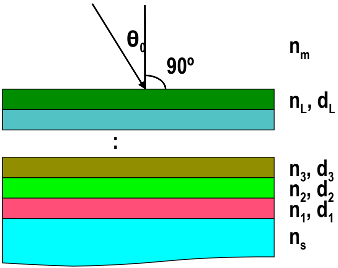
Outline

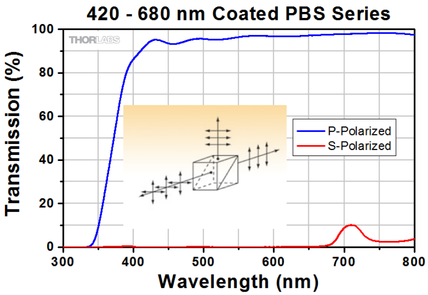
Introduction

- *thin film*:
  - layer with thickness $\lesssim \lambda$
  - extends in 2 other dimensions $\gg \lambda$
- reflection, refraction at all interfaces
- L layers of thin films: *thin film stack*
- layer thickness $d_i \lesssim \lambda \Rightarrow$ interference between reflected and refracted waves
- infinite number of multiple reflections and refractions must be considered
- assumption: *substrate* (index n_s), incident medium (index n_m) have infinite thickness



Thin Films and Polarization

- thin film coatings and polarization are closely coupled
- some polarizers (plate, cube) are based on thin-film coatings
- can dramatically reduce polarizing effects of optical components
- all aluminum mirrors have aluminum oxide thin film



www.thorlabs.com/newgrouppage9.cfm?objectgroup_id=739

Calculating Thin-Film Stack Properties

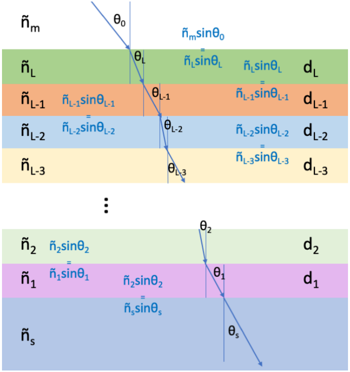
- many layers $\Rightarrow$ consider all interferences between reflected and refracted rays of each interface
- complexity of calculation significantly reduced by matrix approach
- signs and conventions are not conistent in literature

Plane Waves and Thin-Film Stacks

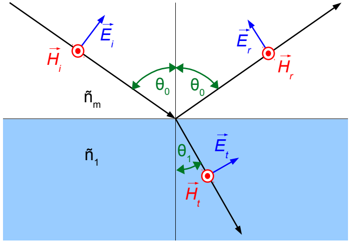
- plane wave $\vec{E} = \vec{E}_0 e^{i\left(\vec{k}\cdot\vec{x}-\omega t\right)}$
- layers numbered from 1 to L with complex index of refraction $\tilde{n}_j$, geometrical thickness d_j
- substrate has refractive index $\tilde{n}_s$
- incident medium has index $\tilde{n}_m$
- angle of incidence in incident medium: θ_0
- Snell's law:

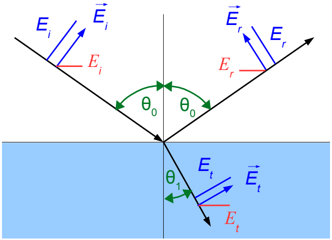
$$\tilde{n}_m \sin\theta_0 = \tilde{n}_L \sin\theta_L = \ldots = \tilde{n}_1 \sin\theta_1 = \tilde{n}_s \sin\theta_s$$

- $\Rightarrow \theta_j$ for every layer j



p/TM Wave: Electric Field at Interface

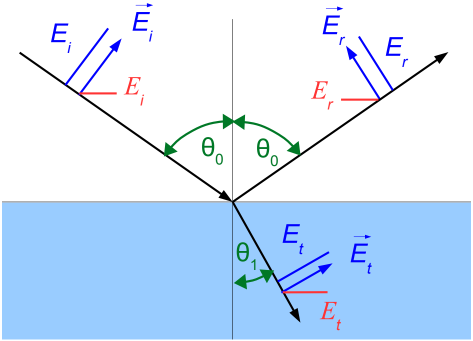




- p/TM wave: electric fields are in (parallel to) plane of incidence
- $E_{i,r,t}$: complex amplitudes of $\vec{E}_{i,r,t}$ of incident, reflected, transmitted electric fields
- $\mathrm{E}_{i,r,t}$: components tangential to interface of complex amplitudes of $\vec{E}_{i,r,t}$ of incident, reflected, transmitted electric fields

$$\mathrm{E}_i = E_i \cos\theta_0, \quad \mathrm{E}_r = E_r \cos\theta_0, \quad \mathrm{E}_t = E_t \cos\theta_1$$

