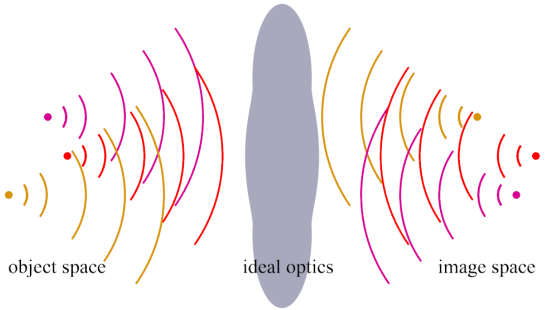
Lecture 2: Geometrical Optics

Outline

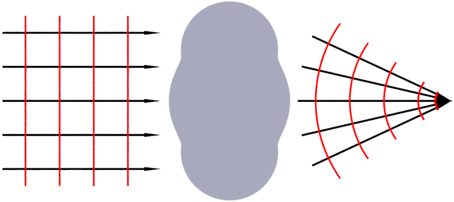
1. Geometrical Approximation
2. Lenses
3. Mirrors
4. Optical Systems
5. Images and Pupils
6. Aberrations

Ideal Optics



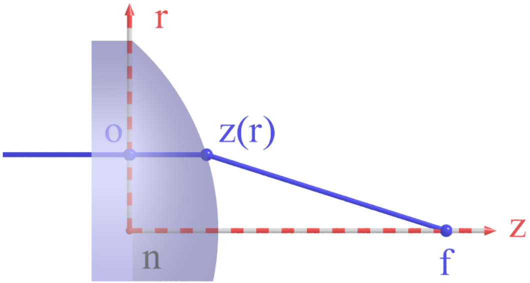
- ideal optics: spherical waves from any point in object space are imaged into points in image space
- corresponding points are called *conjugate points*
- *focal point*: center of converging or diverging spherical wavefront
- object space and image space are reversible

Geometrical Optics



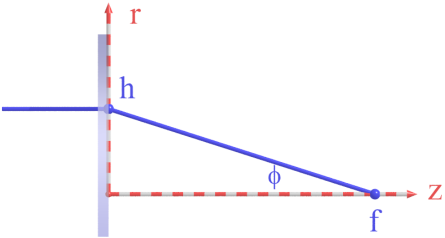
- rays are normal to locally flat wave (locations of constant phase)
- rays are reflected and refracted according to Fresnel equations
- phase is neglected $\Rightarrow$ incoherent sum
- rays can carry polarization information
- optical system is finite $\Rightarrow$ diffraction
- geometrical optics neglects diffraction effects: $\lambda \Rightarrow 0$
- *physical optics* $\lambda > 0$
- simplicity of geometrical optics mostly outweighs limitations

Surface Shape of Perfect Lens



- lens material has index of refraction n
- $\overline{o\,z(r)} \cdot n + \overline{z(r)\,f} = \text{constant}$
- $n \cdot z(r) + \sqrt{r^2 + (f - z(r))^2} = \text{constant}$
- solution $z(r)$ is hyperbola with eccentricity $e = n > 1$

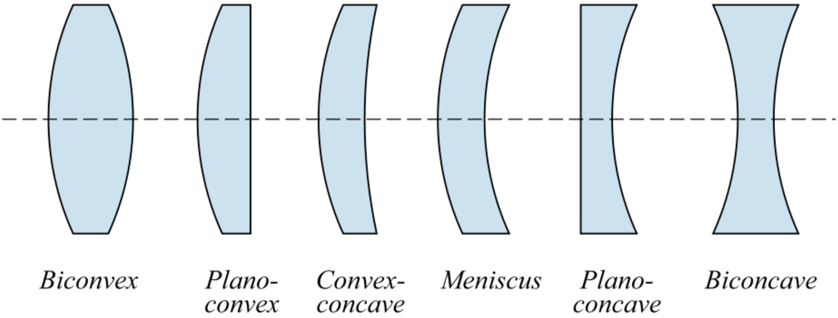
Paraxial Optics



Assumptions:

1. assumption 1: Snell's law for small angles of incidence ($\sin\phi \approx \phi$)
2. assumption 2: ray hight h small so that optics curvature can be neglected (plane optics, ($\cos x \approx 1$))
3. assumption 3: $tan\phi \approx \phi = h/f$
4. decent until about 10 degrees

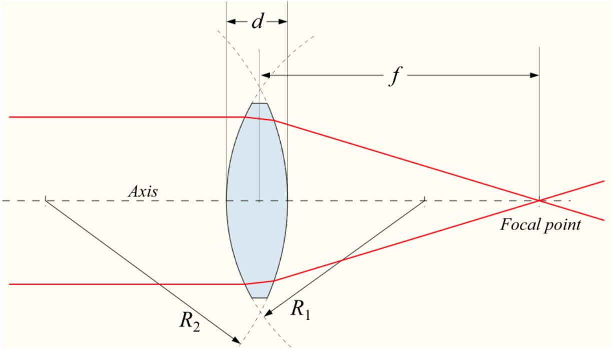
Spherical Lenses



en.wikipedia.org/wiki/File:Lens2.svg

- if two spherical surfaces have same radius, can fit them together
- surface error requirement less than $\lambda/10$
- grinding spherical surfaces is easy $\Rightarrow$ most optical surfaces are spherical

