# Lecture 9: Variable Retarders

## Outline

1. Liquid Crystal Retarders
2. Piezo-Elastic Modulators (PEMs)
3. Achromatic Variable Retarders
4. Comparison of Variable Retarders

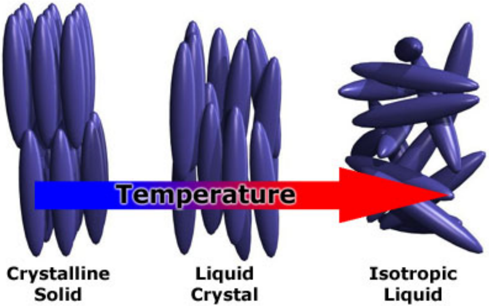
# Variable Retarders

## Introduction

- sensitive polarimeters requires retarders whose properties (retardance, fast axis orientation) can be varied quickly (*modulated*)
- retardance changes (change of birefringence):
  - liquid crystals
  - Faraday, Kerr, Pockels cells
  - piezo-elastic modulators (PEM)
- fast axis orientation changes (change of *c*-axis direction):
  - rotating fixed retarder
  - ferro-electric liquid crystals (FLC)

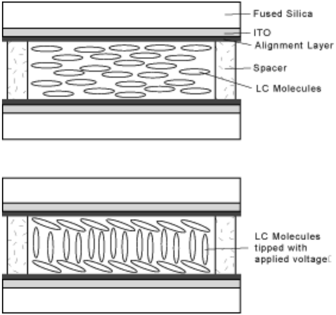
# Liquid Crystal Retarders

## Liquid Crystals



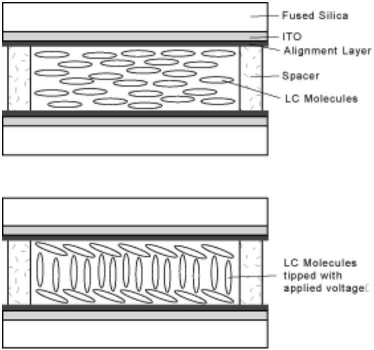
- liquid crystals are fluids whose molecules are elongated
- at high temperatures, liquid crystal is isotropic
- at lower temperature, molecules become ordered in orientation and sometimes also space in one or more dimensions
- liquid crystals can line up parallel or perpendicular to external electrical field

# Liquid Crystal Retarders



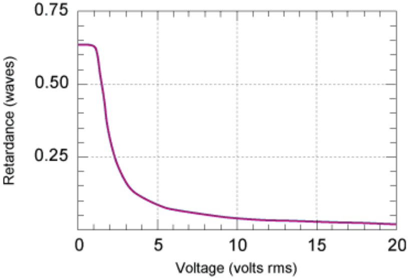
- dielectric constant anisotropy often large ⇒ very responsive to changes in applied electric field
- birefringence $\delta n$ can be very large (larger than typical crystal birefringence)
- liquid crystal layer only a few $\mu$m thick ⇒ true zero-order retarder
- anisotropy, and therefore birefringence, shows strong temperature dependence

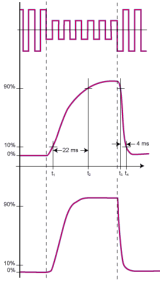
# Nematic Liquid Crystal Variable Retarders



- in *nematic phase*, molecules are randomly positioned but aligned (more or less) in one direction
- with zero voltage applied externally, liquid crystal molecules are parallel to substrates ⇒ maximum retardation
- with electrical field, liquid crystal molecules tip perpendicular to substrate ⇒ reduced effective birefringence/retardance

