Today's course

Chapter 1.5-1.7, 2.2.2 OAF-2

Topics:

measuring moments of a s.p.

Stochastic nature of radiation processes

Recap lecture 2

data sampling: ideal case Nyquist criterium is fulfilled → sampling does not lead to loss of information

Conditions:

- → band-limited response of the detector removes highest noise powers and the sampling is fast enough to cover the band limit of the detector
- → Signal is band-limited also and $\nu_{max,detector} > \nu_{max,signal}$

Filtering:

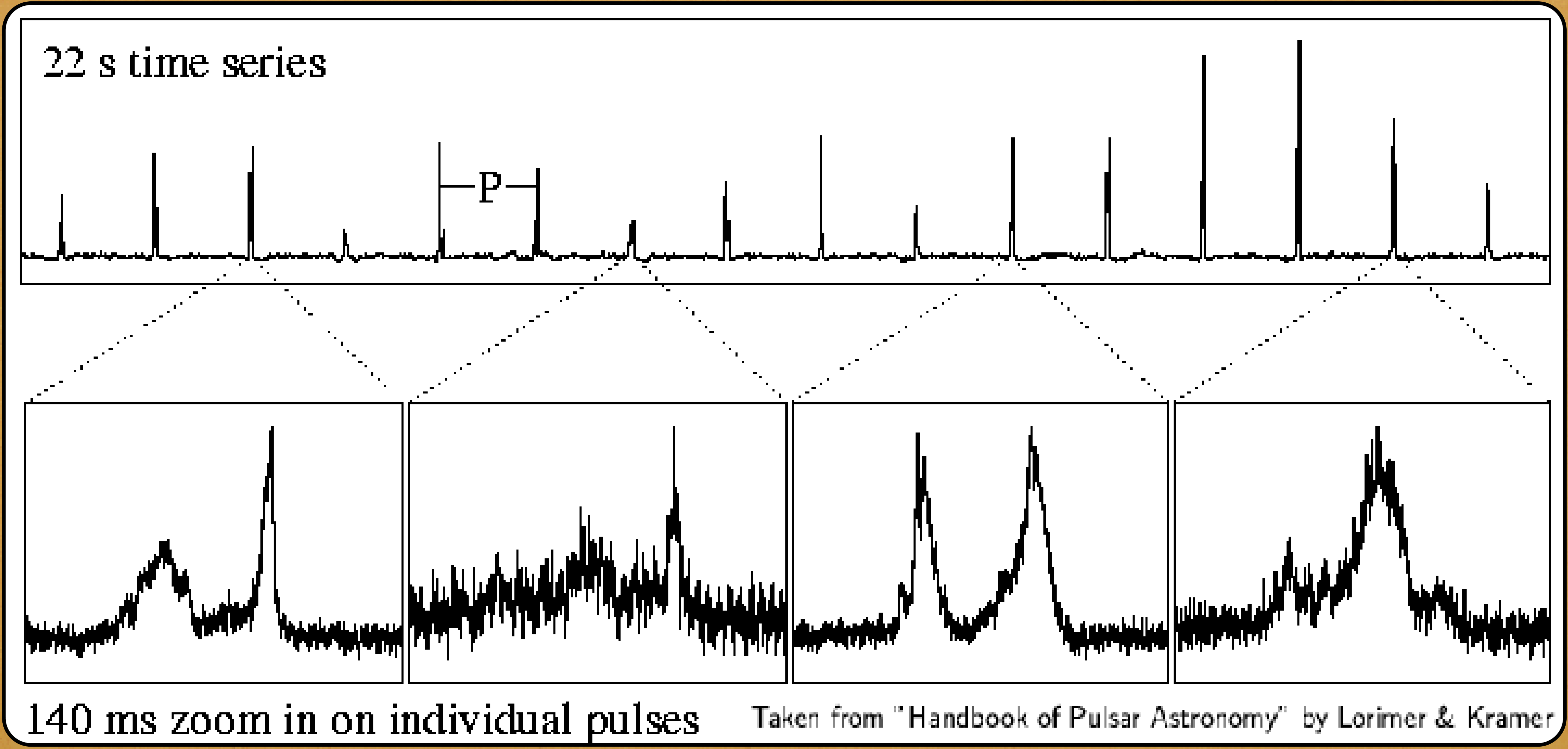
one can design an optimal filter such that the filtered measured data-set is as close as possible (in least-square sense) to the uncorrupted signal

