Lecture 2: Geometrical Optics 1

Outline

1. Spherical Waves
2. From Waves to Rays
3. Lenses
4. Chromatic Aberrations
5. Mirrors

Introduction to Geometrical Optics

Spherical Waves

- wave equations for dielectric

$$\nabla^2 \vec{E} - \frac{\mu\epsilon}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

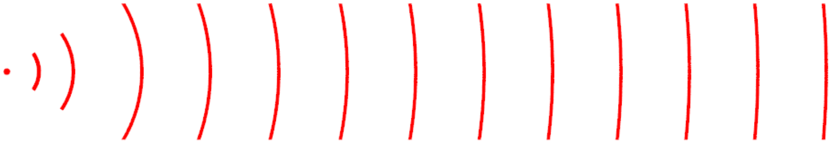
- spherical wave solution

$$\vec{E}(r,t) = \vec{E}_0 \frac{A}{r} e^{i(kr-\omega t)}$$

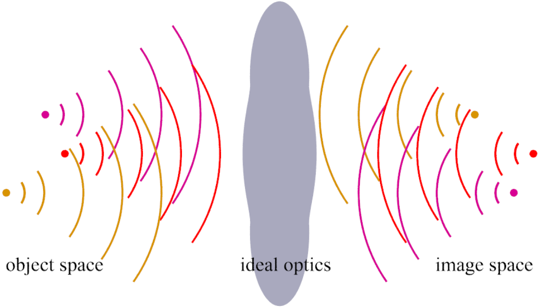
  - $\vec{E}_0$: polarization
  - A: a constant
  - r: radial distance from center/source of wave
- mostly part of spherical wave

Light Sources

- light source: collection of sources of spherical waves
- astronomical sources: almost exclusively incoherent
- lasers, masers: coherent sources
- spherical wave originating at very large distance can be approximated by plane wave

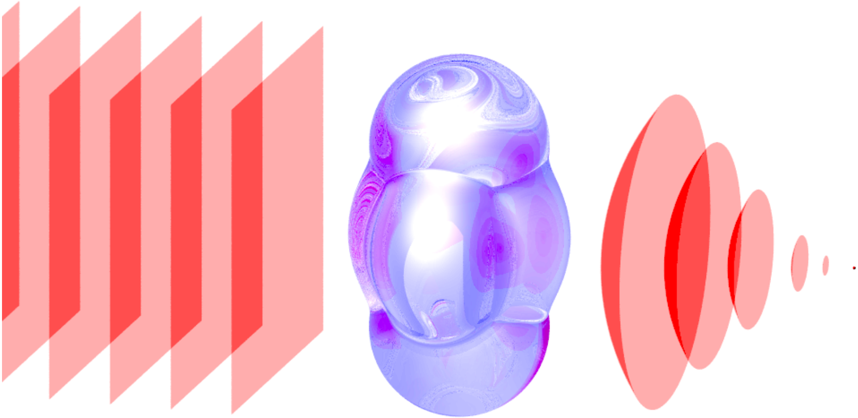


Ideal Optics



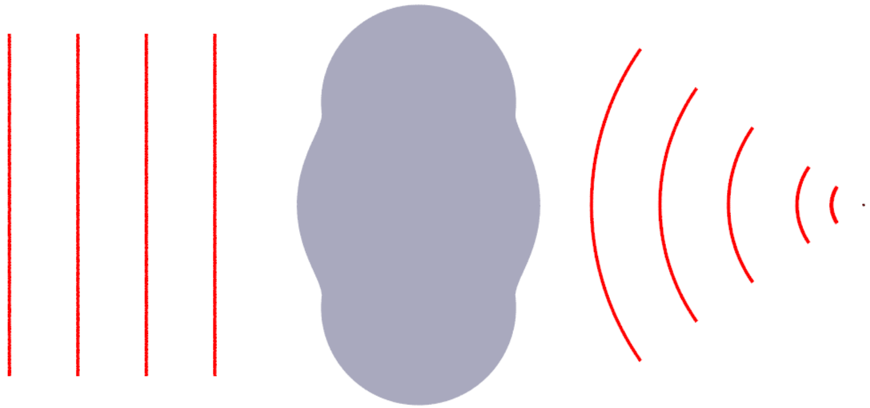
- ideal optics: spherical waves from any point in object space are imaged into points in image space
- corresponding points are called *conjugate points*
- *focal point*: center of converging or diverging spherical wavefront
- object space and image space are reversible

