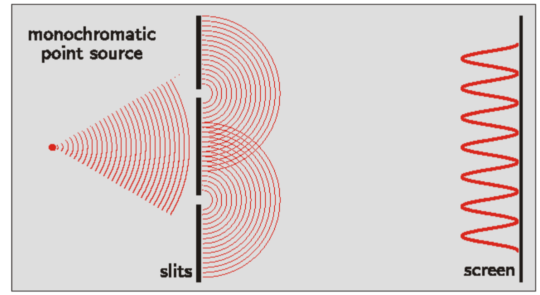
Optical Coherence & Modal decomposition of light

- Intro: optical coherence in Young's double-slit experiment
- Theory: optical coherence versus coherent light
- The Van Cittert-Zernike theorem: creation of spatial coherence
- Optical etendue & Spectral brightness

Coherence in Young's double slit experiment



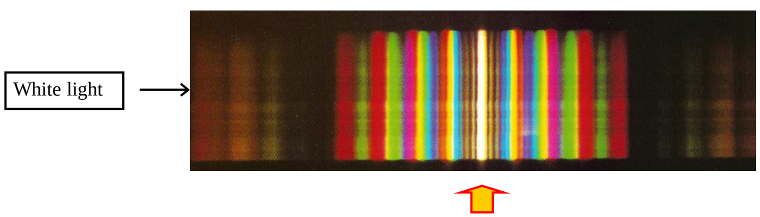
Why only in 1802 ?

Requirements for successful experiment:
1. Monochromatic light (sufficiently 1 color)
2. Point source (What is sufficiently small?)

Requirement 1: monochromatic light

Young's interference pattern is different for each different color

Only in centre (θ=0) is there constructive interference for all colors



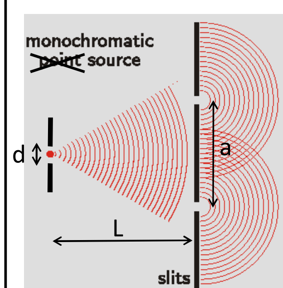
Which quantity in optical field characterizes this?

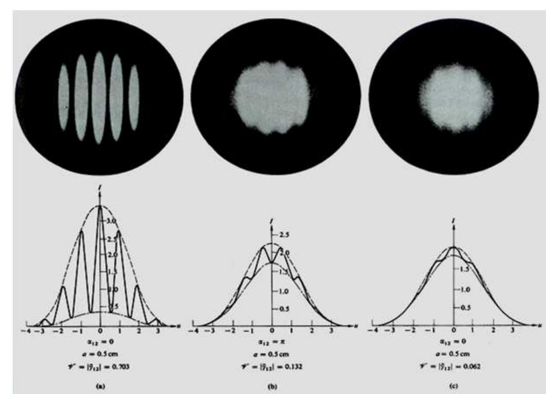
Temporal coherence $\rho(\Delta t) \equiv \langle E^*(t) E(t+\Delta t) \rangle$

$$\rho(\Delta t) = \int I(\omega)\, e^{i\omega\Delta t} d\omega$$

Requirement 2: sufficient spatial coherence

Young's double slit with extended source

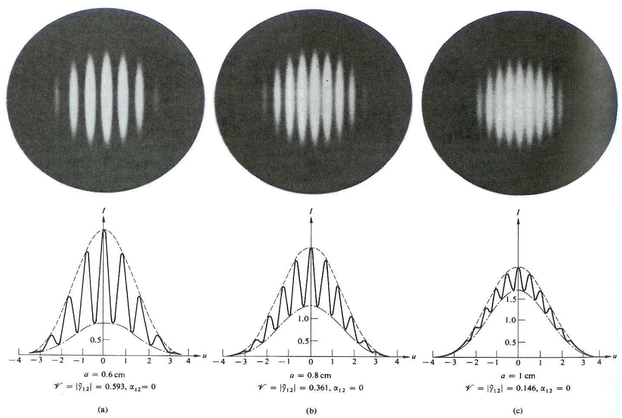




Visibility depends on $\frac{a.d}{L.\lambda}$ → Increase size of source

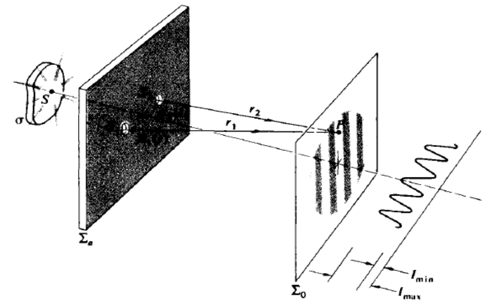
Requirement 2: sufficient spatial coherence

