Lecture 11: Polarized Light

Outline

1. Fundamentals of Polarized Light
2. Descriptions of Polarized Light
3. Scattering Polarization
4. Zeeman Effect
5. Hanle Effect
Fundamentals of Polarized Light

Electromagnetic Waves in Matter

- electromagnetic waves are a direct consequence of Maxwell’s equations
- optics: interaction of electromagnetic waves with matter as described by material equations
- polarization properties of electromagnetic waves are integral part of optics
Plane-Wave Solutions

Plane Vector Wave ansatz

\[
\vec{E} = \vec{E}_0 e^{i(\vec{k} \cdot \vec{x} - \omega t)}
\]

- \(\vec{k}\) spatially and temporally constant \textit{wave vector}
- \(\vec{k}\) normal to surfaces of constant phase
- \(|\vec{k}|\) \textit{wave number}
- \(\vec{x}\) spatial location
- \(\omega\) \textit{angular frequency} \((2\pi \times \text{frequency})\)
- \(t\) time
- \(\vec{E}_0\) a (generally complex) vector independent of time and space
  - damping if \(\vec{k}\) is complex
  - real electric field vector given by real part of \(\vec{E}\)
Polarization

- spatially, temporally constant vector $\vec{E}_0$ lays in plane perpendicular to propagation direction $\vec{k}$
- represent $\vec{E}_0$ in 2-D basis, unit vectors $\vec{e}_1$ and $\vec{e}_2$, both perpendicular to $\vec{k}$
  \[ \vec{E}_0 = E_1 \vec{e}_1 + E_2 \vec{e}_2. \]

$E_1, E_2$: arbitrary complex scalars
- damped plane-wave solution with given $\omega$, $\vec{k}$ has 4 degrees of freedom (two complex scalars)
- additional property is called polarization
- many ways to represent these four quantities
- if $E_1$ and $E_2$ have identical phases, $\vec{E}$ oscillates in fixed plane
Description of Polarized Light

Polarization Ellipse

\[ \vec{E}(t) = \vec{E}_0 e^{i(k \cdot \vec{x} - \omega t)} \]
\[ \vec{E}_0 = E_1 e^{i\delta_1} \vec{e}_x + E_2 e^{i\delta_2} \vec{e}_y \]
- wave vector in \( z \)-direction
- \( \vec{e}_x, \vec{e}_y \): unit vectors in \( x, y \) directions
- \( E_1, E_2 \): (real) amplitudes
- \( \delta_1, \delta_2 \): (real) phases

Polarization Description
- 2 complex scalars not the most useful description
- at given \( \vec{x} \), time evolution of \( \vec{E} \) described by \textit{polarization ellipse}
- ellipse described by axes \( a, b \), orientation \( \psi \)
Jones Formalism

Jones Vectors

\[ \vec{E}_0 = E_x \vec{e}_x + E_y \vec{e}_y \]

- beam in z-direction
- \( \vec{e}_x, \vec{e}_y \) unit vectors in x, y-direction
- complex scalars \( E_x, E_y \)
- Jones vector

\[ \vec{e} = \begin{pmatrix} E_x \\ E_y \end{pmatrix} \]

- phase difference between \( E_x, E_y \) multiple of \( \pi \), electric field vector oscillates in a fixed plane \( \Rightarrow \) linear polarization
- phase difference \( \pm \frac{\pi}{2} \) \( \Rightarrow \) circular polarization
Summing and Measuring Jones Vectors

\[ \vec{E}_0 = E_x \vec{e}_x + E_y \vec{e}_y \]

\[ \vec{e} = \begin{pmatrix} E_x \\ E_y \end{pmatrix} \]

- Maxwell’s equations linear \( \Rightarrow \) sum of two solutions again a solution
- Jones vector of sum of two waves = sum of Jones vectors of individual waves if wave vectors \( \vec{k} \) the same
- addition of Jones vectors: \textit{coherent} superposition of waves
- elements of Jones vectors are not observed directly
- observables always depend on products of elements of Jones vectors, i.e. intensity

\[ I = \vec{e} \cdot \vec{e}^* = e_x e_x^* + e_y e_y^* , \]
Jones matrices

- Influence of medium on polarization described by $2 \times 2$ complex Jones matrix $J$

\[
\vec{e}' = J\vec{e} = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix} \vec{e}.
\]

- Assumes that medium not affected by polarization state

- Different media 1 to $N$ in order of wave direction, combined influence described

\[
J = J_N J_{N-1} \cdots J_2 J_1
\]

- Order of matrices in product is crucial

- Jones calculus describes coherent superposition of polarized light
Quasi-Monochromatic Light

- monochromatic light: purely theoretical concept
- monochromatic light wave always fully polarized
- real life: light includes range of wavelengths ⇒ _quasi-monochromatic light_
- quasi-monochromatic: superposition of mutually incoherent monochromatic light beams whose wavelengths vary in narrow range \( \delta \lambda \) around central wavelength \( \lambda_0 \)

\[
\frac{\delta \lambda}{\lambda} \ll 1
\]

- measurement of quasi-monochromatic light: integral over measurement time \( t_m \)
- amplitude, phase (slow) functions of time for given spatial location
- _slow_: variations occur on time scales much longer than the mean period of the wave
Polarization of Quasi-Monochromatic Light

- The electric field vector for quasi-monochromatic plane wave is the sum of electric field vectors of all monochromatic beams:
  \[
  \vec{E}(t) = \vec{E}_0(t) e^{i(\vec{k} \cdot \vec{x} - \omega t)}.
  \]

- We can write this way because \( \delta \lambda \ll \lambda_0 \).

