Outline

What Does It Mean

The total is more than the sum of its parts.

Polarimetry Systems Engineering

Goal: model and understand performance of polarimeter designs

Answer the following questions:

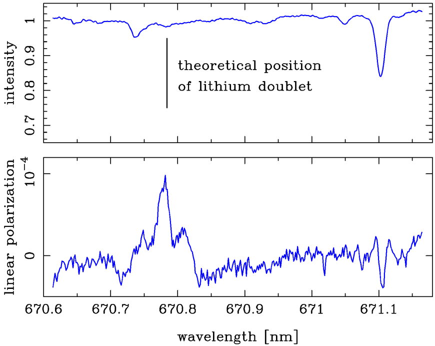
- How to define polarimeter performance?
- How to compare different polarimeter designs?
- How to maximize performance?
- How to do all of this for different polarimeter designs?

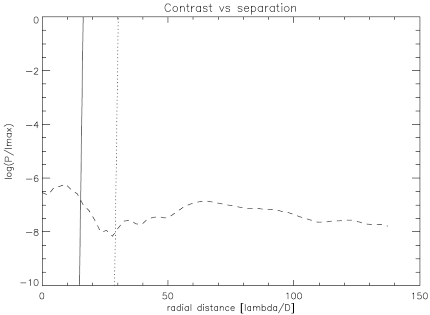
Polarization Sensitivity and Accuracy

Definitions

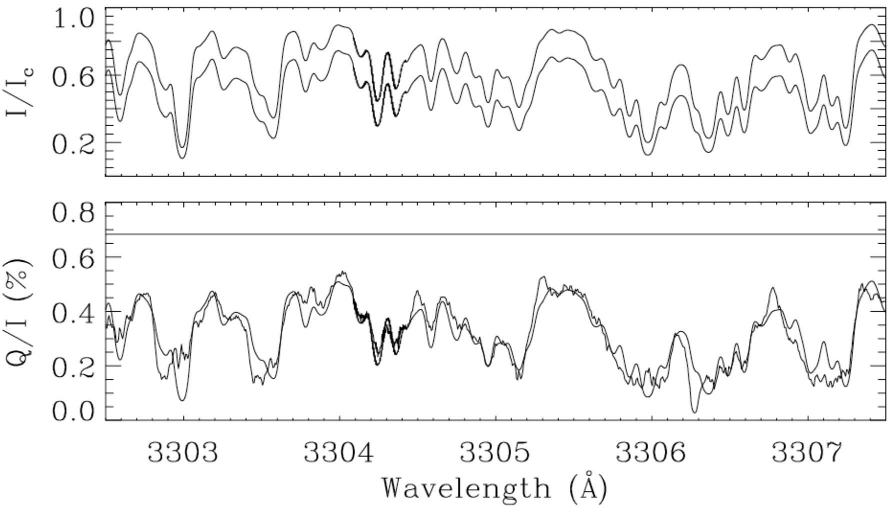
- *polarimetric sensitivity*: smallest detectable polarization signal
  - related to noise levels in Q/I, U/I, V/I (photon noise, read-out noise, spurious polarization signals
  - determined by errors that are not “real” polarization effects
  - expressed as fraction of intensity I
- *polarimetric accuracy*: accuracy with which the magnitude of detected polarization signal can be quantified
  - accuracy of measured polarization as compared to actual Stokes parameter(s) of incoming light without noise, spurious polarization signals
  - parametrized by position of zero point for Q, U, V (*absolute polarimetric accuracy*) and measurement scale (*relative polarimetric accuracy*)

Sensitivity Examples





Accuracy Example



Stenflo (2005) Figure 4

Performance and Errors

- polarimeter design: important to model and understand polarimeter performance
- establish performance goals, then maximize performance
- many ways to define and maximize efficiency of polarization measurement
- often impossible to compare efficiency of different designs
- random *statistical errors* (photon noise, detector read-out noise) and *systematic errors* (instrumental errors) limit performance
- trade-off between these error components needed
- example: does not make sense to build polarimeter with very small systematic errors but so low efficiency that statistical errors completely dominate

