Outline

Fixed Retarders

Introduction

- retarder: splits beam into 2 components, retards phase of one component, combines components at exit into single beam
- ideal retarder does not change intensity of light or degree of polarization
- any retarder is characterized by two (not identical, not trivial) Stokes vectors of incoming light that are not changed by retarder $\Rightarrow$ *eigenvectors* of retarder
- depending on polarization described by eigenvectors, retarder is
  - *linear retarder*
  - *circular retarder*
  - *elliptical retarder*
- linear, circular retarders are special cases of elliptical retarders
- circular retarders sometimes called *rotators*
- linear retarders by far the most common type of retarder

Jones Matrix for Linear Retarders

- linear retarder with fast axis at 0° characterized by Jones matrix

$$\mathsf{J}_r(\delta) = \begin{pmatrix} e^{i\delta} & 0 \\ 0 & 1 \end{pmatrix}, \quad \mathsf{J}_r(\delta) = \begin{pmatrix} e^{i\frac{\delta}{2}} & 0 \\ 0 & e^{-i\frac{\delta}{2}} \end{pmatrix}$$

- δ: phase shift between two linear polarization components (in radians)
- absolute phase does not matter $\Rightarrow$ ‘symmetric’ version avoids absolute phase that depends on retardation
- use of ’asymmetric’ version led to some erroneous theoretical calculations of instrumental Mueller matrix of telescopes

Mueller Matrices for Linear Retarders

- corresponding Mueller matrix is given by

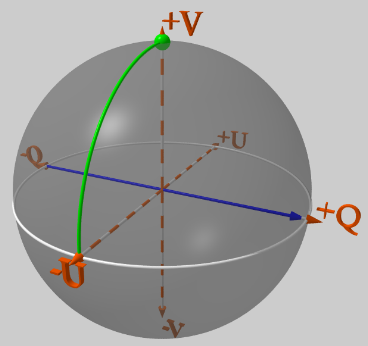
$$M_r = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\delta & -\sin\delta \\ 0 & 0 & \sin\delta & \cos\delta \end{pmatrix}$$

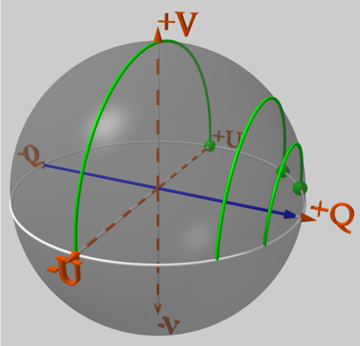
- linear retarder, fast axis angle θ, retardance δ

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos^2 2\theta + \cos\delta \sin^2 2\theta & \cos 2\theta \sin 2\theta - \cos 2\theta \cos\delta \sin 2\theta & \sin 2\theta \sin\delta \\ 0 & \cos 2\theta \sin 2\theta - \cos 2\theta \cos\delta \sin 2\theta & \cos\delta \cos^2 2\theta + \sin^2 2\theta & -\cos 2\theta \sin\delta \\ 0 & -\sin 2\theta \sin\delta & \cos 2\theta \sin\delta & \cos\delta \end{pmatrix}$$

- 2 or more linear retarders in series ⇒ (in general) equivalent to 1 elliptical retarder

Retarders on the Poincaré Sphere





- retarder eigenvector (fast axis) in Poincaré sphere
- points on sphere are rotated around retarder axis by amount of retardation

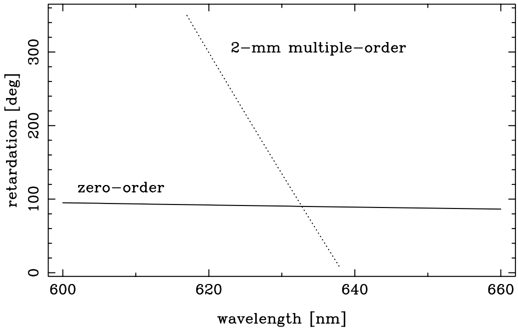
Zero and Multiple-Order Linear Retarders