From Waves to Rays: General Optical System



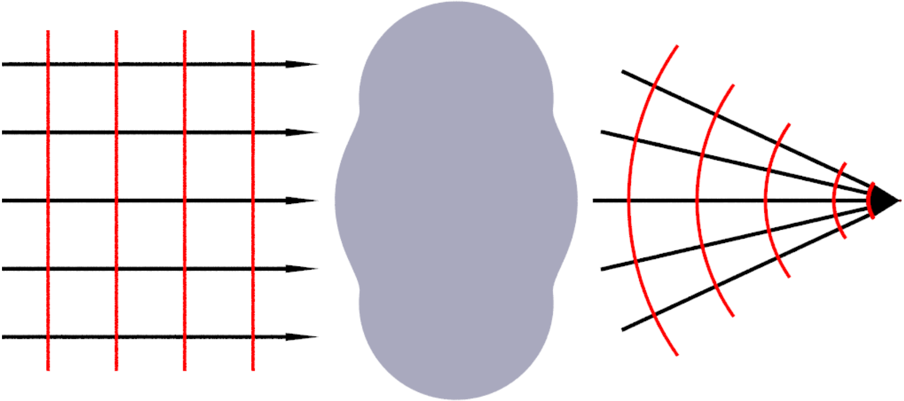
- ideal optical system transforms plane wavefront into spherical, converging wavefront

From Waves to Rays: Azimuthal Symmetry



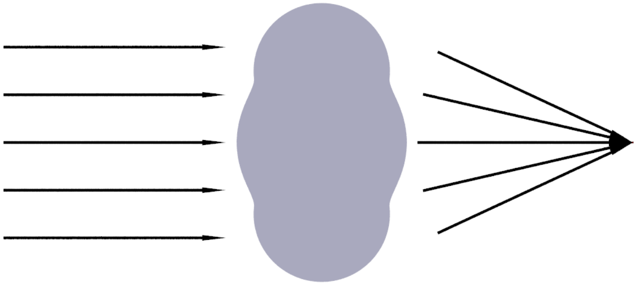
- most optical systems are azimuthally symmetric
- axis of symmetry is *optical axis*

From Waves to Rays: Locally Flat Wavefronts



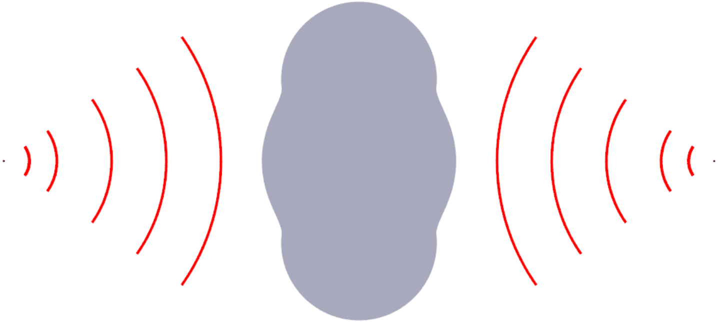
- rays are normal to local wave (locations of constant phase)
- local wave around rays is assumed to be plane wave

From Waves to Rays: Rays



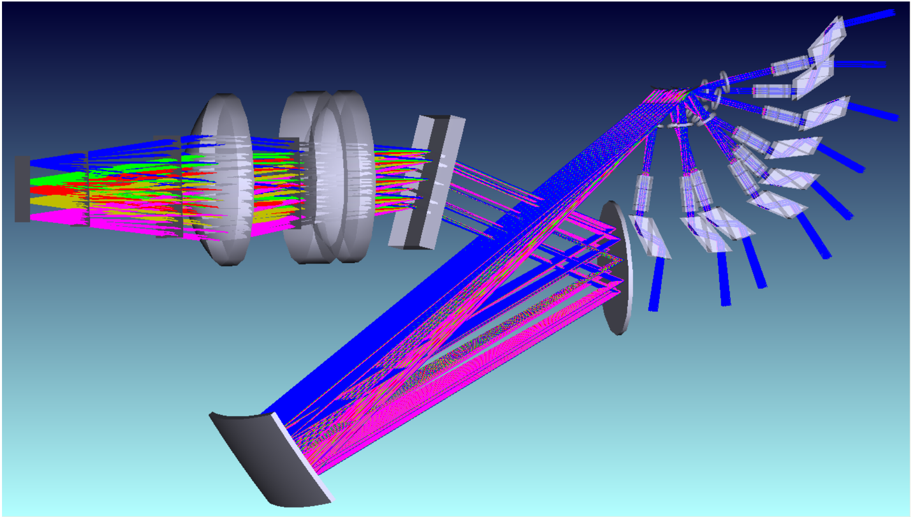
- geomtrical optics works with rays only
- rays are reflected and refracted according to Fresnel equations
- phase is neglected ⇒ incoherent sum
- rays can carry polarization information