Assumptions about Random Noise

Use error propagation to derive statistical errors in measured Stokes vector from noise in individual intensity measurements

assumptions on measurement noise:

- linear relation between Stokes parameters and measured signals
- noise in different measurements is independent
- noise is independent of signal, if
  - noise dominated by signal-independent detector noise (e.g. read-out noise)
  - incoming vector only slightly polarized (measurements will have very similar light levels)
- noise statistic has a Gaussian distribution

Intensity Signals from First Rows of Mueller Matrices

- telescope, instrument, polarimeter described by Mueller matrices
- each measurement j of a particular polarimeter arrangement only measures an intensity signal S_j

$$\begin{pmatrix} S_j \\ - \\ - \\ - \end{pmatrix} = \begin{pmatrix} m_{j,11} & m_{j,12} & m_{j,13} & m_{j,14} \\ m_{j,21} & m_{j,22} & m_{j,23} & m_{j,24} \\ m_{j,31} & m_{j,32} & m_{j,33} & m_{j,34} \\ m_{j,41} & m_{j,42} & m_{j,43} & m_{j,44} \end{pmatrix} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix}$$

Signal Matrix

- $m > 1$ intensity measurements combined into *signal vector* $\vec{S}$
  - $m = 2$ for circular polarimeter
  - $m = 4$ for liquid-crystal-based vector polarimeters
  - $m = 8$ for rotating retarder vector polarimeter
- related to incoming Stokes vector, $\vec{I}$ by $4 \times m$ *signal (or modulation) matrix* X

$$\vec{S} = \mathrm{X}\vec{I}$$

- $\mathrm{X}(\vec{v})$: function of free design parameters $\vec{v}$
- each row of X corresponds to first row of Mueller matrix describing particular intensity measurement

Measuring Stokes Parameters

- estimate $\vec{I}'$ of incoming Stokes vector $\vec{I}$

$$\vec{I}' = \mathrm{Y}\vec{S} = \mathrm{X}^{-1}S$$

- Y: *synthesis* or *demodulation matrix*
- error propagation ⇒ standard deviations of Stokes parameters

$$I'_i = \sum_{j=1}^{4} \mathrm{Y}_{ij} S_j \quad \Rightarrow \quad \sigma_{I'_i} = \sqrt{\sum_{j=1}^{4} \mathrm{Y}_{ij}^2 \sigma_{S_j}^2}$$

- σ_{S_j}: standard deviation of intensity in measurement S_j
- σ_{S_j} often independent of $j \Rightarrow \sigma_{I'_i} = \sigma_S \sqrt{\sum_{j=1}^{4} \mathrm{Y}_{ij}^2}$
- further error propagation ⇒ formal errors for degree of circular, linear polarization, angle of linear polarization, etc.

Polarimetric Efficiency

Characterizing Polarimeters with Respect to Statistical Noise

- need to define quantities that summarize performance of given polarimeter design with respect to statistical noise
- for given photon flux and detector noise, design optimum polarimeter by maximizing *polarimetric efficiency*

Desired Properties for Definition of Polarimetric Efficiency

- comparable between different polarimeter designs and measurement approaches
- larger values should correspond to better designs
- independent of the intensity throughput
- consist of 4 quantities (*Stokes efficiency*)
- theoretical maximum efficiency shall be 1

Noise Propagation

- if all measurements have same noise

$$\sigma_{I'_i} = \sigma_S \sqrt{\sum_{j=1}^{m} \mathrm{Y}_{ij}^2}$$

- minimize square root of sum of squares of rows of Y, but this depends on number of measurements m
- measurements take time T
- individual measurement takes time T/m
- standard deviation of measurement noise proportional to square root of inverse of duration of individual measurement
- need to multiply $\sqrt{\sum_{j=1}^{m} \mathrm{Y}_{ij}^2}$ with $\sqrt{m}$

