Lecture 7: Linear Polarizers

Outline

1. Jones and Mueller Matrices for Linear Polarizers
2. Wire Grid Polarizers
3. Polaroid-type Polarizers
4. Crystal-based Polarizers
5. Thin-Film Polarizers
6. Polarizer Selection Guide

Polarizers

- polarizer: optical element producing (at least partially) polarized light from unpolarized input beam
- linear, circular, or in general, elliptical polarizer
- most common: linear polarizer
- large variety of types

Jones Matrix for Linear Polarizers

- linear polarizer described by transmittance of electrical field in 2 orthogonal directions
- Jones matrix for linear polarizer:

$$\mathrm{J}_p = \begin{pmatrix} p_x & 0 \\ 0 & p_y \end{pmatrix}$$

- real values $0 \leq p_x \leq 1$ and $0 \leq p_y \leq 1$: transmission factors for x and y-components of electric field

$$E'_x = p_x E_x, \quad E'_y = p_y E_y$$

- $p_x = 1,\ p_y = 0$: linear polarizer in $+Q$ direction
- $p_x = 0,\ p_y = 1$: linear polarizer in $-Q$ direction
- $p_x = p_y$: neutral density filter

Mueller Matric for Linear Polarizer

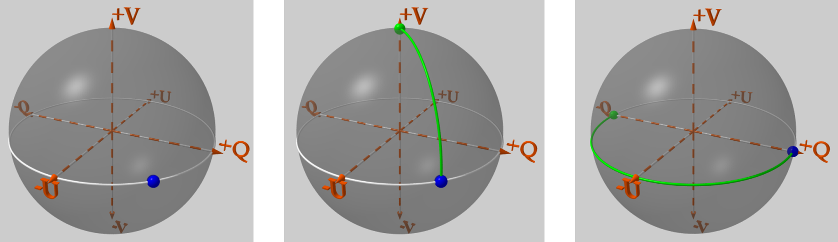
$$\mathrm{M}_p = \frac{1}{2}\begin{pmatrix} p_x^2+p_y^2 & p_x^2-p_y^2 & 0 & 0 \\ p_x^2-p_y^2 & p_x^2+p_y^2 & 0 & 0 \\ 0 & 0 & 2p_xp_y & 0 \\ 0 & 0 & 0 & 2p_xp_y \end{pmatrix}$$

- unpolarized incoming beam always linearly polarized
- emerging Stokes vector only completely polarized if $p_x^2+p_y^2 = \left|p_x^2-p_y^2\right|$
- *partial* linear polarizer produces partially polarized beam from unpolarized light
- real polarizers always only partial polarizers
- polarized incoming beam $\Rightarrow$ elliptically polarized output beam because of non-zero diagonal terms $2p_xp_y$
- totally polarized beam remains totally polarized even when passing partial linear polarizer (no depolarization)

Mueller Matrix for Ideal Linear Polarizer at Angle θ

$$\mathrm{M}_{\mathrm{pol}}\left(\theta\right)=\frac{1}{2}\begin{pmatrix} 1 & \cos 2\theta & \sin 2\theta & 0 \\ \cos 2\theta & \cos^2 2\theta & \sin 2\theta \cos 2\theta & 0 \\ \sin 2\theta & \sin 2\theta \cos 2\theta & \sin^2 2\theta & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Poincare Sphere



- polarizer is a point on the Poincaré sphere
- transmitted intensity: $\cos^2(l/2)$, l is arch length of great circle between incoming polarization and polarizer on Poincaré sphere

Characterizing Linear Polarizers

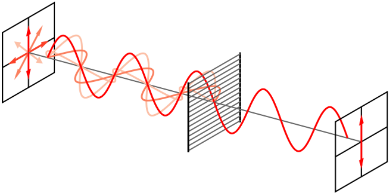
- 2 parameters describing performance:
  - k_1: transmittance for fully linearly polarized beam at angle that maximizes transmitted intensity
  - k_2: minimum transmittance for incoming linearly polarized beam
- $k_1 = p_x^2$, $k_2 = p_y^2$ if $p_x > p_y$
- *extinction ratio* k_1/k_2 *contrast*
- k_1, k_2 depend on wavelength
- determined from transmittances for unpolarized light of parallel and crossed identical polarizers

$$\begin{aligned} T_{\text{parallel}} &= \tfrac{1}{2}\left(k_1^2 + k_2^2\right) \\ T_{\text{crossed}} &= k_1 k_2 \end{aligned}$$

