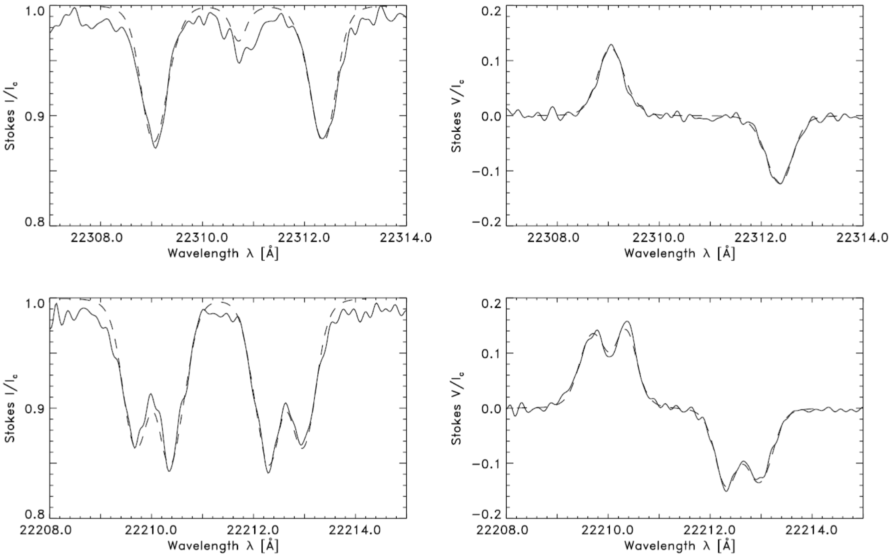


Bernasconi et al. 1998

Zeeman Effect in Solar Physics

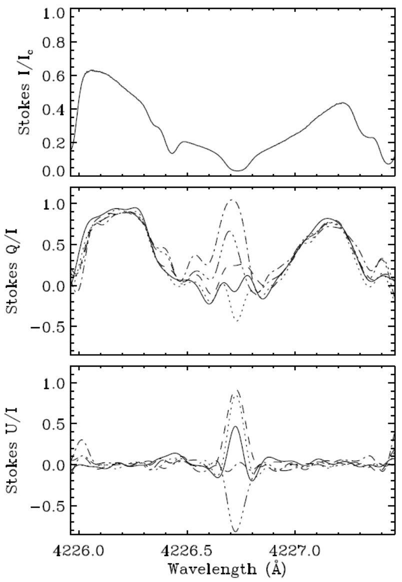
- discovered in sunspots by G.E.Hale in 1908
- splitting small except for in sunspots
- much of intensity profile due to non-magnetic area $\Rightarrow$ filling factor
- a lot of strong fields outside of sunspots
- full Stokes polarization measurements are key to determine solar magnetic fields
- 180 degree ambiguity

Fully Split Titanium Lines at 2.2μm



Rüedi et al. 1998

Hanle Effect



Bianda et al. 1998

Depolarization and Rotation

- scattering polarization modified by magnetic field
- precession around magnetic field depolarizes and rotates polarization
- sensitive $\sim 10^3$ times smaller field strengths that Zeeman effect
- measureable effects even for isotropic field vector orientations