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Exercises on Diffraction

Exercise 1: Three simple exercises with diffraction

Neglect atmospheric effects if not mentioned otherwise and use a standard wavelength of 500 nm.

a) *Aperture Diffraction*: light is incident on a circular aperture with a diameter of 150 μm. Consider Fraunhofer diffraction: what is the angle between the central intensity maximum and the first minimum?

b) *Eye resolution*: if we want to tell two stars apart at a distance of 4 light years, what is their minimum distance at which we can do this with the naked eye? (estimate the relevant quantity describing our eye)

c) *Corona around the sun*: if you see the sun through a thin cloud, a colored corona may appear due to diffraction at small water droplets. Assume that you can model the droplets by opaque discs of the same diameter as the droplet and use Babinet's principle. If the angular diameter of this corona is 10° at 500 nm wavelength, what is the size of the droplets?

Exercise 2: On-axis properties of an aperture

A 0.5 mm aperture is illuminated with a plane wave (λ=500 nm). At some distance behind the aperture the on-axis intensity is larger than the incident intensity.

a) What is the maximum distance from the aperture where the light is focussed to a bright spot?

b) Compare the intensity of this spot to the intensity with the aperture removed

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Exercise 3: Fresnel zone plate

Construct a Fresnel Zone Plate with 5 zones with radii r_i so that the outermost and innermost zones are both opaque and the outermost extends from r_4 to infinity. The plate should produce an intensity maximum on the axis at 1 m distance behind the aperture at the wavelength of 500 nm (i.e. derive the large-radius approximation).

a) Sketch the Fresnel zone plate and calculate r_1

b) Calculate the ratios of the radii $r_1 : r_2 : r_3 : r_4$

c) Calculate the on-axis intensity at 1 m distance in terms of the intensity of the incoming wave.

d) Can you find further intensity maxima on the axis? Where?

Exercise 4: Lunar ranging

We have a 10 cm diameter telescope to send a $\lambda = 500$ nm beam to the moon. On the moon, there is a 1 m diameter retro-reflector.

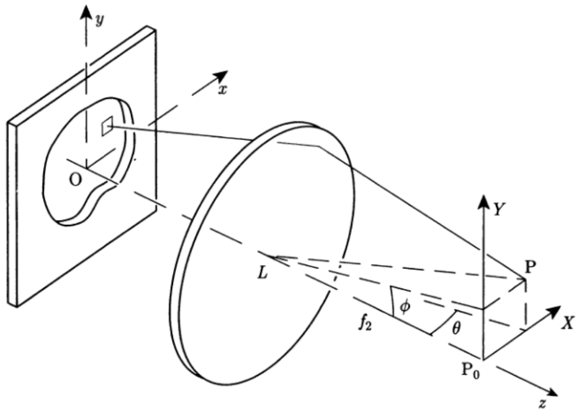
a) If we send a laser pulse with 1 mJ pulse energy, how many photons of the reflected pulse can we detect in the same telescope after travelling to the moon and back? (The distance earth-moon is approximately 4×10^8 m)

b) This is apparently not really helpful, how about using a 1 m telescope?

c) Now we get a problem: How does this change if we consider additional diffraction due to atmospheric turbulences (the atmosphere leads to 1 arcsecond additional beam spread)?

Exercise 5: Fraunhofer pattern of rectangular aperture

We consider a rectangular aperture with sides w_x and w_y in an opaque screen. The aperture is illuminated at normal incidence with a plane wave of amplitude U_0, and behind is a lens with focal length f.



a) Show that the Fourier spectrum of the diffracted field is (up to a constant)

$$U_{dif}(X,Y) \propto \frac{U_0}{i\lambda f} \times w_x w_y \times \mathrm{sinc}(\frac{1}{2}kw_x \sin\theta_x) \times \mathrm{sinc}(\frac{1}{2}kw_y \sin\theta_y)$$

b) Sketch the pattern for $w_x = 10\lambda,\ w_y = 20\lambda$, and discuss the positions of the inner nodal lines.

c) What are the requirements that the diffraction amplitude factorizes nicely in $f(\theta) \times f(\phi)$ like in this case!

d) Finally, consider the situation that the incident plane wave travels at an angle α with respect to the z-axis, so that $\vec{k}_{in} = (k\ sin\ \alpha, 0, k\ cos\alpha)$. Sketch the new situation and explain how U_{diff} changes due to the new propagation direction.

Exercise 6: Fraunhofer diffraction and Gaussian beams

Both diffracted light and Gaussian beams show angular divergence. Using $\lambda = 500nm$, calculate both angular divergences by looking at:

a) the size of the central maximum of diffracted light from a circular aperture of radius R, at a distance L justifying the use of Fraunhofer approximation;

b) the width of a fundamental Gaussian beam with $w_0 = R$ at a big distance z = L from the minimum waist, positioned at z=0;

Calculate the angular spread from the full widths at half maximum (FWHMs) of the intensity profiles in both situations (alternatively choose other quantities describing the size and justify your choice).
