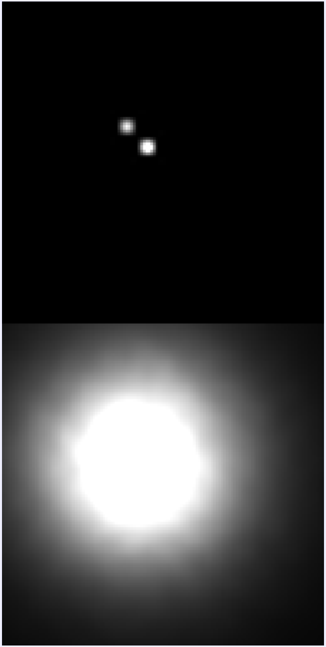
Outline

Full-Aperture Interferometry

Diffraction Limit

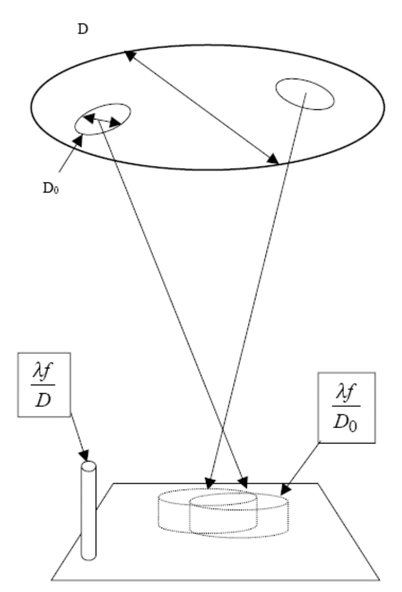
- Theoretical angular resolution of a telescope is proportional to λ/D where λ is the wavelength and D is the diameter of the telescope

Diameter	Wavelength	Diffraction Limit
10 cm	500 nm	$1.0''$
100 cm	500 nm	$0.''1$
800 cm	500 nm	$0.''0125$
100 cm	5000 nm	$1.0''$



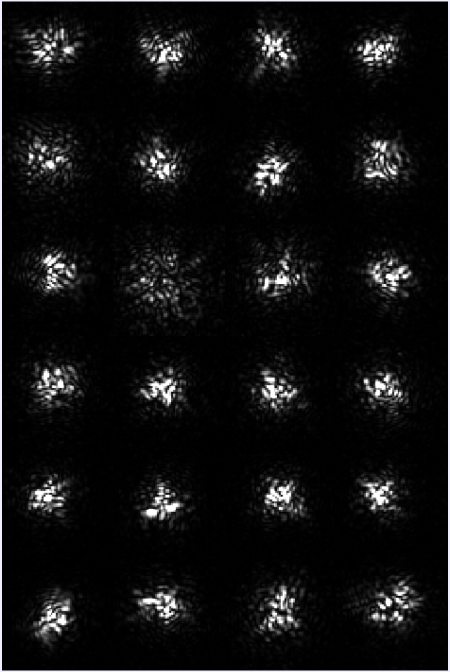
Seeing

- resolution limited by Earth's atmosphere to $\approx 0\overset{\prime\prime}{.}5$ independent of D
- refraction on changes in index of refraction
- index of refraction of air depends on temperature
- atmosphere is turbulent medium with small-scale temperature fluctuations
- *Speckle-Interferometry*: suitable observational method and post-facto reconstruction teqchnique to eliminate the angular smearing due to seeing



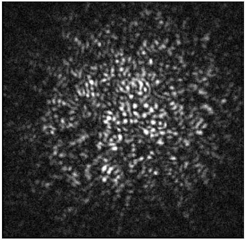
Interference

- telescope combines light that has passed through different parts of of the atmosphere
- Seeing corresponds to many small telescopes that are differently affected by the atmosphere
- quality of seeing described by *Fried's Parameter* r_0, diameter of telescopes that would be diffraction-limited under current seeing conditions
- typical values in the visible r_0: 5–30 cm

