Lecture 4: Polarization

Outline

1. Polarized Light in the Universe
2. Brewster Angle and Total Internal Reflection
3. Descriptions of Polarized Light
4. Polarizers
5. Retarders

Polarized Light in the Universe

Polarization indicates *anisotropy* $\Rightarrow$ not all directions are equal

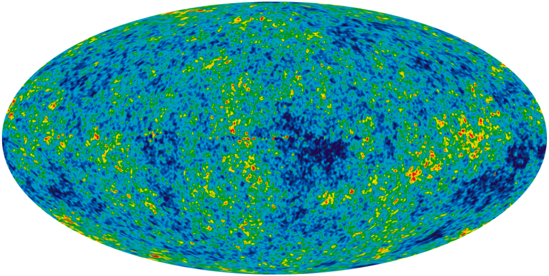
Typical anisotropies introduced by

- geometry (not everything is spherically symmetric)
- temperature gradients
- magnetic fields
- electrical fields

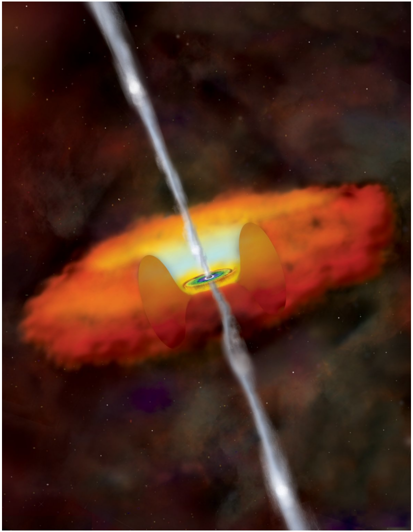
Polarized Light from the Big Bang

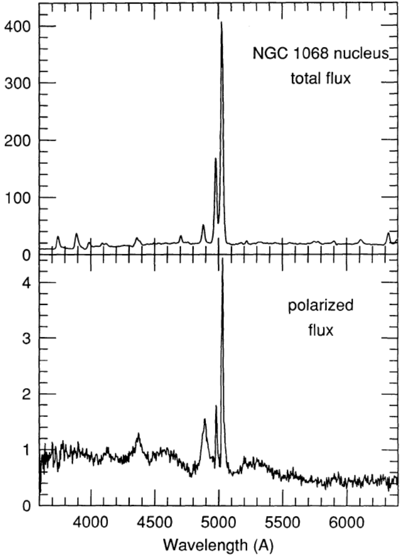
- Cosmic Microwave Background (CMB) is red-shifted radiation from Big Bang 14×10^9 years ago
- age, geometry, density, of universe from CMB intensity pattern
- first 0.1 seconds from polarization pattern of CMB
- inflation $\Rightarrow$ gravitational waves $\Rightarrow$ polarization signals
- polarization expected at (or below) 10^{-6} of intensity