Polarimetric Efficiency

- define polarimetric efficiency (del Toro Iniesta & Collados 2000)

$$\epsilon_i = \left(m \sum_{j=1}^{m} Y_{ij}^2 \right)^{-\frac{1}{2}}$$

- in most cases $X_{i1} = 1$, i.e. all measurements have the same throughput $\Rightarrow$ normalize intensity measurements
- for normalized measurements, it can be shown that

$$\epsilon_1 \leq 1 \; ; \sum_{i=2}^{4} \epsilon_i^2 \leq 1$$

Analytic Optimization

- for certain performance quantities (e.g. polarimetric efficiency), one can derive properties of optimum polarimeter
- *maximum performance*: best performance of all polarimeter designs
- *optimum performance*: best performance that can be achieved with given polarimeter design
- choose $X(\vec{v})$, Y to minimize difference between $\vec{I}'$ and $\vec{I}$, given σ_{S_j}
- 3 steps:
  - derive equation for optimum Y given $X(\vec{v})$
  - using the equation for optimum Y, derive optimum X
  - choose optimum X and Y for maximum performance polarimeter

Analytic Optimization

- formally add additive, random, zero-mean noise $\vec{n}$ in measurements:
$$\vec{S} = \mathrm{X}(\vec{v})\vec{I} + \vec{n}$$
- $\mathrm{X}(\vec{v})$: 4 by m matrix, m number of intensity measurements
- minimum of $m = 4$ to determine all 4 Stokes parameters
- estimate $\vec{I}'$ of Stokes vector $\vec{I}$ from measurements $\vec{S}$ by inversion of X
$$\vec{I}' = \mathrm{Y}\vec{S} = \mathrm{Y}\left(\mathrm{X}(\vec{v})\vec{I} + \vec{n}\right) = \mathrm{YX}(\vec{v})\vec{I} + \mathrm{Y}\vec{n}\,.$$
- goal: choose $\mathrm{X}(\vec{v})$ and Y to minimize difference between $\vec{I}'$ and $\vec{I}$ given standard deviations σ_{S_j} of measurement errors n_j

Optimum Synthesis Matrix

- $m = 4$ linearly independent combinations of Stokes parameters

$$\mathsf{Y} = \mathsf{X}^{-1}$$

- if $m > 4$, matrix inverse not uniquely defined $\Rightarrow$ *generalized inverse*:

$$\mathsf{Y} = \left(\mathsf{X}^T\mathsf{X}\right)^{-1}\mathsf{X}^T$$

- among all possible Y with $\mathsf{YX} = 1$, generalized inverse minimizes sum of squares of rows
- optimum polarimetric efficiencies given by

$$\epsilon_{\mathrm{opt},i} = \sqrt{\frac{1}{m\left(\mathsf{X}^T\mathsf{X}\right)^{-1}_{ii}}}$$

- $\mathsf{X}^T\mathsf{X}$ symmetric, positive definite $\Rightarrow$ can be inverted

Polarimeter with Maximum Polarimetric Efficiency

- maximum efficiency polarimeter has signal matrix X with

$$\mathsf{X}^T\mathsf{X} = m \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \epsilon_{\max,2}^2 & 0 & 0 \\ 0 & 0 & \epsilon_{\max,3}^2 & 0 \\ 0 & 0 & 0 & \epsilon_{\max,4}^2 \end{pmatrix}$$

- intensity efficiency $\epsilon_1 = 1$
- $\sum_{i=2}^{4} \epsilon_i^2 \leq 1 \Rightarrow$ maximum efficiency for measuring polarized Stokes components with equal efficiencies given by $\frac{1}{\sqrt{n}}$, $n =$ number of measured polarized Stokes parameters
- 4 Stokes parameters: maximum efficiency is $\frac{1}{\sqrt{3}} \approx 0.577$
- linear polarization only: maximum efficiency is $\frac{1}{\sqrt{2}} \approx 0.707$
- circular polarization only: maximum efficiency is 1

