Today's course

Chapter 1.3, 1.4 & 2.2 OAF-2
Numerical Recipes Chapter 13.3

Topics:

- Aliasing & Nyquist frequency
- (Optimal) Filtering
- measuring moments of a s.p.

Recap lecture 1

detection of astronomical signals (s.p.) plus noise is influenced by instrument transfer function and data sampling

statistical moments characterise the signal (plus noise)

Noise can be due to the detector, background, and/or intrinsic to the signal

Assume WSS s.p. (mean does not depend on time, or much slower than measuring process, auto-correlation depends on offset only)

Another math tool
Power Spectral Density

(∝ amplitude of individual sinusoids)

(will return in more depth in Chapter 6)

Continuous FT: $$F(f) = \int_{-\infty}^{\infty} f(t)\, e^{-2\pi i f t} dt$$

Continuous PSD: $$P(f) = \tilde{F(f)} F(\tilde{f})^*$$

for WSS signals: $$P(f) = \int_{-\infty}^{\infty} R(\tau) e^{-2\pi i f \tau} d\tau$$

hence:

$$\tilde{F(f)} F(f) = |F(f)|^2 = \int_{-\infty}^{\infty} R(\tau) e^{-2\pi i f \tau} d\tau$$

Data sampling

time domain multiply s.p. with shah function

$$m_{samp,n} = m_s(x) = m(x)\frac{1}{\tau}\text{III}(\frac{x}{\tau}) = \sum_n m(n\tau)\delta(x - n\tau)$$

Discrete FT: $$M_{samp,k} = \sum_{n=0}^{N-1} m_{samp,n}\ e^{2\pi ink/N}$$

Discrete PSD: $$P_j = \frac{2}{a_0}|a_j|^2$$ power ∝ amplitude squared:

$$a_0 = M_{samp,k=0} = \sum_{n=0}^{N-1} m_{samp,n} \equiv N_0$$

$$a_k = M_{samp,k} = \sum_{n=0}^{N-1} m_{samp,n}\ e^{2\pi ink/N}$$

Nyquist frequency

- Nyquist frequency = half the frequency related with the sampling rate

- sharp narrow signal requires more frequencies for its description than broad signal
  - cf. the number of sin+cos necessary to describe the signal

- Optical spectra: range from min to maxi frequency = bandwidth set by the width and shape of the spectral lines

