Lecture 7: Variable Retarders

Outline

- Liquid Crystal Retarders
- Piezo-Elastic Modulators (PEMs)
- Achromatic Variable Retarders
- Comparison of Variable Retarders

Variable Retarders

Introduction

- sensitive polarimeters require retarders whose properties (retardance, fast axis orientation) can be varied quickly
  (modulated)
- retardance changes (change of birefringence):
  - liquid crystals
  - Faraday, Kerr, Pockels cells
  - piezo-elastic modulators (PEM)
- fast axis orientation changes (change of c-axis direction):
  - rotating fixed retarder
  - ferro-electric liquid crystals (FLC)

Liquid Crystal Retarders

Liquid Crystals

- liquid crystals are fluids whose molecules are elongated
- at high temperatures, liquid crystal is isotropic
- at lower temperature, molecules become ordered in orientation and sometimes also space in one or more dimensions
- liquid crystals can line up parallel or perpendicular to external electrical field

- dielectric constant anisotropy often large \( \Rightarrow \) very responsive to changes in applied electric field
- birefringence \( \delta n \) can be very large (larger than typical crystal birefringence)
- liquid crystal layer only a few \( \mu m \) thick \( \Rightarrow \) true zero-order retarder
- anisotropy, and therefore birefringence, shows strong temperature dependence
Nematic Liquid Crystal Variable Retarders

- In nematic phase, molecules are randomly positioned but aligned (more or less) in one direction
- With zero voltage applied externally, liquid crystal molecules are parallel to substrates ⇒ maximum retardation
- With electrical field, liquid crystal molecules tip perpendicular to substrate ⇒ reduced effective birefringence/retardance

Voltage Dependence of Birefringence

- Alignment layer between substrate and liquid crystal prevents molecules at surface to rotate freely
- Residual retardance of about 30 nm even at high voltages (about 20 V)
- Retardance changes by about −0.4% per °C
- Response time of nematic liquid crystal retarders is proportional to square of layer thickness (=total retardance) and of the order of 20 ms

Ferro-Electric Liquid Crystals

- Smectic liquid crystal phases characterized by well-defined layers that can slide over one another
- Molecules are positionally ordered along one direction
- In smectic C phase, molecules are tilted away from layer normal
- Ferroelectric liquid crystals (FLCs) are tilted phases of chiral molecules (smectic C⁺), which have permanent polarization

Ferro-Electric Liquid Crystal Retarders

- Respond much more quickly to externally applied fields than nematic liquid crystals
- Can be used to make fast, bistable electro-optic devices
- FLCs act like retarders with fixed retardation where fast axis direction can be switched by about 45° (switching angle) by alternating sign of applied electrical field
- Achromatic modulators with FLCs in Pancharatnam configuration
- Switching times on the order of 150 µs
- Switching angle is temperature sensitive
- Retardance rather insensitive to temperature variations
**Liquid Crystal Advantages and Disadvantages**

**Advantages:**
- arbitrary (optimized) modulation schemes
- large, uniform apertures available
- retardation or fast axis changes possible
- FLC allow fast modulation (<10 kHz)
- require only low voltages at moderate driving powers (~1 W)

**Disadvantages:**
- degrades quickly under UV irradiation
- requires temperature control
- nematic have slow modulation frequency (<50 Hz)
- FLCs cannot change retardation

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**Piezo-Elastic Modulators (PEMs)**

- stress-induced birefringence, also sometimes called piezo-optical or photo-elastic effect
- block of a few cm in side length of common BK7 glass can be stressed enough by hand such as to introduce a quarter-wave retardation
- stress-induced birefringence is proportional to stress $\sigma$
- retardation (in waves)
  \[ \delta = \frac{1}{\lambda} K d \sigma \]

- $K$ stress optical constant
- $d$ thickness of variable retarder
- $\lambda$ wavelength
- construct variable retarder by compressing optical glass
- requires considerable mechanical power to modulate