- The measured intensity of a quasi-monochromatic beam is:
  \[
  \langle \vec{E}_x \vec{E}_x^* \rangle + \langle \vec{E}_y \vec{E}_y^* \rangle = \lim_{t_m \to \infty} \frac{1}{t_m} \int_{-t_m/2}^{t_m/2} \vec{E}_x(t) \vec{E}_x^*(t) + \vec{E}_y(t) \vec{E}_y^*(t) \, dt,
  \]

- The average \( \langle \cdots \rangle \): averaging over measurement time \( t_m \).

- The measured intensity is independent of time.

- Quasi-monochromatic: frequency-dependent material properties (e.g. index of refraction) are constant within \( \Delta \lambda \).
Stokes and Mueller Formalisms

Stokes Vector

- need formalism to describe polarization of quasi-monochromatic light
- directly related to measurable intensities
- Stokes vector fulfills these requirements

\[ \vec{I} = \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} E_x E_x^* + E_y E_y^* \\ E_x E_x^* - E_y E_y^* \\ E_x E_y^* + E_y E_x^* \\ i(E_x E_y^* - E_y E_x^*) \end{pmatrix} = \begin{pmatrix} E_1^2 + E_2^2 \\ E_1^2 - E_2^2 \\ 2E_1 E_2 \cos \delta \\ 2E_1 E_2 \sin \delta \end{pmatrix} \]

Jones vector elements \( E_{x,y} \), real amplitudes \( E_{1,2} \), phase difference \( \delta = \delta_2 - \delta_1 \)

- \[ I^2 \geq Q^2 + U^2 + V^2 \]
Stokes Vector Interpretation

\[ \vec{I} = \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} \text{intensity} \\ \text{linear } 0^\circ - \text{linear } 90^\circ \\ \text{linear } 45^\circ - \text{linear } 135^\circ \\ \text{circular left} - \text{right} \end{pmatrix} \]

- *degree of polarization*

\[ P = \frac{\sqrt{Q^2 + U^2 + V^2}}{I} \]

1 for fully polarized light, 0 for unpolarized light

- summing of Stokes vectors = *incoherent* adding of quasi-monochromatic light waves
Mueller Matrices

- 4 × 4 real Mueller matrices describe (linear) transformation between Stokes vectors when passing through or reflecting from media

\[ \vec{i}' = M \vec{i}, \]

\[ M = \begin{pmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{pmatrix} \]

- \( N \) optical elements, combined Mueller matrix is

\[ M' = M_N M_{N-1} \cdots M_2 M_1 \]
Magnetic Field Maps from Longitudinal Zeeman Effect
Second Solar Spectrum from Scattering Polarization

- **C₂ lines**
- **1 Na I D₂ and D₁**
- **1 Ba II 4554 Å**
Scattering Polarization

Single Particle Scattering

- light is absorbed and re-emitted
- if light has low enough energy, no energy transferred to electron, but photon changes direction $\Rightarrow$ elastic scattering
- for high enough energy, photon transfers energy onto electron $\Rightarrow$ inelastic (Compton) scattering
- Thomson scattering on free electrons
- Rayleigh scattering on bound electrons
- based on very basic physics, scattered light is linearly polarized
Polarization as a Function of Scattering Angle

- same variation of polarization with scattering angle applies to Thomson and Rayleigh scattering
- scattering angle $\theta$
- projection of amplitudes:
  - 1 for polarization direction perpendicular to scattering plane
  - $\cos \theta$ for linear polarization in scattering plane
- intensities = amplitudes squared
- ratio of $+Q$ to $-Q$ is $\cos^2 \theta$ (to 1)
- total scattered intensity (unpolarized = averaged over all polarization states) proportional to $\frac{1}{2} (1 + \cos^2 \theta)$
Solar Continuum Scattering Polarization

- due to anisotropy of the radiation field
- anisotropy due to limb darkening
- limb darkening due to decreasing temperature with height
- last scattering approximation without radiative transfer

(from Stenflo 2005)
resonance lines exhibit “large” scattering polarization signals
Jupiter and Saturn

Planetary Scattered Light

- Jupiter, Saturn show scattering polarization
- multiple scattering changes polarization as compared to single scattering
- much depends on cloud height
- equivalent effect to study extrasolar planetary systems
- ExPo (Extreme Polarimeter) development here in Utrecht