Nyquist frequency

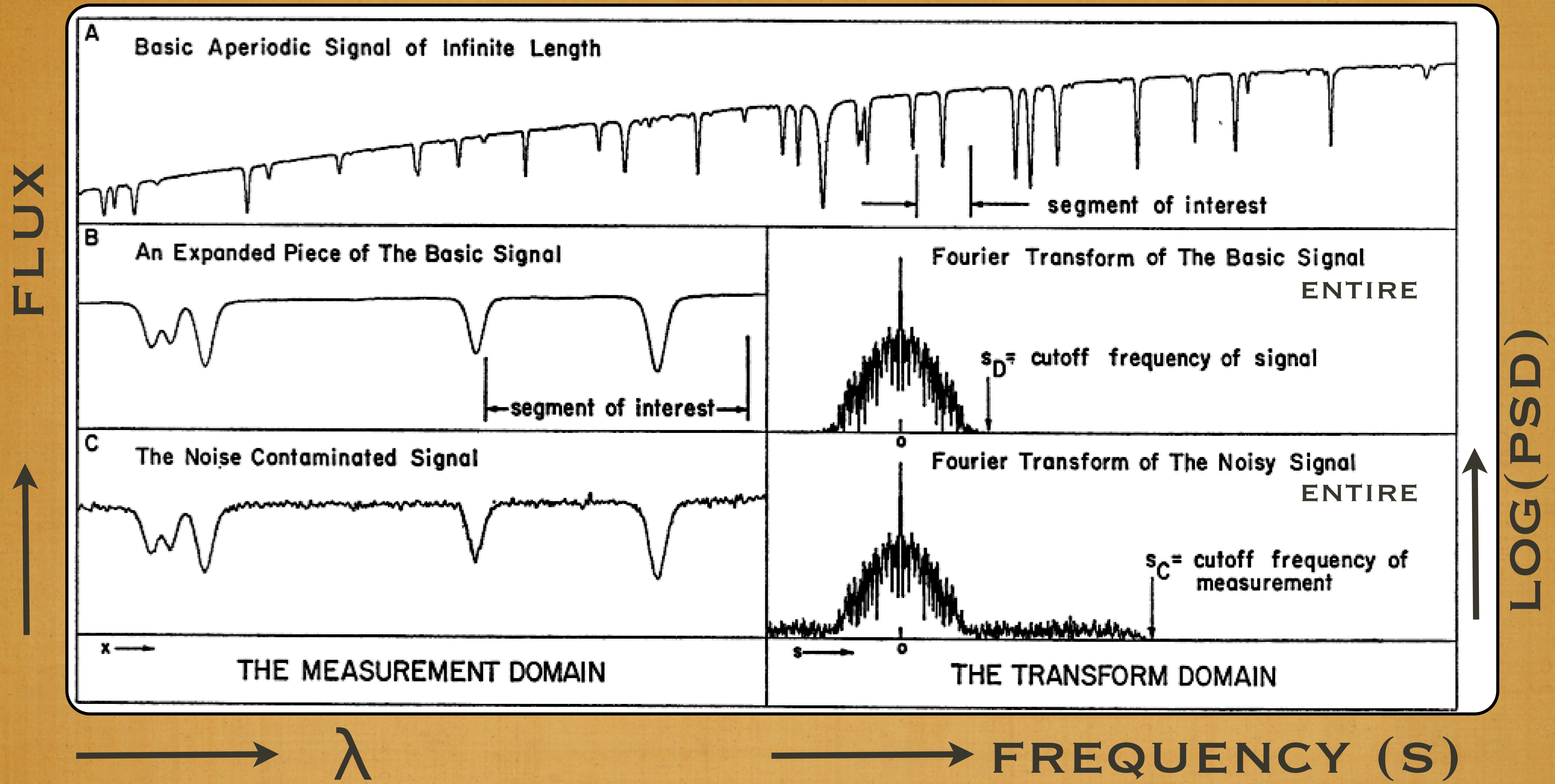
if $f_{nyq} = 2f_{max}$ in signal

Sampling at the Nyquist frequency: no loss of information

Continuous signal h(t) fully described by the samples

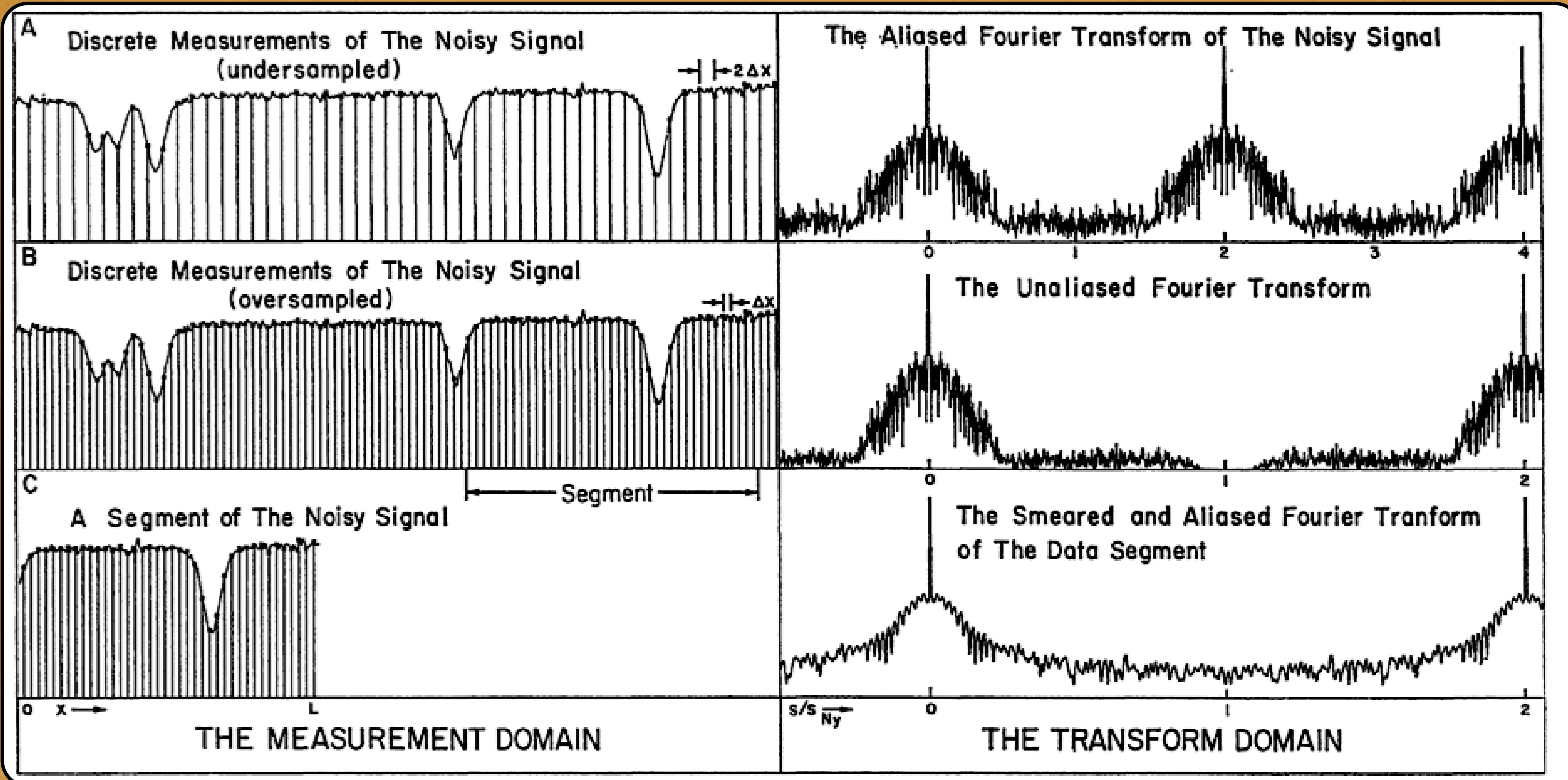
$$h(t) = \Delta t \sum_{n=-\infty}^{\infty} h_n \, \frac{sin[2\pi f_{nyq}(t - n\Delta t)]}{\pi(t - n\Delta t)}$$

Nyquist theorem: cont'd



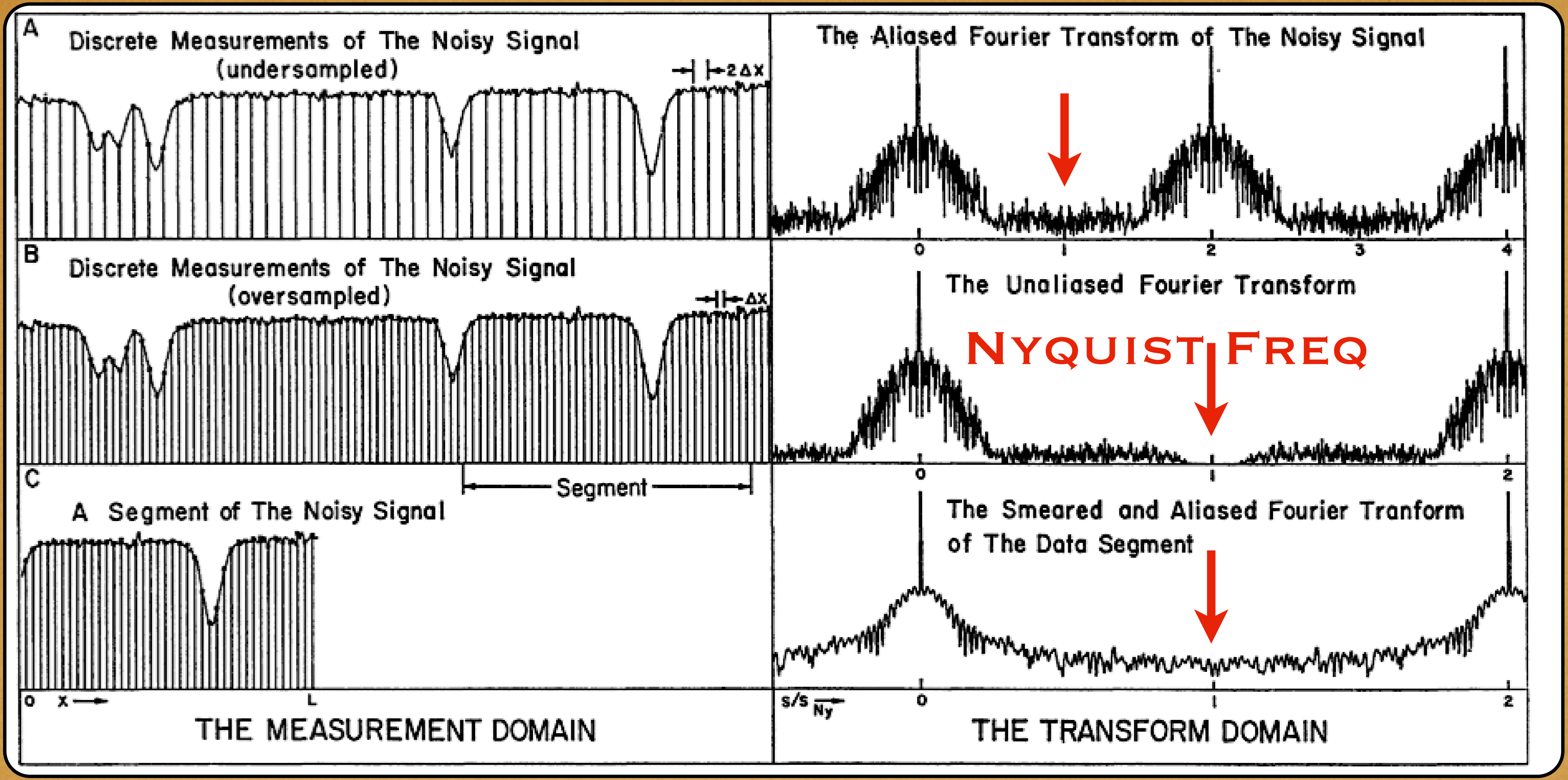
Windowing & noise, Brault & White 1971, A&A

