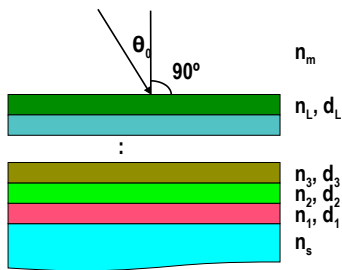


Outline

- 1 Thin Films
- 2 Calculating Thin Film Stack Properties
- 3 Polarization Properties of Thin Films
- 4 Anti-Reflection Coatings
- 5 Interference Filters

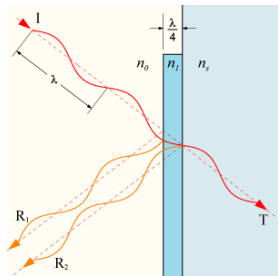
Introduction

- *thin film*:
 - layer with thickness $\lesssim \lambda$
 - extends in 2 other dimensions $\gg \lambda$
- 2 boundary layers to neighbouring media: reflection, refraction at both interfaces
- layer thickness $d_i \lesssim \lambda \Rightarrow$ interference between reflected and refracted waves
- L layers of thin films: *thin film stack*
- *substrate* (index n_s) and incident medium (index n_m) have infinite thickness



Thin Films and Polarization

- some polarizers (plate, cube) based on thin-film coatings
- can dramatically reduce polarizing effects of optical components
- aluminum mirrors have aluminum oxide thin film



Calculating Thin-Film Stack Properties

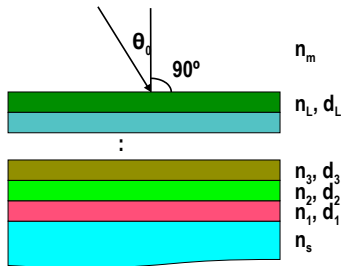
- many layers \Rightarrow consider all interferences between reflected and refracted rays of each interface
- complexity of calculation significantly reduced by matrix approach
- signs and conventions are not consistent in literature
- only isotropic materials here

Plane Waves and Thin-Film Stacks

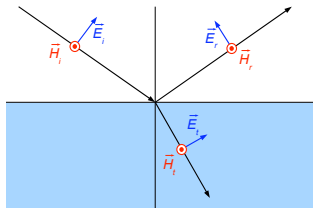
- plane wave $\vec{E} = \vec{E}_0 e^{i(\vec{k} \cdot \vec{x} - \omega t)}$
- layers numbered from 1 to L with complex index of refraction \tilde{n}_j , geometrical thickness d_j
- substrate has refractive index \tilde{n}_s
- incident medium has index \tilde{n}_m
- angle of incidence in incident medium: θ_0
- Snell's law:

$$\tilde{n}_m \sin \theta_0 = \tilde{n}_L \sin \theta_L = \dots = \tilde{n}_1 \sin \theta_1 = \tilde{n}_s \sin \theta_s$$

- $\Rightarrow \theta_j$ for every layer j



p/TM Wave: Electric Field at Interface



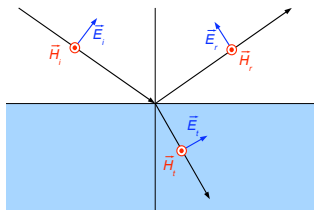
- $E_{i,r,t}$: components parallel to interface of complex amplitudes of \vec{E}_0 of incident, reflected, transmitted electric fields

$$E_i = E_i \cos \theta_0, \quad E_r = E_r \cos \theta_0, \quad E_t = E_t \cos \theta_1$$

- $E_{i,r,t}$: complex amplitudes of \vec{E}_0 of incident, reflected, transmitted electric fields
- continuous electric field \parallel interface: $E_i - E_r = E_t$

$$E_i \cos \theta_0 - E_r \cos \theta_0 = E_t \cos \theta_1$$

p/TM Wave: Electric Field at Interface (continued)