From Waves to Rays: Finite Object Distance



- object may also be at finite distance
- also in astronomy: reimaging within instruments and telescopes

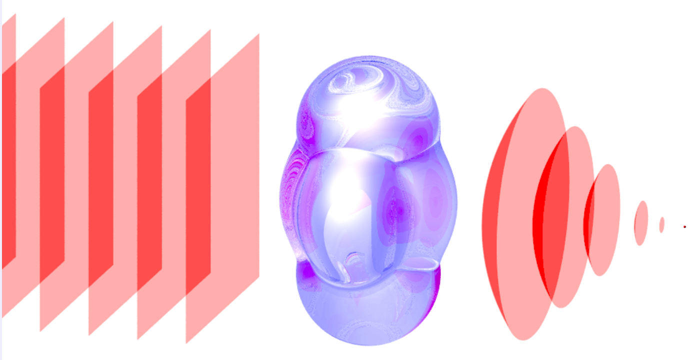
Geometrical Optics Example: SPEX



Limitations of Geometrical Optics

- optical system cannot collect all parts of spherical wavefront $\Rightarrow$ diffraction
- geometrical optics neglects diffraction effects
- *geometrical optics*: $\lambda \Rightarrow 0$
- *physical optics* $\lambda > 0$
- simplicity of geometrical optics mostly outweighs limitations

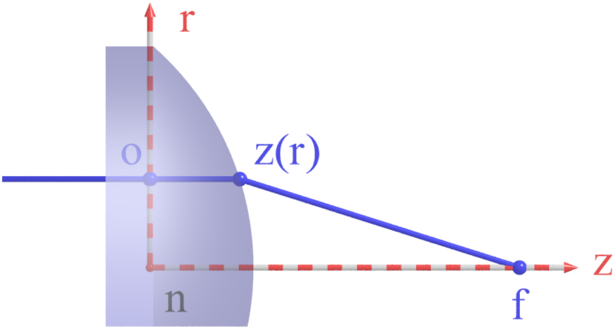
Lenses



Definitions

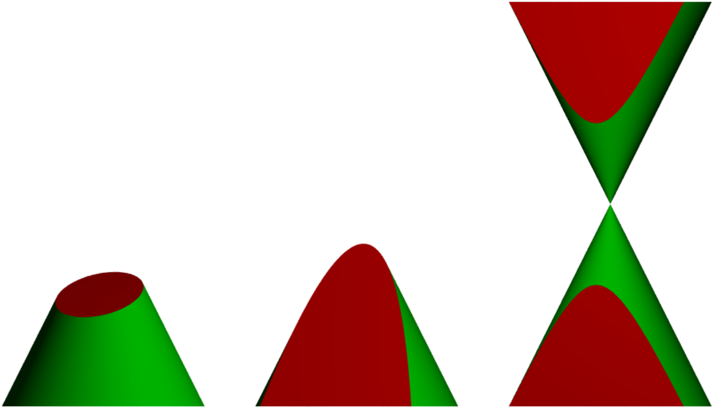
- lens = refracting device, discontinuity in material properties
- perfect lens for infinite object: makes plane wavefront into spherical wavefront material

Surface Shape of Perfect Lens

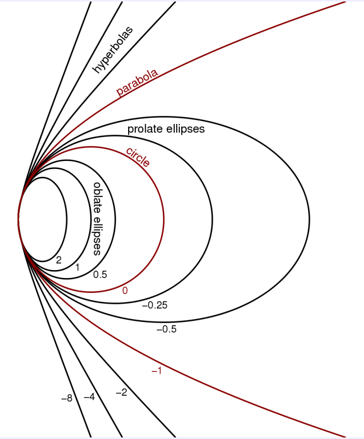


- lens material has index of refraction n
- $\overline{o\,z(r)} \cdot n + \overline{z(r)\,f} = \text{constant}$
- $n \cdot z(r) + \sqrt{r^2 + (f - z(r))^2} = \text{constant}$
- solution $z(r)$ is hyperbola with eccentricity $e = n > 1$