Nyquist theorem: cont'd



Sampling; Brault & White 1971, A&A

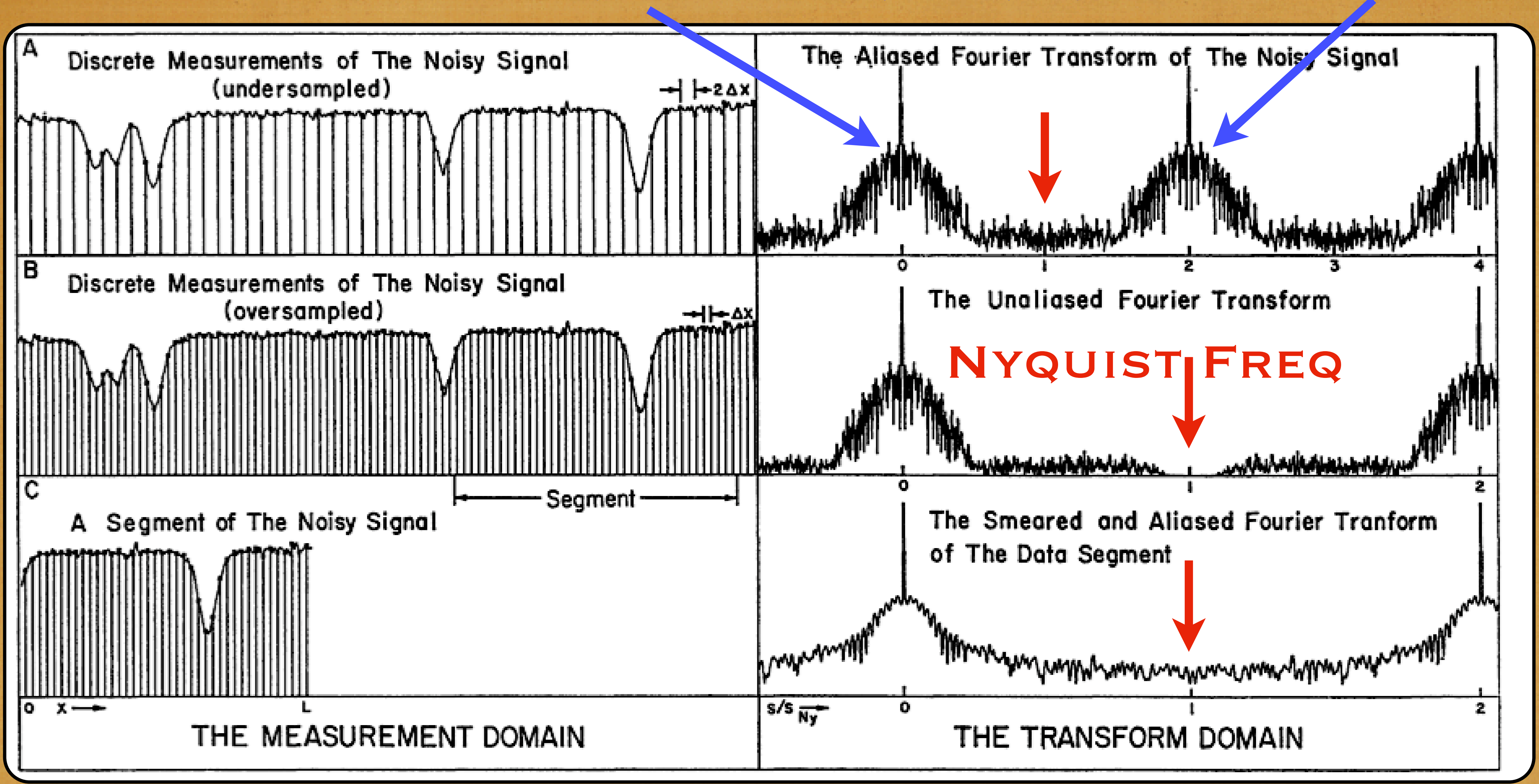
Nyquist theorem: cont'd



Sampling; Brault & White 1971, A&A

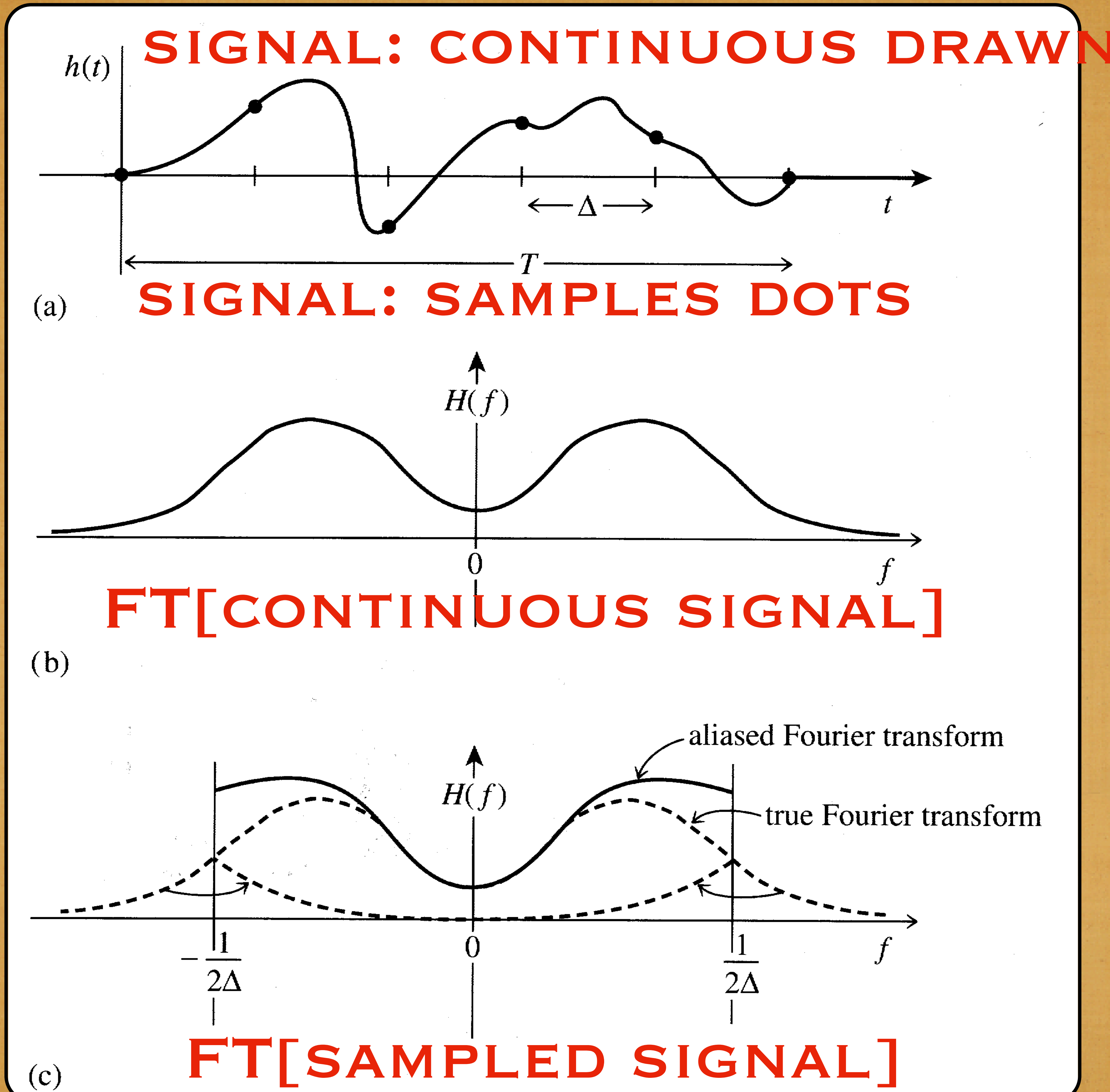
Nyquist theorem: cont'd

Sampling causes replication of signal

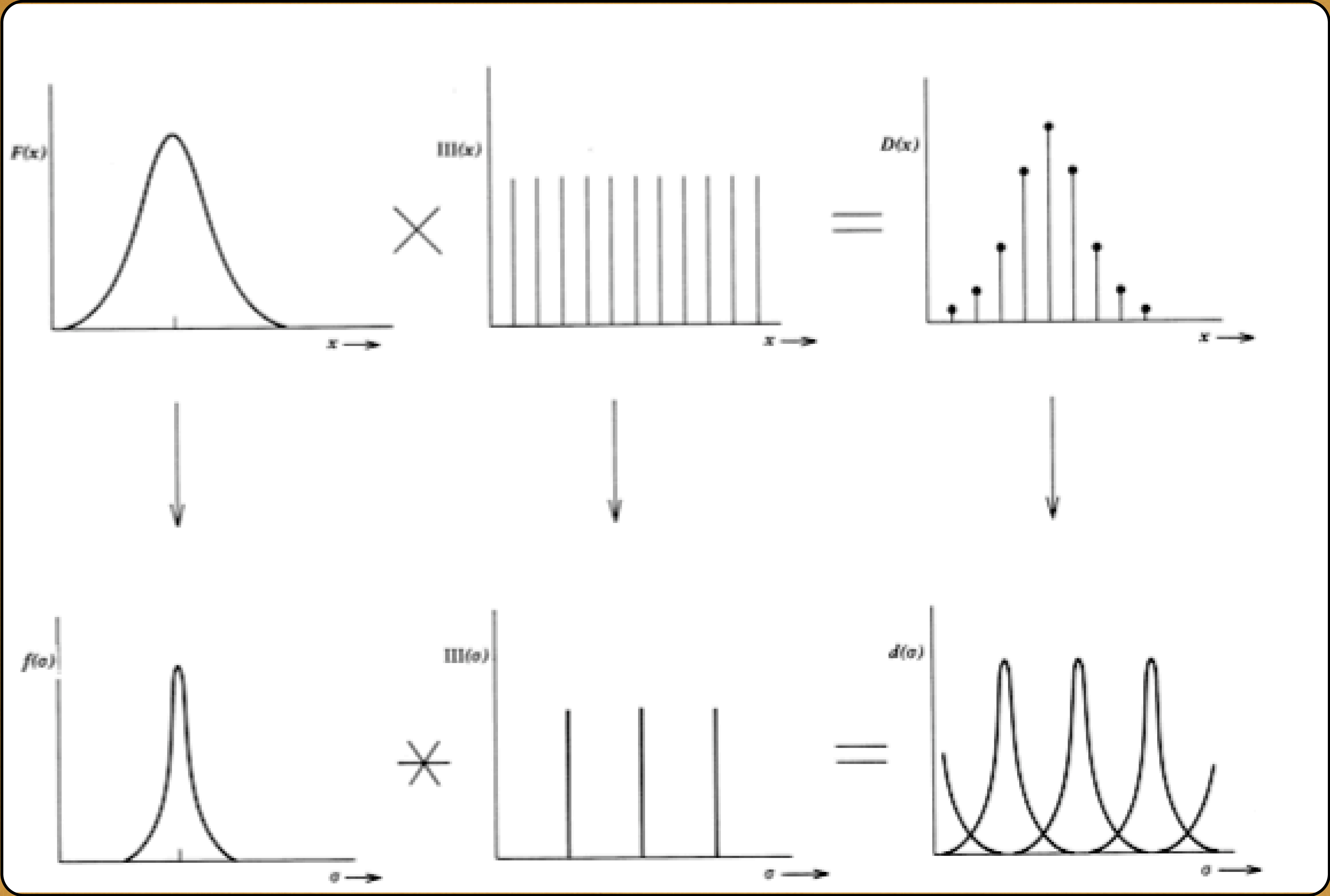


Sampling; Brault & White 1971, A&A

Aliasing


SIGNAL: CONTINUOUS DRAWN
h(t)
Δ
t
T
(a)
SIGNAL: SAMPLES DOTS
H(f)
0
f
FT[CONTINUOUS SIGNAL]
(b)
aliased Fourier transform
H(f)
true Fourier transform
−1/2Δ
0
1/2Δ
f
(c)
FT[SAMPLED SIGNAL]

Aliasing: cont'd



Convolution with shah function: replication

Sampling: high frequencies are filtered out

Window: low frequencies are filtered out

→ leads to band limited data

Filtering

→ Frequency Filtering $Y(f) = X(f)H(f)$

$$y(t) = \int_{-\infty}^{\infty} x(t-\theta)h(\theta)d\theta$$

$$y(t) = x(t) * h(t)$$

Filtering of process x with filter h

→ Time Filtering

measure a process x(t) over interval T assumed zero outside T

$$\equiv y(t) = \Pi(\frac{t}{T})x(t)$$

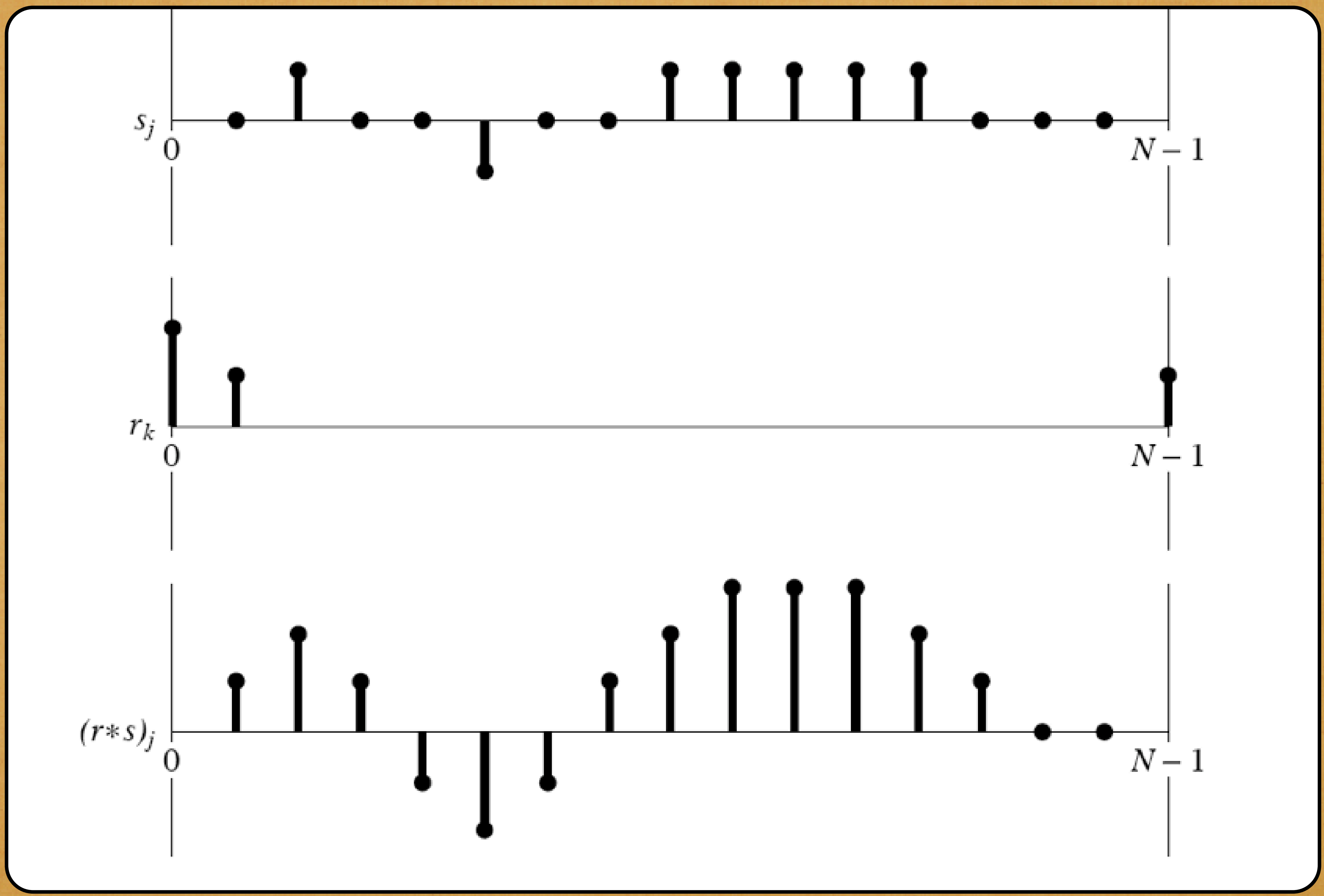
$$Y(f) = X(f) * Tsinc(Tf)$$