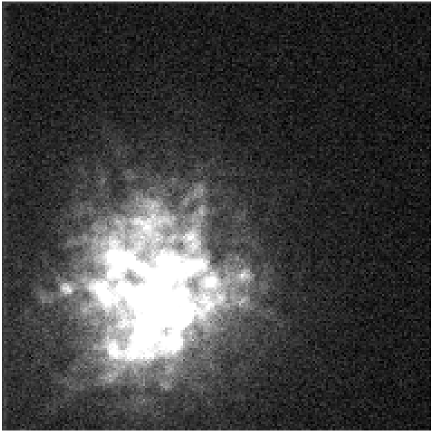


Speckles

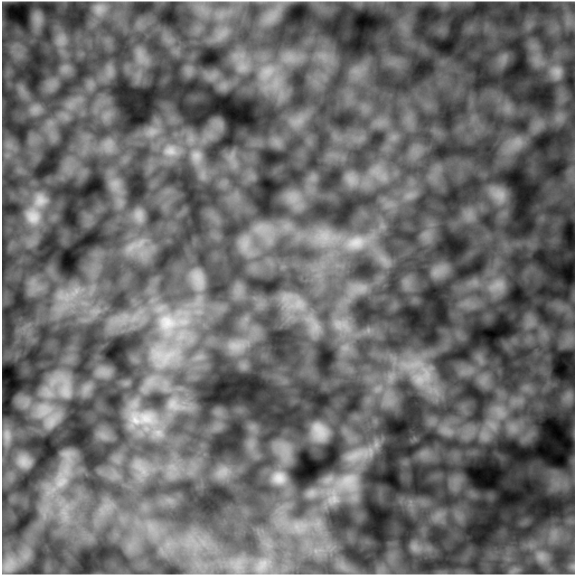
- image of a point source is a cloud of small dots: *Speckles*
- Speckles have diameters corresponding to diffraction limit
- life time of speckles in the visible about 10 ms, longer at larger wavelengths



Seeing for Single and Binary Stars



Seeing and Solar Granulation



Mathematical Description of Seeing

- image of a point source: *Point Spread Function (PSF)*
- image formation due to time-varying PSF

$$i(t) = o * s(t)$$

i	observed image
o	true object, constant in time
s	point spread function
$*$	convolution

Fourier Domain

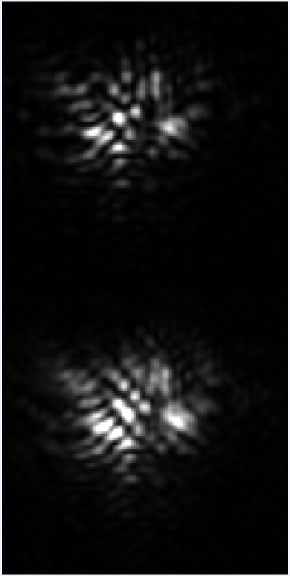
- after Fourier transformation:

$$I(t) = O \cdot S(t)$$

I	observed image
O	true object, constant in time
S	optical transfer function describing seeing and instrument

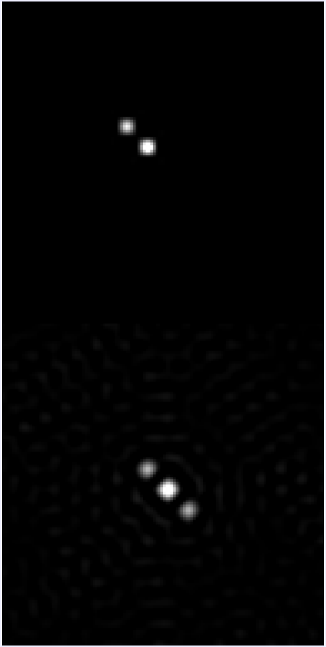
- *isoplanatic patch*: constant PSF over an are of 3–10″
- extended objects as sum of point sources convolved with PSF

Labeyrie Technique



Basic Idea

- developed in 1970 by Antoine Labeyrie
- exposure time shorter than time constant of seeing ($\leq$ 20 ms in the visible)
- clever average of many images that is free from atmospheric influences
- assumption: PSF constant during exposure time
- image of binary star consists of two identical, overlapping speckle clounds



