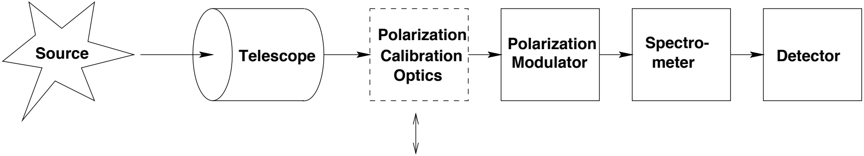
Lecture 11: Polarimetry Systems Engineering 2

Outline

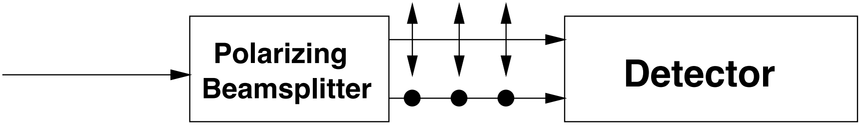
Temporal and Spatial Modulation

General Polarimeters



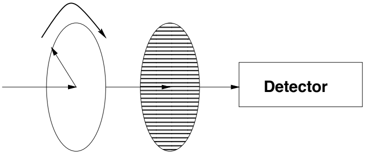
- polarimeters: optical elements (e.g. retarders, polarizers) that change polarization state of incoming light in controlled way
- detectors always measure only intensities
- intensity measurements combined to retrieve polarization state of incoming light
- polarimeters vary by polarization modulation scheme
- polarimeter should also include polarization calibration optics

Spatial Polarization Modulation



- polarizing beam-splitter polarimeter
- simple linear polarimeter: polarizing beam-splitter producing 2 beams corresponding to 2 orthogonal linear polarization states
- full linear polarization information from rotating assembly
- *spatial modulation*: simultaneous measurements of two (or more) Stokes parameters

Temporal Polarization Modulation



- rotating waveplate polarimeter
- rotating retarder, fixed linear polarizer
- measured intensity as function of retardance δ, position angle θ

$$I' = \frac{1}{2}\left(I + \frac{Q}{2}\left((1+\cos\delta) + (1-\cos\delta)\cos 4\theta\right) + \frac{U}{2}(1-\cos\delta)\sin 4\theta - V\sin\delta\sin 2\theta\right)$$

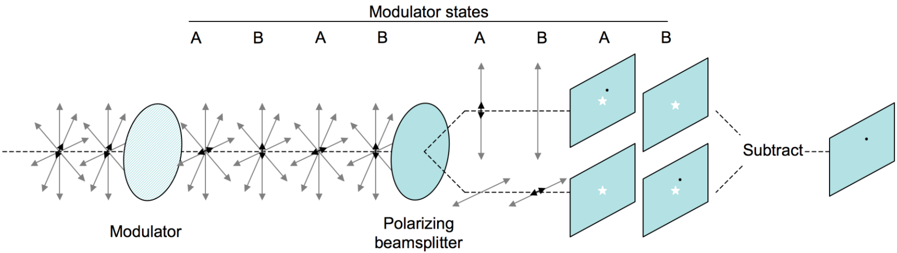
- only terms in θ lead to modulated signal
- equal modulation amplitudes in Q, U, and V for δ=127°
- *temporal modulation*: sequential measurements of $I\pm$ one or more Stokes parameters

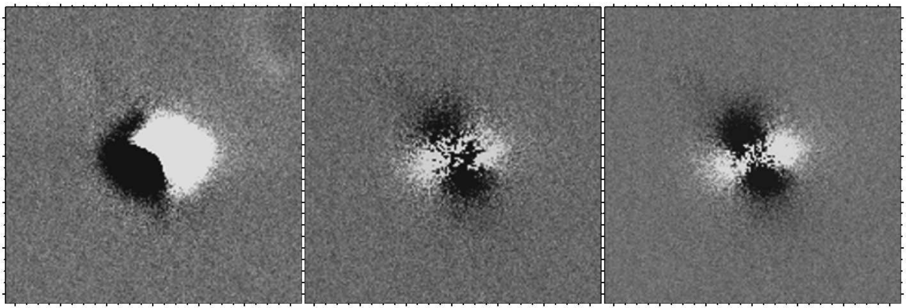
Comparison of Temporal and Spatial Modulation Schemes