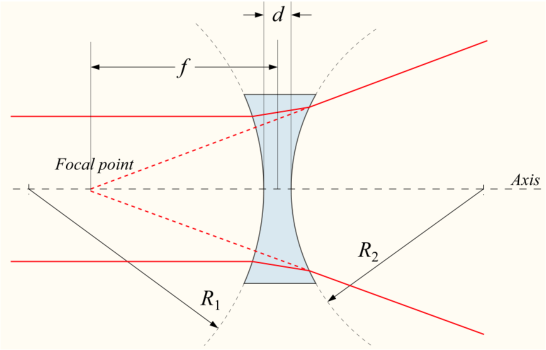
Positive/Converging Spherical Lens Parameters



commons.wikimedia.org/wiki/File:Lens1.svg

- center of curvature and radii with signs: $R_1 > 0, R_2 < 0$
- center thickness: d
- positive focal length $f > 0$

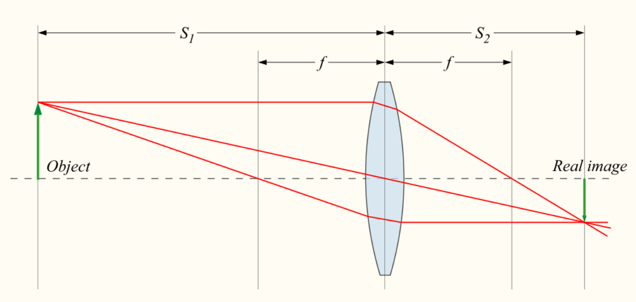
Negative/Diverging Spherical Lens Parameters



commons.wikimedia.org/wiki/File:Lens1b.svg

- note different signs of radii: $R_1 < 0$, $R_2 > 0$
- virtual focal point
- negative focal length ($f < 0$)

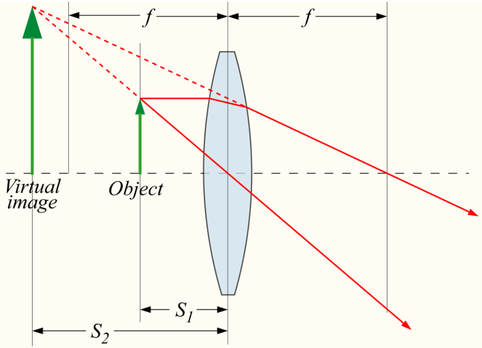
General Lens Setup: Real Image



commons.wikimedia.org/wiki/File:Lens3.svg

- *object distance* S_1, *object height* h_1
- *image distance* S_2, *image height* h_2
- axis through two centers of curvature is *optical axis*
- surface point on optical axis is the *vertex*
- *chief ray* through center maintains direction

General Lens Setup: Virtual Image



commons.wikimedia.org/wiki/File:Lens3b.svg

- note object closer than focal length of lens
- virtual image

Thin Lens Approximation

- thin-lens equation:

$$\frac{1}{S_1} + \frac{1}{S_2} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

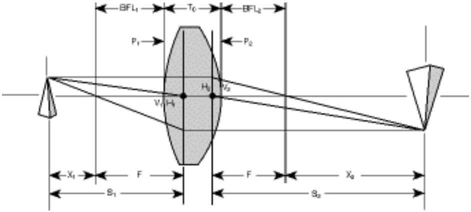
- Gaussian lens formula:

$$\frac{1}{S_1} + \frac{1}{S_2} = \frac{1}{f}$$

Finite Imaging

- rarely image point sources, but extended object
- object and image size are proportional
- orientation of object and image are inverted
- (transverse) magnification perpendicular to optical axis: $M = h_2/h_1 = -S_2/S_1$

Thick Lenses

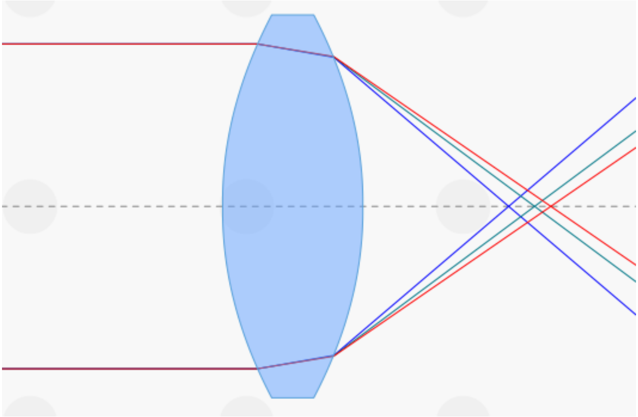


www.newport.com/servicesupport/Tutorials/default.aspx?id=169

- basic thick lens equation $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n-1)d}{nR_1R_2}\right)$
- thin means $d << R_1R_2$
- focal lengths measured from *principal planes*
- distance between vertices and principal planes given by