all information about frequencies <1/T is lost!

(Optimal) Filtering

Num Res Chapter 13.0-13.3

deconvolve measured signal and response function of sampled data



Num Res Chapter 13.1

Discrete convolution theorem

$$(r * s)_j \equiv \sum_{k=-M/2+1}^{M/2} s_{j-k} r_k \Leftrightarrow S_n R_n$$

Discrete deconvolution

$$\frac{\tilde{F}(r * s)_j}{R_n} = S_n$$

However noise and uncertainties is response can make this process unreliable

Noise removal by optimal filtering

$$cs(t) = s(t) + n(t)$$

$s(t)$ is the smeared signal i.e. true $\times$ response

Design an optimal filter φ(t) that produces a signal u(T) as close as possible to u(t)

$$\widetilde{U(f)} = \frac{C(f)\phi(f)}{R(f)}$$

Close in least square sense

$$\int_{-\infty}^{\infty} |\widetilde{U(f)} - U(f)|^2 df$$ is minimised

Noise removal by optimal filtering

$$\int_{-\infty}^{\infty} \left| \frac{[S(f)+N(f)]\phi(f)}{R(f)} - \frac{S(f)}{R(f)} \right|^2 df$$

$\int S(f)N(f)df$ terms are zero since noise and signal are uncorrelated

$$\int_{-\infty}^{\infty} |R(f)|^{-2} \underbrace{\{|S(f)|^2|1-\phi(f)|^2 + |N(f)|^2|\phi(f)|^2\}}_{\Theta} df$$