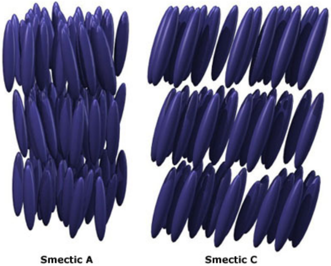
# Voltage Dependence of Birefringence

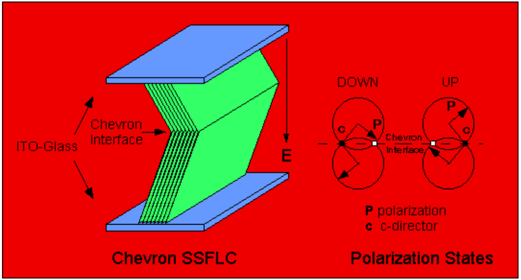




- alignment layer between substrate and liquid crystal prevents molecules at surface to rotate freely
- ⇒ residual retardance of about 30 nm even at high voltages (about 20 V)
- retardance changes by about $-0.4\%$ per °C
- response time of nematic liquid crystal retarders is proportional to square of layer thickness (=total retardance) and of the order of 20 ms

# Ferro-Electric Liquid Crystals





- smectic liquid crystal phases characterized by well-defined layers that can slide over one another
- molecules are positionally ordered along one direction
- in smectic C phase, molecules are tilted away from layer normal
- ferroelectric liquid crystals (FLCs) are tilted phases of chiral molecules (smectic C$^*$), which have permanent polarization

## Ferro-Electric Liquid Crystal Retarders

- respond much more quickly to externally applied fields than nematic liquid crystals
- can be used to make fast, bistable electro-optic devices
- FLCs act like retarders with fixed retardation where fast axis direction can be switched by about $45^\circ$ (switching angle) by alternating sign of applied electrical field
- achromatic modulators with FLCs in Pancharatnam configuration
- switching times on the order of 150 $\mu$s
- switching angle is temperature sensitive
- retardance rather insensitive to temperature variations

## Liquid Crystal Advantages and Disadvantages

**Advantages**:

- arbitrary (optimized) modulation schemes
- large, uniform apertures available
- retardation or fast axis changes possible
- FLC allow fast modulation (<10 kHz
- require only low voltages at moderate driving powers ($\sim$ 1 W)

**Disadvantages**:

- degrades quickly under UV irradiation
- requires temperature control
- nematic have slow modulation frequency (<50 Hz)
- FLCs cannot change retardation

# Piezo-Elastic Modulators (PEMs)

- stress-induced birefringence, also sometimes called piezo-optical or photo-elastic effect
- block of a few cm in side length of common BK7 glass can be stressed enough by hand such as to introduce a quarter-wave retardation
- stress-induced birefringence is proportional to stress $\sigma$
- retardation (in waves)

$$\delta = \frac{1}{\lambda} K d \sigma$$

| | |
|---|---|
| $K$ | stress optical constant |
| $d$ | thickness of variable retarder |
| $\lambda$ | wavelength |

- construct variable retarder by compressing optical glass
- requires considerable mechanical power to modulate

# Mechanical Resonance

- mechanically resonant oscillation reduces power requirement to one over the mechanical $Q$ ($10^3$-$10^4$ for most glasses)
- slab of length $L$ excited at fundamental mode ⇒ standing acoustic wave with wavelength $2L$, frequency $\omega$

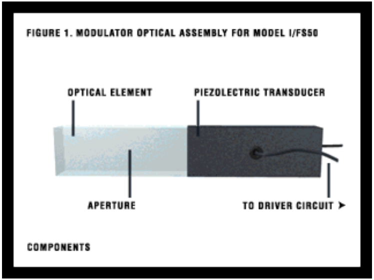
$$\omega = \frac{c_s}{2L}$$

- $c_s$: sound speed in optical material
- 57-mm-long fused silica slab ⇒ resonance frequency is 50 kHz.
- resulting stress, retardance as function of position $x$ and time $t$
- stress-induced birefringence $\delta(x,t)$ given by

$$\delta(x,t) = A \sin \omega t \sin(\frac{\pi x}{L})$$

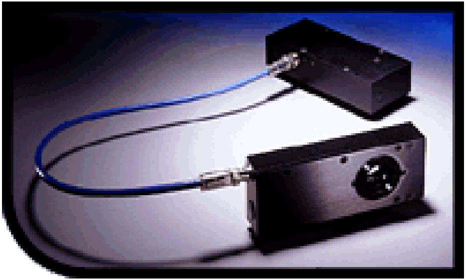
$A$ amplitude of oscillation
$x$ from 0 to $L$