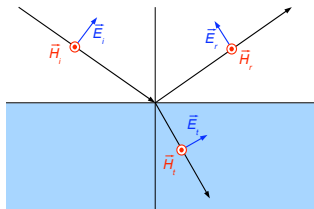


- continuous electric field \parallel interface

$$E_i \cos \theta_0 - E_r \cos \theta_0 = E_t \cos \theta_1$$

- direction of electric field vector fully determined by angle of incidence
- sufficient to look at complex scalar quantities instead of full 3-D vector since the electric field is perpendicular to the wave vector and in the plane of incidence

p/TM Wave: Magnetic Field at Interface



- non-magnetic material ($\mu = 1$): $\vec{H}_0 = \tilde{n} \frac{\vec{k}}{|\vec{k}|} \times \vec{E}_0$
- (complex) magnitudes related by of \vec{E}_0 and \vec{H}_0

$$H_{i,r} = \tilde{n}_m E_{i,r}, \quad H_t = \tilde{n}_1 E_t$$

- parallel component of magnetic field continuous

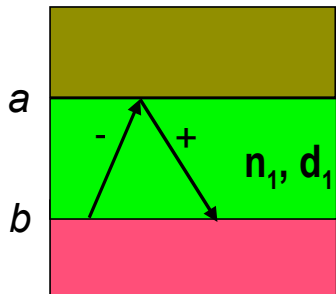
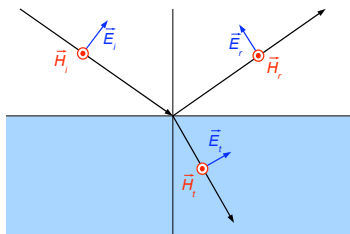
$$\tilde{n}_m E_i + \tilde{n}_m E_r = \tilde{n}_1 E_t$$

Matrix Formalism: Tangential Components in one Medium

- single interface in thin-film stack, combine all waves into
 - wave that travels towards substrate (+ superscript)
 - wave that travels away from substrate (– superscript)
- at interface a , *tangential components* of complex electric and magnetic field amplitudes in medium 1 given by

$$E_a = E_{1a}^+ - E_{1a}^-$$
$$H_a = \frac{\tilde{n}_1}{\cos \theta_1} (E_{1a}^+ + E_{1a}^-)$$

- negative sign for outwards traveling electric field component

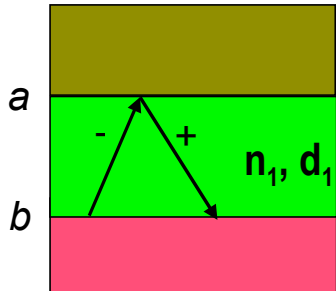


Matrix Formalism: Electric Field Propagation

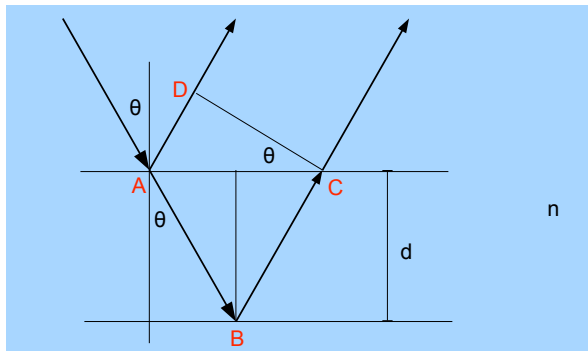
- field amplitudes in medium 1 at (other) interface b from wave propagation with common phase factor
- d_1 : geometrical thickness of layer
- phase factor for forward propagating wave:

$$\delta = \frac{2\pi}{\lambda} \tilde{n}_1 d_1 \cos \theta_1$$

- backwards propagating wave: same phase factor with negative sign



Plane Wave Path Length for Oblique Incidence



- consider theoretical reflections in single medium
- need to correct for plane wave propagation
- path length for “reflected light”: $\bar{AB} + \bar{BC} - \bar{AD}$

$$\frac{2d}{\cos \theta} - 2d \tan \theta \cdot \sin \theta = 2d \frac{1 - \sin^2 \theta}{\cos \theta} = 2d \cos \theta$$

Matrix Formalism

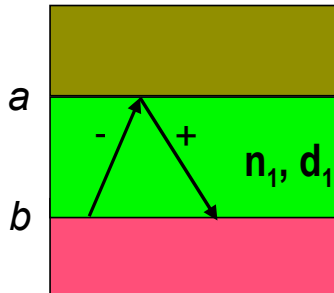
- at interface b in medium 1

$$E_{1b}^+ = E_{1a}^+(\cos \delta + i \sin \delta)$$

$$E_{1b}^- = E_{1a}^-(\cos \delta - i \sin \delta)$$

$$H_{1b}^+ = H_{1a}^+(\cos \delta + i \sin \delta)$$

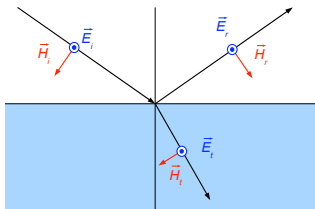
$$H_{1b}^- = H_{1a}^-(\cos \delta - i \sin \delta)$$