Θ minimised with respect to φ

Noise removal by optimal filtering

$$\frac{d\theta}{d\phi} = 0$$

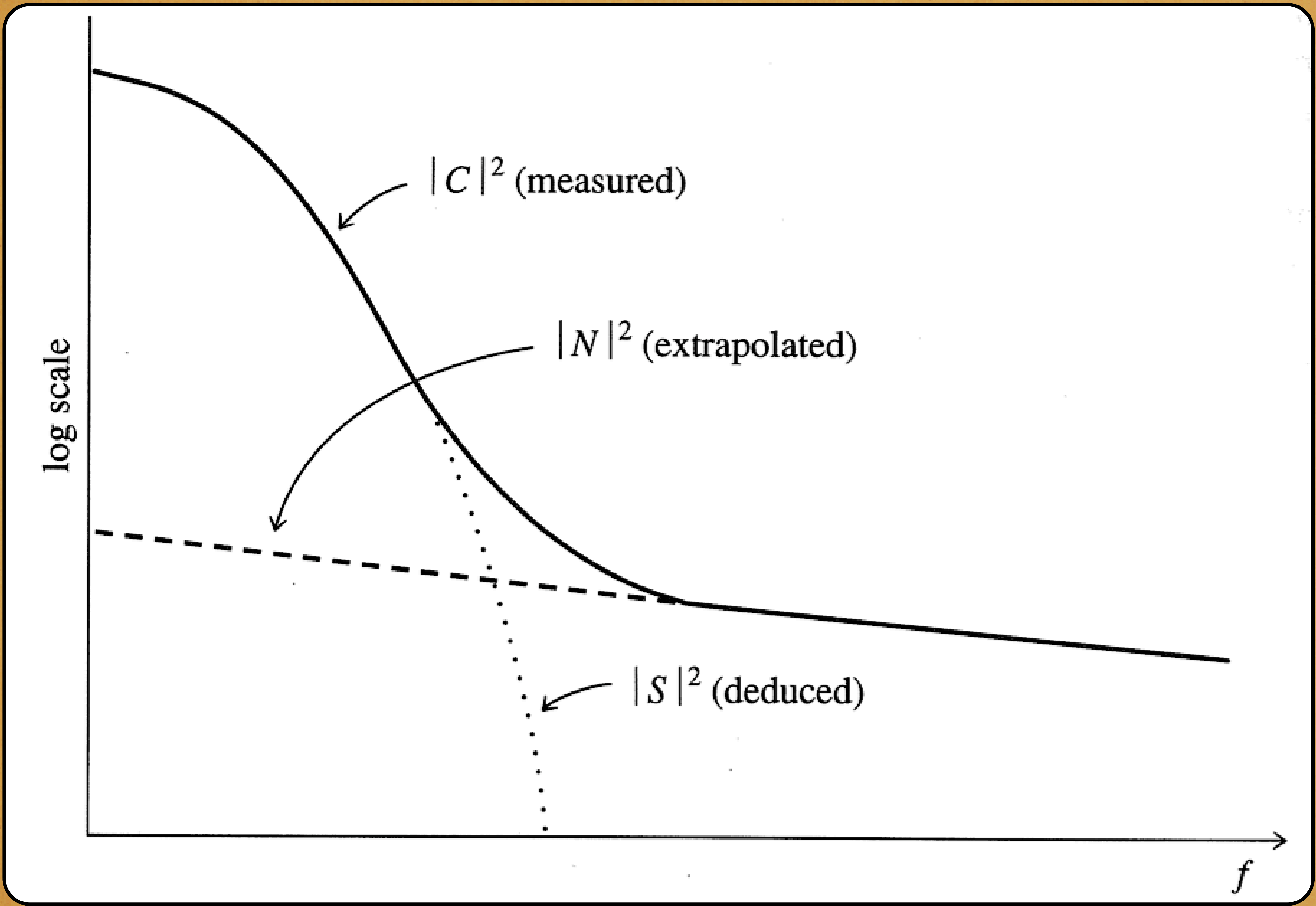
$$-2S^2(1-\phi) + 2N^2\phi = 0$$

Optimal filter $$\phi = \frac{S^2}{S^2 + N^2}$$

Does not contain true signal directly!

$$|S(f)|^2 + |N(f)|^2 = PDS(f) = |CS(f)|^2$$

Noise removal by optimal filtering



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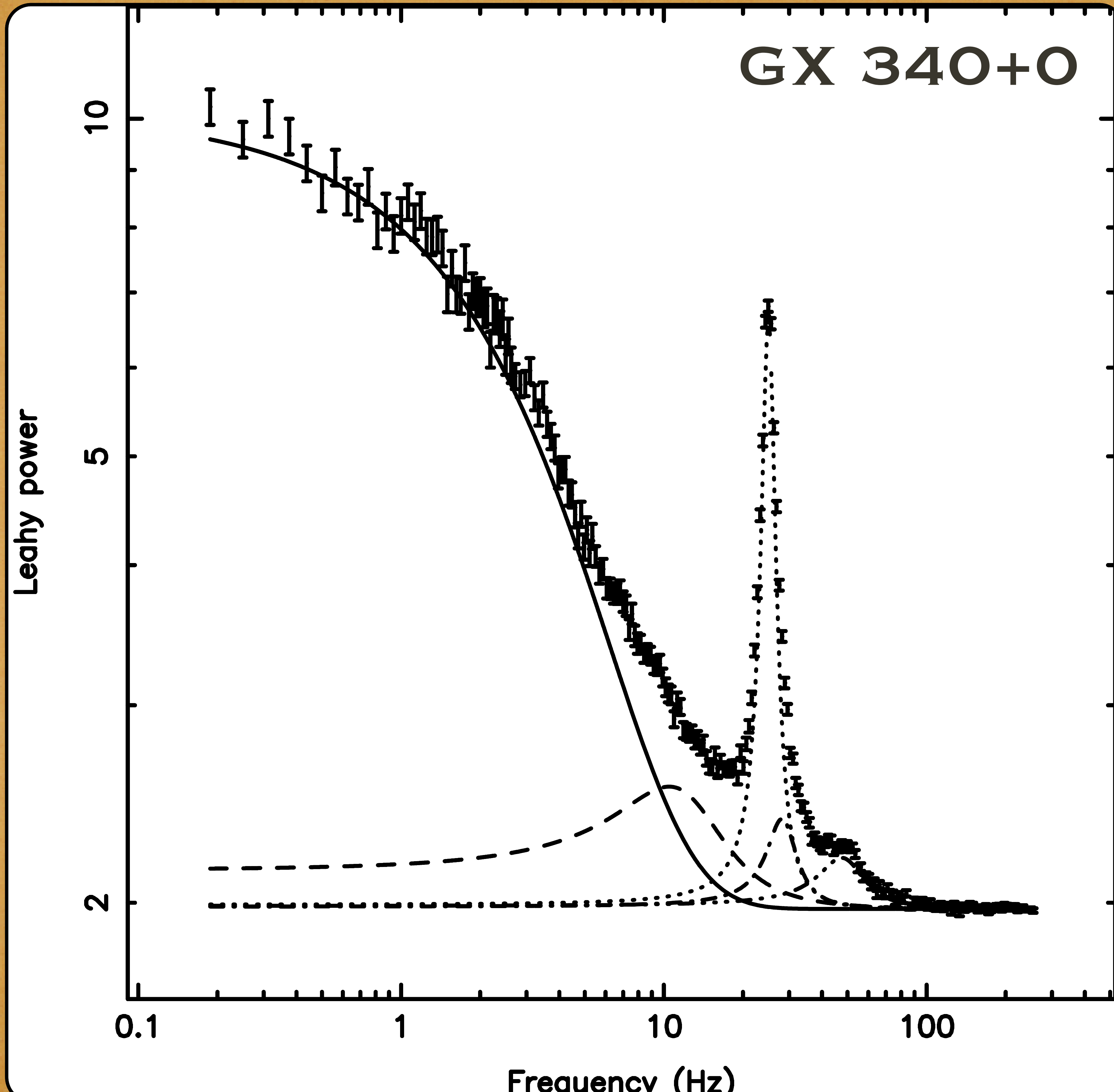
Some Applications

