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Exercises on Ray matrices & Gaussian Beams

Exercise 1: Ray matrices

Study the imaging properties of a system of two lenses with focal lengths f_1 and f_2 calculating the ABCD matrix of the system from the object plane of the first lens to the image plane of the second lens. Do it for a separation length between the two lenses of $L_1 = f_1 + f_2$ and $L_2 = f_1 + f_2 + \Delta L$.

Describe what happens in the second case and, comparing the two cases, say in which respect the first case is superior to the second one.

Exercise 2: Complex beam parameter & ABCD

The field in a Gaussian beam is conveniently described using the beam parameter q, such that

$$E(r) \propto \frac{1}{q} \exp\left(\frac{ikr^2}{q}\right) \exp\left(ikz - i\omega t\right)$$

a) From $q(z) = z - i\, z_0$ (z_0 is sometimes indicated as b), separate $1/q(z)$ in real and imaginary parts to find the functional form of $R(z)$ and $w(z)$ knowing that $\frac{1}{q(z)} = \frac{1}{R(z)} + i\ \frac{\lambda}{\pi\ w(z)^2}$

b) Recall how ABCD matrix can be applied to Gaussian beams and examine, in terms of R and w, the effect on a general Gaussian beam of the optical system in exercise 1 for the case $f_1 = 2 \cdot f_2$ and $\Delta L = 0$.

c) A concrete example: calculate the case of wavelength 500 nm and a spot size of $w_0 = 200\ \mu$m, with $f_1 = 10\ cm$, $f_2 = 5\ cm$ where the incoming-beam waist is at the object plane of the first lens (at $-f_1$). What is the beam waist at the image plane of the 2nd lens ($\Delta L = 0$)? What if ($\Delta L > 0$)?

Exercise 3: Gaussian beam

We use a laser operating at 500 nm, emitting a Gaussian beam with zero curvature and an intensity full-width-at-half-maximum (FWHM) of 5 mm. Treat this as an ideal, non-diffracting beam until it hits an ideal thin lens with focal length 5 cm.

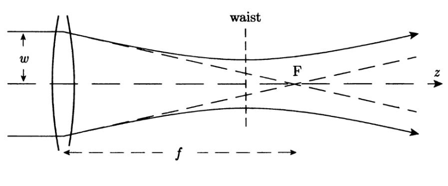
a) First, from FWHM determine the beam waist w at the laser aperture, the waist at the focus $w_{z=f}$ and the Rayleigh range z_0.

b) Determine the beam waist at the Rayleigh range.

c) Show that a factor $w_0/w(z)$ is required for the Gaussian-beam amplitude to keep the total energy constant.

Exercise 4: Wave-optical focus shift

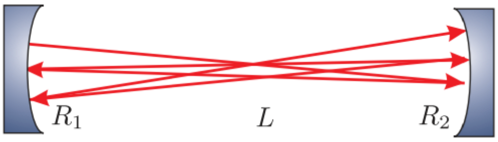
In wave optics, the beam waist is not at the geometric focus position.



a) Using the quantities describing a gaussian beam, determine the distance of the beam waist from the center of the lens as a function of the focal length f, beam "diameter" w and wavelength λ. [use on the left side of the lens a collimated Gaussian beam, i.e. with curvature zero]. Hint: first calculate the Gaussian q-parameter right behind the lens and then propagate it over a distance z.

b) Calculate the distance between the ray optics focus and the beam waist for a beam with w =1 mm, $\lambda = 500\,nm$ and f=50 mm and f=5 m!

Exercise 5: When is a cavity "stable"?



We will calculate under which conditions a cavity is stable. In the ray-optical picture this means that the ray won't leave the cavity after arbitrary many round-trips.

a) Construct the ABCD matrix for one round-trip. Hint: although the physics should be the same, the calculated matrix might be different/easier if you start from the center than if you start from one of the mirrors.

b) What does it require mathematically for the cavity to be stable? Which conditions does this impose? Hint: use the orthonormality of the matrix, which is real valued and has determinant 1. Note: solving this matrix is very hard and requires Sylvester's theorem. A real challenge for mathematically-inclined students who like challenges.

c) Does it make sense to use the ray-optical picture in a clearly wave-optical context? Think of it and briefly discuss.