13.7 billion year old temperature fluctuations from WMAP

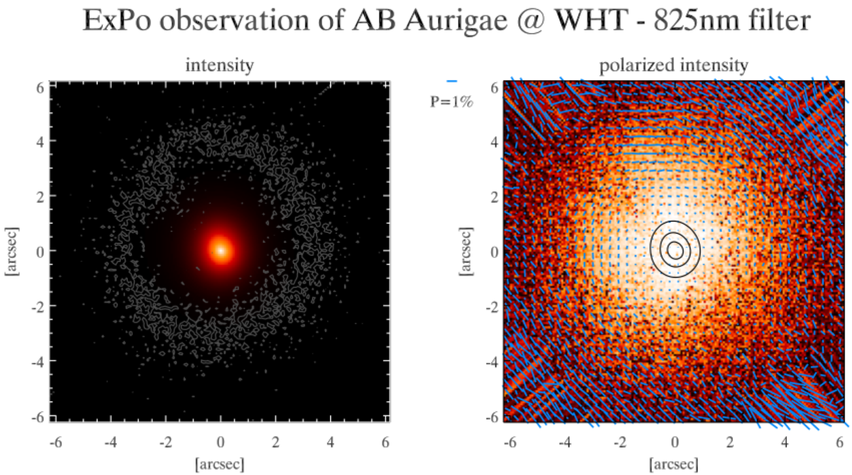


Unified Model of Active Galactic Nuclei

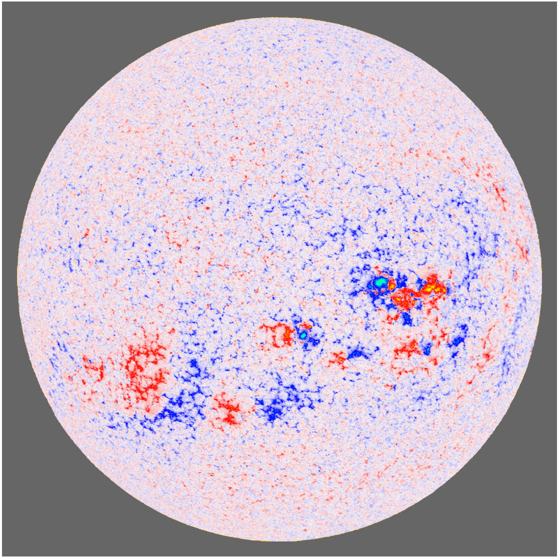




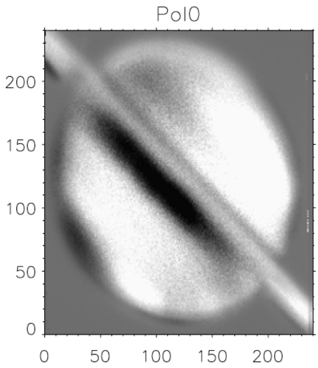
Protoplanetary Disk in Scattered Light

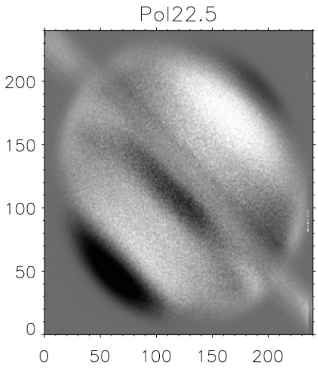


Solar Magnetic Field Maps from Longitudinal Zeeman Effect



Planetary Scattered Light





- solar-system planets show scattering polarization
- much depends on cloud height
- can be used to study exoplanets
- ExPo and EPICS developments at UU

Polarization

- Plane Vector Wave ansatz $\vec{E} = \vec{E}_0 e^{i\left(\vec{k}\cdot\vec{x}-\omega t\right)}$
- spatially, temporally constant vector $\vec{E}_0$ lays in plane perpendicular to propagation direction $\vec{k}$
- represent $\vec{E}_0$ in 2-D basis, unit vectors $\vec{e}_1$ and $\vec{e}_2$, both perpendicular to $\vec{k}$

$$\vec{E}_0 = E_1\vec{e}_1 + E_2\vec{e}_2.$$

  E_1, E_2: arbitrary complex scalars
- damped plane-wave solution with given ω, $\vec{k}$ has 4 degrees of freedom (two complex scalars)
- additional property is called *polarization*
- many ways to represent these four quantities
- if E_1 and E_2 have identical phases, $\vec{E}$ oscillates in fixed plane

