Optics & Instruments 2015/2016: Optical Coherence & Optical Etendue

November 18, 2015

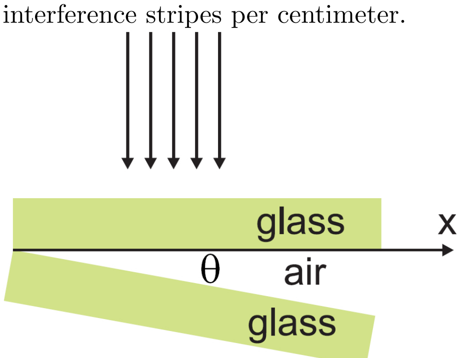
1. **Three simple example for optical coherence**

   (a) C*oherence of a laser:* What is the coherence length of a HeNe laser operating at 632.8 nm with a linewidth of 1 MHz?

   (b) *Angular size of the sun:* We want to do a double-slit experiment using sunlight that we filter spectrally so we deal with approximately monochromatic light at 500 nm. If the opening angle of the light from the sun is 0.5 degree, what is the maximum distance d of the slits at which we can still well resolve the interference pattern?

   (c) *Longitudinal coherence of AM radio waves:* In an AM radio transmission the music signal (100Hz-20kHz) is transmitted as an amplitude modulation of the carrier wave. Recalling the effects in frequency of the sum of two waves, estimate the coherence length of the radiation if the carrier wave is at 538 kHz. Which audio frequency, the lowest or the highest, is relevant?

2. **Optical coherence and interference**
   Two thick glass sheets are put together leaving a wedge-shaped air gap (wedge angle $\theta \ll 1$). This wedge is normal illuminated by parallel light with a wavelength around 600 nm. In reflection you see 14 bright interference stripes per centimeter.



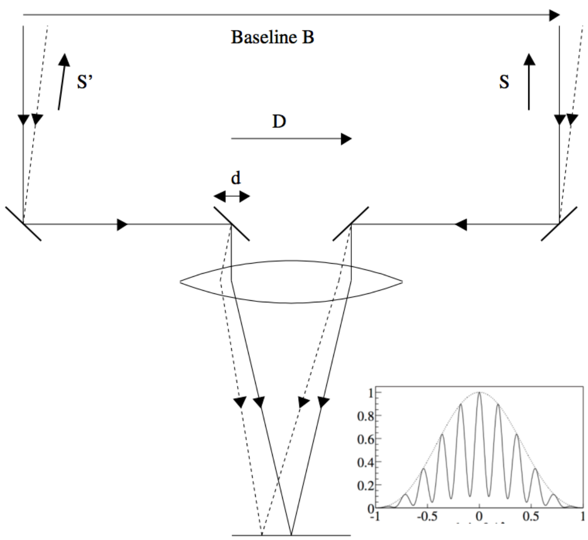
   (a) For which optical path differences Δs do you observe constructive inference between the zeroth and the first reflected light ray?

   (b) Calculate the wedge angle θ

   (c) The interference pattern vanishes 3 cm away from the point of contact of the two glass sheets. What is the coherence length if you consider only the longitudinal (time) coherence?

   (d) What is the (transverse) coherence length if you consider only the transverse coherence?

3. **Michelson stellar interferometer**
Consider the stellar interferometer shown below, where the "slit width" is set by the mirror diameter d and the slit spacing is set by the baseline B. Denote the optical wavelength as λ and the focal length of the telescope as f.



   (a) Looking at the diraction pattern in the inset express the number of fringes and the fringes spacing in terms of the relevant quantities of the interferometer.

   (b) Explain why a Michelson stellar interferometer cannot be used with white light and formulate a criteria for the allowed spectral bandwidth required to observed at least N interference fringes. Note: this number N is different (and typically much smaller) than the number of fringes mentioned under (a).

   (c) At visible frequencies ($\lambda \approx 570$ nm), atmospheric turbulence typically limits the angular resolution to 0.25 arcsec. What is the diameter of an ordinary telescope with this angular resolution?

   (d) Does this limitation also apply to the angular resolution of a stellar interferometer?

   (e) Describe the nature of and the temporal variations in the interference pattern of a stellar interferometer with a large baseline illuminated with spectrally-filtered light from the star. *Hint: to first order, atmospheric turbulence produces an apparent shift of the optical source that varies on a timescale of milliseconds.*

4. **How bright is the star Betelgeuse?**
We want to estimate the amount of light that we can collect from the star Betelgeuse with a large telescope on earth, using simple thermodynamic arguments. For this we consider a telescope with a diameter $D =$ 5 m operated under atmospheric conditions with an angular resolution limited by a seeing of $\theta \approx 1.22\,\lambda/L$ with Freed parameter $L \approx 20$ cm around a wavelength $\lambda \approx 640$ nm. Earlier spectral and interferometeric measurements on the star Betelgeuse have produced estimates of its effective temperature $T \approx 3140 - 3640$ K and angular diameter of $\theta_{star} = 0.043 - 0.056$ arcseconds (source Wikipedia).

   (a) Estimate the power collected by the telescope in the spectral window $\lambda =$ 630-650 nm. Express your results both in Watts as well as in number of photons per second. *Hint: First estimate the average photon number per optical mode and the 'number of spatial mode' that the telescope can collect of light from Betelgeuse.*

   (b) The above estimate is rough, but perfectly OK as an order of magnitude estimate. You can obtain an alternative order of magnitude estimate by comparing the intensity of sunlight on earth, which is specified as ≈ 1.3 kW/m^2, with information on the luminoscity of Betelgeuse $L \approx 0.9 - 1.5 \times 10^5\ L_0$ and its distances of $\approx 500 - 780$ lightyears. Use this information to estimated the power collected by the telescope within the complete optical spectrum and compare this estimate with that obtained in (a).