- most retarders based on birefringent materials
- typical birefringent materials: quartz, mica, and polymer films
- *c*-axis parallel to interface
- retardation (delay between ordinary and extraordinary ray):

$$N\lambda = d\,(n_e - n_o)$$

  - d: geometrical thickness
  - λ: wavelength
  - n_e, n_o: indices of refraction for extraordinary and ordinary rays
  - N: *retardation* expressed in waves
- quarter wave plate is obtained with $N = m + \frac{1}{4}$ and m being an integer
- $m = 0$: *true zero-order* retarder
- $m > 0$: *multi-order* retarder

Wavelength Dependence of Retarders



wavelength dependence of 2-mm multi-order and true zero-order quartz quarter-wave retarder assuming constant n_o and n_e

- retardation of 1.25 waves equivalent 0.25 waves
- the larger d, the faster retardation changes as function of wavelength

Quartz Retarders

- quartz available in large sizes
- can be produced artificially
- most commonly used crystal for high-quality retarders
- true zero-order quarter-wave retarder in visible: 15 μm thick
- very difficult to manufacture
- *compound zero-order retarder*: two $\sim$1-mm thick plates with difference in thickness corresponding to desired zero-order retarder
- plates optically contacted with fast axes at 90°
- plates cancel each other except for small path-length difference
- usable from about 180 nm to 2700 nm

Mica Retarders

- natural mica often used for commercial retarders
- cheap, available in large sizes (20 cm by 20 cm)
- mica crystals easily cleaved into very thin sheets
- quarter-wave plate in visible $\sim$50 μm thick
- transparent from 350 nm to 6 μm, but absorbs even in visible
- thicker at longer wavelengths $\Rightarrow$ larger absorption

Polymer Retarders

- stretched polymers (e.g. polyvinyl alcohol) also birefringent
- fast axis perpendicular to stretch direction
- quarter-wave retarder is $\approx$20 μm in visible
- true zero-order retarders
- highly transparent even in UV
- sizes up to 40 cm

Achromatic Retarders

Different Birefringent Materials

- retarders highly wavelength sensitive due to
  - wavelength itself
  - wavelength dependence of the birefringence
- combine two materials with opposite variations of $\Delta n = n_e - n_o$ with wavelength
- choose appropriate thicknesses for achromatic retarder (perfect retardance at 2 wavelengths)

$$
\begin{aligned}
N\lambda_1 &= d_a \Delta n_{1a} + d_b \Delta n_{1b} \\
N\lambda_2 &= d_a \Delta n_{2a} + d_b \Delta n_{2b}
\end{aligned}
$$

  - N: desired retardance
  - λ_1, λ_2: two wavelengths where correct retardation is achieved
  - d_a, d_b: thicknesses of plates made from materials *a* and *b*
  - Δn_{ij}: birefringence for material *j* at wavelength *i*

Bicrystaline Retarders

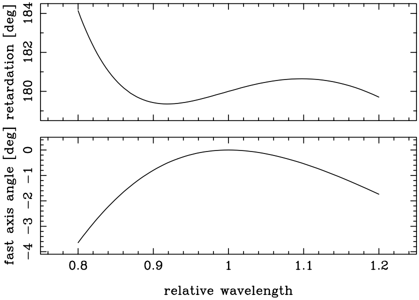
- solve equations for two thicknesses

$$d_a = N \frac{\lambda_1 \Delta n_{2b} - \lambda_2 \Delta n_{1b}}{\Delta n_{1a} \Delta n_{2b} - \Delta n_{1b} \Delta n_{2a}}$$

$$d_b = N \frac{\lambda_2 \Delta n_{1a} - \lambda_1 \Delta n_{2a}}{\Delta n_{1a} \Delta n_{2b} - \Delta n_{1b} \Delta n_{2a}}$$

- achromatic retarder as long as denominator $\neq 0$
- negative thickness $d_{a,b} \Rightarrow$ fast axis at 90°
- if thickness too small $\Rightarrow$ replace with compound retarder
- better: numerically optimize over desired wavelength range
- quartz and MgF_2 used most often in visible
- different materials have widely different off-axis performance
- combined temperature dependence also material dependent
- trade off wavelength versus field-of-view versus temperature variation of retardance

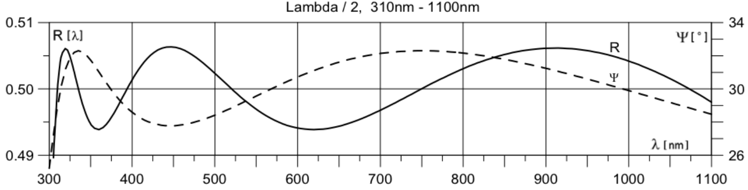
Combinations of Retarders of the Same Material