Extended source, change slit distance



A=0.6 cm, V=0.59 A=0.8 cm, V=0.36 A=1.0 cm, V=0.15

Mutual coherence combines space & time



$$E(t) = E_1(t+\tau) + E_2(t)$$

$$I(t) \propto |E(t)|^2 = |E_1(t+\tau) + E_2(t)|^2 = |E_1|^2 + |E_2|^2 + 2Re[E_1(t+\tau)E_2^*(t)]$$

$$V \equiv \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{2\sqrt{I_1 I_2}}{I_1 + I_2} |\gamma_{12}(\tau)|$$

$$\gamma_{12}(\tau) \equiv \frac{\langle E_1(t+\tau)E_2^*(t)\rangle}{\sqrt{\langle |E_1|^2\rangle\langle |E_2|^2\rangle}}$$

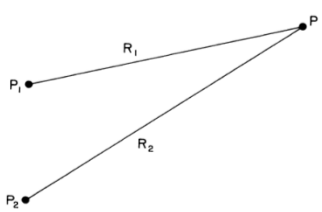
Hecht Fig.12.11

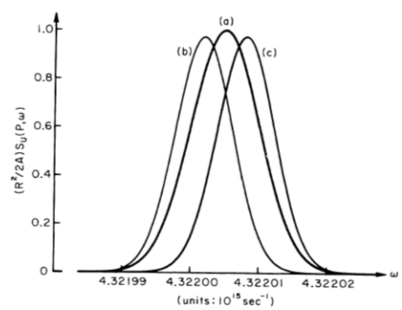
Correlated temporal and spatial coherence can produce surprises

VOLUME 58, NUMBER 25 PHYSICAL REVIEW LETTERS 22 JUNE 1987

Red Shifts and Blue Shifts of Spectral Lines Emitted by Two Correlated Sources

Emil Wolf[a]





Coherence function:

$$S_U(P,\omega) = (2/R^2)S_Q(\omega)[1 + \mathrm{Re}\,\mu_Q(\omega)],$$

"I also showed that under certain circumstances source correlations may produce red shifts or blue shifts of spectral lines in the emitted radiation. This prediction has obviously important implications, particularly for astronomy, and it is therefore desirable to verify it also by experiment."

Coherent optical field versus coherence function of field

$E(x,t)$ versus $\rho(x_1, t_1; x_2, t_2) \equiv \langle E^*(x_1, t_1)E(x_2, t_2)\rangle$

⇩ stationary system

$$\rho(x_1, x_2; \Delta t) \equiv \langle E^*(x_1, t)E(x_2, t+\Delta t)\rangle$$

$$\tilde{\rho}(x_1, x_2; \omega) \equiv \int \rho(x_1, x_2; \Delta t)\, e^{i\omega\Delta t} d\Delta t$$

QM pure state versus QM mixed state

$|\psi\rangle$ $\rho = \overline{|\psi\rangle\langle\psi|}$

Temporal coherence versus spatial coherence

Temporal = longitudinal coherence

$$\rho(\tau) \equiv \langle E^*(t)E(t+\tau)\rangle$$

Fourier related to optical spectrum

$$\gamma(\tau) \equiv \int \left|\tilde{E}(\omega)\right|^2 e^{i\omega\tau} d\omega$$

Spatial = transverse coherence

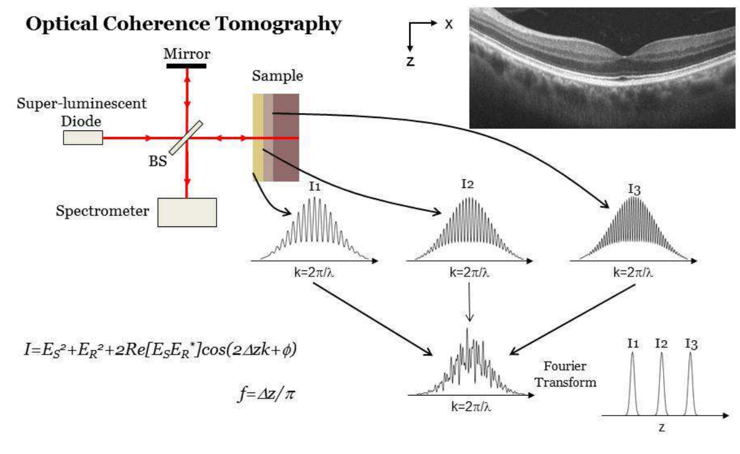
$$\rho\left(x-\tfrac{1}{2}\Delta x, x+\tfrac{1}{2}\Delta x\right) \equiv \left\langle E^*\left(x-\tfrac{1}{2}\Delta x\right)E\left(x+\tfrac{1}{2}\Delta x\right)\right\rangle$$

Fourier related to angular spread

Wigner function: $W(x,p) \equiv \int \rho\left(x-\tfrac{1}{2}\Delta x, x+\tfrac{1}{2}\Delta x\right)e^{ip\Delta x} d\Delta x$