Conic Sections



- circle and ellipses: cuts angle $<$ cone angle
- parabola: angle = cone angle
- hyperbola: cut along axis

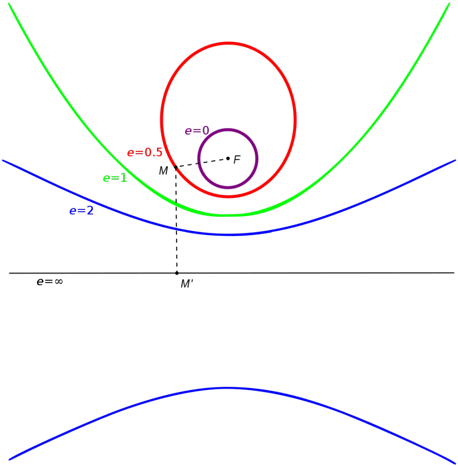


en.wikipedia.org/wiki/Conic_constant

Conic Constant

- $r^2 - 2Rz + (1 + K)z^2 = 0$ for $z(0) = 0$
- $z = \frac{r^2}{R} \frac{1}{1+\sqrt{1-(1+K)\frac{r^2}{R^2}}}$
- R radius of curvature
- K conic constant
- $K = -e^2$, e eccentricity
- prolate elliptical ($K > 0$)
- spherical ($K = 0$)
- oblate elliptical ($0 > K > -1$)
- parabolic ($K = -1$)
- hyperbolic ($K < -1$)
- sphere is good approximation to all conic sections close to origin

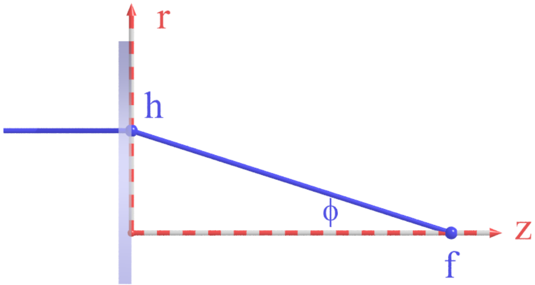
Foci of Conic Sections



en.wikipedia.org/wiki/File:Eccentricity.svg

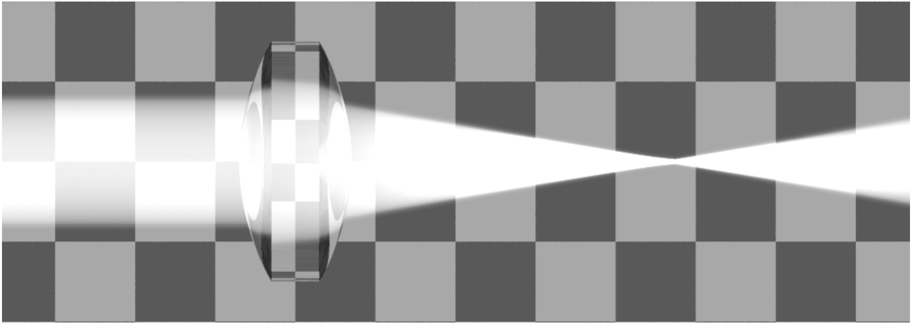
- sphere has single focus
- ellipse has two foci
- parabola (ellipse with $e = 1$) has one focus (and another one at infinity)
- hyperbola ($e > 1$) has two focal points