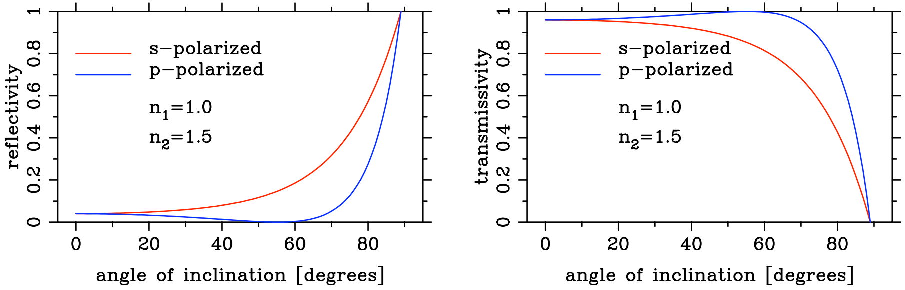
Summary of Fresnel Equations

- electric field amplitude transmission $t_{s,p}$, reflection $r_{s,p}$

$$
\begin{aligned}
t_s &= \frac{2\tilde{n}_1 \cos\theta_i}{\tilde{n}_1 \cos\theta_i + \sqrt{\tilde{n}_2^2 - \tilde{n}_1^2 \sin^2\theta_i}} \\
t_p &= \frac{2\tilde{n}_1 \tilde{n}_2 \cos\theta_i}{\tilde{n}_2^2 \cos\theta_i + \tilde{n}_1 \sqrt{\tilde{n}_2^2 - \tilde{n}_1^2 \sin^2\theta_i}} \\
r_s &= \frac{\tilde{n}_1 \cos\theta_i - \sqrt{\tilde{n}_2^2 - \tilde{n}_1^2 \sin^2\theta_i}}{\tilde{n}_1 \cos\theta_i + \sqrt{\tilde{n}_2^2 - \tilde{n}_1^2 \sin^2\theta_i}} \\
r_p &= \frac{\tilde{n}_2^2 \cos\theta_i - \tilde{n}_1 \sqrt{\tilde{n}_2^2 - \tilde{n}_1^2 \sin^2\theta_i}}{\tilde{n}_2^2 \cos\theta_i + \tilde{n}_1 \sqrt{\tilde{n}_2^2 - \tilde{n}_1^2 \sin^2\theta_i}}
\end{aligned}
$$

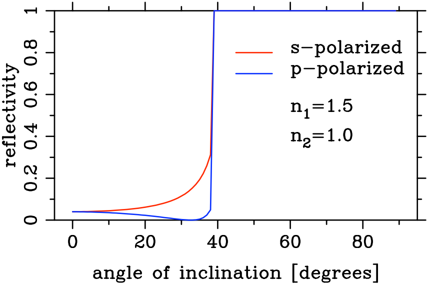
- reflectivity: $R = |r_{s,p}|^2$, transmissivity: $T = \frac{|\tilde{n}_2| \cos\theta_t}{|\tilde{n}_1| \cos\theta_i} |t_{s,p}|^2$

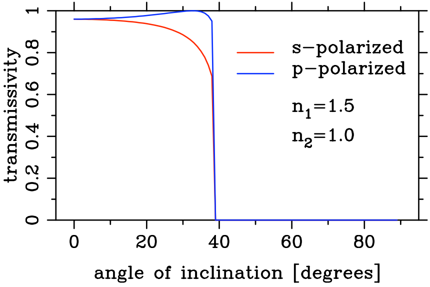
Brewster Angle



- $r_p = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)} = 0$ when $\theta_i + \theta_t = \frac{\pi}{2}$
- corresponds to *Brewster angle* of incidence of $\tan\theta_B = \frac{n_2}{n_1}$
- occurs when reflected wave is perpendicular to transmitted wave
- reflected light is completely s-polarized
- transmitted light is moderately polarized

Total Internal Reflection (TIR)





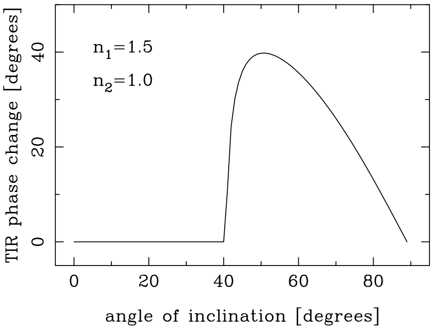
- Snell's law: $\sin\theta_t = \frac{n_1}{n_2}\sin\theta_i$
- wave from high-index medium into lower index medium (e.g. glass to air): $n_1/n_2 > 1$
- right-hand side > 1 for $\sin\theta_i > \frac{n_2}{n_1}$
- all light is reflected in high-index medium $\Rightarrow$ *total internal reflection*
- transmitted wave has complex phase angle $\Rightarrow$ damped wave along interface

Phase Change on Total Internal Reflection (TIR)

- TIR induces phase change that depends on polarization
- complex ratios: $r_{s,p} = |r_{s,p}| e^{i\delta_{s,p}}$
- phase change $\delta = \delta_s - \delta_p$