On optimal detection of point sources in CMB maps
Vio et al, 2002, A&A, 391, 789

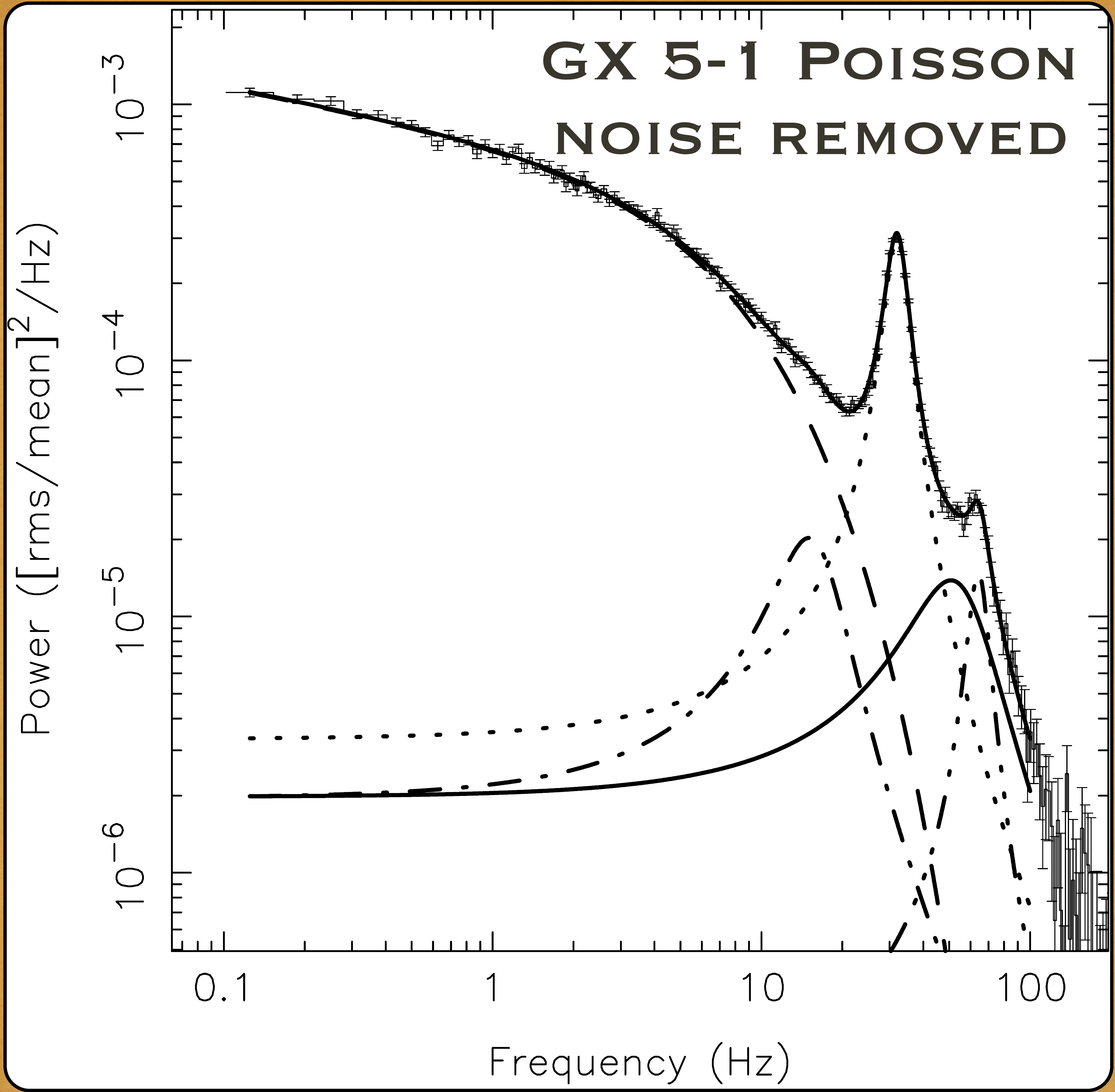
An optimal filter for the detection of galaxy clusters through weak lensing
Maturi, et al. 2005, A&A, 442, 851

The largest scale perturbations: A window on the physics of the beginning
Wandelt, New Astronomy Review, 2006, 11, 900

