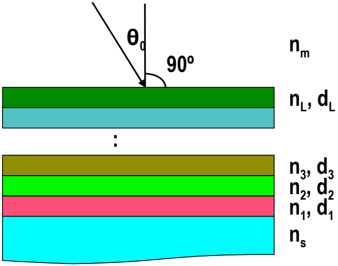
Outline

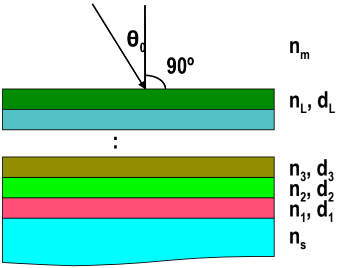
Introduction

- *thin film*:
  - layer with thickness $\lesssim \lambda$
  - extends in 2 other dimensions $\gg \lambda$
- 2 boundary layers to neighbouring media: reflection, refraction at both interfaces
- layer thickness $d_i \lesssim \lambda \Rightarrow$ interference between reflected and refracted waves
- L layers of thin films: *thin film stack*
- *substrate* (index n_s) and incident medium (index n_m) have infinite thickness



Plane Waves and Thin-Film Stacks

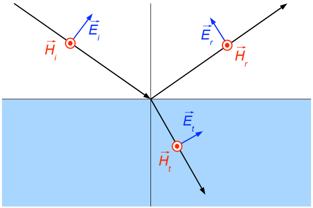
- plane wave $\vec{E} = \vec{E}_0 e^{i\left(\vec{k}\cdot\vec{x}-\omega t\right)}$
- layers numbered from 1 to L with complex index of refraction $\tilde{n}_j$, geometrical thickness d_j
- substrate has refractive index $\tilde{n}_s$
- incident medium has index $\tilde{n}_m$
- angle of incidence in incident medium: θ_0
- Snell's law:



$$\tilde{n}_m \sin\theta_0 = \tilde{n}_L \sin\theta_L = ... = \tilde{n}_1 \sin\theta_1 = \tilde{n}_s \sin\theta_s$$

- $\Rightarrow \theta_j$ for every layer j

p/TM Wave: Electric Field at Interface

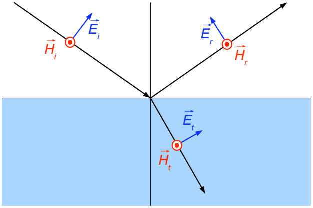


- continuous electric field $\parallel$ interface using $E_{i,r,t}$ instead of $E_0^{i,r,t}$:

$$E_i \cos\theta_0 - E_r \cos\theta_0 = E_t \cos\theta_1$$

- direction of electric field vector fully determined by angle of incidence
- sufficient to look at complex scalar quantities instead of full 3-D vector since electric field is perpendicular to wave vector and in plane of incidence

p/TM Wave: Magnetic Field at Interface



- from before: $\vec{H}_0 = \tilde{n}\frac{\vec{k}}{|\vec{k}|} \times \vec{E}_0$
- (complex) magnitudes also related by $\vec{E}_0$ and $\vec{H}_0$

$$H_{i,r} = \tilde{n}_m E_{i,r}, \quad H_t = \tilde{n}_1 E_t$$

- parallel component of magnetic field continuous

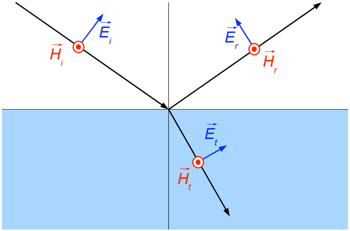
$$\tilde{n}_m E_i + \tilde{n}_m E_r = \tilde{n}_1 E_t$$

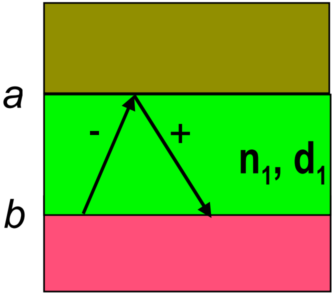
Matrix Formalism: Tangential Components in one Medium