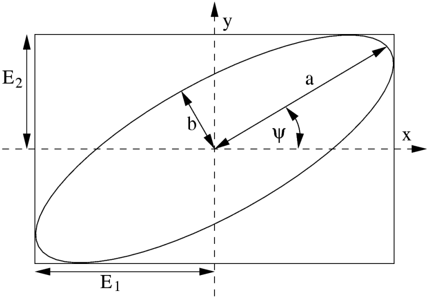
$$\tan\frac{\delta}{2} = \frac{\cos\theta_i \sqrt{\sin^2\theta_i - \left(\frac{n_2}{n_1}\right)^2}}{\sin^2\theta_i}$$

- relation valid between critical angle and grazing incidence
- for critical angle and grazing incidence, phase difference is zero



Polarization Ellipse

Polarization Ellipse



Polarization

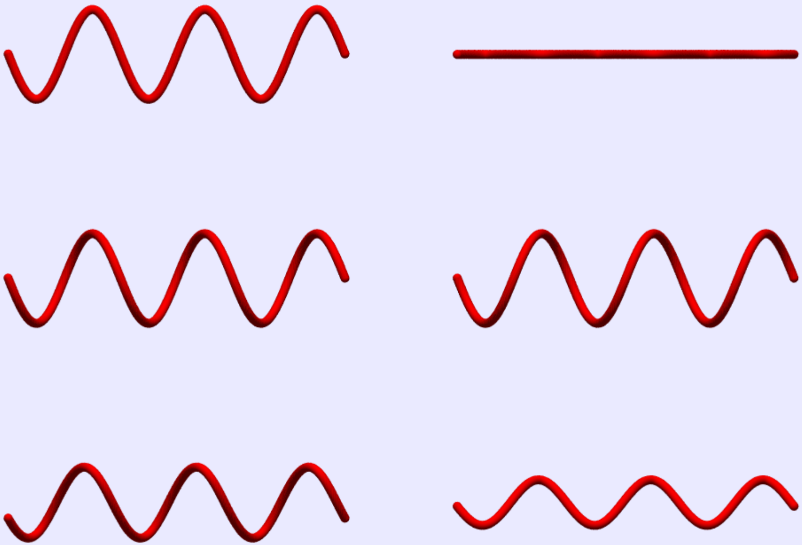
$$\vec{E}(t) = \vec{E}_0 e^{i\left(\vec{k}\cdot\vec{x}-\omega t\right)}$$

$$\vec{E}_0 = E_1 e^{i\delta_1}\vec{e}_x + E_2 e^{i\delta_2}\vec{e}_y$$

- wave vector in z-direction
- $\vec{e}_x$, $\vec{e}_y$: unit vectors in x, y
- E_1, E_2: (real) amplitudes
- $\delta_{1,2}$: (real) phases

Polarization Description

- 2 complex scalars not the most useful description
- at given $\vec{x}$, time evolution of $\vec{E}$ described by *polarization ellipse*
- ellipse described by axes a, b, orientation ψ



Jones Formalism

Jones Vectors

$$\vec{E}_0 = E_x \vec{e}_x + E_y \vec{e}_y$$

- beam in z-direction
- $\vec{e}_x$, $\vec{e}_y$ unit vectors in x, y-direction
- complex scalars $E_{x,y}$
- Jones vector

$$\vec{e} = \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

- phase difference between E_x, E_y multiple of π, electric field vector oscillates in a fixed plane $\Rightarrow$ *linear polarization*
- phase difference $\pm\frac{\pi}{2} \Rightarrow$ *circular polarization*

Summing and Measuring Jones Vectors

$$\vec{E}_0 = E_x \vec{e}_x + E_y \vec{e}_y$$

$$\vec{e} = \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

- Maxwell's equations linear $\Rightarrow$ sum of two solutions again a solution
- Jones vector of sum of two waves $=$ sum of Jones vectors of individual waves if wave vectors $\vec{k}$ the same
- addition of Jones vectors: *coherent* superposition of waves
- elements of Jones vectors are not observed directly
- observables always depend on products of elements of Jones vectors, i.e. intensity

$$I = \vec{e} \cdot \vec{e}^* = e_x e_x^* + e_y e_y^*$$

Jones matrices

- influence of medium on polarization described by 2×2 complex *Jones matrix* J

$$\vec{e}' = \mathrm{J}\vec{e} = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix} \vec{e}$$