- also used: *degree of polarizability* or *polarizance* defined by

$$P = \frac{k_1 - k_2}{k_1 + k_2}$$

Wire Grid Polarizers



- parallel conducting wires, spacing $d \lesssim \lambda$ act as polarizer
- electric field component parallel to wires induces electrical currents in wires, attenuates transmitted electric field parallel to wires
- induced electrical current ’reflects’ polarization parallel to wires $\Rightarrow$ polarizing beam-splitter

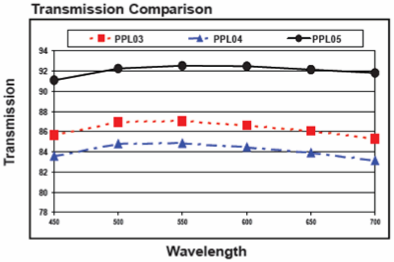
Wire Grid Polarizers for the Infrared

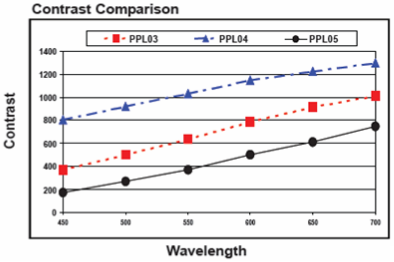
- rule of thumb:
    - $d < \lambda/2 \Rightarrow$ strong polarization
    - $d \gg \lambda \Rightarrow$ high transmission of both polarization states (weak polarization)
- mostly used in infrared because wire spacing becomes very small at visible wavelengths
- made by depositing thin-film metallic grid on substrate
- free-standing wire grid for longer wavelengths

Wire-Grid Polarizers for the Visible

- excellent performance at high light/energy levels
- 90% of energy reflected
- good in high heat/flux environment
- trade-off between transmission and contrast

MOXTEK Inc. ProFlux

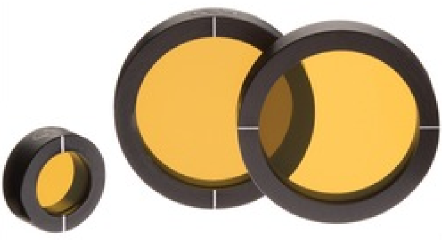

Transmission Comparison

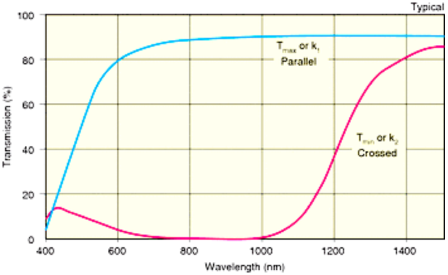

Contrast Comparison

Polarcor (Corning)

- glass polarizer with high performance for 600 to 2300 nm
- glass with aligned silver nano-particles in surface layers
- elongated, conducting silver particles act as small wires
- maximum diameter currently limited to 20 mm
- contrast ratio > 10000

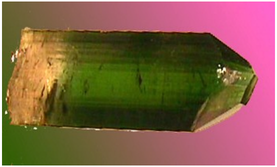
Corning Polarcor

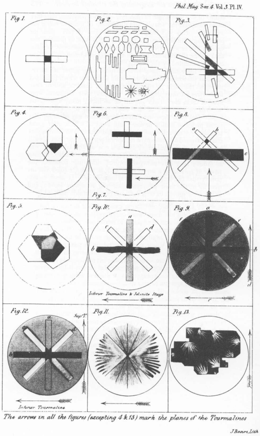




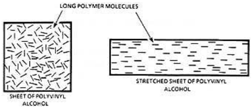
Dichroic Materials

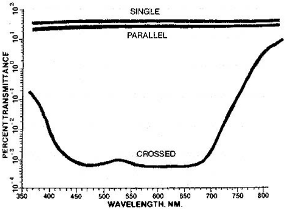
- absorb one polarization state
- absorption depends on wavelength ⇒ different colors depending on angles of illumination and viewing
- arises from anisotropy of complex index of refraction
- natural dichroic crystals: tourmaline, herapathite
- W.B. Herapath discovered in 1852 salt of quinine with polarizing properties; made first artificial crystals large enough to study under microscope
- difficult to produce uniform, large dichroic crystals

