Outline

Electromagnetic Waves in Matter

- *Maxwell's equations* $\Rightarrow$ electromagnetic waves
- optics: interaction of electromagnetic waves with matter as described by *material equations*
- polarization of electromagnetic waves are integral part of optics

Maxwell's Equations in Matter

$$\nabla \cdot \vec{D} = 4\pi\rho$$

$$\nabla \times \vec{H} - \frac{1}{c}\frac{\partial \vec{D}}{\partial t} = \frac{4\pi}{c}\vec{j}$$

$$\nabla \times \vec{E} + \frac{1}{c}\frac{\partial \vec{B}}{\partial t} = 0$$

$$\nabla \cdot \vec{B} = 0$$

Symbols

$\vec{D}$	*electric displacement*
ρ	*electric charge density*
$\vec{H}$	*magnetic field*
c	*speed of light in vacuum*
$\vec{j}$	*electric current density*
$\vec{E}$	*electric field*
$\vec{B}$	*magnetic induction*
t	time

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Fundamentals of Polarized Light

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Linear Material Equations

$$\vec{D} = \epsilon \vec{E}$$
$$\vec{B} = \mu \vec{H}$$
$$\vec{j} = \sigma \vec{E}$$

Symbols

- ϵ *dielectric constant*
- μ *magnetic permeability*
- σ *electrical conductivity*

Isotropic and Anisotropic Media

- isotropic media: ϵ and μ are scalars
- anisotropic media: ϵ and μ are tensors of rank 2
- isotropy of medium broken by
  - anisotropy of material itself (e.g. crystals)
  - external fields (e.g. Kerr effect)

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Wave Equation in Matter

- for most materials: $\rho = 0$, $\mu = 1$
- combine Maxwell, material equations ⇒ differential equations for damped (vector) wave

$$\nabla^2 \vec{E} - \frac{\epsilon}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} - \frac{4\pi\sigma}{c^2}\frac{\partial \vec{E}}{\partial t} = 0$$

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- damping controlled by conductivity σ
- $\vec{E}$ and $\vec{H}$ are equivalent ⇒ sufficient to consider $\vec{E}$
- interaction with matter almost always through $\vec{E}$
- but: at interfaces, boundary conditions for $\vec{H}$ are crucial

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Plane-Wave Solutions

- Plane Vector Wave ansatz $\vec{E} = \vec{E}_0 e^{i\left(\vec{k}\cdot\vec{x}-\omega t\right)}$

$\vec{k}$	spatially and temporally constant *wave vector*
$\vec{k}$	normal to surfaces of constant phase
$\lvert\vec{k}\rvert$	*wave number*
$\vec{x}$	spatial location
ω	*angular frequency* ($2\pi\times$ frequency)
t	time
$\vec{E}_0$	(generally complex) vector independent of time and space

- could also use $\vec{E} = \vec{E}_0 e^{-i\left(\vec{k}\cdot\vec{x}-\omega t\right)}$
- damping if $\vec{k}$ is complex
- real electric field vector given by real part of $\vec{E}$

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Complex Index of Refraction

- temporal derivatives $\Rightarrow$ Helmholtz equation

$$\nabla^2 \vec{E} + \frac{\omega^2}{c^2}\left(\epsilon + i\frac{4\pi\sigma}{\omega}\right)\vec{E} = 0$$

- *dispersion relation* between $\vec{k}$ and ω

$$\vec{k}\cdot\vec{k} = \frac{\omega^2}{c^2}\left(\epsilon + i\frac{4\pi\sigma}{\omega}\right)$$

- *complex index of refraction*

$$\tilde{n}^2 = \epsilon + i\frac{4\pi\sigma}{\omega}, \quad \vec{k}\cdot\vec{k} = \frac{\omega^2}{c^2}\tilde{n}^2$$

- split into real (n: *index of refraction*) and imaginary parts (k: *extinction coefficient*)

$$\tilde{n} = n + ik$$

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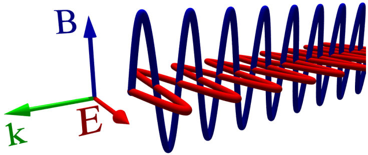

- plane-wave solution must also fulfill Maxwell's equations

$$\vec{E}_0 \cdot \vec{k} = 0, \quad \vec{H}_0 \cdot \vec{k} = 0, \quad \vec{H}_0 = \tilde{n} \frac{\vec{k}}{|\vec{k}|} \times \vec{E}_0$$

- isotropic media: electric, magnetic field vectors normal to wave vector $\Rightarrow$ transverse waves
- $\vec{E}_0$, $\vec{H}_0$, and $\vec{k}$ orthogonal to each other, right-handed vector-triple
- conductive medium $\Rightarrow$ complex $\tilde{n}$, $\vec{E}_0$ and $\vec{H}_0$ out of phase
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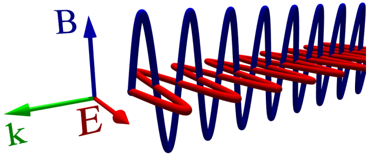