- assumes that medium not affected by polarization state
- different media 1 to N in order of wave direction $\Rightarrow$ combined influence described by

$$\mathrm{J} = \mathrm{J}_N \mathrm{J}_{N-1} \cdots \mathrm{J}_2 \mathrm{J}_1$$

- order of matrices in product is crucial
- Jones calculus describes coherent superposition of polarized light

Linear Polarization

- horizontal: $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
- vertical: $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$
- 45°: $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Circular Polarization

- left: $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}$
- right: $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix}$

Notes on Jones Formalism

- Jones formalism operates on amplitudes, not intensities
- coherent superposition important for coherent light (lasers, interference effects)
- Jones formalism describes 100% polarized light

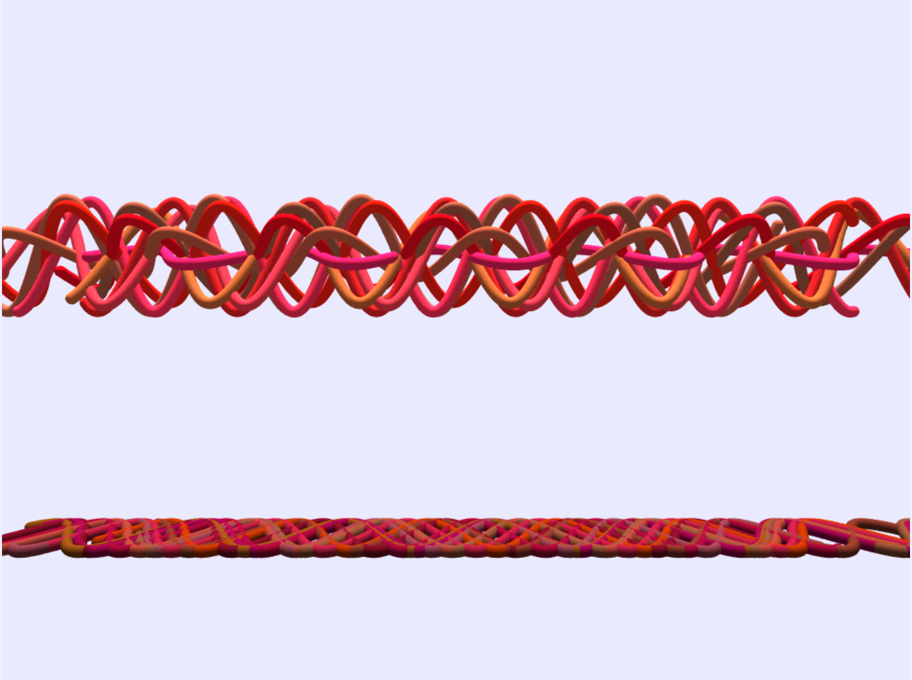
Stokes and Mueller Formalisms

Stokes Vector

- formalism to describe polarization of quasi-monochromatic light
- directly related to measurable intensities
- Stokes vector fulfills these requirements

$$\vec{I} = \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} E_x E_x^* + E_y E_y^* \\ E_x E_x^* - E_y E_y^* \\ E_x E_y^* + E_y E_x^* \\ i\left(E_x E_y^* - E_y E_x^*\right) \end{pmatrix} = \begin{pmatrix} E_1^2 + E_2^2 \\ E_1^2 - E_2^2 \\ 2E_1 E_2 \cos\delta \\ 2E_1 E_2 \sin\delta \end{pmatrix}$$

  Jones vector elements $E_{x,y}$, real amplitudes $E_{1,2}$, phase difference $\delta = \delta_2 - \delta_1$
- $I^2 \geq Q^2 + U^2 + V^2$
- can describe unpolarized ($Q = U = V = 0$) light



Stokes Vector Interpretation

$$\vec{I} = \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} \text{intensity} \\ \text{linear } 0^\circ - \text{linear } 90^\circ \\ \text{linear } 45^\circ - \text{linear } 135^\circ \\ \text{circular left} - \text{right} \end{pmatrix}$$