- single interface in thin-film stack, combine all waves into
  - wave that travels towards substrate ($+$ superscript)
  - wave that travels away from substrate ($-$ superscript)
- at interface *a*, *tangential components* of complex electric and magnetic field amplitudes in medium 1 given by

$$\begin{aligned} E_a &= E_{1a}^{+} - E_{1a}^{-} \\ H_a &= \frac{\tilde{n}_1}{\cos\theta_1}\left(E_{1a}^{+} + E_{1a}^{-}\right) \end{aligned}$$

- negative sign for outwards traveling electric field component



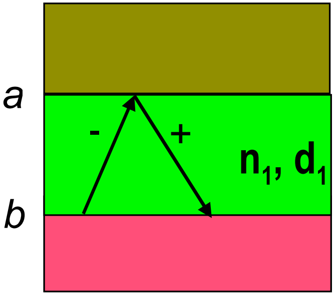


Matrix Formalism: Electric Field Propagation

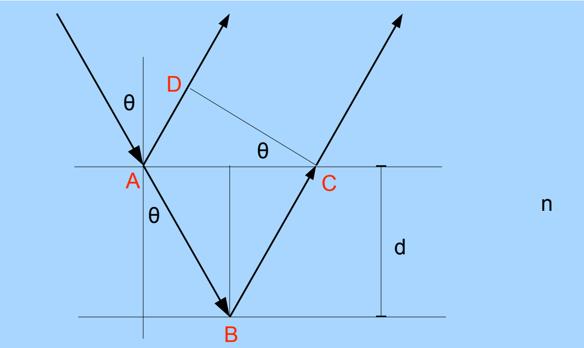
- field amplitudes in medium 1 at (other) interface *b* from wave propagation with common phase factor
- d_1: geometrical thickness of layer
- phase factor for forward propagating wave:

$$\delta = \frac{2\pi}{\lambda} \tilde{n}_1 d_1 \cos\theta_1$$

- backwards propagating wave: same phase factor with negative sign



Plane Wave Path Length for Oblique Incidence



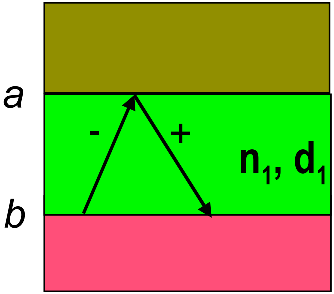
- consider theoretical reflections in single medium
- need to correct for plane wave propagation
- path length for “reflected light”: $\bar{AB} + \bar{BC} - \bar{AD}$

$$\frac{2d}{\cos\theta} - 2d\tan\theta\cdot\sin\theta = 2d\frac{1-\sin^2\theta}{\cos\theta} = 2d\cos\theta$$

Matrix Formalism

- at interface *b* in medium 1

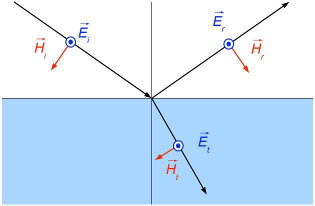
$$\begin{aligned} E_{1b}^{+} &= E_{1a}^{+}(\cos\delta + i\sin\delta) \\ E_{1b}^{-} &= E_{1a}^{-}(\cos\delta - i\sin\delta) \\ H_{1b}^{+} &= H_{1a}^{+}(\cos\delta + i\sin\delta) \\ H_{1b}^{-} &= H_{1a}^{-}(\cos\delta - i\sin\delta) \end{aligned}$$