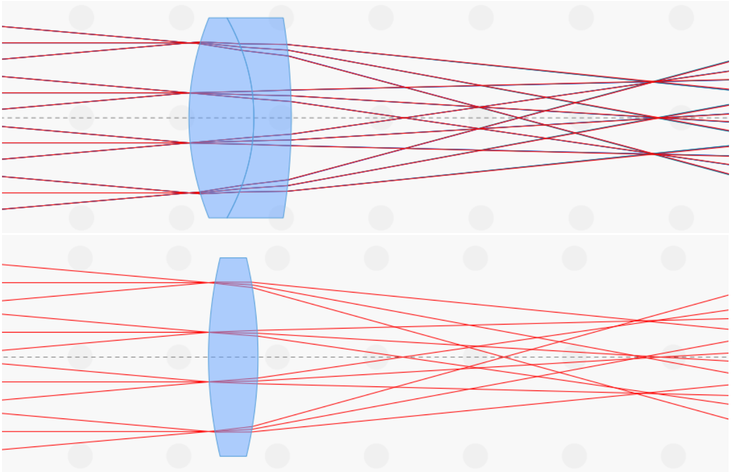
$$H_{1,2} = -\frac{f(n-1)d}{R_{2,1}n}$$

Chromatic Aberration



- due to wavelength dependence of index of refraction
- higher index in the blue $\Rightarrow$ shorter focal length in blue

Achromatic Lens



- combination of 2 lenses, different glass dispersion
- also less spherical aberration

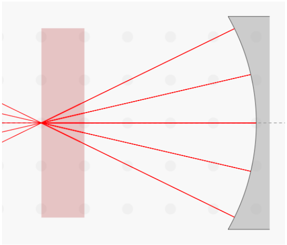
Mirrors

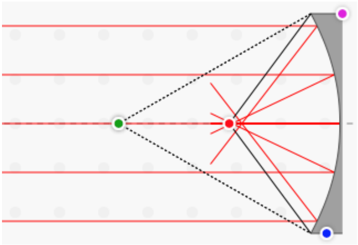
Mirrors vs. Lenses

- mirrors are completely achromatic
- reflective over very large wavelength range (UV to radio)
- can be supported from the back
- can be segmented
- wavefront error is twice that of surface, lens is (n-1) times surface
- only one surface to 'play' with

Plane Mirrors: Fold Mirrors and Beamsplitters

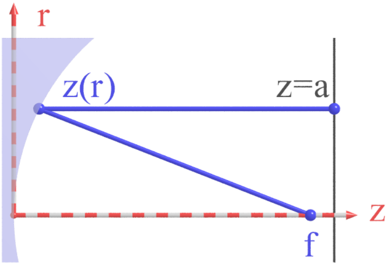
Spherical Mirrors





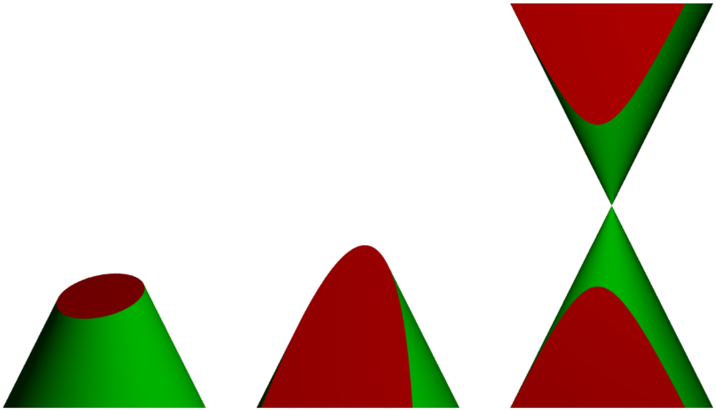
- easy to manufacture
- focuses light from center of curvature onto itself
- focal length is half of curvature: $f = R/2$
- tip-tilt misalignment does not matter
- has no optical axis
- does not image light from infinity correctly (spherical aberration)

Parabolic Mirrors



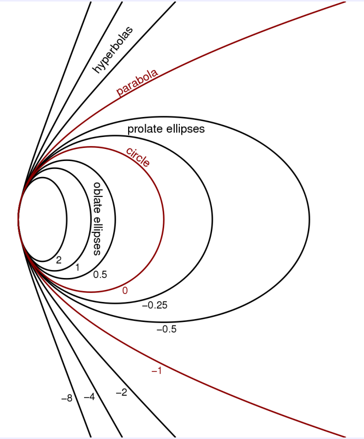
- want to make flat wavefront into spherical wavefront
- distance $\overline{az(r)} + \overline{z(r)f}$ = const.
- $z(r) = r^2/2R$
- perfect image of objects at infinity
- has clear optical axis

Conic Sections



- circle and ellipses: cuts angle $<$ cone angle
- parabola: angle = cone angle
- hyperbola: cut along axis