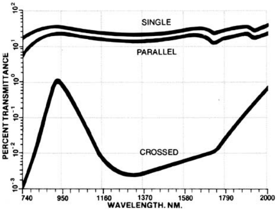




Polaroid-type Polarizers







- H-type sheet polarizers: stretched polyvynil alcohol (PVA) sheet, laminated to sheet of cellulose acetate butyrate, treated with iodine
- different H-type polarizers have different amounts of iodine in PVA
- PVA-iodine complex analogous to short, conducting wire
- Polaroid names (e.g. HN-38) identify overall type (H), color (N=neutral), approximate transmittance for unpolarized light
- K-type similar to H-type, but environmentally more stable
- HR-type based on a PVA-polyvinylene-iodine complex, works well from 0.7 to 2.3 μm

Field-of-View of Dichroic Polarizers

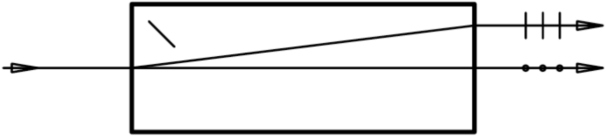
- conducting, needle-like particles aligned on common axis parallel to surface
- uniaxially anisotropic medium with complex indices of refraction
- transmitted polarization state sees real index of refraction
- absorbed polarization state sees complex index of refraction with imaginary part large enough to reduce intensity by orders of magnitude
- dichroic polarizers have limited field of view, largely a geometrical effect inherent to uniaxial medium

Crystal-Based Polarizers

- highest quality polarizers
- precise arrangement of atom/molecules and anisotropy separate incoming beam into two beams with precisely orthogonal polarization
- *polarizing beam-splitters* (both beams usable) and *polarizing prisms* (only one useful state)
- prisms can be *cemented* or *air spaced*
- air-spaced: good for short wavelengths, high power densities
- cemented: much better optical quality
- calcite is most often used in crystal-based polarizers because of very large birefringence, low absorption in visible
- many other suitable materials

Beam Displacer, Savart and Modified Savart Plates

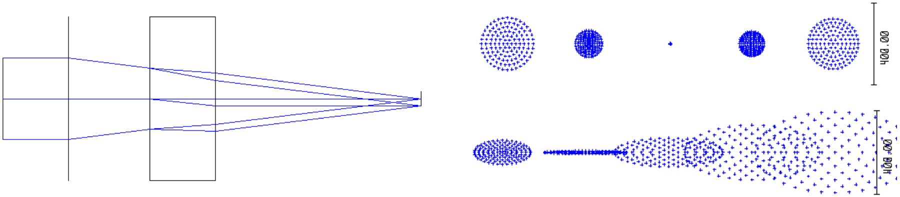
Beam Displacer



- most simple crystal polarizer: *polarizing beam displacer*
- single uniaxial crystal block with optic axis at $\sim$45°
- ordinary ray passes without deflection
- extraordinary ray deflected by dispersion angle α
- beam separation d as function of block length D:

$$d = D \tan \alpha = D \frac{(n_e^2 - n_o^2) \tan \theta}{n_e^2 + n_o^2 \tan^2 \theta}$$

Beam Displacer Problems



- ordinary and extraordinary beams have different path lengths
- extraordinary ray suffers from crystal astigmatism

Focus Difference and Astigmatism

- converging beam focus difference ϵ between ordinary and extraordinary rays:

$$\epsilon = \frac{\sin 2\theta \left(n_e^2 - n_o^2\right)}{2\left(n_o^2 \sin^2\theta + n_e^2 \cos^2\theta\right)}$$