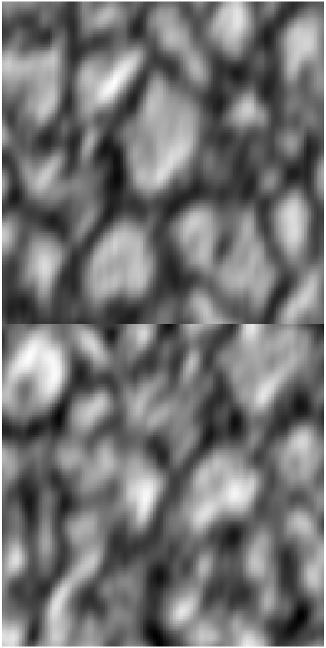
Average Autocorrelation

- *Autocorrelation* can detect identical, but shifted speckle clouds
- Labeyrie technique: average autocorrelation contains diffraction-limited information
- autocorrelation in image space corresponds to power-spectrum in Fourier Space

$$\left\langle |I|^2 \right\rangle = |O|^2 \left\langle |S|^2 \right\rangle$$

- *speckle transfer function* $\left\langle |S|^2 \right\rangle$ from calibration point source
- Labeyrie technique allows reconstruction of Fourier amplitudes
- average auto-correlation is not image

Autocorrelation $\neq$ Image



Reconstructing the Fourier Phases

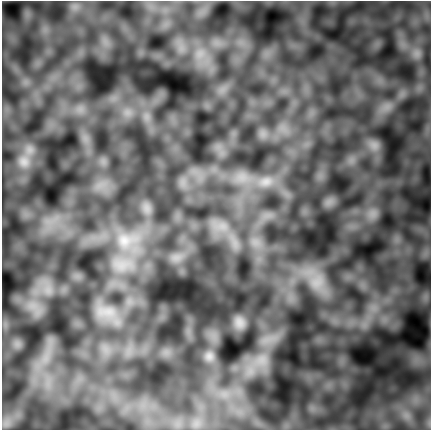
- Knox and Thompson (1973)
- phases are crucial for extended objects such as star clusters, galaxies, the Sun
- phases $\Psi\left(O\right)$ with Knox and Thompson (1974) technique:

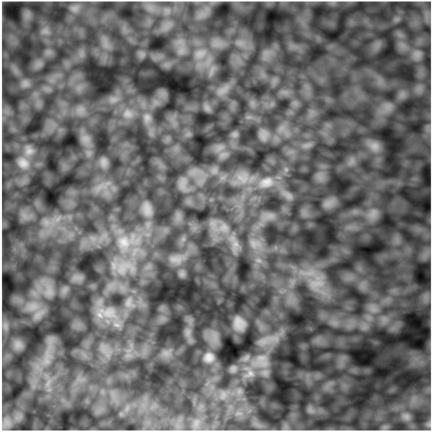
$$\left\langle I(\vec{f})I^*(\vec{f}-\vec{\delta f})\right\rangle = O(\vec{f})O^*(\vec{f}-\vec{\delta f})\left\langle S(\vec{f})S^*(\vec{f}-\vec{\delta f})\right\rangle$$

$$\Psi\left(O\left(\vec{f}\right)O^*\left(\vec{f}-\vec{\delta f}\right)\right) = -\Psi\left(O\left(\vec{f}\right)\right)+\Psi\left(O\left(\vec{f}-\vec{\delta f}\right)\right)\sim\frac{\partial\Psi}{\partial\vec{f}}$$