Modulation	Advantages	Disadvantages
temporal	negligible effects of flat field and optical aberrations	influence of seeing if modulation is slow
	potentially high polarimetric sensitivity	limited read-out rate of array detectors
spatial	off-the-shelf array detectors	requires up to four times larger sensor
	high photon collection efficiency	influence of flat field
	allows post-facto reconstruction	influence of differential aberrations

schemes rather complementary ⇒ modern, sensitive polarimeters use both to combine advantages and minimize disadvantages

Combined Spatial and Temporal Modulation





Double-Ratio Technique

- combination of spatial and temporal modulation
- data reduction minimizes effects of many artifacts
- rotatable quarter-wave plate, polarizing beam-splitter
- consider case of circularly polarized light
- quarter-wave plate switches between +45° or -45° to polarizing beam-splitter
- both beams recorded simultaneously
- four measurements are combined to obtain estimate of Stokes V/I ratio largely free of effects from seeing and gain variations between different detector areas
- excellent if polarization signal is small
- frequently used in stellar polarimetry
- can be applied to any polarized Stokes parameter
- works very well for solar applications where the spectrum in the first and the second exposures are different

Double-Ratio Technique (continued)

- measured intensities in two beams in first exposure

$$S_1^l = g_l\alpha_1(I_1 + V_1), \;\; S_1^r = g_r\alpha_1(I_1 - V_1)$$

  - subscript 1: first exposure
  - subscripts l, r: left, right beams
  - S: measured signal
  - g: gain in particular beam
  - α: transmission of atmosphere, instrument
- second exposure

$$S_2^l = g_l\alpha_2(I_2 - V_2), \;\; S_2^r = g_r\alpha_2(I_2 + V_2)$$

- incoming I and V in second exposure may be completely different from first exposure
- also includes beam-wobble induced by rotation of wave plate

Double-Ratio Technique (continued)

- combination of 4 measured intensities removes effect of transmission changes and differential gain variations of different detector areas

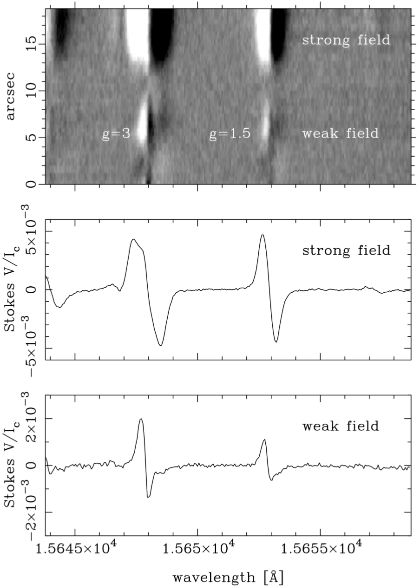
$$\frac{1}{4}\left(\frac{S_1^l}{S_2^l}\frac{S_2^r}{S_1^r}-1\right)=\frac{1}{2}\frac{I_2V_1+I_1V_2}{I_1I_2-I_2V_1-I_1V_2+V_1V_2}$$

- if $V \ll I$

$$\frac{1}{2}\left(\frac{V_1}{I_1}+\frac{V_2}{I_2}\right)$$

- obtain average V/I signal of two exposures
- no spurious polarization signals are introduced

Example of Solar Double Ratio at 1.56 μm (NIM at McMath-Pierce)



Calibration

Basic Approach

- optimum way to calibrate polarimeter, i.e. measure X

$$\vec{S} = \mathsf{X}\vec{I}$$

- at least 16 measurements of signal vector elements $\vec{S}_i, i = 1..m$
- at least 4 different input Stokes vectors $\vec{I}_i^c, i = 1..m$
- group calibration input Stokes vectors, signal vectors into 4 by 4 matrices (Azzam et al. 1988)