Paraxial Optics



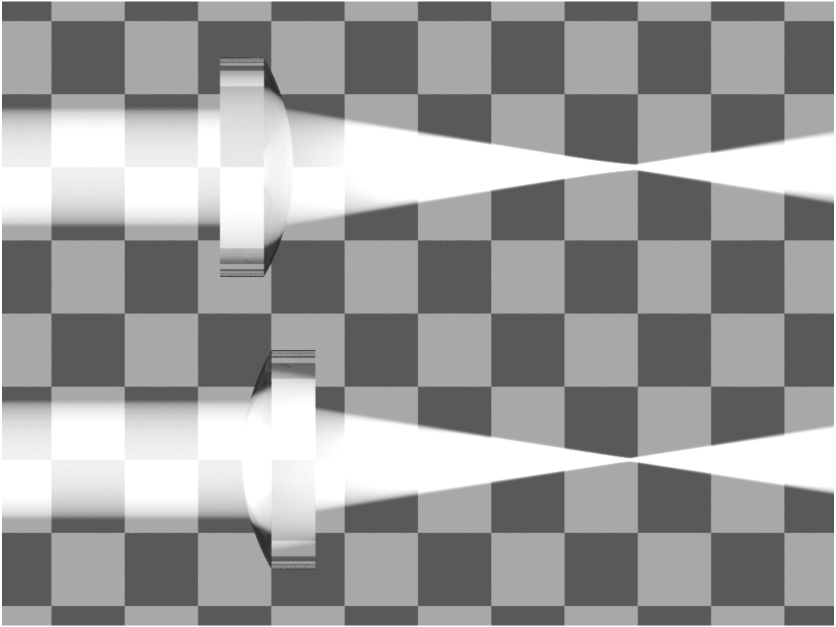
- assumption 1: Snell's law for small angles of incidence ($\sin x \approx x$): $n \cdot \phi = \phi'$
- assumption 2: ray hight h small so that optics curvature can be neglected (plane optics, ($\cos x \approx 1$))
- assumption 3: $tan\phi \approx \phi = h/f$

Spherical Lenses

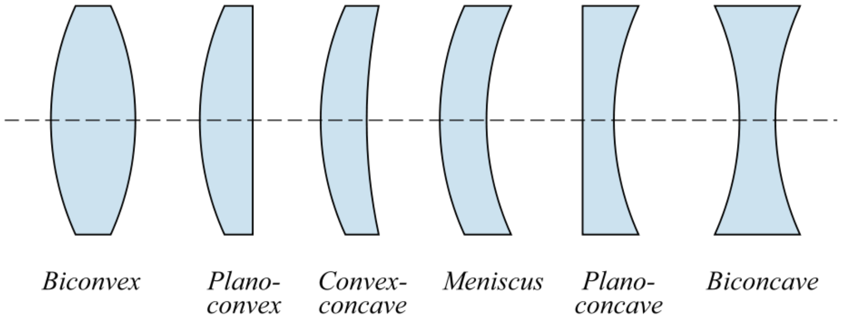


- if two spherical surfaces have same radius, can fit them together
- surface error requirement less than $\lambda/10$
- 5cm diameter lens, 500 nm wavelength $\Rightarrow$ 1ppm accuracy
- grinding spherical surfaces is easy $\Rightarrow$ most optical surfaces are spherical

Planoconvex Lenses

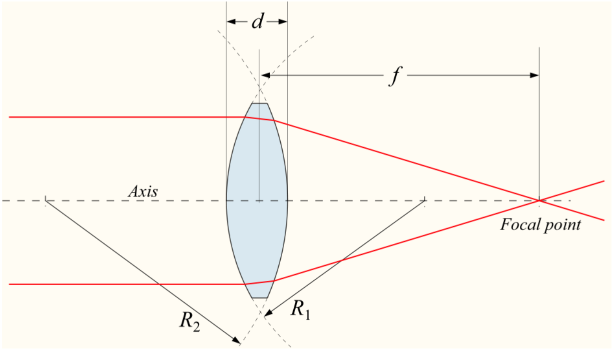


Types of Lenses



en.wikipedia.org/wiki/File:Lens2.svg

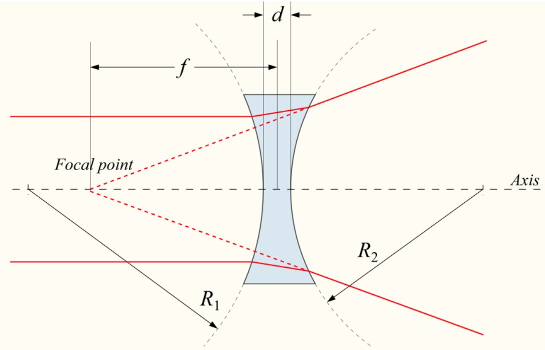
Positive/Converging Spherical Lens Parameters



commons.wikimedia.org/wiki/File:Lens1.svg

- center of curvature and radii with signs: $R_1 > 0, R_2 < 0$
- center thickness: d
- with material index of refraction ⇒ positive focal length f