- from before $E_{a,b} = E_{1a,b}^{+} - E_{1a,b}^{-}$, $H_{a,b} = \frac{\tilde{n}_1}{\cos\theta_1}\left(E_{1a,b}^{+} + E_{1a,b}^{-}\right)$
- propagation of tangential components from *a* to *b*

$$\begin{pmatrix} E_b \\ H_b \end{pmatrix} = \begin{pmatrix} \cos\delta & \frac{\cos\theta_1}{\tilde{n}_1} i\sin\delta \\ i\frac{\tilde{n}_1}{\cos\theta_1}\sin\delta & \cos\delta \end{pmatrix} \begin{pmatrix} E_a \\ H_a \end{pmatrix}$$

s/TE waves: Electric and Magnetic Fields at Interface



- parallel electric field component continuous: $E_i + E_r = E_t$
- parallel magnetic field component continuous

$$H_i \cos\theta_0 - H_r \cos\theta_0 = H_t \cos\theta_1$$

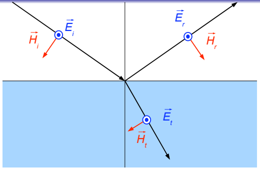
- and using relation between H and E

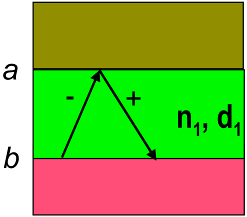
$$\tilde{n}_m \cos\theta_0 E_i - \tilde{n}_m \cos\theta_0 E_r = \tilde{n}_1 \cos\theta_1 E_t$$

s-Polarized Waves in one Medium

- at boundary a:

$$\begin{aligned} E_a &= E_{1a}^{+} + E_{1a}^{-} \\ H_a &= (E_{1a}^{+} - E_{1a}^{-})\tilde{n}_1 \cos\theta_1 \end{aligned}$$





- propagation of tangential components from a to b

$$\begin{pmatrix} E_b \\ H_b \end{pmatrix} = \begin{pmatrix} \cos\delta & \frac{1}{\tilde{n}_1\cos\theta_1} i \sin\delta \\ i\tilde{n}_1 \cos\theta_1 \sin\delta & \cos\delta \end{pmatrix} \begin{pmatrix} E_a \\ H_a \end{pmatrix}$$

Summary of Matrix Method

for each layer *j* calculate:

- θ_j using Snell's law: $n_m \sin\theta_0 = n_j \sin\theta_j$
- s-polarization: $\eta = n \cos\theta_j$
- p-polarization: $\eta = \frac{n}{\cos\theta_j}$
- phase delays: $\delta_j = \frac{2\pi}{\lambda} n_j d_j \cos\theta_j$
- characteristic matrix:

$$\mathrm{M}_j = \begin{pmatrix} \cos\delta_j & \frac{i}{\eta_j}\sin\delta_j \\ i\eta_j \sin\delta_j & \cos\delta_j \end{pmatrix}$$

Summary of Matrix Method (continued)

- total characteristic matrix M is product of all characteristic matrices

$$\mathrm{M} = \mathrm{M}_L \mathrm{M}_{L-1} ... \mathrm{M}_2 \mathrm{M}_1$$

- fields in incident medium given by

$$\left(\begin{array}{c} E_m \\ H_m \end{array} \right) = \mathrm{M} \left(\begin{array}{c} 1 \\ \eta_s \end{array} \right)$$

- complex reflection and transmission coefficients

$$r = \frac{\eta_m E_m - H_m}{\eta_m E_m + H_m}, \quad t = \frac{2\eta_m}{\eta_m E_m + H_m}$$

Jones Matrices for Thin Film Stacks

- in a coordinate system where p-polarized light has $\vec{E} \parallel x$-axis and s-polarized light has $\vec{E} \parallel y$-axis
- reflection