- *degree of polarization*

$$P = \frac{\sqrt{Q^2 + U^2 + V^2}}{I}$$

  1 for fully polarized light, 0 for unpolarized light
- summing of Stokes vectors = *incoherent* adding of quasi-monochromatic light waves

Linear Polarization

- horizontal: $\begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix}$
- vertical: $\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$
- 45°: $\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$

Circular Polarization

- left: $\begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$
- right: $\begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix}$

Mueller Matrices

- 4×4 real Mueller matrices describe (linear) transformation between Stokes vectors when passing through or reflecting from media

$$\vec{I}' = \mathrm{M}\vec{I} \, ,$$

$$\mathrm{M} = \begin{pmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{pmatrix}$$

- N optical elements, combined Mueller matrix is

$$\mathrm{M}' = \mathrm{M}_N \mathrm{M}_{N-1} \cdots \mathrm{M}_2 \mathrm{M}_1$$

Rotating Mueller Matrices

- optical element with Mueller matrix M
- Mueller matrix of the same element rotated by θ around the beam given by

$$\mathrm{M}(\theta) = \mathrm{R}(-\theta)\mathrm{M}\mathrm{R}(\theta)$$

with

$$\mathrm{R}(\theta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\theta & \sin 2\theta & 0 \\ 0 & -\sin 2\theta & \cos 2\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Vertical Linear Polarizer

$$\mathrm{M_{pol}}(\theta) = \frac{1}{2}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

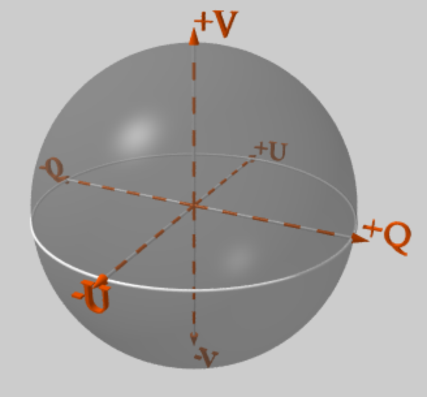
Horizontal Linear Polarizer

$$\mathrm{M_{pol}}(\theta) = \frac{1}{2}\begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Mueller Matrix for Ideal Linear Polarizer at Angle θ

$$\mathrm{M_{pol}}(\theta) = \frac{1}{2}\begin{pmatrix} 1 & \cos 2\theta & \sin 2\theta & 0 \\ \cos 2\theta & \cos^2 2\theta & \sin 2\theta \cos 2\theta & 0 \\ \sin 2\theta & \sin 2\theta \cos 2\theta & \sin^2 2\theta & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Poincaré Sphere



Relation to Stokes Vector

- fully polarized light: $I^2 = Q^2 + U^2 + V^2$
- for $I^2 = 1$: sphere in Q, U, V coordinate system
- point on Poincaré sphere represents particular state of polarization
- graphical representation of fully polarized light

Polarizers

- polarizer: optical element that produces polarized light from unpolarized input light
- linear, circular, or in general elliptical polarizer, depending on type of transmitted polarization
- linear polarizers by far the most common
- large variety of polarizers

Jones Matrix for Linear Polarizers

- Jones matrix for linear polarizer:

$$\mathrm{J}_p = \begin{pmatrix} p_x & 0 \\ 0 & p_y \end{pmatrix}$$

- $0 \leq p_x \leq 1$ and $0 \leq p_y \leq 1$, real: transmission factors for x, y-components of electric field: $E'_x = p_x E_x, \quad E'_y = p_y E_y$
- $p_x = 1,\ p_y = 0$: linear polarizer in $+Q$ direction
- $p_x = 0,\ p_y = 1$: linear polarizer in $-Q$ direction
- $p_x = p_y$: neutral density filter