Estimating the moments of a stochastic process

Chapter 2.2.2

See also Appendix B3.2, Lena ea.

How representative is a measurement of a S.P.?



ONE SAMPLE

1 MEASUREMENT OF X(T) IN A TIME T → WINDOWING AND AVERAGING OVER TIME ΔT

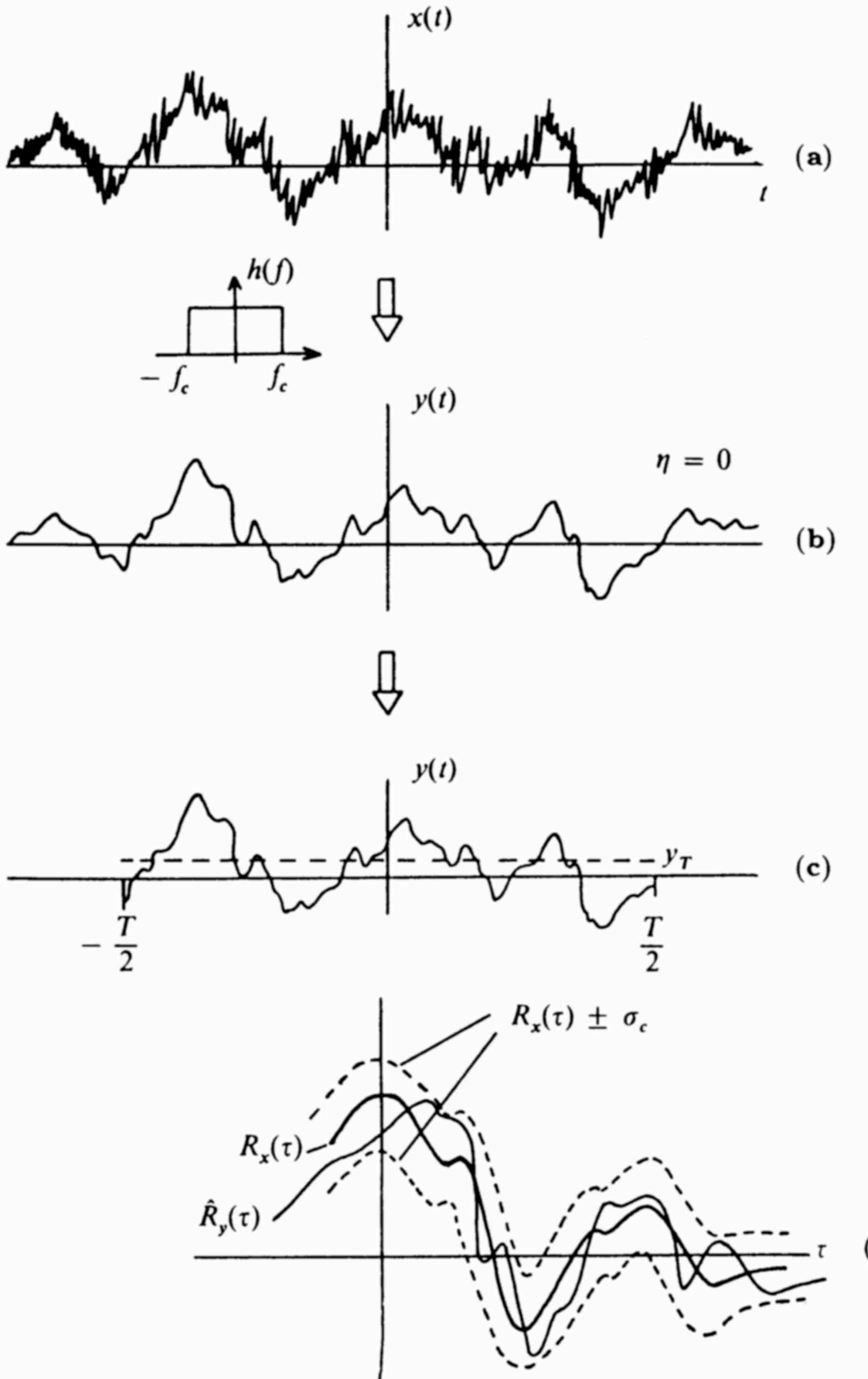
WINDOWING: $$y(t) = \Pi(\frac{t}{T})x(t)$$