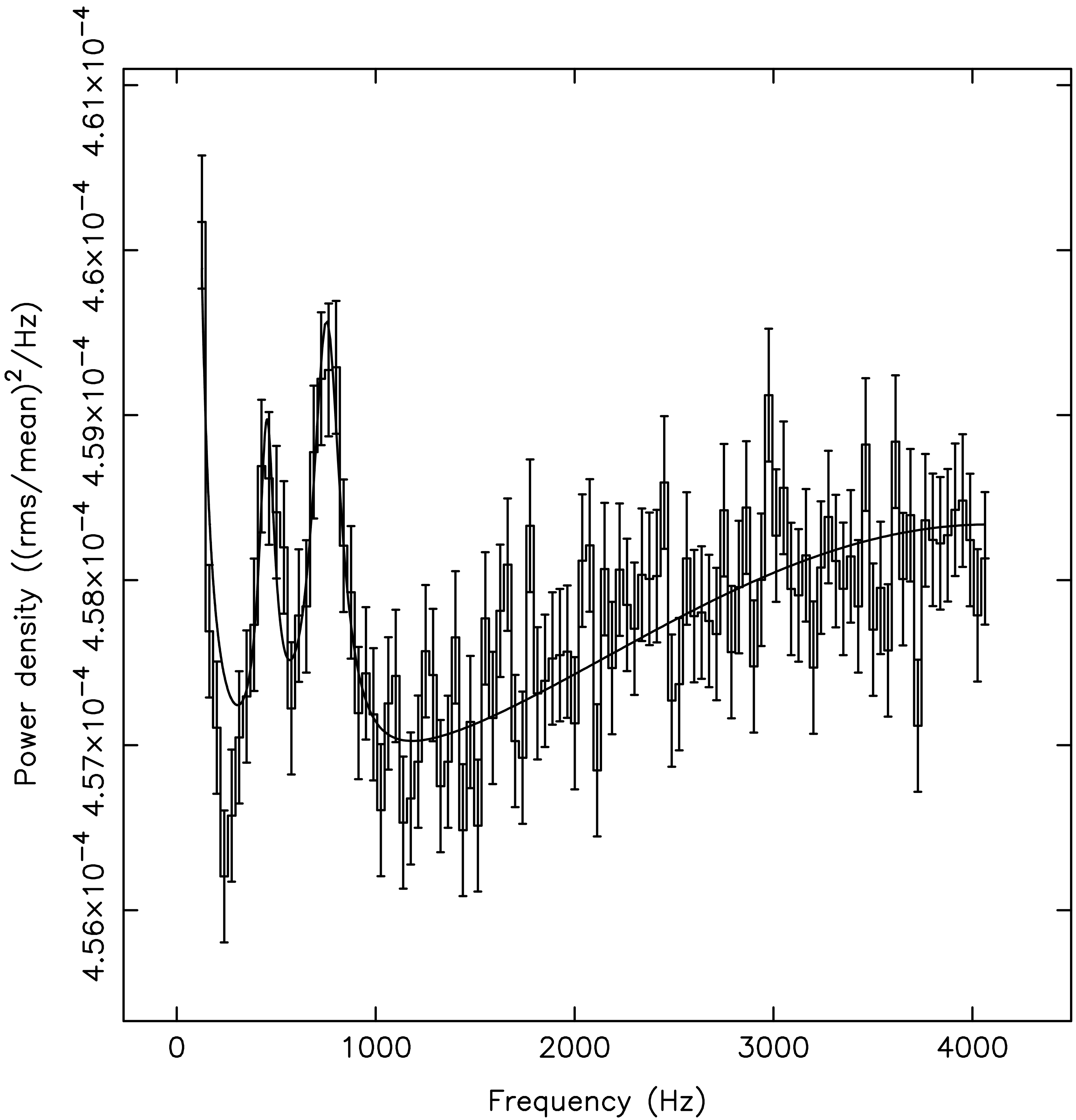
C.F. X-ray Timing experiments



X-ray Timing experiments



X-ray Timing experiments

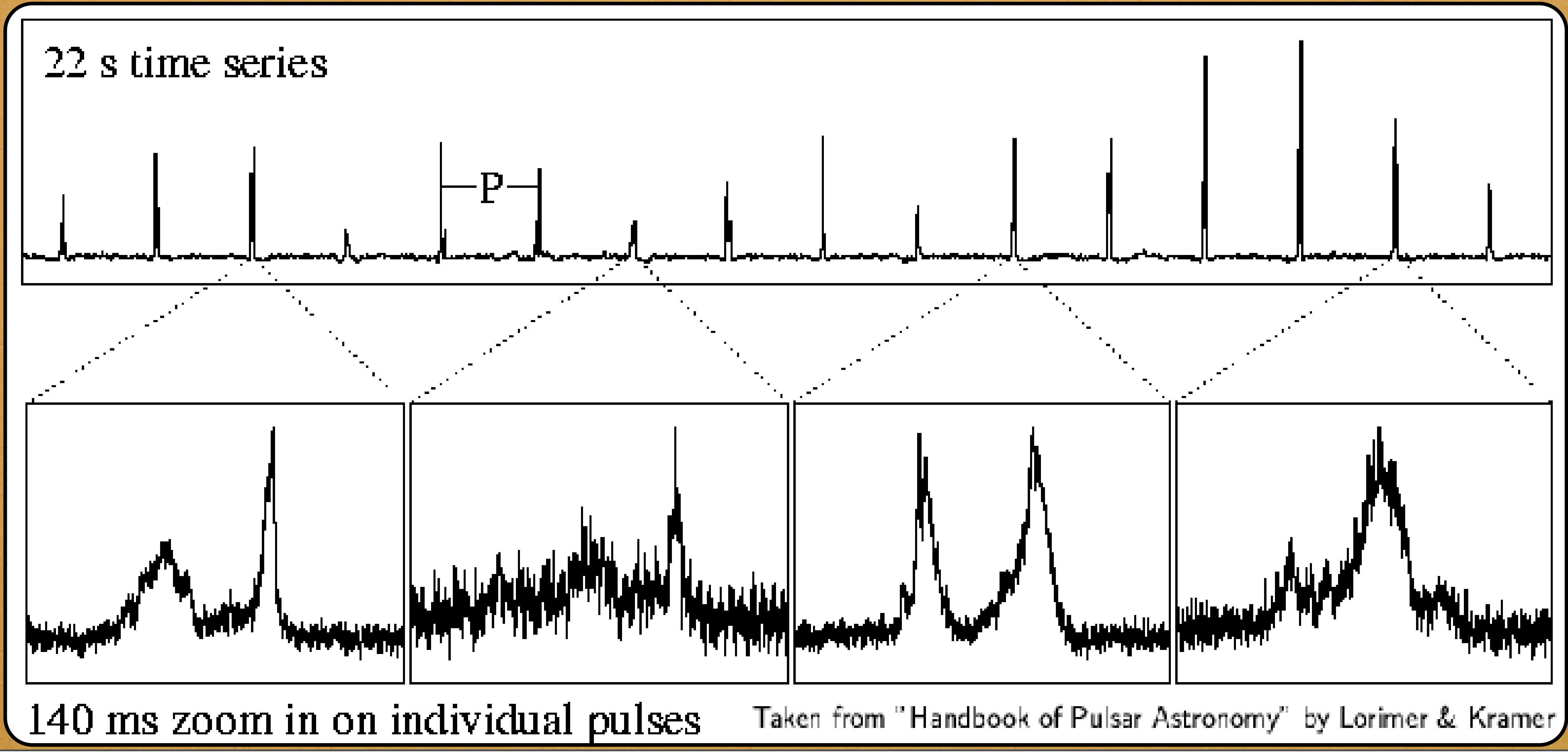


ESTIMATING THE MOMENTS OF A STOCHASTIC PROCESS

CHAPTER 2.2.2

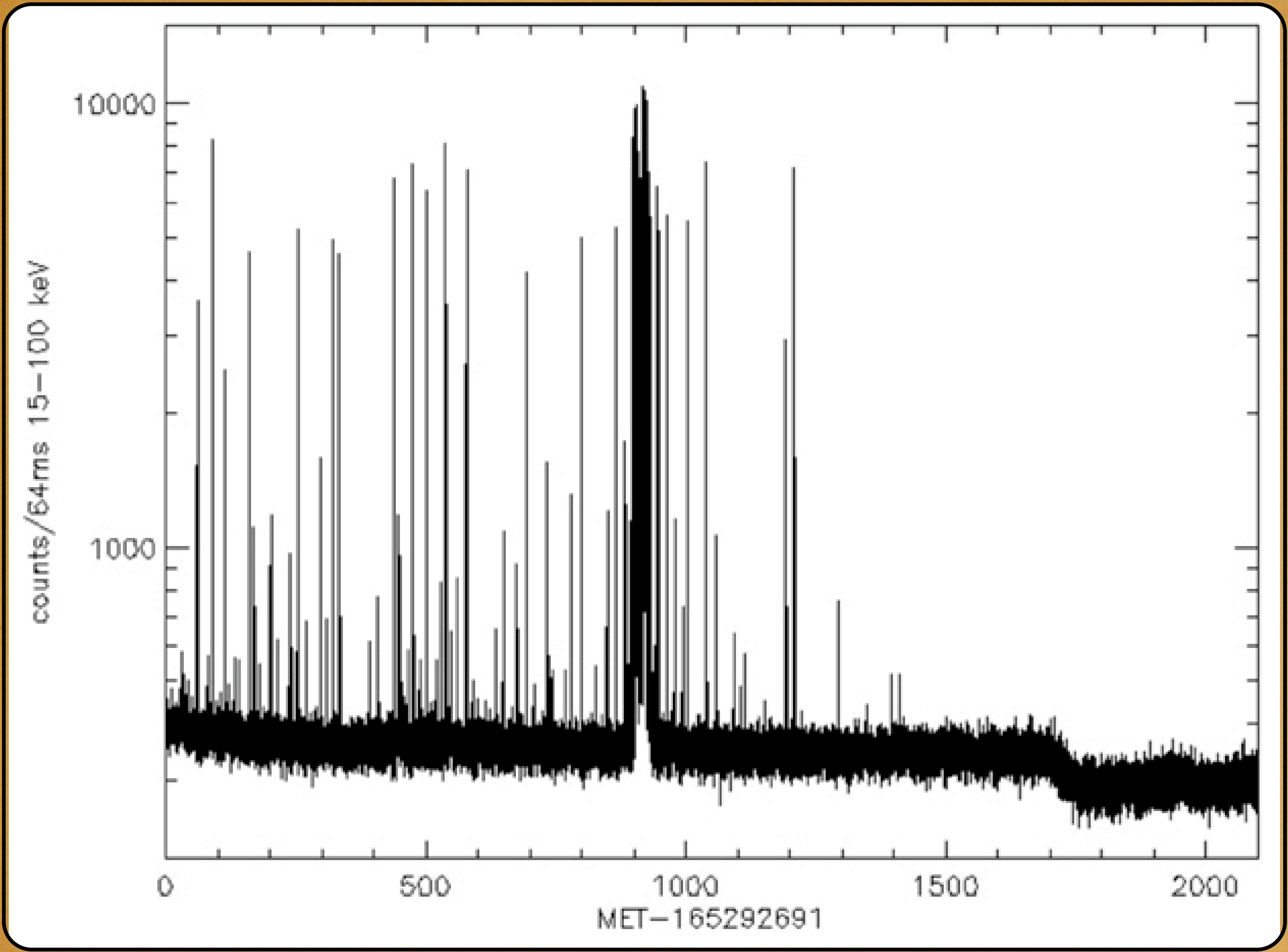
SEE ALSO APPENDIX B3.2, LENA EA.

HOW REPRESENTATIVE IS A MEASUREMENT OF A S.P.?