Circular Polarimeter with Maximum Polarimetric Efficiency

- polarimeter measuring only circular polarization with maximum efficiency with two measurements has signal matrix

$$X = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 \end{pmatrix}$$

- signal matrix can be achieved with
  - quarter-wave retarder and polarizing beamsplitter
  - rotating quarter-wave plate and linear polarizer where measurements are performed with quarter-wave plate at $\pm 45^\circ$ with respect to linear polarizer
- equivalent matrices and setups can be derived for only one component of linear polarization

Full Stokes Polarimeter with Maximum Polarimetric Efficiency

- minimum of 4 measurements
- best to make measurements corresponding to points on Poincaré sphere building the corners of an equilateral tetrahedron
- tetrahedron can be arbitrarily rotated within Poincaré sphere
- possible signal matrix:

$$\mathrm{X} = \begin{pmatrix} 1 & x & x & x \\ 1 & x & -x & -x \\ 1 & -x & x & -x \\ 1 & -x & -x & x \end{pmatrix}, \quad x = \frac{1}{\sqrt{3}}$$

6-Measurement Maximum Efficiency Full Stokes Polarimeter

- six-measurement matrix conceptually easier to implement

$$X = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & -1 \end{pmatrix}$$

- can be implemented with quarter-wave plate and linear polarizer that both rotate
- two variable retarders (at 0° and 22.5°) and linear polarizer at 45°:

δ_1	δ_2	X_{i1}	X_{i1}	X_{i1}	X_{i1}
0	0	1	1	0	0
0	180	1	-1	0	0
180	0	1	0	1	0
180	180	1	0	-1	0
0	90	1	0	0	-1
180	90	1	0	0	-1

Photon Budget

Quantity	wavelength (nm)			units	comment
	630.2	**854.2**	**1083**		
Solar flux in continuum	175	101	63.2	$erg/cm^2/A$	from Allen
Photon energy	0	0	0	erg/photon	
Atmospheric transmission	0.86	0.95	0.97		approx. 7000ft
Primary diameter	50	50	50	cm	effective diameter
Central obscuration diameter	15.6	15.6	15.6	cm	this is the aperture stop
Effective aperture	1772.36	1772.36	1772.36	cm^2	
Surface	**transmission/reflectivity**				
Entrance window	0.965	0.961	0.954		fused silica and MgF_2
Primary mirror	0.960	0.960	0.960		protected silver
Secondary mirror	0.960	0.970	0.980		SilverStar, measured
Corrector lens 1	0.990	0.990	0.960		
Total transmission	0.21	0.22	0.23		unpolarized
HyViSI quantum efficiency	0.87	0.83	0.04	e-/photon	Rockwell, measured
Spatial sample size	4.37E-07	4.37E-07	4.37E-07	solar fraction	1.125" by 1.125" sample size
Spectral sample size	2.59E-02	3.94E-02	4.50E-02	A	Ming's report
Photoelectric flux	1.72E+08	2.29E+08	1.07E+07	e-/s	per pixel and second
Polarization channels	2	2	1		
Readout rate	1.00E+02	1.00E+02	1.00E+02	Hz	
Full well capacity	2.00E+06	2.00E+06	2.00E+06	e-	Rockwell
Maximum detection	4.00E+08	4.00E+08	2.00E+08	e-/s	
Detected flux	1.72E+08	2.29E+08	1.07E+07	e-/s	
Stokes Q modulation efficiency	0.58				
Stokes V modulation efficiency	0.50	1.00			
Stokes I noise in 0.5s	1.08E-04	9.35E-05	4.33E-04		
Stokes Q,U noise in 0.5s	1.87E-04				
Stokes V noise in 0.5s	2.16E-04	9.35E-05			

Some Systematic Errors (Instrumental Errors)

- atmospheric seeing and guiding errors
- instrumental polarization due to
  - telescope and instrument optics
  - polarized scattered light in telescope and instrument
  - spectrograph slit polarization
  - angle, wavelength, temperature dependence of retarders
  - crystal aberrations
  - polarized fringes
- ghost images
- variable sky background
- unpolarized scattered light in atmosphere and optics
- limited calibration accuracy
- ...