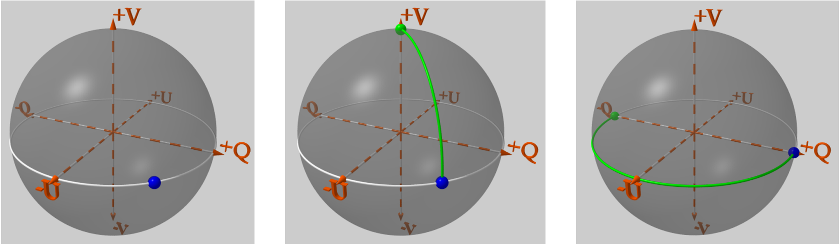
Mueller Matrix for Linear Polarizers

$$\mathrm{M}_p = \frac{1}{2} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Mueller Matrix for Ideal Linear Polarizer at Angle θ

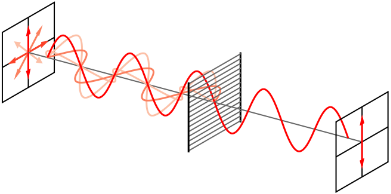
$$M_{\text{pol}}(\theta) = \frac{1}{2}\begin{pmatrix} 1 & \cos 2\theta & \sin 2\theta & 0 \\ \cos 2\theta & \cos^2 2\theta & \sin 2\theta \cos 2\theta & 0 \\ \sin 2\theta & \sin 2\theta \cos 2\theta & \sin^2 2\theta & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Poincare Sphere



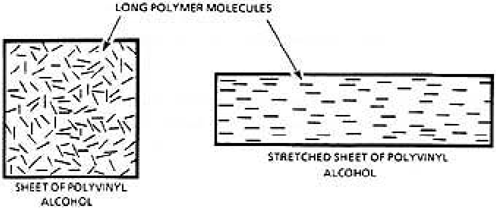
- polarizer is a point on the Poincaré sphere
- transmitted intensity: $\cos^2(l/2)$, l is arch length of great circle between incoming polarization and polarizer on Poincaré sphere

Wire Grid Polarizers



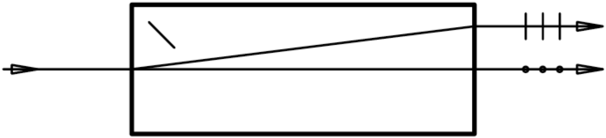
- parallel conducting wires, spacing $d \lesssim \lambda$ act as polarizer
- electric field parallel to wires induces electrical currents in wires
- induced electrical current reflects polarization parallel to wires
- polarization perpendicular to wires is transmitted
- rule of thumb:
  - $d < \lambda/2 \Rightarrow$ strong polarization
  - $d \gg \lambda \Rightarrow$ high transmission of both polarization states (weak polarization)
- mostly used in infrared

Polaroid-type Polarizers



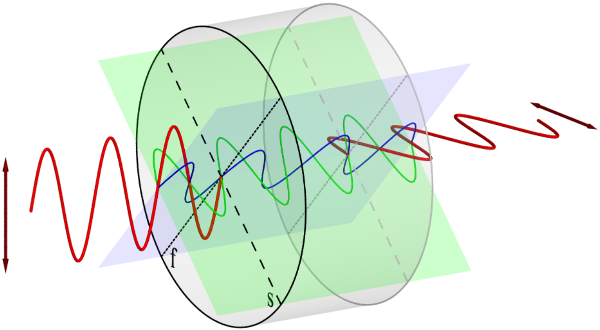
- developed by Edwin Land in 1938 $\Rightarrow$ Polaroid
- sheet polarizers: stretched polyvynil alcohol (PVA) sheet, laminated to sheet of cellulose acetate butyrate, treated with iodine
- PVA-iodine complex analogous to short, conducting wire
- cheap, can be manufactured in large sizes

Crystal-Based Polarizers



- crystals are basis of highest-quality polarizers
- precise arrangement of atom/molecules and anisotropy of index of refraction separate incoming beam into two beams with precisely orthogonal linear polarization states
- work well over large wavelength range
- many different configurations
- calcite most often used in crystal-based polarizers because of large birefringence, low absorption in visible
- many other suitable materials

Retarders or Wave Plates



- retards (delays) phase of one electric field component with respect to orthogonal component
- anisotropic material (crystal) has index of refraction that depends on polarization