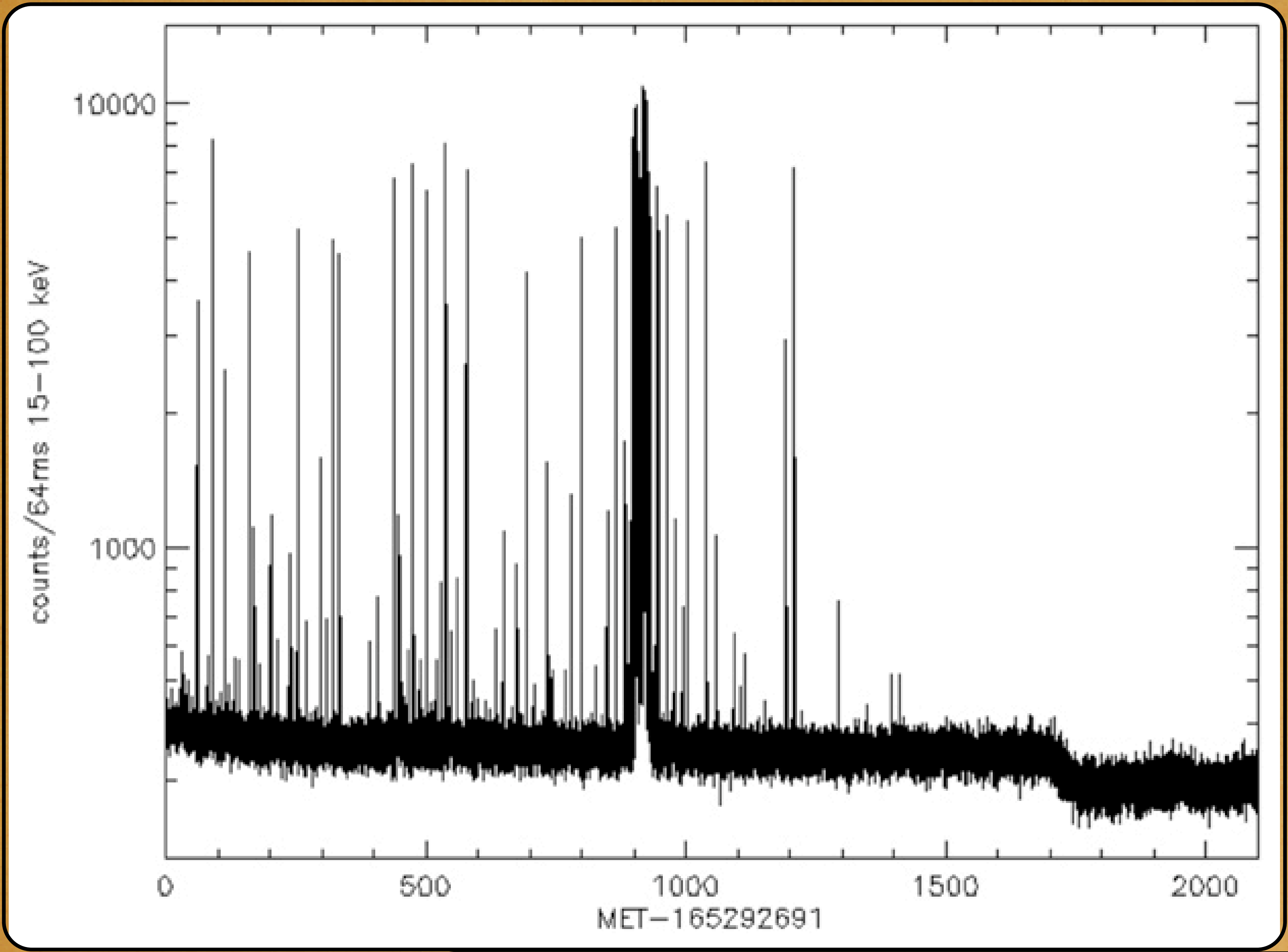
How representative is a measurement?

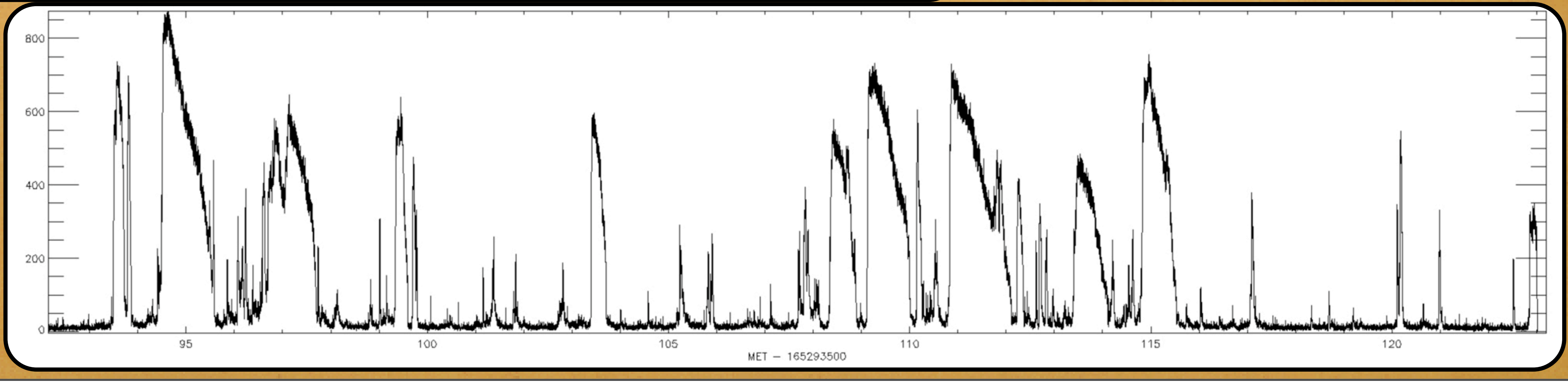
SGR 1900+14



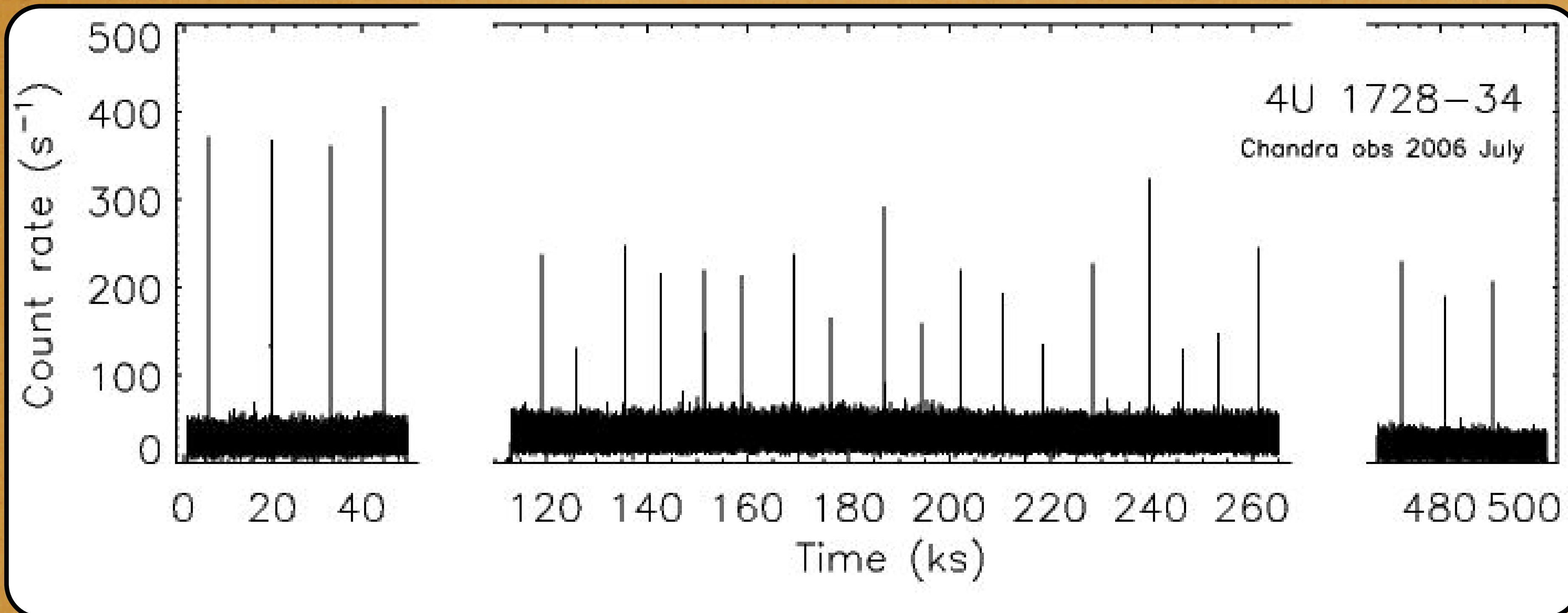
How representative is a measurement?

SGR 1900+14





Different for the case of type I X-ray bursts



Galloway private communication

1 Measurement of x(t) in a time T => windowing and averaging over time ΔT

windowing: $$y(t) = \Pi(\frac{t}{T})x(t)$$

averaging:

$$z(t) \equiv y_{\Delta T}(t) = \frac{1}{\Delta T}\int_{t-\Delta T/2}^{t+\Delta T/2} y(t')dt' = \frac{1}{\Delta T}\int_{-\infty}^{\infty} \Pi(\frac{t-t'}{\Delta T})y(t')dt'$$

= low-pass filter, remember Nyquist theorem