Optical Coherence Tomography = OCT

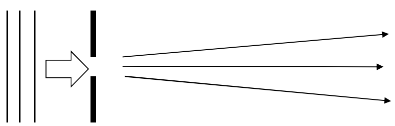
Wide spectrum = short coherence length = good longitudinal resolution



Courtesy R. Nick Graf

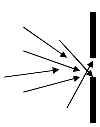
The Van Cittert-Zernike theorem

How optical coherence builds up upon propagation



$$\tilde{E}(\theta) \propto \int E_{in}(x)e^{ikx\theta} dx$$

Coherent illumination

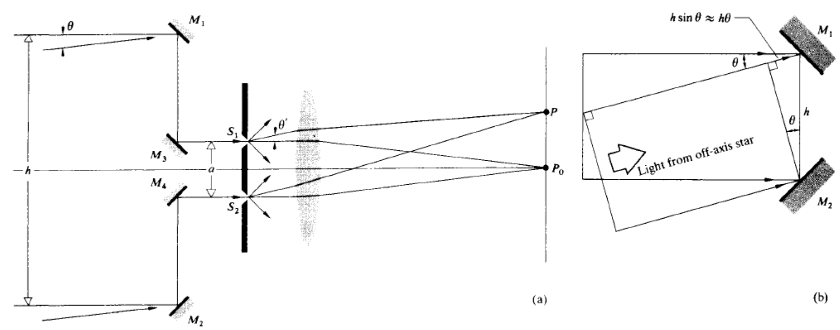


$$\langle \tilde{E}^*(\theta)\tilde{E}(\theta+\Delta\theta)\rangle \propto \int I_{in}(x)e^{ikx\Delta\theta} dx$$

Fully incoherent illumination

$$\langle E_{in}^*(x_1)E_{in}(x_2)\rangle \propto I_{in}\left(\frac{x_1+x_2}{2}\right)\delta(x_1-x_2)$$

Stellar interferometry

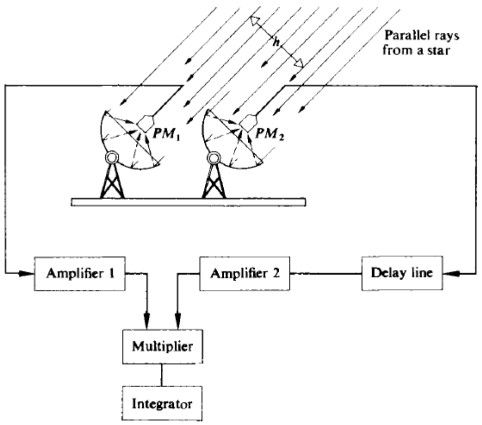


Michelson used this method on star Betelgeuse (Orion) to find:

$\Delta x = 3$ m $\Rightarrow \Delta\theta = 0.23\ \mu$rad

Hecht Fig.12.13

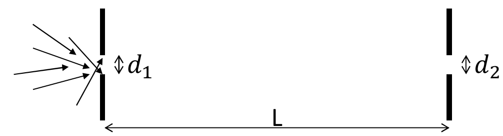
Correlation interferometry (with star light)



$$\langle I_1(t) I_2(t+\tau)\rangle \propto |\langle E_1^*(t) E_2(t+\tau)\rangle|^2 \propto |\gamma_{12}(\tau)|^2$$

Hecht Fig.12.15

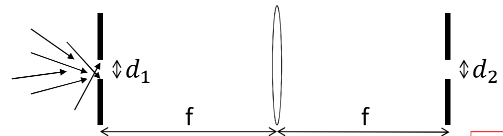
Create a spatially coherent source with two apertures



$$\theta_{coh} = \frac{\lambda}{d_1} \Rightarrow d_{coh} = \frac{L\lambda}{d_1}$$

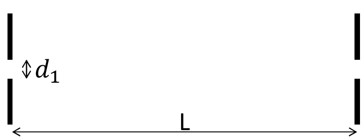
≈ coherent light when

$$N \equiv \frac{d_2}{d_{coh}} = \frac{d_1 d_2}{L\lambda} = \frac{\Delta x . \Delta\theta}{\lambda} < 1$$



$$N = \frac{d_1 d_2}{f\lambda}$$

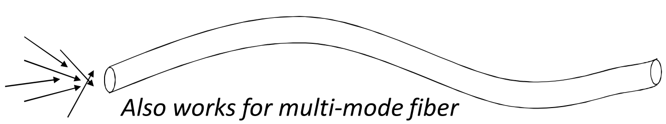
Optical Etendue in two dimensions $N \equiv \Delta S . \Delta\Omega / \lambda^2$



$$\rho_{out}(\vec{x}_1, \vec{x}_2) \equiv \langle E^*(\vec{x}_1) E(\vec{x}_2)\rangle$$