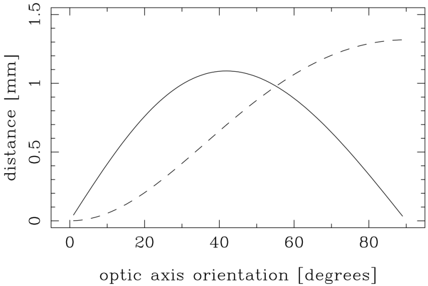
θ is angle of optic axis with normal to interface

- astigmatism leads to two focus positions *longitudinal astigmatism*

$$\epsilon_l = \frac{D \tan\theta \tan\alpha n_o}{n_e \sqrt{n_o^2 \sin^2\theta + n_e^2 \cos^2\theta}}$$

- for imaging system, 'smearing' of image given by *transverse astigmatism* $\epsilon_t = \frac{\epsilon_l}{F}$ where F is the F-number of the beam
- rule of thumb for calcite: astigmatic focus difference ϵ_l ~5% of thickness

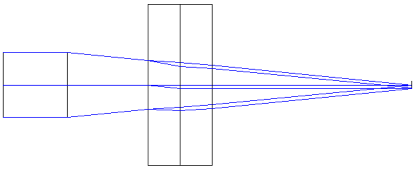
Beam Separation and Astigmatism

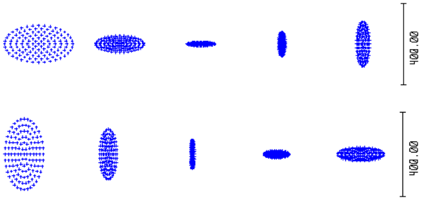


beam separation d (solid), longitudinal astigmatism ϵ_l (dashed) of extraordinary beam for 10-mm calcite beam displacer at 630 nm

- optic axis orientation of 45° close to optimum
- beam separation increases almost linearly for small optic axis angles
- astigmatism increases much more slowly
- possible to trade off beam separation versus astigmatism

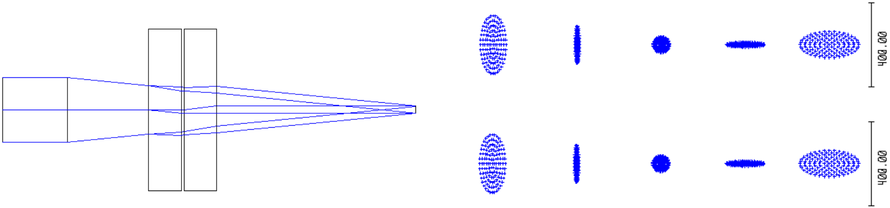
Savart Plate





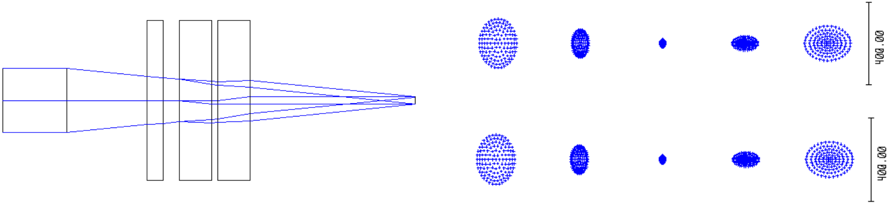
- avoid difference of focal point position problem by splitting calcite into two pieces crossed at 90°
- exchanges ordinary and extraordinary beams half-way ⇒ both beams have same optical path length
- splitting reduced by $\sqrt{2}$ for same total length of calcite
- both beams now show crystal astigmatism, oriented in opposite directions; amount of astigmatism half of single-piece beam displacer

Modified Savart Plate



- both calcite blocks in same orientation
- half-wave plate at 45° between them to exchange ordinary and extraordinary beams
- same direction of astigmatism in both beams

Cylinder Lens Compensating Crystal Astigmatism



- additional cylindrical lens compensates astigmatism
- limited wavelength range because of wavlength dependence of half-wave retarder
- deviation from half-wave retardance or 45° orientation leads to ghost beam between oppositely polarized beams
- other aberrations such as spherical aberration occur also
- developed for SOLIS VSM, excellent performance