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**Mechanical Resonance**

- mechanically resonant oscillation reduces power requirement to one over the mechanical $Q$ ($10^3-10^4$ for most glasses)
- slab of length $L$ excited at fundamental mode $\Rightarrow$ standing acoustic wave with wavelength $2L$, frequency $\omega$
  \[ \omega = \frac{c_s}{2L} \]
- $c_s$: sound speed in optical material
- 57-mm-long fused silica slab $\Rightarrow$ resonance frequency is 50 kHz.
- resulting stress, retardance as function of position $x$ and time $t$
- stress-induced birefringence $\delta(x, t)$ given by
  \[ \delta(x, t) = A \sin \omega t \sin \left( \frac{\pi x}{L} \right) \]
- $A$ amplitude of oscillation
- $x$ from 0 to $L$

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**PEM Driver**

- to make slab oscillate, quartz crystal with electrodes on its surfaces is forced to oscillate by externally applied electrical field via piezo effect
- quartz slab mechanically coupled to modulator slab
- electrical field driven at mechanical resonance frequency
- oscillation amplitude $A$ regulated with electronic feedback circuit
PEM Power

- Oscillation dampened by friction losses within modulator material
- Energy loss inversely proportional to mechanical $Q$
- $Q$ very large $\Rightarrow$ little energy loss in modulator small (0.1 to 1 W)
- Material with high $Q$ (fused silica, $Q \approx 10^4$) desirable
- Typical glass has $Q \sim 10^3$
- Required drive power does not depend on length of slab

PEM Birefringence

- Stress-induced birefringence
  \[ \delta(x, t) = A \sin \omega t \sin \left( \frac{\pi x}{L} \right) \]
- Combine $\sin \left( \frac{\pi x}{L} \right)$, amplitude $A$ into spatially varying amplitude $A(x)$
- Birefringence becomes $\delta(x, t) = A(x) \sin(\omega t)$
- A small $\Rightarrow$ PEM is true zero-order retarder
- PEM Mueller matrix corresponds to retarder with time-dependent retardation
- Retarder Mueller matrix contains elements with $\sin \delta(x, t)$ and $\cos \delta(x, t)$
- Expand $\sin(\sin(x))$ and $\cos(\sin(x))$ in terms of Bessel functions

Bessel Functions

- Mueller matrix elements become
  \[ \sin \delta(x, t) = 2J_1(A(x)) \sin \omega t + \cdots, \]
  \[ \cos \delta(x, t) = J_0(A(x)) + 2J_2(A(x)) \cos 2\omega t + \cdots, \]
- $J_{0,1,2}$: Bessel functions of order 0,1 and 2

PEM Advantages and Disadvantages

Advantages:
- PE Ms are stable in operation
- Show no degrading at high intensity levels and/or UV irradiation
- Have good optical properties
- Large spatial and angular aperture
- Require only low voltages at moderate driving powers (\(< 1\) W)

Disadvantages:
- Sinusoidal modulation (as compared to more efficient square-wave modulation possible with liquid crystals)
- Very high modulation frequency (20 to 50 kHz), which requires specialized array detectors (ZIMPOL)
Achromatic Variable Retarders

Some Thoughts

- Achromatic variable retarders would provide major advantages.
- Achromatic retarders using two different materials very difficult because wavelength-dependence of birefringence needs to be very different for the two materials.
- Bi-liquid-crystal achromatic retarders have been built.
- Pancharatnam approach looks more feasible.
- Variable birefringence retarders (nematic liquid crystals, PEMs) do not work because Pancharatnam approach works minimizes dependence on retardance of individual components.
- Three half-wave FLCs in Pancharatnam configuration provide excellent achromatic half-wave plate with switchable fast axis orientation.
- Can obtain achromatic performance without achromatic variable retarder.

Comparison of Variable Retarders

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<thead>
<tr>
<th>Modulator</th>
<th>Advantages</th>
<th>Disadvantages</th>
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<tbody>
<tr>
<td>rotating retarder</td>
<td>high stability, large wavelength range</td>
<td>relatively slow modulation, beam motion, needs 8 measurements for all Stokes parameters</td>
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<tr>
<td>liquid crystal</td>
<td>relatively fast modulation, narrow simultaneous wavelength range, only 4 measurements for all Stokes parameters, no moving parts</td>
<td>limited temporal stability, damaged by strong UV light</td>
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<tr>
<td>PEM</td>
<td>very fast modulation, high stability, no moving parts</td>
<td>narrow simultaneous wavelength range, needs special CCD camera, spatial retardance variation</td>
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