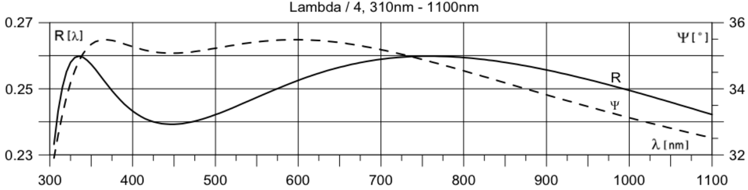


Theoretical variation of retardation and fast axis orientation as a function of relative wavelength for a Pancharatnam achromatic half-wave plate

- several retarders made from same material (Pancharatnam 1955)
- half-wave plate: outer two plates parallel fast axes, inner plate rotated by $\approx 60^\circ$
- fast axis direction of combined retarder depends on wavelength
- also achromatic quarter-wave plates, but not as good

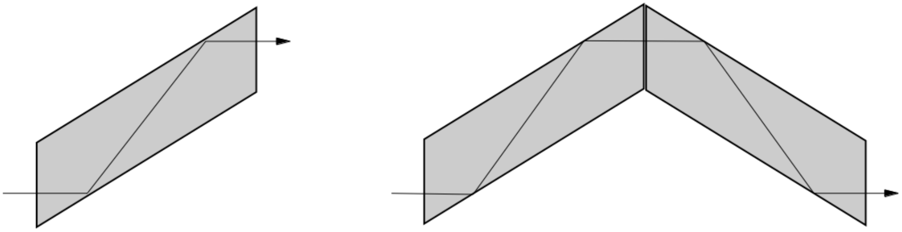
Superachromatic Retarders





- 3 bicrystaline retarders in Pancharatnam configuration
- fast axis direction depends on wavelength
- angular acceptance angle very limited for crystal achromats
- much better angular performance with plastic retarders

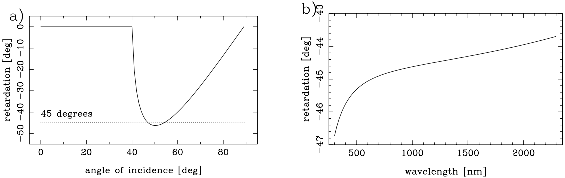
Fresnel Rhombs



traditional arrangements for quarter-wave (left) and half-wave (right) Fresnel rhombs

- phase shift on total internal reflection (TIR) on interface between dielectrica
- in the visible: not possible to achieve 90° phase shift on single reflection
- several reflections can produce $\lambda/4$ and $\lambda/2$ retardation

Fresnel Rhomb Performance



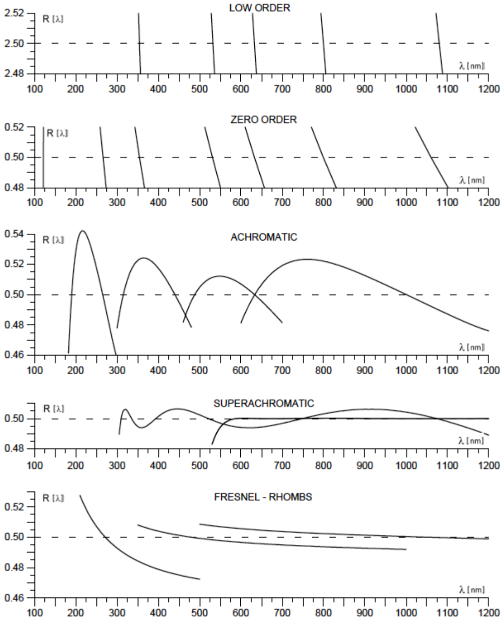
BK7 at 632.8 nm; retardance at 55.08°

- total internal reflection on glass (n_i) air interface for $n_i \sin\beta > 1$
- β: (internal) angle of incidence, phase shift δ

$$\tan\delta/2 = -\frac{\cos\beta\sqrt{n_i^2\sin^2\beta - 1}}{n_i\sin^2\beta}$$

- retardation strongly depends on angle $\Rightarrow$ small acceptance angle
- variation of retardance with wavelength purely due to variation of index of refraction with wavelength