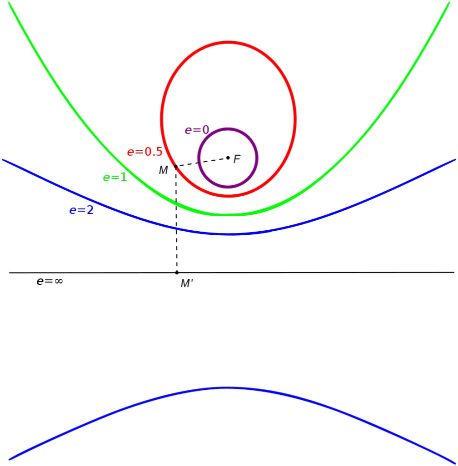
Conic Constant K



en.wikipedia.org/wiki/Conic_constant

- $r^2 - 2Rz + (1+K)z^2 = 0$ for $z(r=0) = 0$
- $z = \frac{r^2}{R} \frac{1}{1+\sqrt{1-(1+K)\frac{r^2}{R^2}}}$
- R radius of curvature
- $K = -e^2$, e eccentricity
- prolate ellipsoid ($K > 0$)
- sphere ($K = 0$)
- oblate ellipsoid ($0 > K > -1$)
- parabola ($K = -1$)
- hyperbola ($K < -1$)
- all conics are almost spherical close to origin
- analytical ray intersections

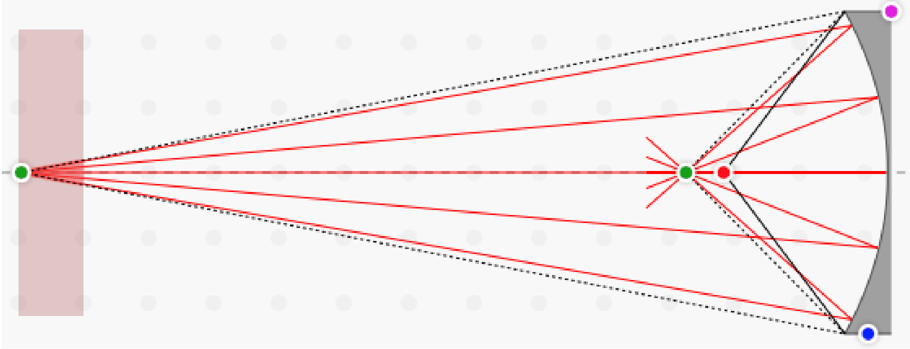
Foci of Conic Sections



en.wikipedia.org/wiki/File:Eccentricity.svg

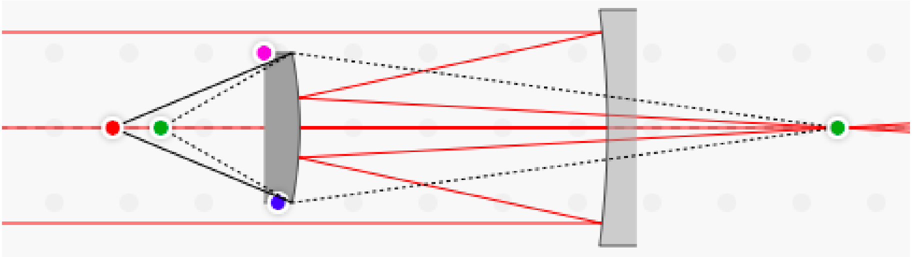
- sphere has single focus
- ellipse has two foci
- parabola (ellipse with $e = 1$) has one focus (and another one at infinity)
- hyperbola ($e > 1$) has two focal points

Elliptical Mirrors



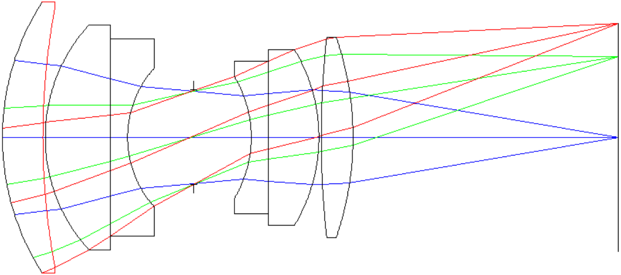
- have two foci at finite distances
- perfectly reimage one focal point into another

Hyperbolic Mirrors



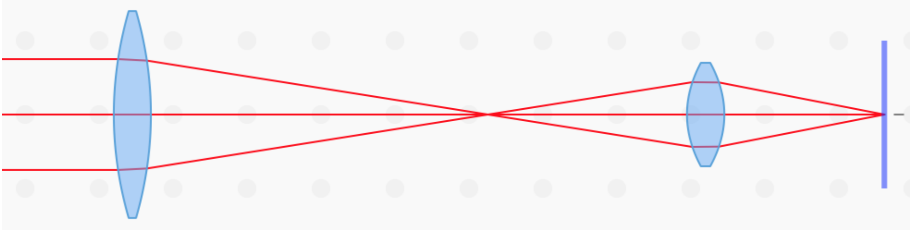
- have a real focus and a virtual focus (behind mirror)
- perfectly reimage one focal point into another