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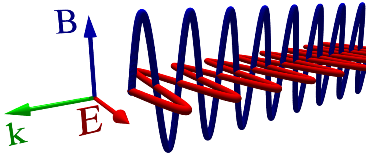

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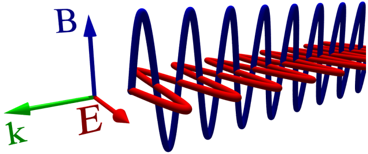

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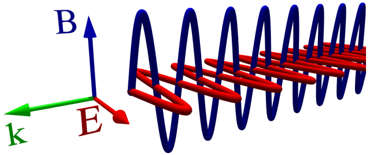

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Energy Propagation in Isotropic Media

- *Poynting vector*

$$\vec{S} = \frac{c}{4\pi}\left(\vec{E} \times \vec{H}\right)$$

- $|\vec{S}|$: energy through unit area perpendicular to $\vec{S}$ per unit time
- direction of $\vec{S}$ is direction of energy flow
- time-averaged Poynting vector given by

$$\left\langle \vec{S} \right\rangle = \frac{c}{8\pi}\mathrm{Re}\left(\vec{E}_0 \times \vec{H}_0^*\right)$$

Re real part of complex expression
* complex conjugate
$\langle . \rangle$ time average

- energy flow parallel to wave vector (in isotropic media)

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Quasi-Monochromatic Light

- monochromatic light: purely theoretical concept
- monochromatic light wave always fully polarized
- real life: light includes range of wavelengths ⇒ *quasi-monochromatic light*
- quasi-monochromatic: superposition of mutually incoherent monochromatic light beams whose wavelengths vary in narrow range $\delta\lambda$ around central wavelength λ_0

$$\frac{\delta\lambda}{\lambda} \ll 1$$

- measurement of quasi-monochromatic light: integral over measurement time t_m
- amplitude, phase (slow) functions of time for given spatial location
- *slow*: variations occur on time scales much longer than the mean period of the wave

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Polarization of Quasi-Monochromatic Light

- electric field vector for quasi-monochromatic plane wave is sum of electric field vectors of all monochromatic beams

$$\vec{E}(t) = \vec{E}_0(t)\, e^{i\left(\vec{k}\cdot\vec{x}-\omega t\right)}$$

- can write this way because $\delta\lambda \ll \lambda_0$
- measured intensity of quasi-monochromatic beam

$$\left\langle \vec{E}_x \vec{E}_x^* \right\rangle + \left\langle \vec{E}_y \vec{E}_y^* \right\rangle = \lim_{t_m->\infty} \frac{1}{t_m} \int_{-t_m/2}^{t_m/2} \vec{E}_x(t)\vec{E}_x^*(t) + \vec{E}_y(t)\vec{E}_y^*(t) dt$$

$\langle\cdots\rangle$: averaging over measurement time t_m

- measured intensity independent of time
- quasi-monochromatic: frequency-dependent material properties (e.g. index of refraction) are constant within $\Delta\lambda$

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Polychromatic Light or White Light

- wavelength range comparable wavelength ($\frac{\delta\lambda}{\lambda} \sim 1$)
- incoherent sum of quasi-monochromatic beams that have large variations in wavelength
- cannot write electric field vector in a plane-wave form
- must take into account frequency-dependent material characteristics
- intensity of polychromatic light is given by sum of intensities of constituting quasi-monochromatic beams

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Polychromatic Light or White Light

- wavelength range comparable wavelength ($\frac{\delta\lambda}{\lambda} \sim 1$)
- incoherent sum of quasi-monochromatic beams that have large variations in wavelength
- cannot write electric field vector in a plane-wave form
- must take into account frequency-dependent material characteristics
- intensity of polychromatic light is given by sum of intensities of constituting quasi-monochromatic beams

Fields at Interfaces

- classical optics due to interfaces between 2 different media
- from Maxwell's equations in integral form at interface from medium 1 to medium 2

$$\left(\vec{D}_2 - \vec{D}_1\right) \cdot \vec{n} = 0$$
$$\left(\vec{B}_2 - \vec{B}_1\right) \cdot \vec{n} = 0$$
$$\left(\vec{E}_2 - \vec{E}_1\right) \times \vec{n} = 0$$
$$\left(\vec{H}_2 - \vec{H}_1\right) \times \vec{n} = 0$$

$\vec{n}$ normal on interface, points from medium 1 to medium 2

- normal components of $\vec{D}$ and $\vec{B}$ are continuous across interface
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Plane of Incidence

- plane wave onto interface
- incident (^{i}), reflected (^{r}), and transmitted (^{t}) waves

$$\vec{E}^{i,r,t} = \vec{E}_0^{i,r,t} e^{i\left(\vec{k}^{i,r,t}\cdot\vec{x}-\omega t\right)}$$

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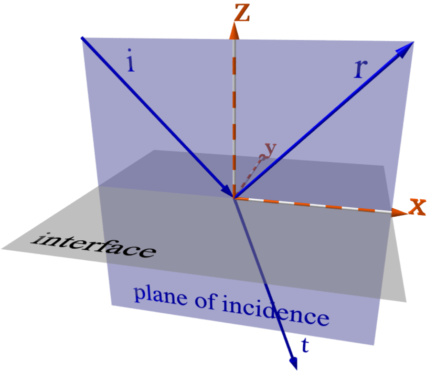
- interface normal $\vec{n} \parallel z$-axis