- from before $E_{a,b} = E_{1a,b}^+ - E_{1a,b}^-$, $H_{a,b} = \frac{\tilde{n}_1}{\cos \theta_1} (E_{1a,b}^+ + E_{1a,b}^-)$
- propagation of tangential components from a to b

$$\begin{pmatrix} E_b \\ H_b \end{pmatrix} = \begin{pmatrix} \cos \delta & \frac{\cos \theta_1}{\tilde{n}_1} i \sin \delta \\ i \frac{\tilde{n}_1}{\cos \theta_1} \sin \delta & \cos \delta \end{pmatrix} \begin{pmatrix} E_a \\ H_a \end{pmatrix}$$

s/TE waves: Electric and Magnetic Fields at Interface



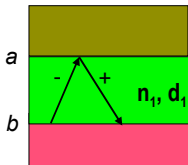
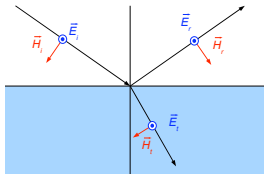
- parallel electric field component continuous: $E_i + E_r = E_t$
- parallel magnetic field component continuous

$$H_i \cos \theta_0 - H_r \cos \theta_0 = H_t \cos \theta_1$$

- and using relation between H and E

$$\tilde{n}_m \cos \theta_0 E_i - \tilde{n}_m \cos \theta_0 E_r = \tilde{n}_1 \cos \theta_1 E_t$$

s-Polarized Waves in one Medium



- at boundary a:

$$E_a = E_{1a}^+ + E_{1a}^-$$

$$H_a = (E_{1a}^+ - E_{1a}^-) \tilde{n}_1 \cos \theta_1$$

- propagation of tangential components from a to b

$$\begin{pmatrix} E_b \\ H_b \end{pmatrix} = \begin{pmatrix} \cos \delta & \frac{1}{\tilde{n}_1 \cos \theta_1} i \sin \delta \\ i \tilde{n}_1 \cos \theta_1 \sin \delta & \cos \delta \end{pmatrix} \begin{pmatrix} E_a \\ H_a \end{pmatrix}$$

Summary of Matrix Method

for each layer j calculate:

- θ_j using Snell's law: $n_m \sin \theta_0 = n_j \sin \theta_j$
- s-polarization: $\eta_j = n_j \cos \theta_j$
- p-polarization: $\eta_j = \frac{n_j}{\cos \theta_j}$
- phase delays: $\delta_j = \frac{2\pi}{\lambda} n_j d_j \cos \theta_j$
- characteristic matrix:

$$M_j = \begin{pmatrix} \cos \delta_j & \frac{i}{\eta_j} \sin \delta_j \\ i\eta_j \sin \delta_j & \cos \delta_j \end{pmatrix}$$

Summary of Matrix Method (continued)

- total characteristic matrix M is product of all characteristic matrices

$$M = M_L M_{L-1} \dots M_2 M_1$$

- fields in incident medium given by

$$\begin{pmatrix} E_m \\ H_m \end{pmatrix} = M \begin{pmatrix} 1 \\ \eta_s \end{pmatrix}$$

- complex reflection and transmission coefficients

$$r = \frac{\eta_m E_m - H_m}{\eta_m E_m + H_m}, \quad t = \frac{2\eta_m}{\eta_m E_m + H_m}$$

Materials and Refractive Indices

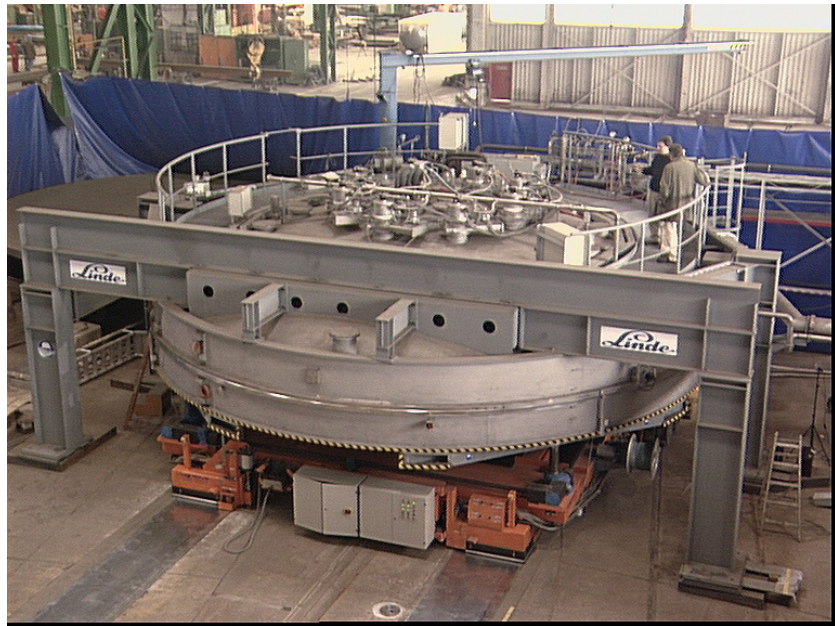
- dielectric materials:

Material	n(550 nm)	transparency range in nm
MgF ₂	1.38	210-10000
TiO ₂	2.2-2.7	350-12000
HfO ₂	2.0	220-12000
SiO	2.0	500-8000
SiO ₂	1.46	190-8000
ZrO ₂	2.1	340-12000

Al

- metals: Ag
Au

VLT Coating Chamber



VLT Coating Chamber with Magnetogron



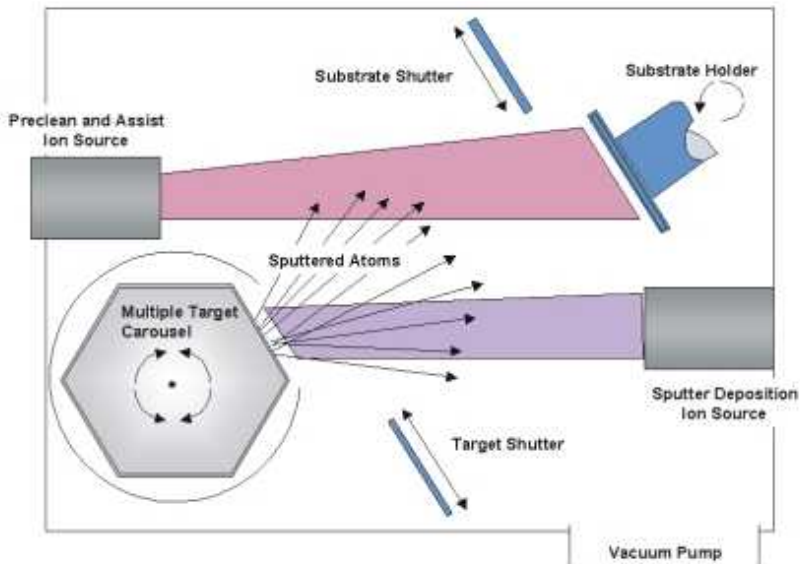
Evaporation

- evaporation of solid material through high electric current (e.g. classic Al mirror coatings)
- sputtering: plasma glow discharge ejects material from solid substance

Deposition

- uncontrolled ballistic flights, mechanical shields to homogenize beam
- ion-assisted deposition

Ion-Beam Deposition



Jones Matrices for Thin Film Stacks

- in a coordinate system where p-polarized light has $\vec{E} \parallel x$ -axis and s-polarized light has $\vec{E} \parallel y$ -axis
- reflection

$$J_r = \begin{pmatrix} r_p & 0 \\ 0 & r_s \end{pmatrix}$$

- transmission

$$J_t = \begin{pmatrix} t_p & 0 \\ 0 & t_s \end{pmatrix}$$

- other orientations through rotation of Jones matrices

Mueller Matrices for Thin Film Stacks

- reflection

$$M_r = \frac{1}{2} \begin{pmatrix} |r_p|^2 + |r_s|^2 & |r_p|^2 - |r_s|^2 & 0 & 0 \\ |r_p|^2 - |r_s|^2 & |r_p|^2 + |r_s|^2 & 0 & 0 \\ 0 & 0 & 2\text{Re}(r_p^* r_s) & 2\text{Im}(r_p^* r_s) \\ 0 & 0 & -2\text{Im}(r_p^* r_s) & 2\text{Re}(r_p^* r_s) \end{pmatrix}$$

- transmission

$$M_t = \frac{\eta_s}{2\eta_m} \begin{pmatrix} |t_p|^2 + |t_s|^2 & |t_p|^2 - |t_s|^2 & 0 & 0 \\ |t_p|^2 - |t_s|^2 & |t_p|^2 + |t_s|^2 & 0 & 0 \\ 0 & 0 & 2\text{Re}(t_p^* t_s) & 2\text{Im}(t_p^* t_s) \\ 0 & 0 & -2\text{Im}(t_p^* t_s) & 2\text{Re}(t_p^* t_s) \end{pmatrix}$$