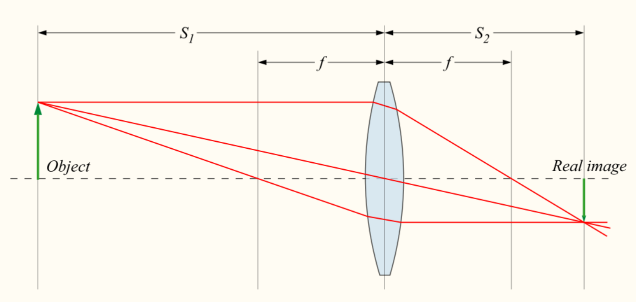
Negative/Diverging Spherical Lens Parameters



commons.wikimedia.org/wiki/File:Lens1b.svg

- note different signs of radii: $R_1 < 0$, $R_2 > 0$
- virtual focal point
- negative focal length

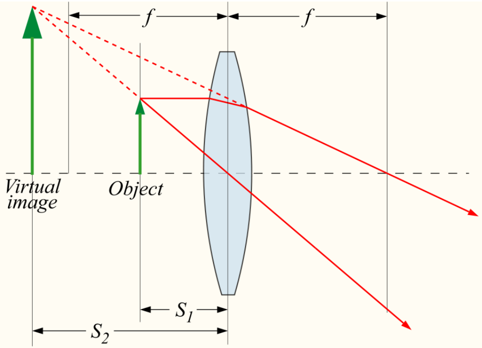
General Lens Setup: Real Image



commons.wikimedia.org/wiki/File:Lens3.svg

- *object distance S_1, object height h_1*
- *image distance S_2, image height h_2*
- axis through two centers of curvature is *optical axis*
- surface point on optical axis is the *vertex*
- *chief ray* through center maintains direction

General Lens Setup: Virtual Image



commons.wikimedia.org/wiki/File:Lens3b.svg

- note object closer than focal length of lens
- virtual image

Thin Lens Approximation

- thin-lens equation:

$$\frac{1}{S_1} + \frac{1}{S_2} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

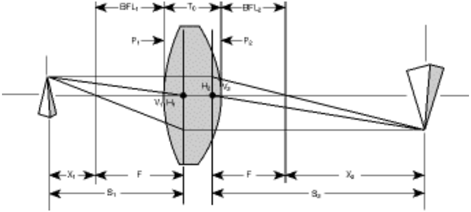
- Gaussian lens formula:

$$\frac{1}{S_1} + \frac{1}{S_2} = \frac{1}{f}$$

Finite Imaging

- rarely image point sources, but extended object
- object and image size are proportional
- orientation of object and image are inverted
- (transverse) magnification perpendicular to optical axis:
  $M = h_2/h_1 = -S_2/S_1$