Overview



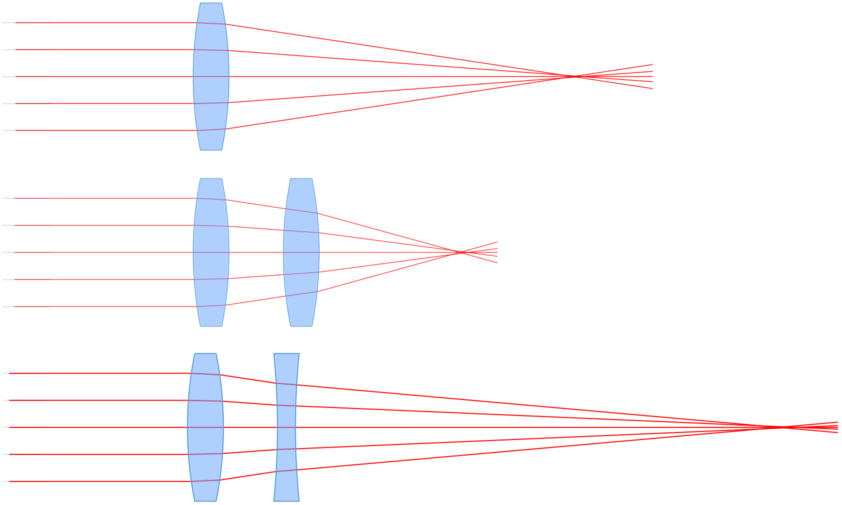
- combinations of several optical elements (lenses, mirrors, stops)
- examples: camera “lens”, microscope, telescopes, instruments
- thin-lens combinations can be treated analytically
- effective focal length: $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$

Simple Thin-Lens Combinations



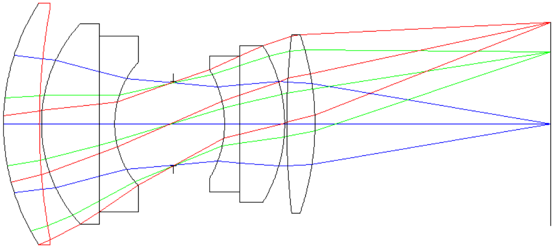
- distance $>$ sum of focal lengths $\Rightarrow$ real image between lenses
- apply single-lens equation successively

Second Lens Adds Convergence or Divergence



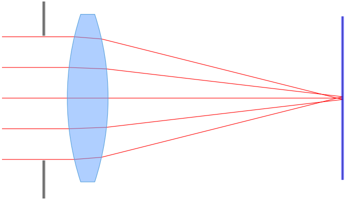
F-number and Numerical Aperture

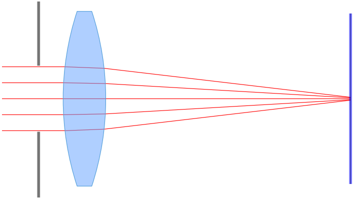
Aperture



- all optical systems have a place where 'aperture' is limited
- main mirror of telescopes
- aperture stop in photographic lenses
- aperture typically has a maximum diameter
- aperture size is important for diffraction effects

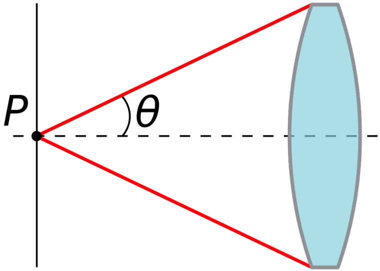
F-number

f/2: 

f/4: 

- describes the light-gathering ability of the lens
- f-number given by $F = f/D$
- also called focal ratio or f-ratio, written as: f/F
- the bigger F, the better the paraxial approximation works
- fast system for $F < 2$, slow system for $F > 2$

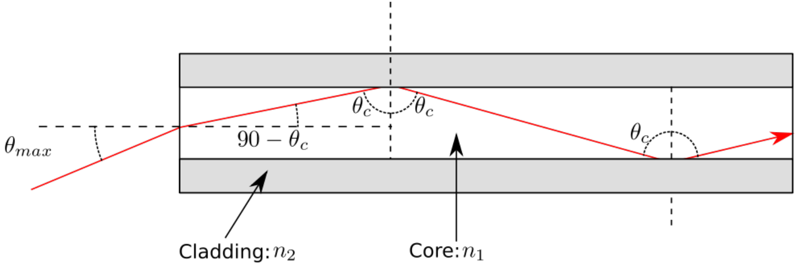
Numerical Aperture



en.wikipedia.org/wiki/File:Numerical_aperture.svg

- numerical aperture (NA): $n \sin\theta$
- n index of refraction of working medium
- θ half-angle of maximum cone of light that can enter or exit lens
- important for microscope objectives (n often not 1)

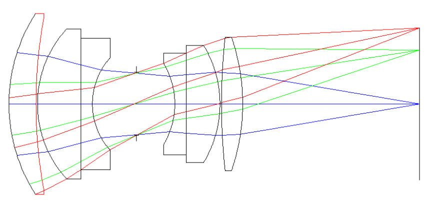
Numerical Aperture in Fibers



en.wikipedia.org/wiki/File:OF-na.svg

- acceptance cone of the fiber determined by materials
- $NA = n \sin\theta = \sqrt{n_1^2 - n_2^2}$
- *n* index of refraction of working medium