- M_t includes correction for projection

Mueller Matrix for Dielectric

- reflection

$$M_r = \frac{1}{2} \left(\frac{\tan \alpha_-}{\sin \alpha_+} \right)^2 \begin{pmatrix} c_-^2 + c_+^2 & c_-^2 - c_+^2 & 0 & 0 \\ c_-^2 - c_+^2 & c_-^2 + c_+^2 & 0 & 0 \\ 0 & 0 & -2c_+c_- & 0 \\ 0 & 0 & 0 & -2c_+c_- \end{pmatrix}$$

- transmission

$$M_t = \frac{1}{2} \frac{\sin 2i \sin 2r}{(\sin \alpha_+ \cos \alpha_-)^2} \begin{pmatrix} c_-^2 + 1 & c_-^2 - 1 & 0 & 0 \\ c_-^2 - 1 & c_-^2 + 1 & 0 & 0 \\ 0 & 0 & 2c_- & 0 \\ 0 & 0 & 0 & 2c_- \end{pmatrix}$$

- $\alpha_{\pm} = \theta_i \pm \theta_t$, $c_{\pm} = \cos \alpha_{\pm}$, and Snell: $\sin \theta_i = n \sin \theta_t$

Mueller Matrix for Reflection off Metal

$$M_r = \frac{1}{2} \begin{pmatrix} \rho_s^2 + \rho_p^2 & \rho_s^2 - \rho_p^2 & 0 & 0 \\ \rho_s^2 - \rho_p^2 & \rho_s^2 + \rho_p^2 & 0 & 0 \\ 0 & 0 & 2\rho_s\rho_p \cos \delta & 2\rho_s\rho_p \sin \delta \\ 0 & 0 & -2\rho_s\rho_p \sin \delta & 2\rho_s\rho_p \cos \delta \end{pmatrix}$$

- real quantities $\rho_s, \rho_p, \delta = \phi_s - \phi_p$ defined by

$$\rho_s e^{i\phi_s} = -\frac{\sin(i-r)}{\sin(i+r)} = \frac{\cos i - \tilde{n} \cos r}{\cos i + \tilde{n} \cos r}$$

$$\rho_p e^{i\phi_p} = \frac{\tan(i-r)}{\tan(i+r)} = \left(\frac{\tilde{n} \cos r - \cos i}{\cos i + \tilde{n} \cos r} \right) \left(\frac{\tilde{n} \cos r \cos i - \sin^2 i}{\tilde{n} \cos r \cos i + \sin^2 i} \right)$$

Example: Coated Metal Mirror

- mirror with $n = 1.2$, $k = 7.5$ (aluminum at 630 nm) at 45°

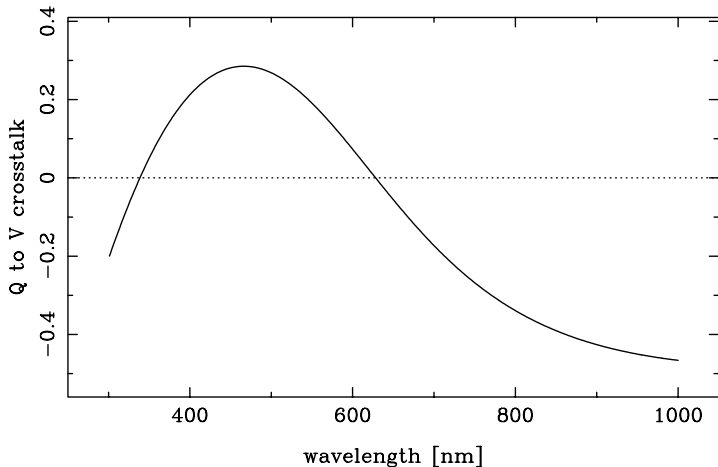
$$\begin{pmatrix} 1.000 & 0.028 & 0.000 & 0.000 \\ 0.028 & 1.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.983 & 0.180 \\ 0.000 & 0.000 & -0.180 & 0.983 \end{pmatrix}$$

- 126-nm thick dielectric layer with $n = 1.4$ on top

$$\begin{pmatrix} 1.000 & -0.009 & 0.000 & 0.000 \\ -0.009 & 1.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 1.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 1.000 \end{pmatrix}$$