Thick Lenses

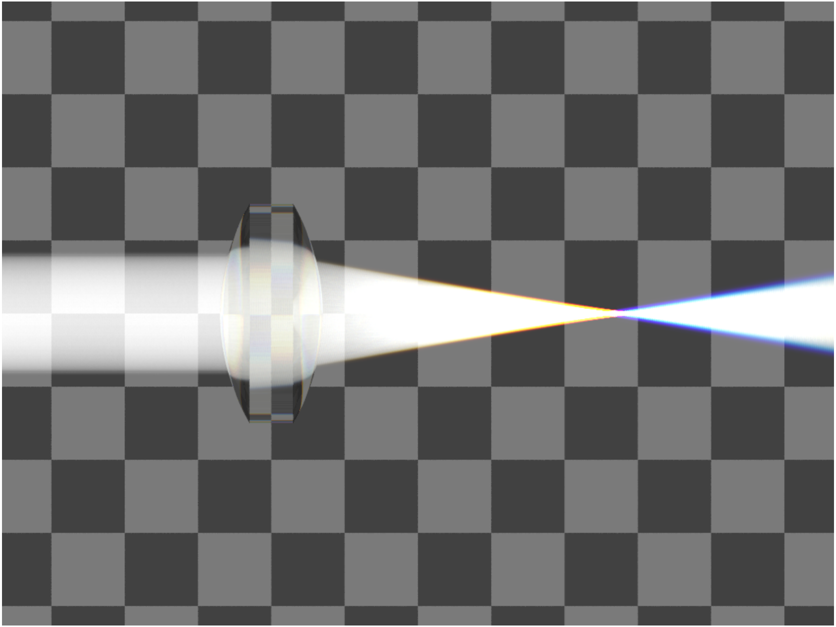


www.newport.com/servicesupport/Tutorials/default.aspx?id=169

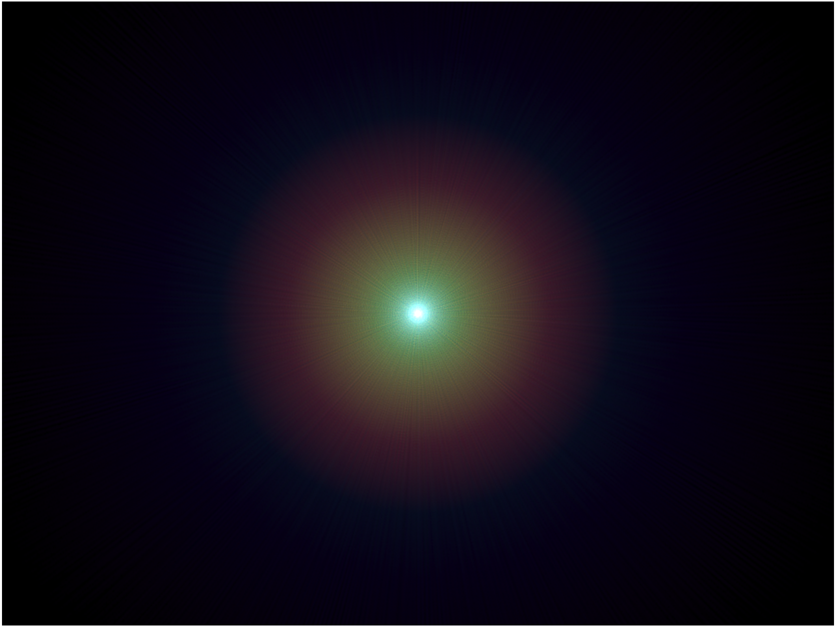
- basic thick lens equation $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n-1)d}{nR_1R_2}\right)$
- thin means $d << R_1R_2$
- focal lengths measured from *principal planes*
- distance between vertices and principal planes given by

$$H_{1,2} = -\frac{f(n-1)d}{R_{2,1}n}$$

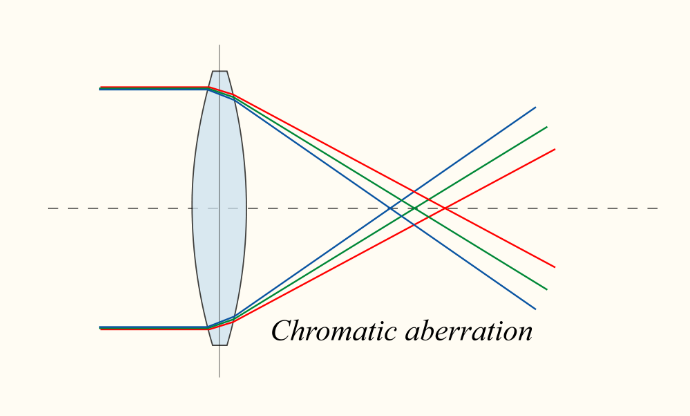
Chromatic Aberration 1



Chromatic Aberration 2



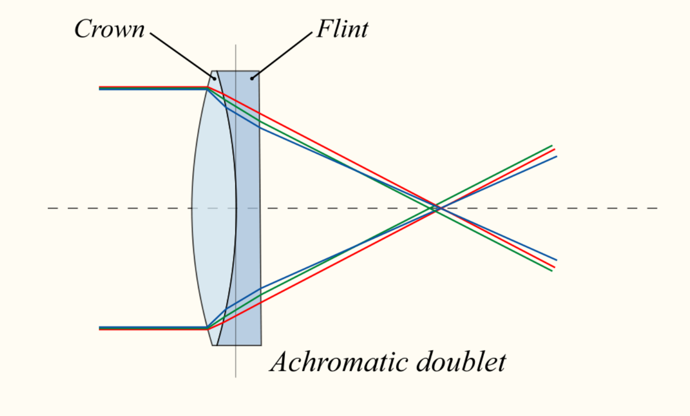
Chromatic Aberration 3



en.wikipedia.org/wiki/File:Lens6a.svg

- due to wavelength dependence of index of refraction
- higher index in the blue ⇒ shorter focal length in blue