Retarder Properties

- does not change intensity or degree of polarization
- characterized by two (not identical, not trivial) Stokes vectors of incoming light that are not changed by retarder $\Rightarrow$ *eigenvectors* of retarder
- depending on polarization described by eigenvectors, retarder is
  - *linear retarder*
  - *circular retarder*
  - *elliptical retarder*
- linear, circular retarders are special cases of elliptical retarders
- circular retarders sometimes called *rotators* since they rotate the orientation of linearly polarized light
- linear retarders by far the most common type of retarder

Jones Matrix for Linear Retarders

- linear retarder with fast axis at 0° characterized by Jones matrix

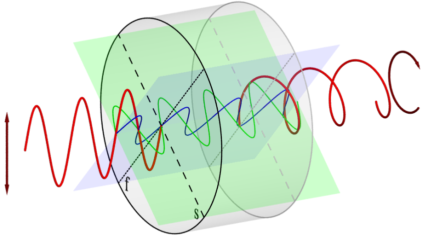
$$\mathrm{J}_r(\delta) = \begin{pmatrix} e^{i\delta} & 0 \\ 0 & 1 \end{pmatrix}, \quad \mathrm{J}_r(\delta) = \begin{pmatrix} e^{i\frac{\delta}{2}} & 0 \\ 0 & e^{-i\frac{\delta}{2}} \end{pmatrix}$$

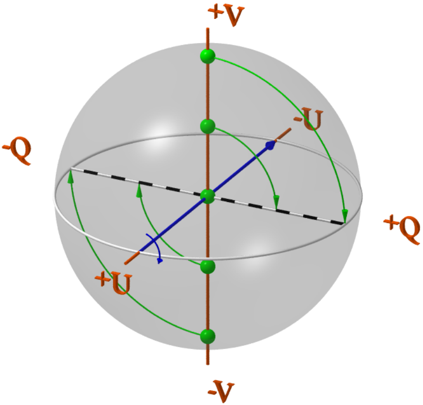
- δ: phase shift between two linear polarization components (in radians)
- absolute phase does not matter

Mueller Matrix for Linear Retarder

$$\mathrm{M}_r = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\delta & -\sin\delta \\ 0 & 0 & \sin\delta & \cos\delta \end{pmatrix}$$

Quarter-Wave Plate on the Poincaré Sphere



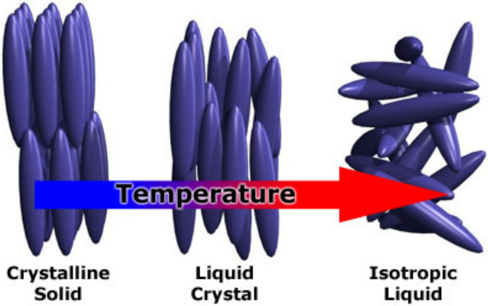


- retarder eigenvector (fast axis) in Poincaré sphere
- points on sphere are rotated around retarder axis by amount of retardation

Variable Retarders

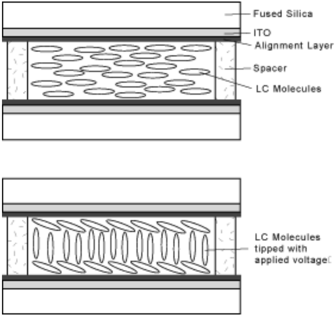
- sensitive polarimeters requires retarders whose properties (retardance, fast axis orientation) can be varied quickly (*modulated*)
- retardance changes (change of birefringence):
  - liquid crystals
  - Faraday, Kerr, Pockels cells
  - piezo-elastic modulators (PEM)
- fast axis orientation changes (change of *c*-axis direction):
  - rotating fixed retarder
  - ferro-electric liquid crystals (FLC)

Liquid Crystals



- liquid crystals: fluids with elongated molecules
- at high temperatures: liquid crystal is isotropic
- at lower temperature: molecules become ordered in orientation and sometimes also space in one or more dimensions
- liquid crystals can line up parallel or perpendicular to external electrical field

Liquid Crystal Retarders



- dielectric constant anisotropy often large $\Rightarrow$ very responsive to changes in applied electric field
- birefringence δn can be very large (larger than typical crystal birefringence)
- liquid crystal layer only a few μm thick
- birefringence shows strong temperature dependence