(courtesy H.M.Schmid and D.Gisler)
Zeeman Effect

Splitting/Polarization of Spectral Lines

- discovered in 1896 by Dutch physicist Pieter Zeeman
- different spectral lines show different splitting patterns
- splitting proportional to magnetic field
- split components are polarized
- *normal Zeeman effect* with 3 components explained by H.A. Lorentz using classical physics
- splitting of sodium D doublet could not be explained by classical physics (*anomalous Zeeman effect*)
- quantum theory and electron’s intrinsic spin led to satisfactory explanation
Quantum-Mechanical Hamiltonian

- classical interaction of magnetic dipol moment $\vec{\mu}$ and magnetic field given by magnetic potential energy
  $$U = -\vec{\mu} \cdot \vec{B}$$

- $\vec{\mu}$ the magnetic moment and $\vec{B}$ the magnetic field vector
- magnetic moment of electron due to orbit and spin
- Hamiltonian for quantum mechanics
  $$H = H_0 + H_1 = H_0 + \frac{e}{2mc} \left( \vec{L} + 2\vec{S} \right) \vec{B}$$

$H_0$ Hamiltonian of atom without magnetic field
$H_1$ Hamiltonian component due to magnetic field
$e$ charge of electron
$m$ electron rest mass
$\vec{L}$ the orbital angular momentum operator
$\vec{S}$ the spin operator
Energy States in a Magnetic Field

- energy state $\langle E_{NLSJ} \rangle$ characterized by
  - main quantum number $N$ of energy state
  - $L(L + 1)$, the eigenvalue of $\vec{L}^2$
  - $S(S + 1)$, the eigenvalue of $\vec{S}^2$
  - $J(J + 1)$, the eigenvalue of $\vec{J}^2$,
    $\vec{J} = \vec{L} + \vec{S}$ being the total angular momentum
  - $M$, the eigenvalue of $J_z$ in the state $\langle NLSJM \rangle$

- for the magnetic field in the z-direction, the change in energy is given by

$$\Delta E_{NLSJ}(M) = \langle NLSJM | H_1 | NLSJM \rangle$$
The Landé g Factor

- based on pure mathematics (group theory, Wiegner Eckart theorem), one obtains

\[ \Delta E_{NLSJ}(M) = \mu_0 g_L BM \]

with \( \mu_0 = \frac{e\hbar}{2m} \) the Bohr magneton, and \( g_L \) the Landé g-factor

- in LS coupling where \( B \) sufficiently small compared to spin-orbit splitting field

\[ g_L = 1 + \frac{J(J+1) + L(L+1) - S(S+1)}{2J(J+1)} \]
spectral lines are due to transitions between energy states:

- **lower** level with \(2J_l + 1\) sublevels \(M_l\)
- **upper** level with \(2J_u + 1\) sublevels \(M_u\)

not all transitions occur
Selection rule

- not all transitions between two levels are allowed
- assuming dipole radiation, quantum mechanics gives us the selection rules:
  - \( L_u - L_l = \Delta L = \pm 1 \)
  - \( M_u - M_l = \Delta M = 0, \pm 1 \)
  - \( M_u = 0 \) to \( M_l = 0 \) is forbidden for \( J_u - J_l = 0 \)
- total angular momentum conservation: photon always carries \( J_{\text{photon}} = 1 \)
- *normal Zeeman effect*: line splits into three components because
  - Landé g-factors of upper and lower levels are identical
  - \( J_u = 1 \) to \( J_l = 0 \) transition
- *anomalous Zeeman effect* in all other cases
Effective Landé Factor and Polarized Components

- Each component can be assigned an effective Landé g-factor, corresponding to how much the component shifts in wavelength for a given field strength.
- Components are also grouped according to the linear polarization direction for a magnetic field perpendicular to the line of sight:
  - $\pi$ components are polarized parallel to the magnetic field ($\pi$ for parallel)
  - $\sigma$ components are polarized perpendicular to the magnetic field ($\sigma$ for German senkrecht)
- For a field parallel to the line of sight, the $\pi$-components are not visible, and the $\sigma$ components are circularly polarized.
Zeeman Effect in Solar Physics

- discovered in sunspots by G.E. Hale in 1908
- splitting small except for in sunspots
- much of intensity profile due to non-magnetic area ⇒ filling factor
- a lot of strong fields outside of sunspots
- full Stokes polarization measurements are key to determine solar magnetic fields
- 180 degree ambiguity
Fully Split Titanium Lines at 2.2\,\mu m

Rüedi et al. 1998
Hanle Effect

Depolarization and Rotation

- scattering polarization modified by magnetic field
- precession around magnetic field depolarizes and rotates polarization
- sensitive $\sim 10^3$ times smaller field strengths than Zeeman effect
- measurable effects even for isotropic field vector orientations

Bianda et al. 1998