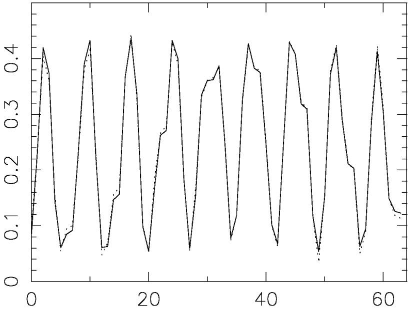
$$\mathsf{S} = \mathsf{X}\mathsf{I}^c$$
$$\mathsf{S} = \left(\vec{S}_1 \vec{S}_2 ... \vec{S}_m\right)$$
$$\mathsf{I}^c = \left(\vec{I}_1^c \vec{I}_2^c ... \vec{I}_m^c\right)$$

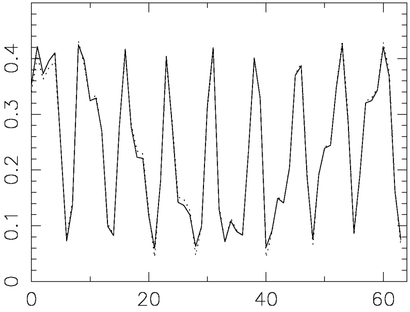
- signal matrix $\mathsf{X} = \mathsf{S}\mathsf{J}$ with $\mathsf{I}^c\mathsf{J} = 1$

Equivalence with Maximum Polarimetric Performance

- choose calibration Stokes vectors $\vec{I}_i^c$ to minimize error in X, given errors in measurements S
- equivalent to optimizing polarimetric efficiency
- calibration input Stokes vectors $\vec{I}_i^c$ equivalent to rows of the signal matrix X of maximum performance polarimeter
- vector-polarimeter: corners of a tetrahedron as suggested by Azzam et al. (1988)
- only true if there are no systematic errors (input polarization is known completely)
- better to use many more measurements and also model non-ideal calibration optics

Back-Calibration

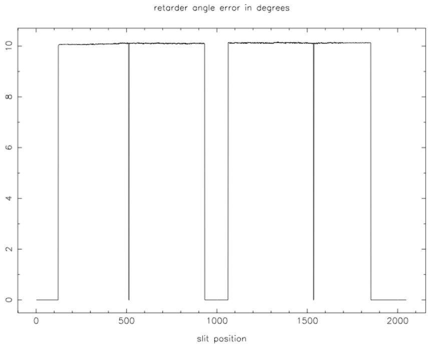


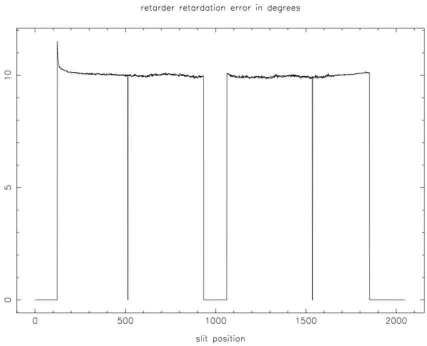


- use calibration matrix to calibrate calibration observations
- as calibration polarization is known ⇒ obtain error estimate of calibration

Calibration Input Polarization Errors

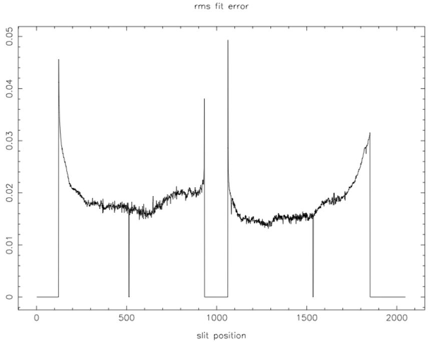
- orientation of calibration optics not know precisely
- retardation of calibration retarders not know precisely
- consider as unknown in fitting of calibration data

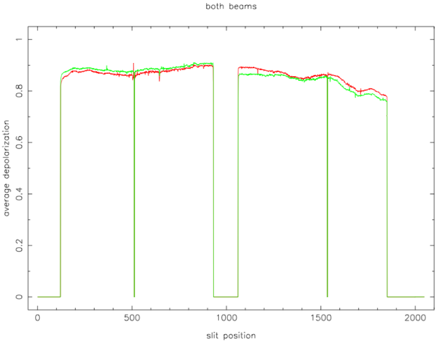




Measuring Modulation Efficiency

- polarization modulator and detector should not depolarize
- should measure fully polarized light on detector
- implies that $(X_2^2 + X_3^2 + X_4^2)/X_1^2 = 1$
- can empirically determine this ratio from calibration data





