Lecture 6: Anisotropic Media

Outline

1. Dichroism
2. Stress Birefringence
3. Electro-Optics

Dichroism in Uniaxial Crystals

- some materials exhibit different absorption properties for orthogonal polarization states (e.g. tourmaline, sheet polarizers)
- for uniaxial crystals can assume complex indices of refraction $\tilde{n}_{x,y,z}$ to describe anisotropic absorption (dichroism)
- choose prinicpal coordinates where both tensors are diagonal
- repeat derivations as before
- Fresnel equation ⇒ ratio of electric field components k,l

$$\frac{E_k}{E_l} = \frac{s_k\left(\tilde{n}^2 - \tilde{\epsilon}_l\right)}{s_l\left(\tilde{n}^2 - \tilde{\epsilon}_k\right)}$$

- same as for non-absorbing materials, but different interpretion
- $\tilde{n}$, $\tilde{\epsilon}_{l,k}$ complex ⇒ ratios are (in general) complex
- ordinary and extraordinary waves are elliptically polarized
- finite conductivity ⇒ non-vanishing current density $\vec{j}$
- displacement vectors not orthogonal to wave vector

Ordinary and Extraordinary Waves in Uniaxial Dichroic Materials

- for uniaxial media with small conductivities, two solutions to Fresnel equation have analogous form

$$\tilde{n}_1^2 = \tilde{n}_o^2, \quad \frac{1}{\tilde{n}_2^2} = \frac{\cos^2\theta}{\tilde{n}_o^2} + \frac{\sin^2\theta}{\tilde{n}_e^2}$$

- polarization states of 2 waves almost linear because ratios of electric field components dominated by real parts
- attenuation of ordinary wave independent of wave direction
- attenuation of extraordinary wave depends on angle between wave vector and optic axis θ
- separate equation into real and imaginary parts using $n \gg k$

$$\frac{1}{n_2^2} = \frac{\cos^2\theta}{n_o^2} + \frac{\sin^2\theta}{n_e^2}, \quad \frac{k}{n_2^3} = \frac{k_o\cos^2\theta}{n_o^3} + \frac{k_e\sin^2\theta}{n_e^3}$$

Extraordinary Wave in Uniaxial Dichroic Materials

- separate equations for real (*n*) and imaginary (*k*) parts

$$
\begin{aligned}
\frac{1}{n_2^2} &= \frac{\cos^2\theta}{n_o^2} + \frac{\sin^2\theta}{n_e^2} \\
\frac{k}{n_2^3} &= \frac{k_o\cos^2\theta}{n_o^3} + \frac{k_e\sin^2\theta}{n_e^3}
\end{aligned}
$$

- real part of index obeys same law as for non-absorbing material
- extinction coefficient of extraordinary wave shows similar dependence on angle between wave vector and optic axis
- will understand behavior of sheet polarizers at non-normal incidence with these equations

Stress Birefringence

- mechanical stress induces anisotropy in isotropic material
- stress (force per area) can be external (mechanical mounting, inhomogeneous temperature) or intrinsic (due to manufacturing)
- glass has no fixed melting temperature ⇒ stress can be frozen in
- 100 to 1000 hours of *annealing* reduces stress
- refractive indices for polarization parallel and perpendicular to stress

$$n_{||} = n_0 + K_{||}\sigma, \quad n_\perp = n_0 + K_\perp\sigma$$

  - n_0: index of refraction of unstressed matieral
  - σ: stress
  - $K_{||,\perp}$: *stress optical coefficients*
- for polarimetry: interested in birefringence $n_{||} - n_\perp = (K_{||} - K_\perp)\sigma = K\sigma$

Stress Optical Coefficients

typical values for K for different materials

material	stress optical coefficien	thermal expansion coefficient
N-BK7	$2.77 \cdot 10^{-6}$ mm^2/N	$8.30{\cdot}10^{-6}$/K
fused silica	$3.50 \cdot 10^{-6}$ mm^2/N	$0.55{\cdot}10^{-6}$/K
SF57	$0.02 \cdot 10^{-6}$ mm^2/N	$9.20{\cdot}10^{-6}$/K
N-SF57	$2.78 \cdot 10^{-6}$ mm^2/N	$9.88{\cdot}10^{-6}$/K