Modal decomposition: $\rho_{out}(\vec{x}_1, \vec{x}_2) = \sum_i P_i \langle E_i^*(\vec{x}_1) E_i(\vec{x}_2)\rangle$

Effective number of modes = Schmidt number: $N_{eff} = \frac{(\sum P_i)^2}{\sum P_i^2}$

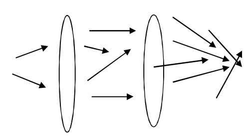


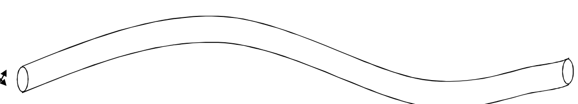
Also works for multi-mode fiber

Brightness of source is limited by its effective temperature

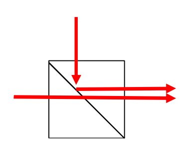
Transmission line theory: $\bar{n}$ photons/s per s^{-1} =Hz bandwidth

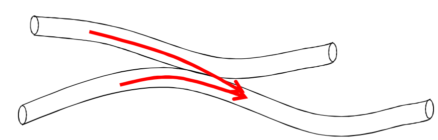
$$\bar{n} = \frac{1}{\exp\left(\frac{h\nu}{k_B T}\right) - 1} \rightarrow \frac{k_B T}{h\nu} \quad \text{for } k_B T > h\nu$$



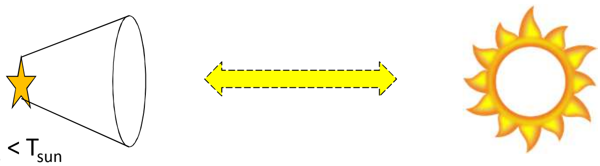


Brightness is limited by T_{eff},
- whatever optics you use
- whatever sources you try to combine





How hot can it be at the focus of sunlight?

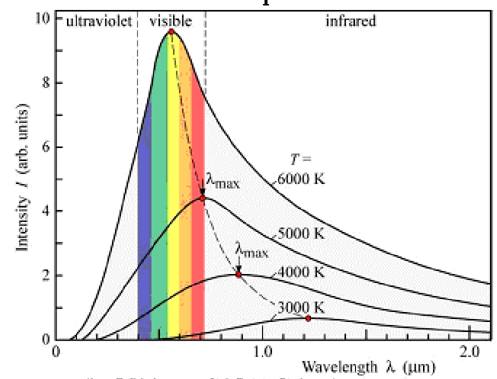


$T_{focus} < T_{sun}$

Planck's expression for brightness of Blackbody radiation

Spectral brightness $B_\nu = \frac{dP}{dS_\perp d\Omega\, d\nu} = 2\frac{\nu^2}{c^2}h\nu\bar{n}$ $\quad \bar{n} = \frac{1}{\exp\left(\frac{h\nu}{k_B T}\right) - 1}$

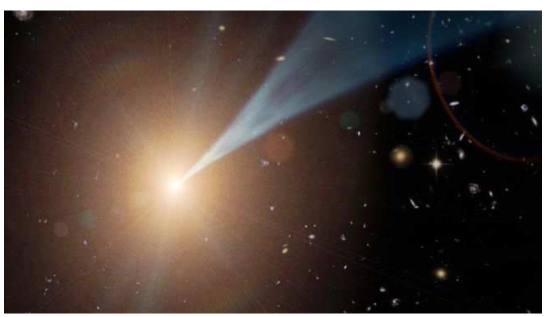
Planck spectrum



(from E. Schubert; www.LightEmittingDiodes.org)

How bright is light from a star?

Starlight can be brighter than any incandescent lamp on earth
if you can observe it in a diffraction-limited way



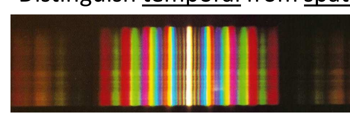
- Night sky is dark because universe is finite (and not ever-lasting)
- Diffraction-limited image of star can be as bright as the sun

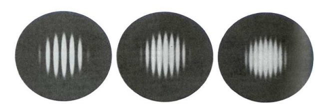
Summary

- Optical coherence function to describe incoherent light

$$\rho(x_1, t_1; x_2, t_2) \equiv \langle E^*(x_1, t_1) E(x_2, t_2) \rangle$$

- Distinguish temporal from spatial coherence





- The Van Cittert-Zernike theorem: Coherence created by propagation

$$N = \frac{d_1 d_2}{L\lambda} = \frac{\Delta x . \Delta\theta}{\lambda}$$

- Optical etendue characterizes the 'number of spatial modes'