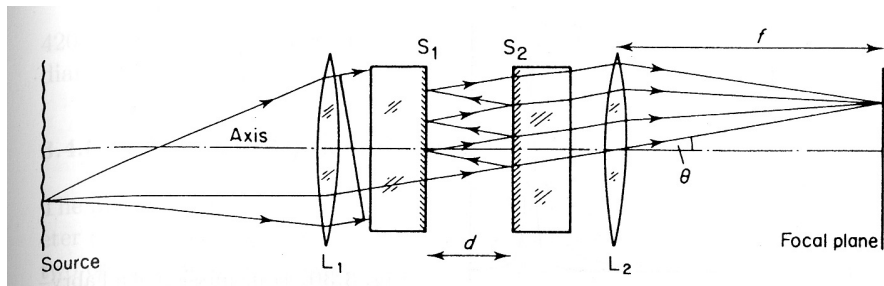
- $Q \Rightarrow V$ disappears, $I \Rightarrow Q$ reduced
- special mirrors minimize cross-talk (e.g. GONG turret)

Wavelength-Dependence of $Q \Rightarrow V$ Cross-Talk



only limited wavelength range can be corrected

Tunable Filter



- invented by Fabry and Perot in 1899
- interference between partially transmitting plates containing medium with index of refraction n
- angle of incidence in material θ , distance d
- path difference between successive beams: $\Delta = 2nd \cos \theta$

Fabry Perot continued

- path difference between successive beams: $\Delta = 2nd \cos \theta$
- phase difference: $\delta = 2\pi\Delta/\lambda = 4\pi nd \cos \theta/\lambda$
- incoming wave: $e^{i\omega t}$
- intensity transmission at surface: T
- intensity reflectivity at surface: R
- outgoing wave is the *coherent sum* of all beams

$$Ae^{i\omega t} = Te^{i\omega t} + TRe^{i(\omega t + \delta)} + TR^2e^{i(\omega t + 2\delta)} + \dots$$

- write this as

$$A = T(1 + Re^{i\delta} + R^2e^{i2\delta} + \dots) = \frac{T}{1 - Re^{i\delta}}$$

Fabry Perot continued

- emerging amplitude

$$A = \frac{T}{1 - Re^{i\delta}}$$

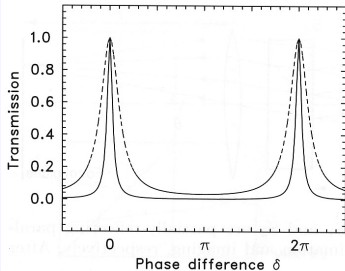
- emerging intensity is therefore

$$I = AA^* = \frac{T^2}{1 - 2R \cos \delta + R^2} = \frac{T^2}{(1 - R)^2 + 4R \sin^2 \frac{\delta}{2}}$$

- with $I_{\max} = T^2 / (1 - R)^2$

$$I = I_{\max} \frac{1}{1 + \frac{4R}{(1-R)^2} \sin^2 \frac{\delta}{2}}$$

Fabry Perot Properties



$$I = I_{\max} \frac{1}{1 + \frac{4R}{(1-R)^2} \sin^2 \frac{\delta}{2}}$$

- transmission is periodic
- distance between transmission peaks, *free spectral range*

$$\text{FSR} = \frac{\lambda_0^2}{2nd \cos \theta}$$

- Full-Width at Half Maximum (FWHM)
 $\Delta\lambda$

$$\Delta\lambda = \text{FSR}/F$$

- *finesse* F

$$F = \frac{\pi\sqrt{R}}{1-R}$$

Reflection Off Uncoated Substrate

- reflectivity of bare substrate:

$$R = \left(\frac{n_m - n_s}{n_m + n_s} \right)^2$$

- fused silica at 600 nm: $n_s = 1.46 \Rightarrow R = 0.035$
- extra-dense flint glass at 600 nm: $n_s = 1.75 \Rightarrow R = 0.074$
- silicon at 600 nm: $n_s = 3.96 \Rightarrow R = 0.6$
- loss in transparency
- ghost reflections