- K only slightly dependent on wavelength in visible
- intrinsic birefringence
  - $<$2 nm/cm for the very best glass
  - $<$1 nm/cm for fused silica
  - 5 nm/cm is typical for precision optics

Electro-Optics

Introduction

- external fields ⇒ isotropic media can become anisotropic
- *electro-optics*: effects of external electric field applied to isotropic media and crystals
- for isotropic crystals (same n for all 3 principal axes)

$$\delta n = n_e - n_o \approx \frac{n^3}{2}\left(rE + RE^2\right)$$

- r: Pockels coefficient
- R: Kerr coefficient
- quadratic (Kerr) effect occurs in all crystals, isotropic media
- linear (Pockels) effect only in some crystals

Modification of Dielectric Tensor

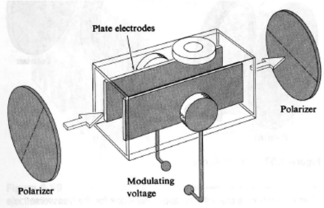
- electric field ⇒ change in elements of the inverse dielectric tensor $\left(\frac{1}{\epsilon}\right)_{ij}$
- linear and quadratic electro-optical effects described by tensors

$$\delta\left(\frac{1}{\epsilon}\right)_{ij} = \delta\left(\frac{1}{n^2}\right)_{ij} = r_{ijk}E_k + R_{ijkl}E_kE_l \; .$$

- many elements of r_{ijk},R_{ijkl} are zero in crystallographic coordinate system because of symmetries of crystal, non-zero elements are tabulated
- determine new principal axis system in which dielectric tensor is diagonal again
- in piezoelectric crystals (quartz, KDP) and ferroelectric crystals ($LiNbO_3$), linear effect much larger than quadratic effect
- electro-optical coefficients depend strongly on frequency because of mechanical resonances

Kerr effect

- discovered 1875 by John Kerr in a piece of glass
- also seen in liquids, gases
- isotropic medium becomes uniaxially anisotropic with optic axis parallel to electric field ⇒ field perpendicular to line of sight.
- Kerr cells not used much anymore
    - quadratic dependence on electric field strength ⇒ very high voltages
    - some of media are toxic or explosive
- birefringence $\Delta n = B\lambda E^2$ with wavelength λ, B Kerr constant



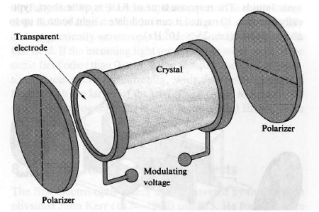
Kerr Constant

Kerr constant, index of refraction for liquids at 550 nm and 22°C

liquid	n	B in 10^{-16} V/m^2
nitrobenzene ($C_6H_5NO_2$)	1.55	44400
acetone (CH_3COCH_3)	1.36	2000
carbon disulfide (CS_2)	1.63	400
chloroform ($CHCl_3$)	1.45	-333

- fast light switches, of the order of a few ps
- effect about one order of magnitude smaller in glasses, often also induced elasto-optical effect due to field-induced deformation.
- effect about 3 orders of magnitude smaller in gasses

Pockels effect



- in KDP crystallographic and principal coordinate systems coincide
- electric field along beam, waves polarized at ±45° propagate with different speeds
- phase retardation ϕ (units of waves) and voltage V

$$\phi = \frac{n_o^3 r_{63} V}{\lambda}$$

- retardation is independent of distance between electrodes

Pockels Cell Properties

- to minimize field-of-view limitations, distance should be as small as possible
- to minimize sparking, distance should be as large as possible
- depending on needs of application, optimize distance

Crystal Aberrations

Crystal Astigmatism: A converging beam passing a uniaxial, plane-parallel plate is subject to crystal astigmatism. The two focal points along the center ray are separated by

$$\Delta x = \frac{D n_o \sin^2 \Omega \left(n_e^2 - n_o^2\right)}{n_e \left(n_o^2 \sin^2 \Omega + n_e^2 \cos^2 \Omega\right)^{\frac{3}{2}}} .$$

Δx	Longitudinal astigmatism in units of the plate thickness
D	Plate thickness
Ω	Angle between optic axis and interface normal
n_o	Ordinary index of refraction
n_e	Extraordinary index of refraction