$$\mathrm{J}_r = \begin{pmatrix} r_p & 0 \\ 0 & r_s \end{pmatrix}$$

- transmission

$$\mathrm{J}_r = \begin{pmatrix} t_p & 0 \\ 0 & t_s \end{pmatrix}$$

- other orientations through rotation of Jones matrices

Mueller Matrices for Thin Film Stacks

- reflection

$$\mathrm{M}_r = \frac{1}{2}\begin{pmatrix} |r_p|^2+|r_s|^2 & |r_p|^2-|r_s|^2 & 0 & 0 \\ |r_p|^2-|r_s|^2 & |r_p|^2+|r_s|^2 & 0 & 0 \\ 0 & 0 & 2\mathrm{Re}\left(r_p^* r_s\right) & 2\mathrm{Im}\left(r_p^* r_s\right) \\ 0 & 0 & -2\mathrm{Im}\left(r_p^* r_s\right) & 2\mathrm{Re}\left(r_p^* r_s\right) \end{pmatrix}$$

- transmission

$$\mathrm{M}_t = \frac{\eta_s}{2\eta_m}\begin{pmatrix} |t_p|^2+|t_s|^2 & |t_p|^2-|t_s|^2 & 0 & 0 \\ |t_p|^2-|t_s|^2 & |t_p|^2+|t_s|^2 & 0 & 0 \\ 0 & 0 & 2\mathrm{Re}\left(t_p^* t_s\right) & 2\mathrm{Im}\left(t_p^* t_s\right) \\ 0 & 0 & -2\mathrm{Im}\left(t_p^* t_s\right) & 2\mathrm{Re}\left(t_p^* t_s\right) \end{pmatrix}$$

- M_t includes correction for projection

Mueller Matrix for Dielectric

- reflection

$$M_r = \frac{1}{2}\left(\frac{\tan\alpha_-}{\sin\alpha_+}\right)^2 \begin{pmatrix} c_-^2 + c_+^2 & c_-^2 - c_+^2 & 0 & 0 \\ c_-^2 - c_+^2 & c_-^2 + c_+^2 & 0 & 0 \\ 0 & 0 & -2c_+c_- & 0 \\ 0 & 0 & 0 & -2c_+c_- \end{pmatrix}$$

- transmission

$$M_t = \frac{1}{2}\frac{\sin 2i \sin 2r}{(\sin\alpha_+ \cos\alpha_-)^2} \begin{pmatrix} c_-^2 + 1 & c_-^2 - 1 & 0 & 0 \\ c_-^2 - 1 & c_-^2 + 1 & 0 & 0 \\ 0 & 0 & 2c_- & 0 \\ 0 & 0 & 0 & 2c_- \end{pmatrix}$$

- $\alpha_\pm = \theta_i \pm \theta_t$, $c_\pm = \cos\alpha_\pm$, and Snell: $\sin\theta_i = n\sin\theta_t$

Mueller Matrix for Reflection off Metal

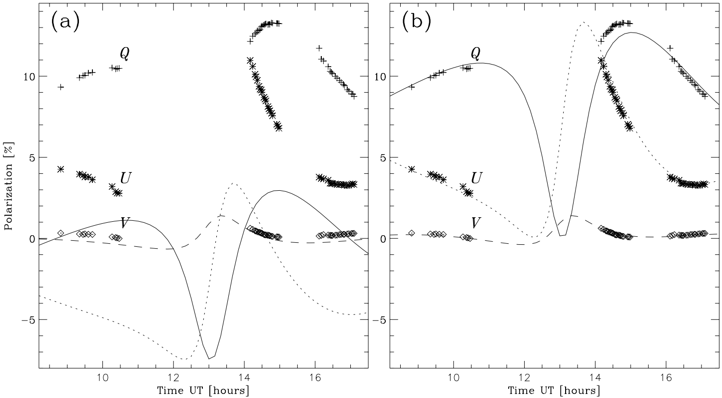
$$
\mathrm{M}_r = \frac{1}{2}\begin{pmatrix} \rho_s^2+\rho_p^2 & \rho_s^2-\rho_p^2 & 0 & 0 \\ \rho_s^2-\rho_p^2 & \rho_s^2+\rho_p^2 & 0 & 0 \\ 0 & 0 & 2\rho_s\rho_p\cos\delta & 2\rho_s\rho_p\sin\delta \\ 0 & 0 & -2\rho_s\rho_p\sin\delta & 2\rho_s\rho_p\cos\delta \end{pmatrix}
$$