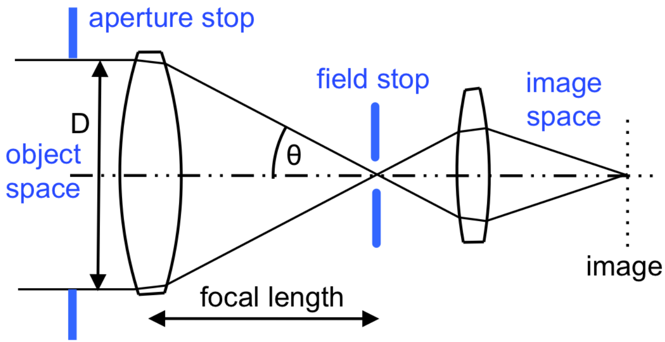
Images and Pupils



Images and Pupils

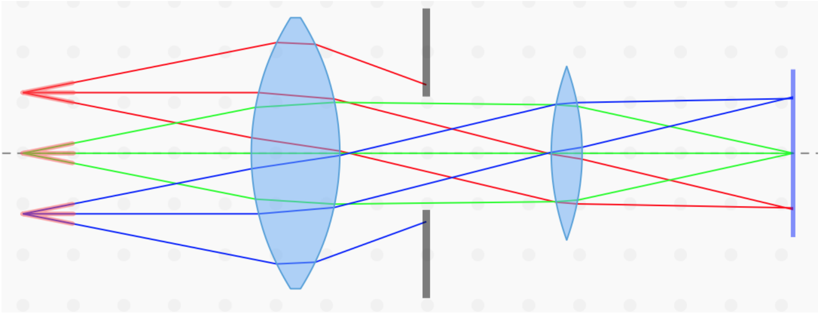
- image
  - every object point comes to a focus in an image plane
  - light in one image point comes from pupil positions
  - object information is encoded in position, not in angle
- pupil
  - all object rays are smeared out over complete aperture
  - light in one pupil point comes from different object positions
  - object information is encoded in angle, not in position

Aperture and Field Stops



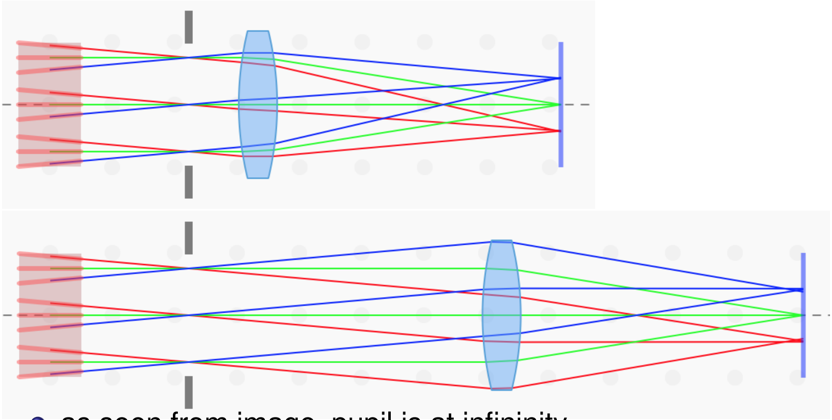
- aperture stop limits the amount of light reaching the image
- aperture stop determines light-gathering ability of optical system
- field stop limits the image size or angle

Vignetting



- effective aperture stop depends on position in object
- image fades toward its edges

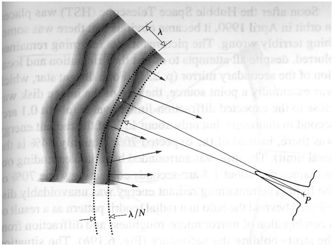
Telecentric Arrangement



- as seen from image, pupil is at infininity
- easy: lens is its focal length away from pupil (image)
- magnification does not change with focus positions
- ray cones for all image points have the same orientation

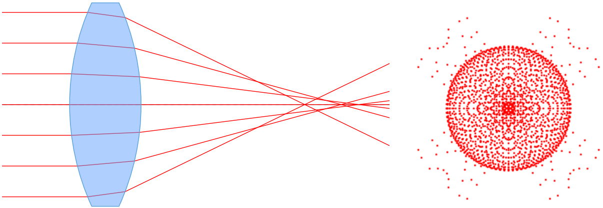
Aberrations

Spot Diagrams and Wavefronts



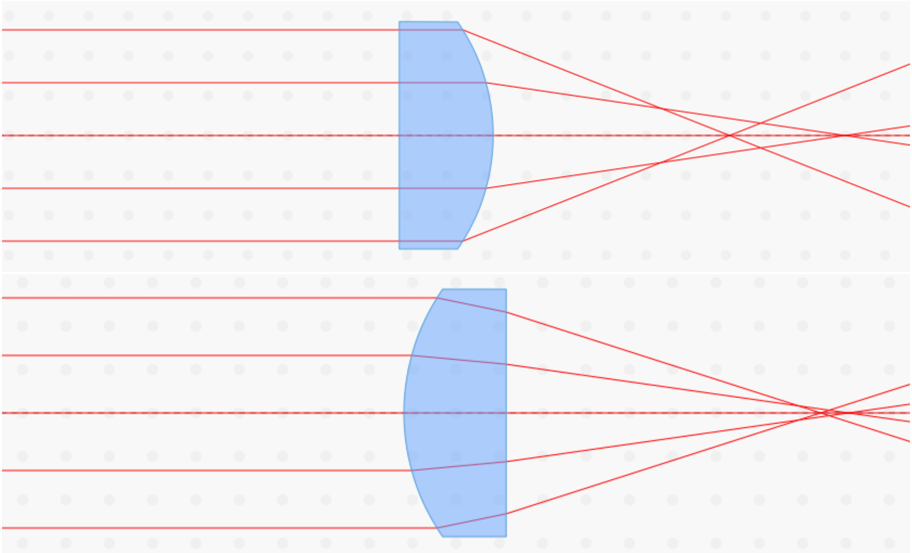
- plane of least confusion is location where image of point source has smallest diameter
- spot diagram: shows ray locations in plane of least confusion
- spot diagrams are closely connected with wavefronts
- aberrations are deviations from spherical wavefront