Analytical treatment of single layer

- assume single-layer coating
- determine optimum thickness and index of material
- amplitude reflection from coating/air interface

$$A_1 = \frac{n_1 - 1}{n_1 + 1}$$

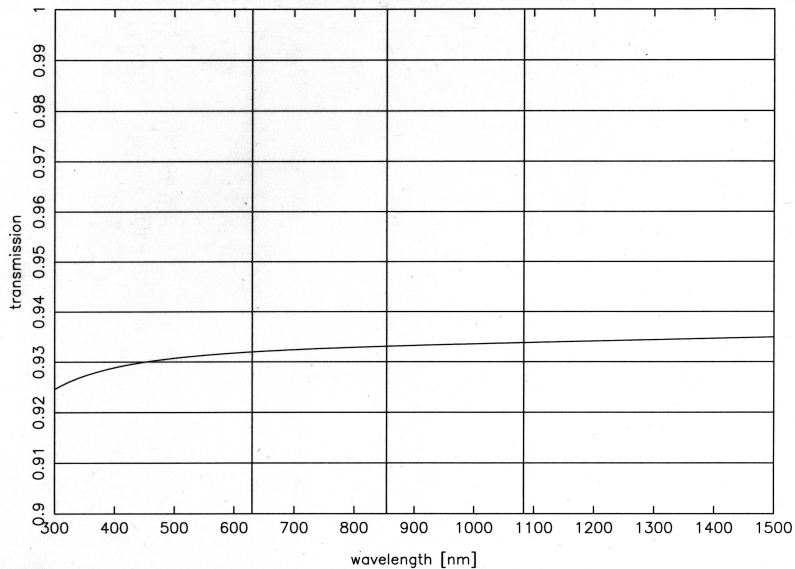
- amplitude reflection from coating/substrate interface

$$A_1 = \frac{n_s - n_1}{n_s + n_1}$$

- amplitudes subtract for 180 degree phase difference \Rightarrow coating should have optical path length $\lambda/4$ thick
- best cancellation for $n_1 = \sqrt{n_s}$

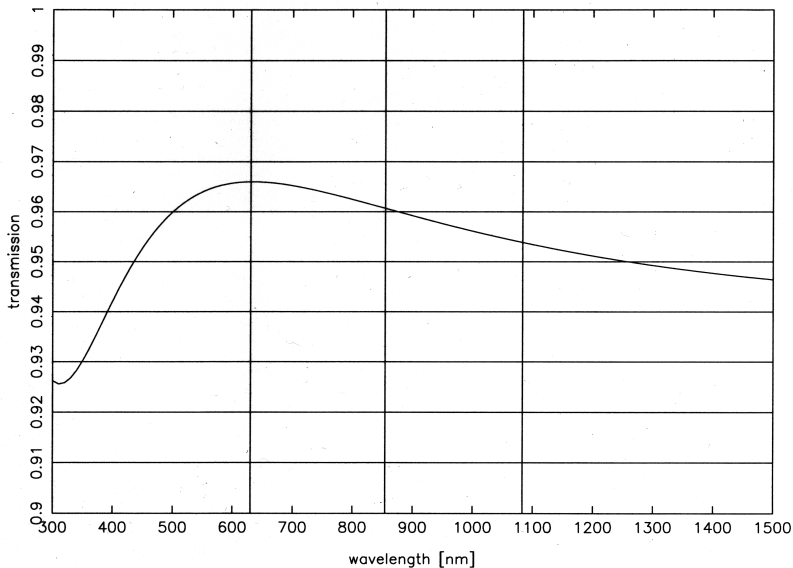
Uncoated Fused Silica

FS substrate, no coating



MgF₂ Coating on Fused Silica

FS substrate, 114.8 nm MgF₂



Multi-Layer Coatings on Fused Silica

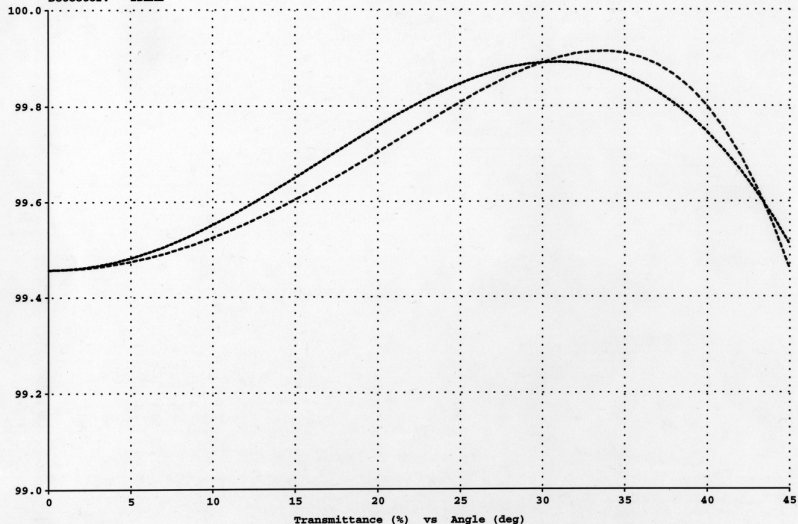
Software Spectra, Inc.