- real quantities ρ_s, ρ_p, $\delta = \phi_s - \phi_p$ defined by

$$
\rho_s e^{i\phi_s} = -\frac{\sin(i-r)}{\sin(i+r)} = \frac{\cos i - \tilde{n}\cos r}{\cos i + \tilde{n}\cos r}
$$

$$
\rho_p e^{i\phi_p} = \frac{\tan(i-r)}{\tan(i+r)} = \left(\frac{\tilde{n}\cos r - \cos i}{\cos i + \tilde{n}\cos r}\right)\left(\frac{\tilde{n}\cos r\cos i - \sin^2 i}{\tilde{n}\cos r\cos i + \sin^2 i}\right)
$$

Instrumental Polarization of Swedish Vacuum Solar Telescope



Wavelength: 525 nm; a) best-fit theoretical model of telescope polarization; b) same model plus empirical offsets (Courtesy Pietro Bernasconi)

Instrumental Polarization

- oblique reflections off metal surfaces introduce polarization and retardation
- oblique reflections off and transmission through dielectrics introduce polarization, but no retardation
- accurate modeling not easy because of multi-layer coatings, oxide layers, and sometimes oil layers
- best to measure instrumental effects of optics

Example: Coated Metal Mirror

- mirror with $n = 1.2$, $k = 7.5$ (aluminum at 630 nm) at 45°

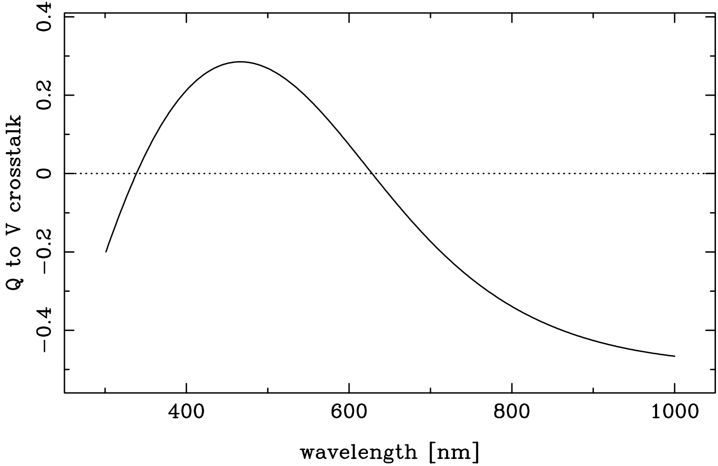
$$\begin{pmatrix} 1.000 & 0.028 & 0.000 & 0.000 \\ 0.028 & 1.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.983 & 0.180 \\ 0.000 & 0.000 & -0.180 & 0.983 \end{pmatrix}$$

- 126-nm thick dielectric layer with $n = 1.4$ on top

$$\begin{pmatrix} 1.000 & -0.009 & 0.000 & 0.000 \\ -0.009 & 1.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 1.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 1.000 \end{pmatrix}$$

- $Q \Rightarrow V$ disappears, $I \Rightarrow Q$ reduced
- special mirrors minimize cross-talk (e.g. GONG turret)

Wavelength-Dependence of $Q \Rightarrow V$ Cross-Talk



only limited wavelength range can be corrected

Oil Layers, Real Mirrors

- 65-nm thick dielectric layer with $n = 1.4$ on top

$$\begin{pmatrix} 1.000 & 0.014 & 0.000 & 0.000 \\ 0.014 & 1.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.858 & 0.514 \\ 0.000 & 0.000 & -0.514 & 0.858 \end{pmatrix}$$

- maximizes crosstalk between Q and V
- thin oil film on aluminum mirror can make a big change in telescope Mueller matrix
- aluminum is covered with thin layer of aluminum oxide that needs to be taken into account