Glan-Thompson Prism

- two calcite prisms with cement (index n_c) in between
- surrounding dielectric medium with refractive index n
- ordinary beam undergoes TIR, absorbed by black paint on side
- critical cut angles for TIR for ordinary and extraordinary beams

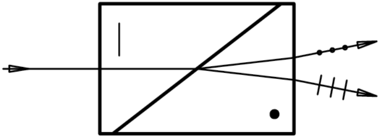
$$\sin \Omega_o = \frac{n_c}{n_o}, \quad \sin \Omega_e = \frac{n_c}{n_e}$$

- cut angle Ω such that acceptable range of angles of incidence is symmetric around normal incidence

Glan-Thompson Properties

- 2 versions differ in prism angles, index of cement
  - long version: acceptance angle of $\sim 26^\circ$
  - short form: acceptance angle of $\sim 15^\circ$
- typical extinction ratios 10^5 to 10^7
- somewhat reduced extinction rations when used from “wrong” side
- if prism is illuminated with convergent light, different parts of beam will have slightly different orientations of linear polarization $\Rightarrow$ strong reduction in extinction ratio for convergent beam

Wollaston Prism



- Wollaston prisms most common beam-deviating polarizers
- 2 beams with orthogonal linear polarizations, parallel and perpendicular to refracting edge
- both rays refracted away from normal of exit face
- first half: ordinary and extraordinary rays see their respective indices of refraction
- indices are reversed in second half of prism
- made of calcite or quartz prisms cemented together
- usable spectral range typically 300 nm to 2200 nm
- angle of divergence determined by wedge angle ($\sim 15 - 45^\circ$)

Wollaston Beam Deviation

- at refracting interface, Snell's law for the ordinary and extraordinary beams

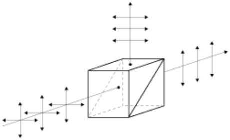
$$n_o \sin\Omega = n_e \sin\theta_o \ \text{ and } \ n_e \sin\Omega = n_o \sin\theta_e$$

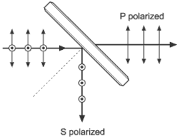
- at exit face, two beams are refracted differently; with Snell's law

$$n_e \sin(\Omega - \theta_o) = n \sin\theta_o' \ \text{ and } \ n_o \sin(\Omega - \theta_e) = n \sin\theta_e'$$

- n is index of external dielectric medium
- equations are not completely symmetrical $\Rightarrow$ two beams will have slightly different absolute angles upon exiting
- beam deviation depends on wavelength-dependent birefringence
- in converging beam, two beams do not come to focus at the same distance behind the exit face
- each beam is astigmatic

Thin-Film Polarizers





- thin-film polarizers mostly used in cube beam-splitters
- 2 orthogonally polarized beams emerge at right angles
- thin-film stack between 2 cemented glass prisms
- total internal reflection at Brewster angle within thin film
- limited extinction ratio and wavelength range
- extinction ratio of transmitted beam much better than reflected beam
- produced cheaply even for large apertures (5–10 cm)
- also possible on surface of oblique glass plate

Polarizer Selection Guide

type	extinction ratio	transmission (polarized)	wavelength range (nm)	bandpass (nm)	acceptance angle (°)	size mm	cost
Glan	$> 10^5$	$> 84\%$	300-2700	full	8	< 40	$$$
Glan-Thompson	$> 10^6$	$> 92\%$	300-2700	full	15-25	< 30	$$$
Wollaston	$> 10^6$	$> 92\%$	300-2200	full	20	< 50	$$
Polarcor	$> 10^4$	$> 80\%$	630-2300	150	> 20	< 25	$$
Polaroid	$150 - 10^4$	$> 75\%$	310-2000	200	> 20	> 200	$
Polarizing cube	> 500	$> 90\%$	400-1600	200-400	10	70	$$
Wire Grid	> 100	$> 90\%$	$4 \cdot 10^2 - 10^6$	$\sim \lambda$	> 20	> 70	$$