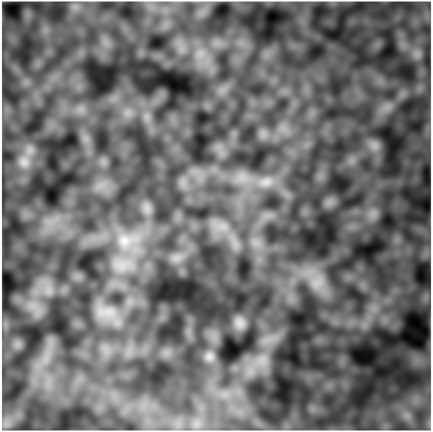
- can integrate phase differences in two directions to recover phases
- iterative approach that minimizes sum of squares of phase differences

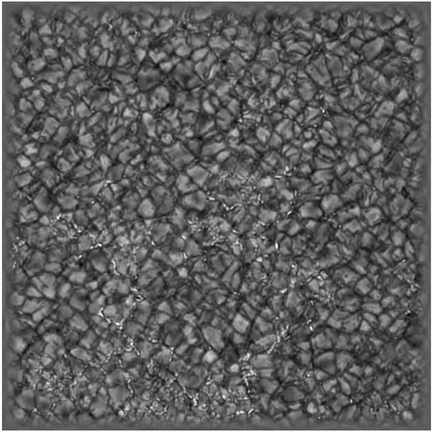
Average and Best Frame of Image Series



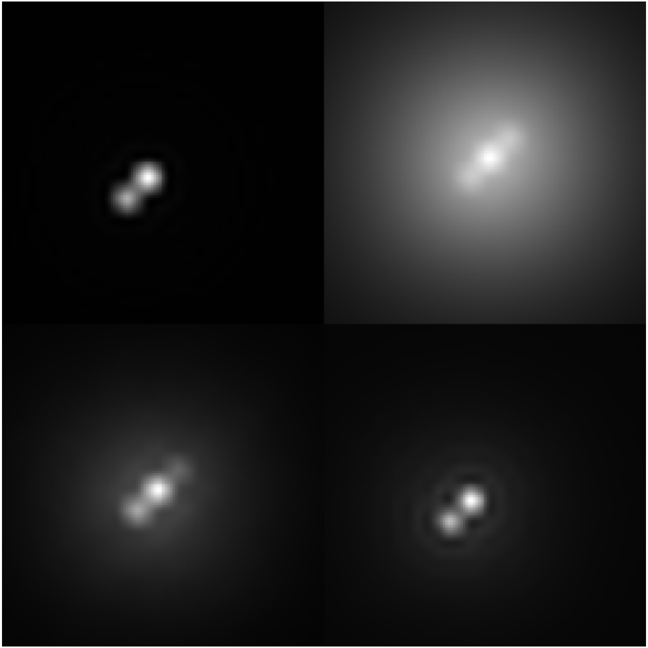


Average and Knox-Thompson Reconstruction





Comparison of Techniques

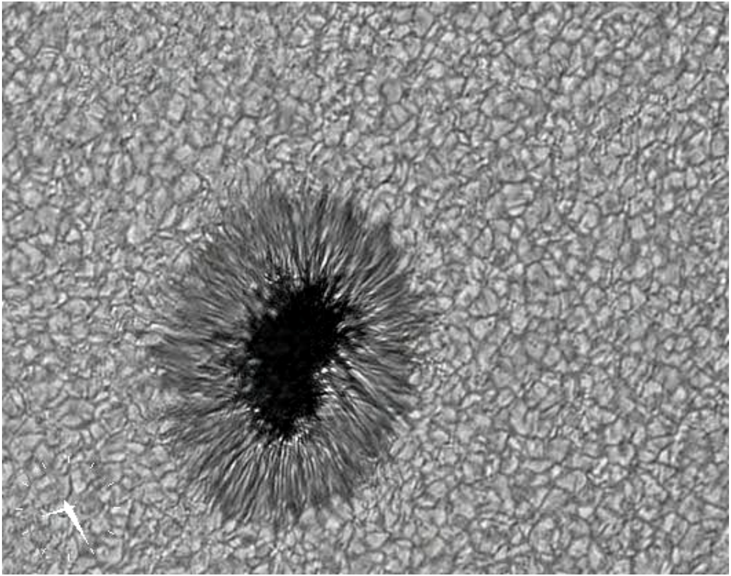


Bispectrum Technique

Weigelt 1977

- product of three Fourier components $\left\langle I(\vec{f}_1)I(\vec{f}_2)I^*(\vec{f}_1+\vec{f}_2)\right\rangle$
- phase of average atmospheric factor is again zero
- phase of bispectrum: $\Psi\left(\vec{f}_1\right)+\Psi\left(\vec{f}_2\right)-\Psi\left(\vec{f}_1+\vec{f}_2\right)$
- many more correlations between Fourier components
- bispectrum is 4-dimensional $\Rightarrow$ large computational effort