Overview of Wavelength-Dependence



Angle-Dependence of Birefringent Retarders

- retardation by uniaxial medium depends on angle of incidence and orientation of plane of incidence with respect to optic axis
- retardation changes because index of extraodinary ray depends on direction
- apparent plate thickness changes for both rays
- for $\sin^2\theta << n_o^2, n_e^2$

$$\delta \approx \delta(\theta = 0)\left[1 + \frac{\sin^2\theta}{2n_o}\left(\frac{\sin^2\phi}{n_e} - \frac{\cos^2\phi}{n_o}\right)\right]$$

θ angle of incidence
ϕ angle between plane of incidence and optic axis
$\delta(\theta = 0)$ retardation at normal incidence

Angle-Dependence of Birefringent Retarders (continued)

$$\delta \approx \delta(\theta = 0)\left[1 + \frac{\sin^2\theta}{2n_o}\left(\frac{\sin^2\phi}{n_e} - \frac{\cos^2\phi}{n_o}\right)\right]$$

- δ decreases when optic axis in plane of incidence ($\phi = 0$)
- δ increases when optic axis perpendicular to plane of incidence ($\phi = \pi/2$)
- linear retarder with slightly wrong retardance can be tipped or tilted to achieve required retardance
- retardation error proportional to retardance $\Rightarrow$ multiple-order waveplates much worse than zero-order retarders
- compound zero-order retarders also much worse performance
  - second retarder has retardation with opposite sign
  - but retardation error also has opposite sign because azimuth changes by 90°
- retardation error increases quadratically with angle and is inversely proportional to index squared

Temperature Dependence of Fixed Retarders

- retardation depends on temperature
- optical path difference variation in linear approximation

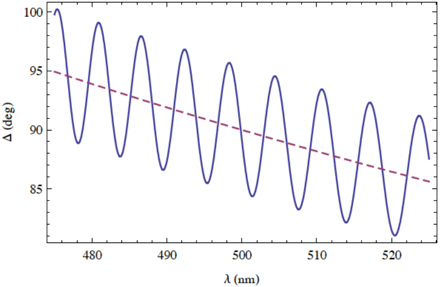
$$\delta_T d \left(n_e - n_o\right) + d \left(\delta_T n_e - \delta_T n_o\right)$$

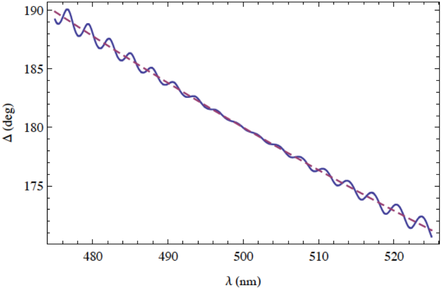
- δ_T indicates variations with temperature
- with coefficient of thermal expansion (CTE) $\alpha = \delta d / d$

$$\delta N = N \left(\alpha + \frac{\delta n_e - \delta n_o}{n_e - n_o}\right)$$

- quartz: $\delta N = N \left(-1.0 \times 10^{-4}\right)$ K^{-1} at 632.8 nm
- 2 mm thick $\lambda/4$ retarder $\Rightarrow$ retardation variation $\approx 1^\circ$per Kelvin
- compound zero-order same as true zero-order retarders
- achromatic retarders made from different materials show stronger temperature dependence

Spectral Fringes in Retarders





ret

for true zero-order quartz retarders using Berreman calculus

- retarders are not ideal because of interference between reflected and transmitted beams at 2 interfaces (Fabry-Perot)
- optical thinkness is different for ordinary and extraordinary beam
- retardation (and transmittance) show spectral fringes

Linear Retarder Selection Guide

Comparison of various types of commercially available zero-order retarders; quartz and MgF_2 are compound zero-order retarders; accuracy in percent refers to half-wave plate

type	accuracy (%)	wavelength range (nm)	bandpass (nm)	acceptance angle (°)
quartz	0.4	180-2700	100	3
MgF_2	0.4	140-6200	100	3
mica	4	350-1550	100	10
polymer	0.6	400-1800	100	10
Fresnel	2	240-2000	330-1000	2