# PEM Driver



- to make slab oscillate, quartz crystal with electrodes on its surfaces is forced to oscillate by externally applied electrical field via piezo effect
- quartz slab mechanically coupled to modulator slab
- electrical field driven at mechanical resonance frequency
- oscillation amplitude $A$ regulated with electronic feedback circuit

# PEM Power



- oscillation dampened by friction losses within modulator material
- energy loss inversely proportional to mechanical $Q$
- $Q$ very large $\Rightarrow$ little energy loss in modulator small (0.1 to 1 W)
- material with high $Q$ (fused silica, $Q \approx 10^4$) desirable
- typical glass has $Q \sim 10^3$
- required drive power does not depend on length of slab

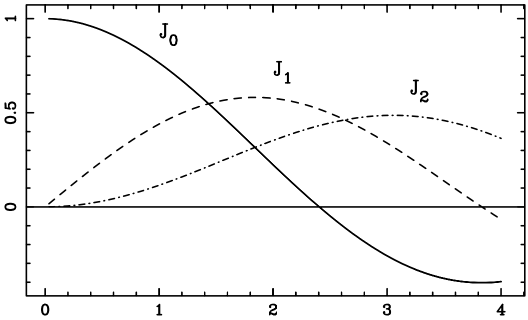
## PEM Birefringence

- stress-induced birefringence

$$\delta(x,t) = A \sin \omega t \sin(\frac{\pi x}{L})$$

- combine $\sin(\frac{\pi x}{L})$, amplitude $A$ into spatially varying amplitude $A(x)$
- birefringence becomes $\delta(x,t) = A(x)\sin(\omega t)$
- $A$ small $\Rightarrow$ PEM is true zero-order retarder
- PEM Mueller matrix corresponds to retarder with time-dependent retardation
- retarder Mueller matrix contains elements with $\sin \delta(x,t)$ and $\cos \delta(x,t)$
- expand $\sin(\sin(\cdot))$ and $\cos(\sin(\cdot))$ in terms of Bessel functions

# Bessel Functions



- Mueller matrix elements become

$$\begin{aligned} \sin\delta(\mathrm{x},\mathrm{t}) &= 2J_1(A(x))\sin\omega t + \cdots \ , \\ \cos\delta(x,t) &= J_0(A(x)) + 2J_2(A(x))\cos 2\omega t + \cdots \ , \end{aligned}$$

- $J_{0,1,2}$: Bessel functions of order 0,1 and 2

## PEM Advantages and Disadvantages

**Advantages**:

- PEMs are stable in operation
- show no degrading at high intensity levels and/or UV irradiation
- have good optical properties
- large spatial and angular aperture
- require only low voltages at moderate driving powers ($< 1$ W)

**Disadvantages**:

- sinusoidal modulation (as compared to more efficient square-wave modulation possible with liquid crystals)
- very high modulation frequency (20 to 50 kHz), which requires specialized array detectors (ZIMPOL)

# Achromatic Variable Retarders

## Some Thoughts

- achromatic variable retarders would provide major advantages
- achromatic retarders using two different materials very difficult because wavelength-dependence of birefringence needs to be very different for the two materials
- bi-liquid-crystal achromatic retarders have been built
- Pancharatnam approach looks more feasible
- variable birefringence retarders (nematic liquid crystals, PEMs) do not work because Pancharatnam approach works minimizes dependence on retardance of individual components
- three half-wave FLCs in Pancharatnam configuration provide excellent achromatic half-wave plate with switchable fast axis orientation
- can obtain achromatic performance without achromatic variable retarder

## Comparison of Variable Retarders

| Modulator | Advantages | Disadvantages |
|---|---|---|
| rotating retarder | high stability | relatively slow modulation |
| | large wavelength range | beam motion |
| | | needs 8 measurements for all Stokes parameters |
| liquid crystal | relatively fast modulation | narrow simultaneous wavelength range |
| | only 4 measurements for all Stokes parameters | limited temporal stability |
| | no moving parts | damaged by strong UV light |
| PEM | very fast modulation | narrow simultaneous wavelength range |
| | high stability | needs special CCD camera |
| | no moving parts | spatial retardance variation |