- today most-often used speckle technique

Dutch Open Telescope Speckle Movie



DOT Blue Continuum Movie of AR10425

Differential Speckle Imaging

The Idea

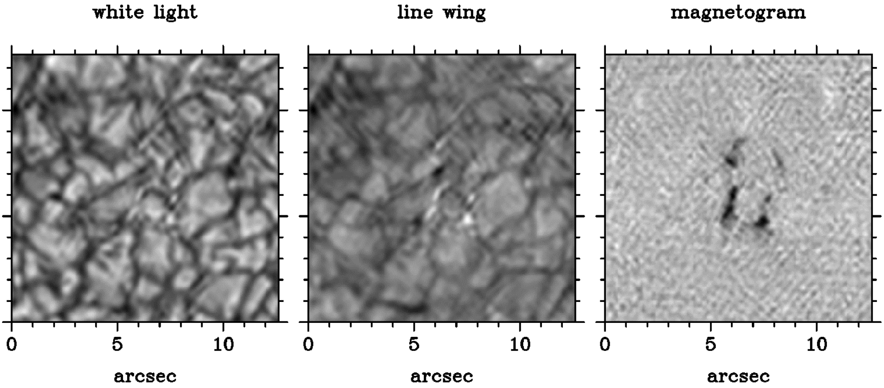
- narrow-band filter images do not have enough signal to do direct speckle imaging
- measure PSF in broad-band channel (b) to deconvolve simultaneous exposures in narrow-band channel (n)
- $I_n = O_n S, \quad I_b = O_b S$
- approximation of O_n:

$$O'_n = \left\langle \frac{I_n}{S} \right\rangle = \left\langle \frac{I_n}{I_b} \right\rangle O_b.$$

- avoid division by 0 via:

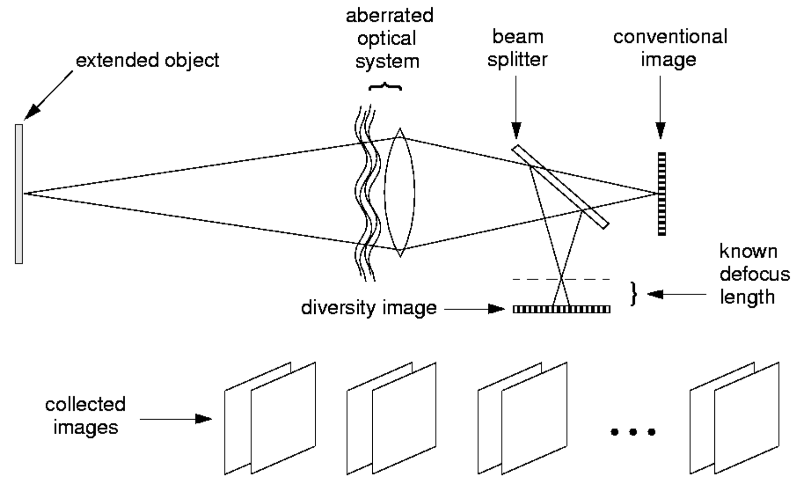
$$O'_n = \frac{\left\langle (I_n/I_b)|I_b|^2 \right\rangle}{\left\langle |I_b|^2 \right\rangle} O_b = \frac{\left\langle I_n I_b^* \right\rangle}{\left\langle |I_b|^2 \right\rangle} O_b$$