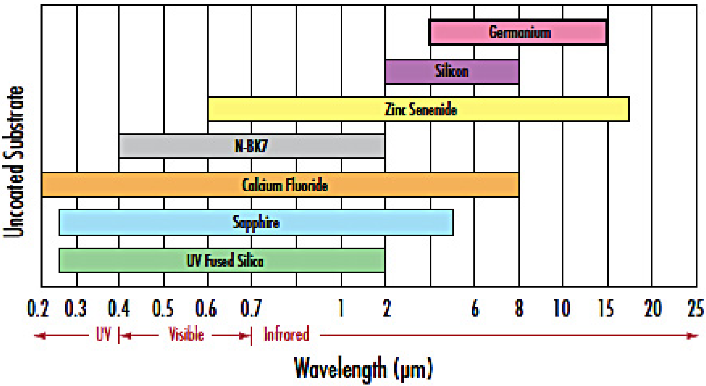
Achromatic Lens



en.wikipedia.org/wiki/File:Lens6b.svg

- combination of positive and negative lens made from materials with differents dispersions

Transmission of Transparent Materials



Wavelength range for N-BK7 is representative for the majority of substrates used for visible wavelengths such as B270, N-SF11, Borofloat™, etc.

www.edmundoptics.com/technical-support/technical-library/frequently-asked-questions/

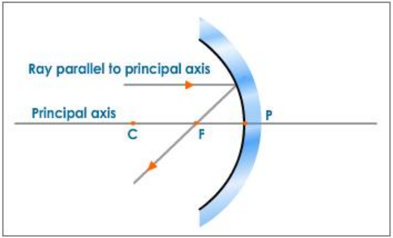
Mirrors

Mirrors vs. Lenses

- mirrors are totally achromatic
- reflective over very large wavelength range (UV to radio)
- can be supported from the back
- wavefront error is twice that of surface, lens is (n-1) times surface
- only one surface to 'play' with

Plane Mirrors: Fold Mirrors and Beamsplitters

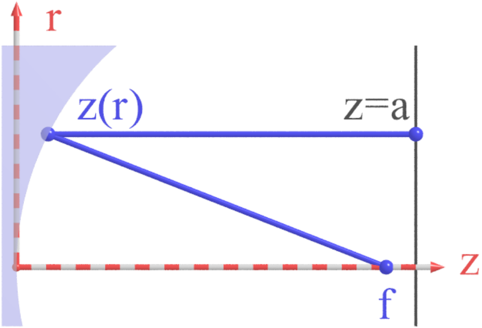
Spherical Mirrors



www.tutorvista.com/content/science/science-ii/reflection-light/formation-spherical-mirrors.php

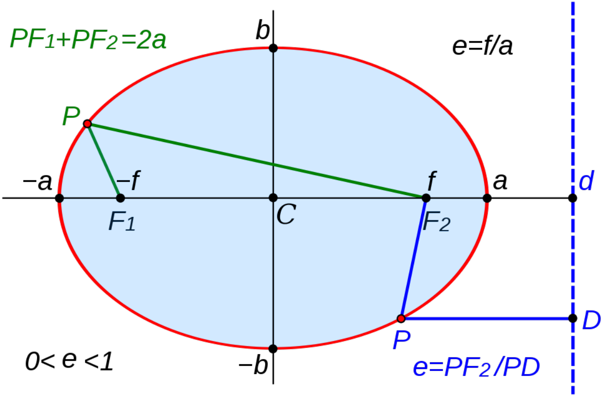
- easy to manufacture
- focuses light from center of curvature onto itself
- focal length is half of curvature
- tip-tilt misalignment does not matter
- has no optical axis
- does not image light from infinity correctly (spherical aberration)

Parabolic Mirrors



- want to make flat wavefront into spherical wavefront
- distance $\overline{az(r)} + \overline{z(r)f}$ = const.
- $z(r) = r^2/2R$
- perfect image of objects at infinity
- has clear optical axis

Elliptical Mirrors



en.wikipedia.org/wiki/Ellipse

- have two foci at finite distances
- reimage one focal point into another