AVERAGING:

$$z(t) \equiv y_{\Delta T}(t) = \frac{1}{\Delta T}\int_{t-\Delta T/2}^{t+\Delta T/2} y(t')dt' = \frac{1}{\Delta T}\int_{-\infty}^{\infty} \Pi(\frac{t-t'}{\Delta T})y(t')dt'$$

= LOW-PASS FILTER

$$with\ \Pi(\frac{t-t'}{\Delta T}) = 1\ \ within\ \ and\ \ 0$$
$$outside\ \ t - \Delta T/2 \leq t' \leq t + \Delta T/2$$



TIME DOMAIN, MULTIPLICATION WITH BOX → CONVOLVING HENCE SMOOTHING WITH SINC IN FREQ DOMAIN

TIME DOMAIN, AVERAGING SAMPLE ΔT → LOW-PASS FILTER

HOW WILL THE DETERMINED MEAN AND AUTOCORRELATION RELATE TO THE VALUES OF THE PARENT DISTRI?

Derivation on black board page 25,26,27 lecture notes

$$\sigma_{x_T}^2 = \sigma^2/N \text{ for T} >> \tau_0$$

τ_0 depends on transfer function which in order to avoid aliasing should be suited for the system under study

Estimating σ if the probability density function of the s.p. is known

→

Propagation of Errors chapter 5.2

Estimating σ if the probability density function of the s.p. is not known

→

e.g.

bootstrap method

Jackknife method

more on this later

Stochastic description of radiation fields

this is also in chapter 6.2 of OA lena

Bose-Einstein statistics

For each energy bin there are

n particles, Z boxes ≡ Z+1 boundaries

of which Z-1 are "movable"

$$W(n_i) = \frac{(n_i + Z_i - 1)!}{n_i!(Z_i - 1)!}$$

WHEN CONSIDERING ALL ENERGIES i

$$N = \sum_{i=1}^{\infty} n_i$$ TOTAL NUMBER OF PARTICLES

$$W = \Pi_{i=1}^{\infty} W(n_i)$$ TOTAL NUMBER OF POSSIBLE DISTRIBUTIONS

$$S \equiv k \ln(W)$$

MAXIMISE ENTROPY S

HENCE

$$\frac{d \ln W}{d n_i} = 0$$

Remember Taylor expansion:

$$W(x+\Delta x) = W(x) + \frac{dW(x)}{dx}\Delta x + \frac{1}{2}\frac{d^2W(x)}{d^2x}\Delta x^2$$

cf. equation 1.28 & 1.37 Lecture notes

Derivation of eq. 1.41 on black board

NOTE THE DEFINITIONS

n_i NUMBER OF PHOTONS WITH ENERGY i

n_{ν_k} OCCUPATION FRACTION

$n(\nu)$ SPECIFIC PHOTON FLUX (PHOTONS PER SECOND PER HERTZ)

$N(\nu)$ VOLUME PHOTON DENSITY (PHOTONS PER SECOND PER HERTZ PER UNIT VOLUME)