corrlens

1/20/99 3:20:04 PM

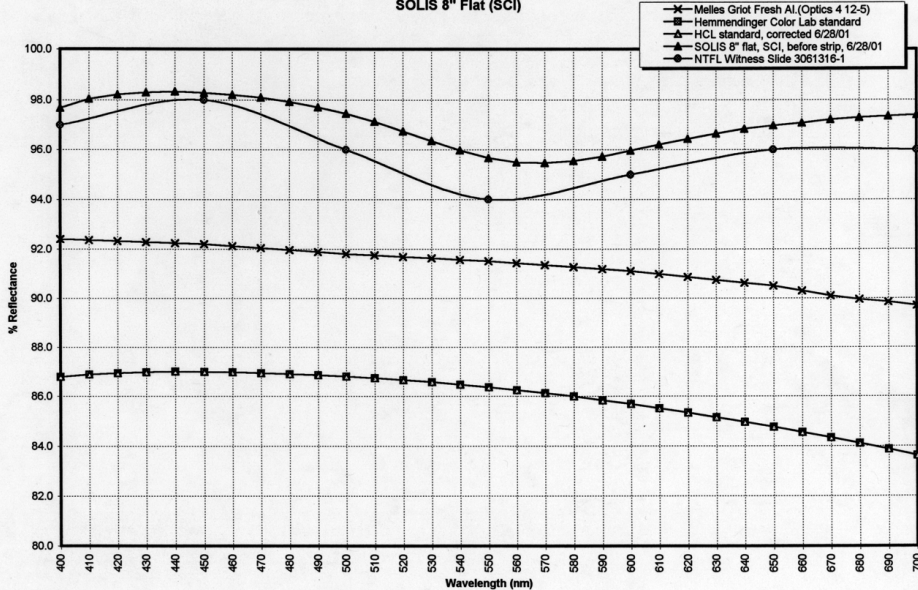
Illuminant: WHITE
Medium: AIR
Substrate: GLASS
Exit: GLASS
Detector: IDEAL

Wavelength: 630.2 (nm)
Reference: 632.8 (nm)
Polarization: S --- P —



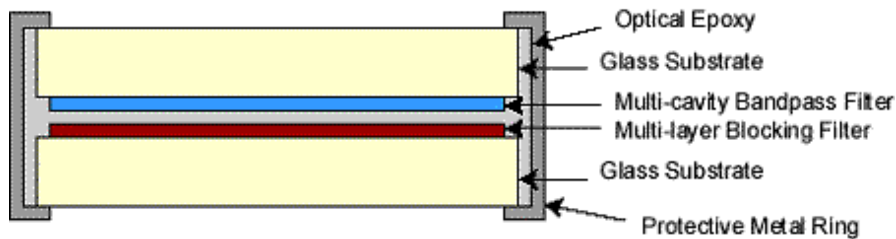
Highly Reflective Coatings

SOLIS 8" Flat (SCI)



Overview

Complete Interference Filter

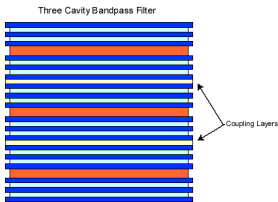


- many layers can achieve many things
- band-pass, long-pass, short-pass, dichroic filters
- colored glass substrates often used in addition to coatings
- sensitive to angle of incidence
- evaporated coatings are very temperature sensitive

Terminology (adapted from www.fluorescence.com/tutorial/int-filt.htm)

- **Bandpass:** range of wavelengths passed by filter
- **Bandpass Interference Filter:** interference filter designed to transmit specific wavelengths band
- **Blocking:** degree of attenuation outside of passband
- **Center Wavelength (CWL):** wavelength at midpoint of passband FWHM
- **Filter Cavity:** Fabry-Perot-like thin-film arrangement
- **Full-width Half-Maximum (FWHM):** width of bandpass at one-half of maximum transmission
- **Peak Transmission:** maximum percentage transmission within passband

Cavities



- basic design element: Fabry-Perot cavity
- cavity: $\lambda/2$ spacer and reflective multi-layer coatings on either side
- stacks of cavities provide much better performance

www.fluorescence.com/tutorial/int-filt.htm