Spherical Aberration of Spherical Lens

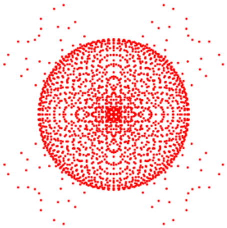


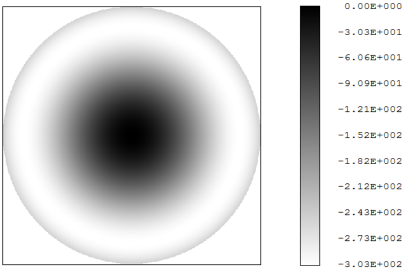
- different focal lengths of paraxial and marginal rays
- longitudinal spherical aberration along optical axis
- transverse (or lateral) spherical aberration in image plane
- much more pronounced for short focal ratios
- foci from paraxial beams are further away than marginal rays
- spot diagram shows central area with fainter disk around it

Minimizing Spherical Aberrations



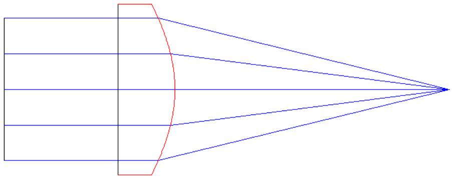
Spherical Aberration Spots and Waves

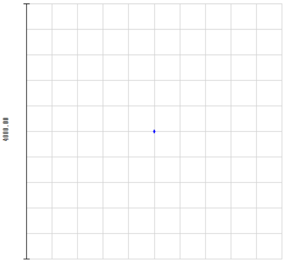




- spot diagram shows central area with fainter disk around it
- wavefront has peak and turned-up edges

Aspheric Lens

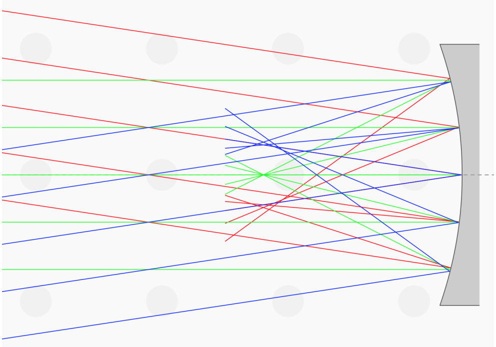




- conic constant $K = -1 - \sqrt{n}$ makes perfect lens
- difficult to manufacture
- but possible these days

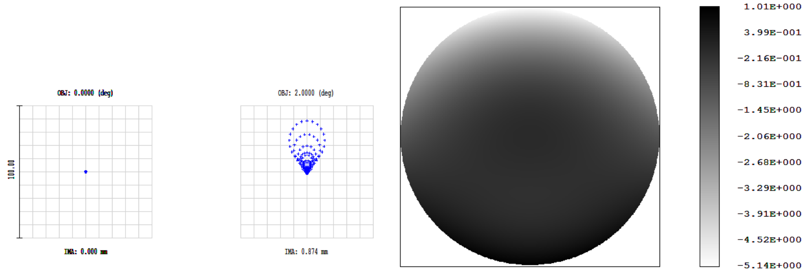
HST

Coma



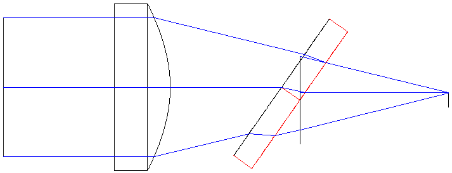
- typically seen for object points away from optical axis
- leads to ’tails’ on stars

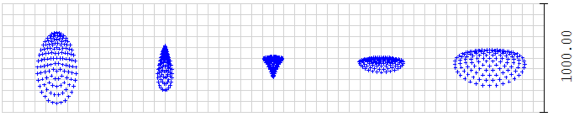
Coma Spots and Waves

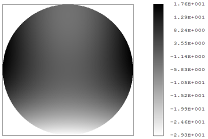


- parabolic mirror with perfect on-axis performance
- spots and wavefront for off-axis image points
- wavefront is tilted in inner part