Application to Solar Zeeman Polarimetry

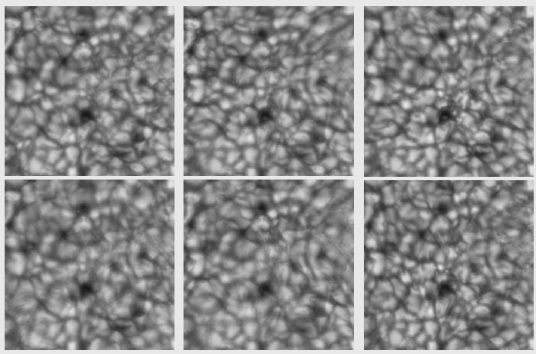


- BUT, in reality $I_n = O_n S + N_n, \;\; I_b = O_b S + N_b$
- account for random photon noise with appropriate optimum filter
- account for anisoplanatism by overlapping segmentation with segment sizes on the order of the isoplanatic patch
- high spatial resolution at good spectral resolution can be achieved

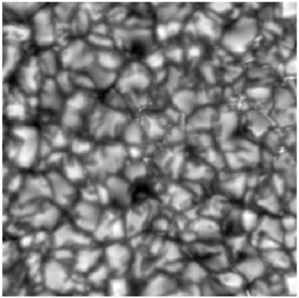
Phase-Diverse Speckle



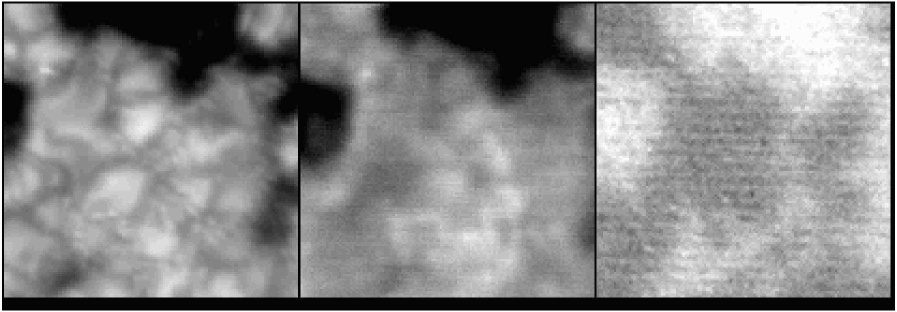
Raw Data



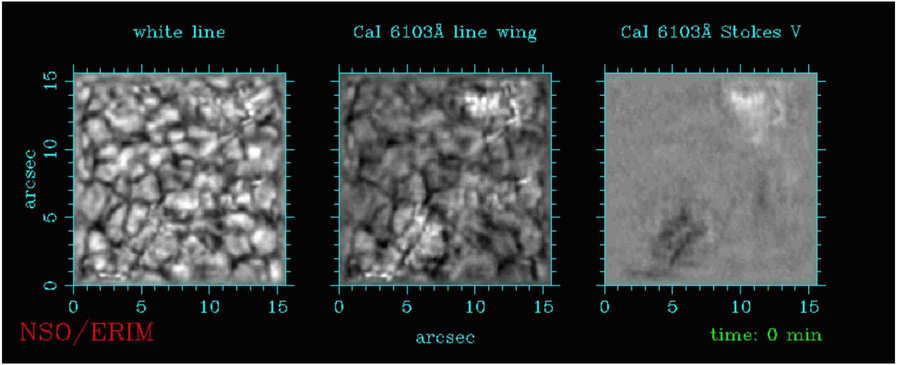
Reconstruction



Adaptive Optics Real-Time Wavefront Correction



Evolution of Small-Scale Fields in the Quiet Sun



Speckle Interferometry

Summary

- Earth’s atmosphere limits angular resolution of optical telescopes
- speckle techniques use clever non-linear averages that preserve information out to the diffraction limit
- speckle imaging is the standard data reduction technique at the Dutch Open Telescope
- Adaptive Optics (AO) can correct some of the aberrations
- highest resolution achieved with combination of AO and reconstruction technique