Nomenclature of Systematic Effects

- instrumental effects (polarizing telescope, instrument) can often be described by Mueller matrices

$$\begin{pmatrix} I \rightarrow I & Q \rightarrow I & U \rightarrow I & V \rightarrow I \\ I \rightarrow Q & Q \rightarrow Q & U \rightarrow Q & V \rightarrow Q \\ I \rightarrow U & Q \rightarrow U & U \rightarrow U & V \rightarrow U \\ I \rightarrow V & Q \rightarrow V & U \rightarrow V & V \rightarrow V \end{pmatrix}$$

- 3 categories of instrumental errors described by Mueller matrix
  - $I \rightarrow X, X = Q, U, V$: instrumentally induced *polarization*
  - $X_1 \rightarrow X_{2 \neq 1}$: instrumentally introduced *cross-talk*
  - $X \rightarrow X$: instrumentally introduced *depolarization*
- polarization *sensitivity*: magnitude of small polarization signal on top of background that can just be detected, expressed as fraction of intensity
- polarization *accuracy*: magnitude of absolute error in polarization measurement, expressed as percentage

Seeing and Guiding Errors

- air not birefringent $\Rightarrow$ seeing no intrinsic problem
- if polarization measurements are not carried out simultaneously, seeing and guiding errors can introduce spurious polarization signals (sequential images are differently distorted)
- sequential polarization measurements $\gtrsim$100 Hz
- best to carry out measurements simultaneously and modulate

Polarizing Telescopes

- every telescope introduces some (small) polarization
- no telescope is free of polarization, although polarization may be so small that it can be neglected (THEMIS, SOLIS VSM)
- intrinsic stress birefringence in glass amounts to cross-talk of up to 1% between V, Q, U for every cm of glass
- temperature-induced stress leads to time-dependent birefringence (use fused silica)

Mitigating Systematic Errors

various approaches to reducing effect of instrumental polarization:

- avoiding oblique reflections;
- compensating instrumental polarization with:
  - retarders
  - partial polarizers such as tilted glass plates
  - crossing mirrors at 90° (see above)
  - use two or four mirror arrangements to compensate for single oblique reflection off mirror
- measure and take into account in data reduction

Data Reduction

- typical reduction steps:
  - subtraction of dark current and bias
  - division by flat field
  - calculation of fractional polarization
  - subtraction of polarization bias
  - removal of polarized fringes
  - calibration with polarization efficiency (polarization flat field)
  - if required, multiplication with calibrated intensity I to obtain V, Q, and U
- best data reduction strategy comes from understanding all relevant instrumental effects
- all data reduction steps should be based on physical model of data collection process (theory of observing process and model of instrument)
- once theory is available, solve it for parameters that should be determined as a function of measured quantities
- this solution helps to identify necessary calibrations

Polarimeter Design

An Overview

- no automatic system can generate optimum polarimeter design for a given application
- human experience and intuition along with careful analysis of potential designs needed to produce good design
- clearly specify requirements for polarimeter without limiting design choices
- after selecting optical components, approximate properties, and order of placement, performance can be optimized automatically
- check whether optimum design can be manufactured with *tolerance analysis*
  - sensitivity analysis to determine most sensitive parameters
  - select maximum allowed performance deviation
  - inverse sensitivity analysis to determine reasonable errors
  - Monte Carlo analysis to make sure that design has high likelihood of achieving requirements