- spatial, temporal behavior at interface the same for all 3 waves

$$(\vec{k}^i \cdot \vec{x})_{z=0} = (\vec{k}^r \cdot \vec{x})_{z=0} = (\vec{k}^t \cdot \vec{x})_{z=0}$$

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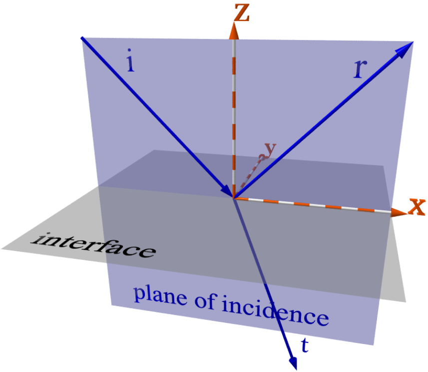

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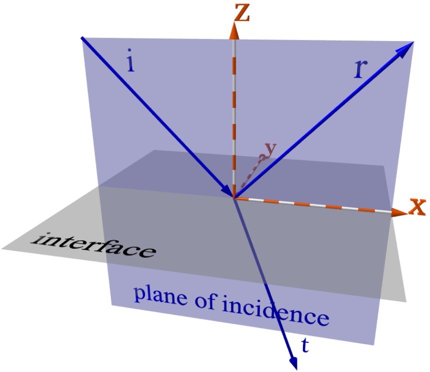

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- $\left|\vec{k}\right| = \frac{\omega}{c}\tilde{n}$
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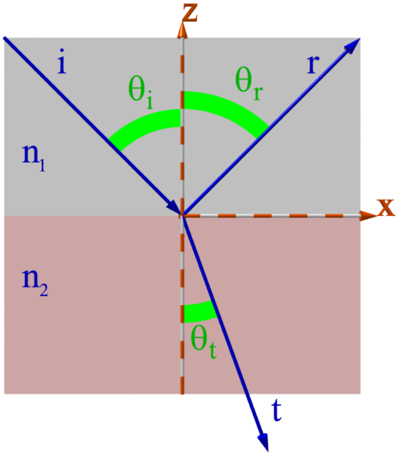

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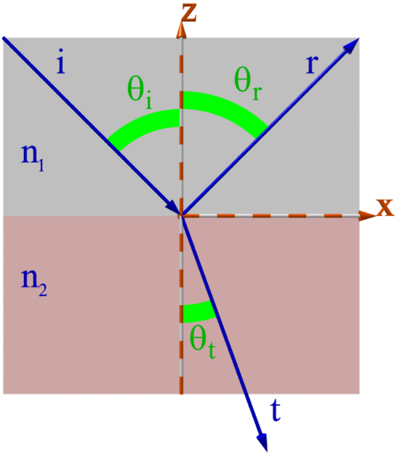

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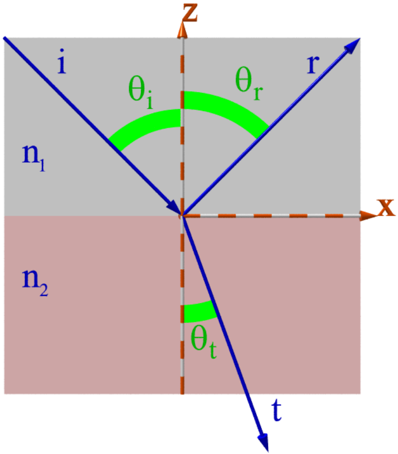

Monochromatic Wave at Interface

- $$\vec{H}_0^{i,r,t} = \frac{c}{\omega}\vec{k}^{i,r,t} \times \vec{E}_0^{i,r,t}, \quad \vec{B}_0^{i,r,t} = \frac{c}{\omega}\vec{k}^{i,r,t} \times \vec{E}_0^{i,r,t}$$
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- 4 equations are not independent
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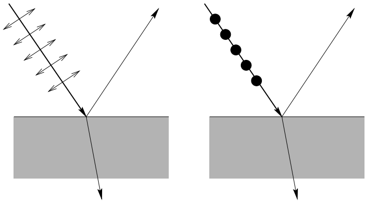
Two Special Cases



1. electric field **p**arallel to plane of incidence ⇒ magnetic field is transverse to plane of incidence (TM)
2. electric field particular (German: **s**enkrecht) or transverse to plane of incidence (TE)

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- choose direction of magnetic field vector such that Poynting vector parallel, same direction as corresponding wave vector

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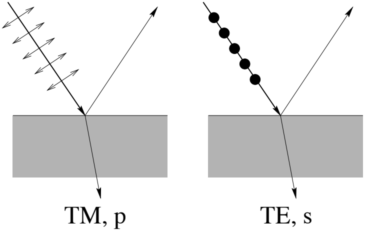

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