Optics and Instruments 2015: Exercises on Geometrical Optics (Due on 23 September 2015 at 13:45)

C.U.Keller

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1 Paper Exercises

1.1 Perfect Lens

Show that the surface of a perfect lens is a hyperbola with eccentricity $e = n$ where n is the index of refraction of the lens material.

Hint: A hyperbola is defined by $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, and its eccentricity is $e = \frac{\sqrt{a^2+b^2}}{a}$. Assume that the origin of the coordinate system is at the vertex of the lens.

1.2 Minimum Distance between Object and Image

Show that the minimum separation between conjugate points in the object and the image space is $4f$ for a thin, positive lens with focal length f.

1.3 Petzval Condition

Petzval field curvature of a lens system can be avoided by obeying the Petzval condition $\sum_i n_i f_i = 0$ where n_i and f_i are the focal lengths and indices of refraction of the individual lenses. An obvious approach for a positive two-lens system to fufil the Petzval condition is to choose a positive follwed by a negative lens with the same asbolute value of the focal length but opposite signs. Show that this combination can still act as a positive lens.

Hint: Use the thick-lens equation.

1.4 Parabolic Mirror

An amateur astronomer polished a spherical mirror with a focal length of 1200 mm and a diameter of 200 mm. Calculate the amount of glass that needs to be removed at the edge of the mirror to make it into a parabolic mirror.

2 Computer Exercises

To answer the practicum questions, you will need to use a web-based optical design software that you can find at https://dl.dropboxusercontent.com/u/2291790/TOD/tod.xhtml.

If you have used a previous version of TOD, you will have to reload the page, delete the design (cross in upper left) and reload again. Note that this web app works offline, too, like Gmail. TOD also works on iPads. TOD requires the web browser to be Safari on OS X, Chrome and Firefox on Windows, Linux and OS X. Internet Explorer is not supported. Any bugs should be reported to c.u.keller@icloud.com.

2.1 Spherical Aberration

For a single, biconvex lens that images an object at infinity on the optical axis, show that the transverse spherical aberration is about proportional to F^{-3}, where F is the f-number, and is independent of the field location. Use the optical design software and determine the rms spot size in the best focus for a number of different beam diameters illuminating a single lens with a fixed focal length. Note that the location of best focus depends on the beam diameter, even if the lens has a constant focal length.

2.2 Distortion

A single lens made of fused silica (FS) with an aperture diameter of 60 mm and a focal length of 100 mm shows image distortion when imaging an object at infinity.

1. For a lens that has the optimum shape (minimizing the spherical aberration), determine the radii of curvature of the two spherical surfaces of the lens.
2. Determine the type of distortion (barrel or pin-cushion) and the amount of distortion for a field angle of 10 degrees.

   Hint: Determine the image location for a field of 1 degree and a field of 10 degrees and determine the relative change with respect to the expected ratio of 10.

2.3 Field Curvature

Make a simple Newtonian telescope (single parabolic primary mirror with 200 mm aperture diameter and 200 mm focal length) and use a flat secondary mirror to bring the focus behind the parabolic mirror (note that the code ignores the secondary mirror on the first pass through the backside of the flat mirror). Design a single lens, located close to the focal plane, that improves the performance at an angle of 8 degrees away from the optical axis. What are the parameters of the lens? Hint: the average rms spot size can be improved by about at least a factor of 3.