Polarized Raytracing

Introduction

- "non-polarimetric" optical elements influence polarization
- characteristics of "polarimetric" components depend on angle of incidence
- "polarizing" components influence optical performance in unexpected ways (e.g. crystal astigmatism)
- need methods to predict optical (and polarimetric) performance
- geometrical optics: $\lambda \ll$ diameter of lenses, apertures etc.
- locally treat electromagnetic field as plane wave
- evaluate using *rays* (plane waves with very limited transverse extensions)
- trace monochromatic rays through optical system
- use Fresnel equations etc. to determine polarization characteristics of individual rays

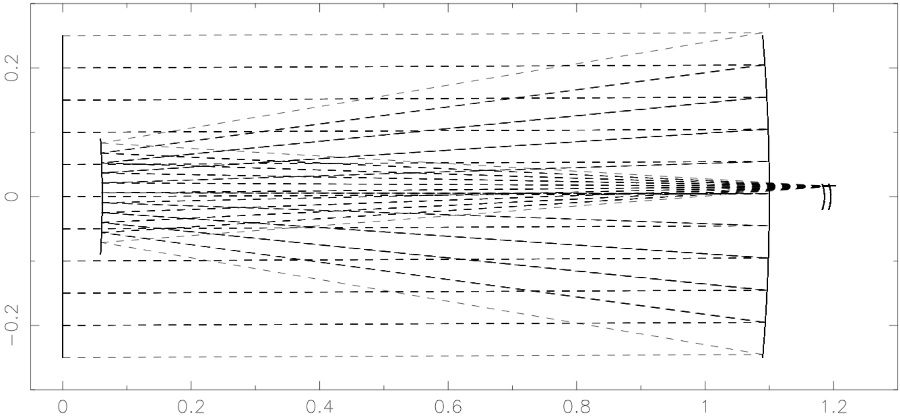
Image Formation

- image of a point source: *point-spread function*
- image of extended object is convolution of object with PSF
- *geometrical PSF* (*spot diagram*) obtained by tracing many rays from point source that uniformly fill entrance aperture
- geometrical PSF of perfect optical system is a point
- diffraction at aperture stop makes PSF with finite extension
- PSF in case of Frauenhofer diffraction (focal plane far away from aperture stop) is given by magnitude squared of Fourier transform of electric field over aperture)

Stokes and Jones Vector Raytracing

- highest-end version of commercial software packages (e.g. ZEMAX) can do (limited) calculations with polarized light
- can correctly propagate light in birefringent media and propagate Jones vectors
- averaging Stokes parameters in focal plane corresponds to Stokes vector of the average over the PSF
- average of Jones vector in focal plane corresponds to Jones vector at center of PSF

Example: Ritchie-Chretien telescope with field corrector



- SOLIS Vector-Spectromagnetograph
- not diffraction-limited $\Rightarrow$ Stokes/Mueller calculus appropriate
- without excellent anti-reflection coating on lenses, about 1% linear polarization towards edge of field

Polarization Ray Tracing

- combines conventional geometric ray tracing and Jones formalism in elegant way (Chipman, 1989)
- $\vec{E}_0$ in plane-wave ansatz

$$\vec{E} = \vec{E}_0 e^{i\left(\vec{k}\cdot\vec{x}-\omega t\right)}$$

  describes polarization of light
- vector has three components E_x, E_y, E_z
- in isotropic medium, $\vec{E}_0$ is perpepdnicular to $\vec{k}$
- amplitude matrices P operate on incident polarization vectors $\vec{E}_0$ to produce emerging polarization vector $\vec{E}_0'$
- complex components of P describe influence of interface between two materials on wave vector direction and polarization

Adding polarization vectors

- parallel beams: Jones and Stokes vectors can be added
- if beams propagate in different directions, needs to transform into common coordinate system before adding
- polarization vector approach operates in global coordinate system where $\vec{E}_0$ also contains information about propagation direction
- polarization vectors can be added directly
- corresponding Jones vector extracted from two components of $\vec{E}_0$ that are perpendicular to propagation direction