Astigmatism due to Tilted Glass Plate in Converging Beam

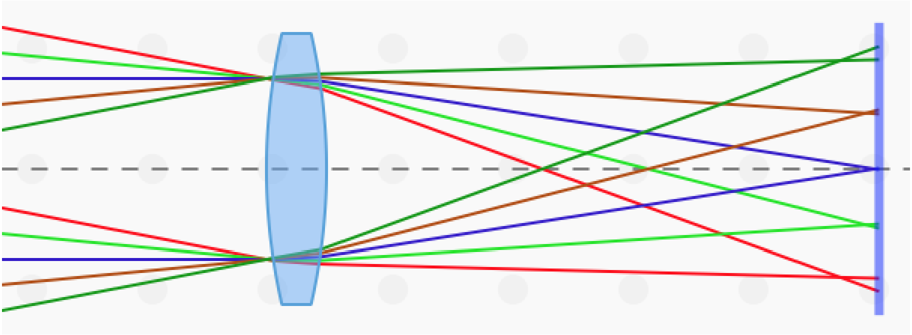






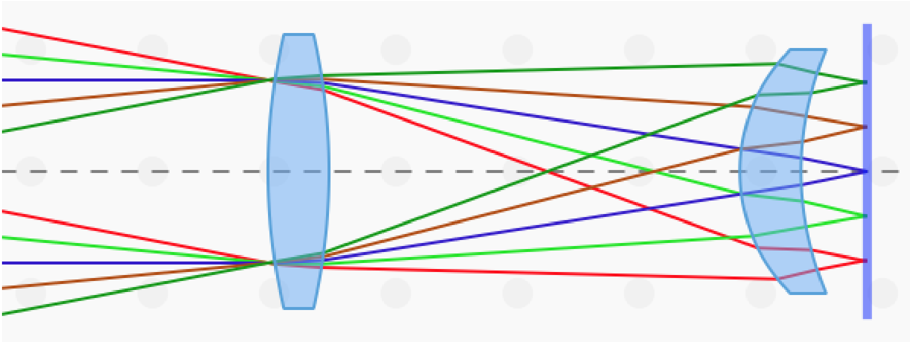
- astigmatism: focus in two orthogonal directions, but not in both at the same time
- tilted glass-plate: astigmatism, spherical aberration, beam shift
- tilted plates: beam shifters, filters, beamsplitters
- difference of two parabolae with different curvatures

Field Curvature



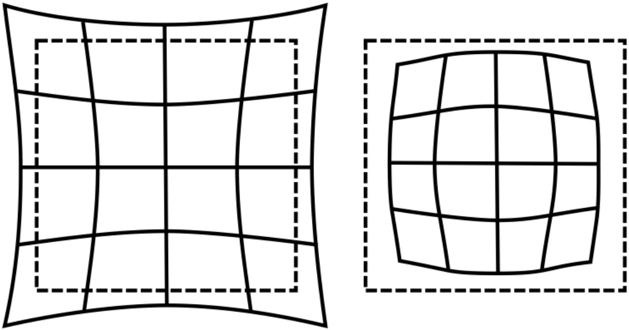
- field (Petzval) curvature: image lies on curved surface
- curvature comes from lens thickness variation across aperture
- problems with flat detectors (e.g. CCDs)
- potential solution: field flattening lens close to focus

Petzval Field Flattening



- Petzval curvature only depends on index of refraction and focal length of lenses
- Petzval curvature is independent of lens position!
- field flattener also makes image much more telecentric

Distortion



- image is sharp but geometrically distorted
- (a) object
- (b) positive (or pincushion) distortion
- (c) negative (or barrel) distortion

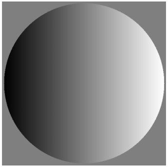
Aberration Descriptions

Seidel Aberrations

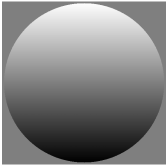
- Ludwig von Seidel (1857)
- Taylor expansion of $\sin\phi$
- $\sin\phi = \phi - \frac{\phi^3}{3!} + \frac{\phi^5}{5!} - ...$
- paraxial: first-order optics
- Seidel optics: third-order optics
- Seidel aberrations: spherical, astigmatism, coma, field curvature, distortion

Zernike Polynomials

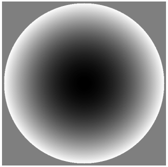
tip



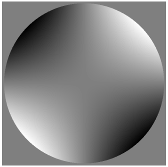
tilt



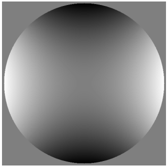
focus



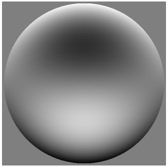
astigmatism (45 deg)



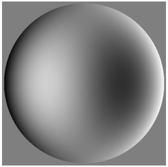
astigmatism 0 deg



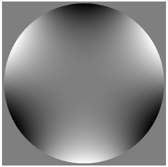
coma (0 deg)



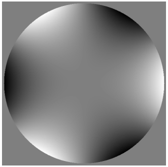
coma (90 deg)



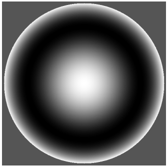
trefoil (0 deg)



trefoil (30 deg)



third-order spherical



- low orders equal Seidel aberrations
- form orthonormal basis on unit circle