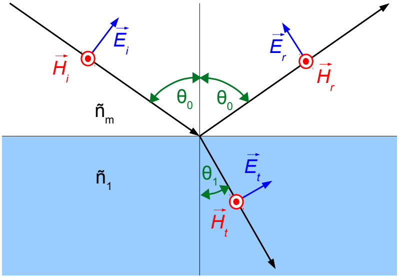
p/TM Wave: Electric Field at Interface (continued)



- continuous electric field tangential to interface:

$$E_i \cos\theta_0 - E_r \cos\theta_0 = E_t \cos\theta_1$$

p/TM Wave: Electric Field at Interface (continued)

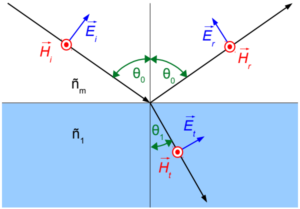


- thin-film formalism uses field components parallel to interface
- continuous electric field tangential to interface

$$\mathrm{E}_i - \mathrm{E}_r = \mathrm{E}_t$$

- electric field direction fully determined by angle of incidence
- complex scalar quantities are sufficient since the electric field is perpendicular to wave vector and in plane of incidence

p/TM Wave: Magnetic Field at Interface

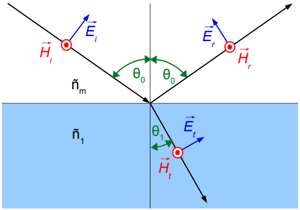


- non-magnetic material ($\mu = 1$): $\vec{H} = \tilde{n}\frac{\vec{k}}{|\vec{k}|} \times \vec{E}$
- $\vec{k}$, $\vec{E}$ perpendicular, cross-product becomes multiplication
- (complex) magnitudes of $\vec{E}_{i,r,t}$ and $\vec{H}_{i,r,t}$ related by

$$H_{i,r} = \tilde{n}_m E_{i,r}, \quad H_t = \tilde{n}_1 E_t$$

- tangential component of $\vec{H}$ continuous: $\mathrm{H}_i + \mathrm{H}_r = \mathrm{H}_t$
- $\vec{H}_{i,r,t}$ is already tangential to interface: $\vec{H}_{i,r,t} = \mathrm{H}_{i,r,t}$

p/TM Wave: Magnetic Field at Interface (continued)



- tangential component of $\vec{H}$ continuous: $H_i + H_r = H_t$
- expressed in electric field

$$\tilde{n}_m E_i + \tilde{n}_m E_r = \tilde{n}_1 E_t$$

- expressed in terms of tangential electric field components

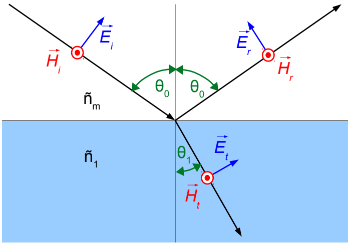
$$\frac{\tilde{n}_m}{\cos\theta_0}\mathrm{E}_i + \frac{\tilde{n}_m}{\cos\theta_0}\mathrm{E}_r = \frac{\tilde{n}_1}{\cos\theta_1}\mathrm{E}_t$$

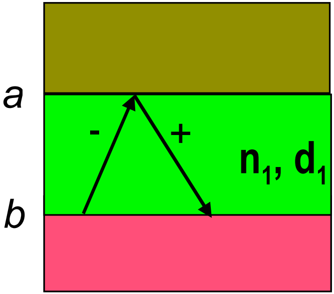
Matrix Formalism: Tangential Components in one Medium

- single interface in thin-film stack, combine all waves into
  - wave that travels towards substrate (+ superscript)
  - wave that travels away from substrate (− superscript)
- at interface *a*, *tangential components* of complex electric and magnetic field amplitudes in medium 1 given by

$$
\begin{aligned}
E_a &= E_{1a}^{+} - E_{1a}^{-} \\
H_a &= \frac{\tilde{n}_1}{\cos\theta_1}\left(E_{1a}^{+} + E_{1a}^{-}\right)
\end{aligned}
$$

- negative sign for outwards traveling electric field component



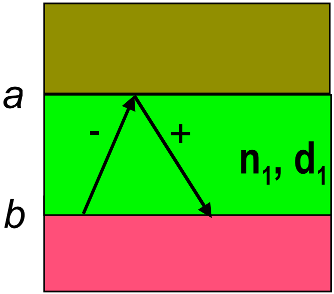


Matrix Formalism: Electric Field Propagation

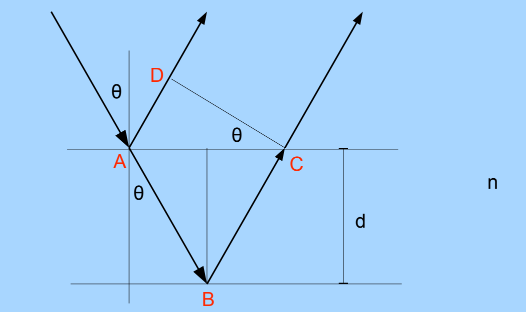
- field amplitudes in medium 1 at (other) interface *b* from wave propagation with common phase factor
- d_1: geometrical thickness of layer
- phase factor for forward propagating wave:

$$\delta = \frac{2\pi}{\lambda} \tilde{n}_1 d_1 \cos\theta_1$$

- backwards propagating wave: same phase factor with negative sign



Plane Wave Path Length for Oblique Incidence



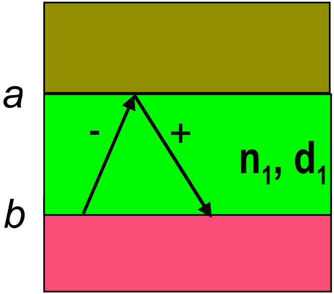
- consider theoretical reflections in single medium
- need to correct for plane wave propagation
- path length for “reflected light”: $\bar{AB} + \bar{BC} - \bar{AD}$

$$\frac{2d}{\cos\theta} - 2d\tan\theta \cdot \sin\theta = 2d\frac{1-\sin^2\theta}{\cos\theta} = 2d\cos\theta$$

Matrix Formalism

- at interface b in medium 1

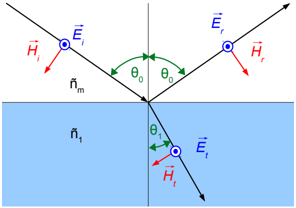
$$
\begin{aligned}
E_{1b}^{+} &= E_{1a}^{+}(\cos\delta + i\sin\delta)\\
E_{1b}^{-} &= E_{1a}^{-}(\cos\delta - i\sin\delta)\\
H_{1b}^{+} &= H_{1a}^{+}(\cos\delta + i\sin\delta)\\
H_{1b}^{-} &= H_{1a}^{-}(\cos\delta - i\sin\delta)
\end{aligned}
$$



- from before $E_{a,b} = E_{1a,b}^{+} - E_{1a,b}^{-}$, $H_{a,b} = \frac{\tilde{n}_1}{\cos\theta_1}\left(E_{1a,b}^{+} + E_{1a,b}^{-}\right)$
- propagation of tangential components from a to b

$$
\begin{pmatrix} E_b \\ H_b \end{pmatrix} = \begin{pmatrix} \cos\delta & \frac{\cos\theta_1}{\tilde{n}_1} i\sin\delta \\ i\frac{\tilde{n}_1}{\cos\theta_1}\sin\delta & \cos\delta \end{pmatrix} \begin{pmatrix} E_a \\ H_a \end{pmatrix}
$$

s/TE waves: Electric and Magnetic Fields at Interface



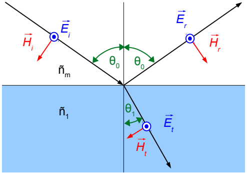
- tangential electric field component continuous: $E_i + E_r = E_t$
- tangential magnetic field component continuous

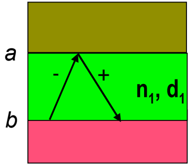
$$H_i \cos\theta_0 - H_r \cos\theta_0 = H_t \cos\theta_1$$

- and using relation between H and E

$$\tilde{n}_m \cos\theta_0 E_i - \tilde{n}_m \cos\theta_0 E_r = \tilde{n}_1 \cos\theta_1 E_t$$

s-Polarized Waves in one Medium





- at boundary a:

$$
\begin{aligned}
E_a &= E_{1a}^{+} + E_{1a}^{-} \\
H_a &= (E_{1a}^{+} - E_{1a}^{-})\tilde{n}_1 \cos\theta_1
\end{aligned}
$$

- propagation of tangential components from *a* to *b*

$$
\begin{pmatrix} E_b \\ H_b \end{pmatrix} = \begin{pmatrix} \cos\delta & \frac{1}{\tilde{n}_1 \cos\theta_1} i \sin\delta \\ i\tilde{n}_1 \cos\theta_1 \sin\delta & \cos\delta \end{pmatrix} \begin{pmatrix} E_a \\ H_a \end{pmatrix}
$$

Summary of Matrix Method

for each layer j calculate:

- θ_j using Snell's law: $n_m \sin\theta_0 = n_j \sin\theta_j$
- s-polarization: $\eta_j = n_j \cos\theta_j$
- p-polarization: $\eta_j = \frac{n_j}{\cos\theta_j}$
- phase delays: $\delta_j = \frac{2\pi}{\lambda} n_j d_j \cos\theta_j$
- characteristic matrix:

$$\mathrm{M}_j = \begin{pmatrix} \cos\delta_j & \frac{i}{\eta_j}\sin\delta_j \\ i\eta_j \sin\delta_j & \cos\delta_j \end{pmatrix}$$

Summary of Matrix Method (continued)

- total characteristic matrix M is product of all characteristic matrices

$$\mathrm{M} = \mathrm{M}_L \mathrm{M}_{L-1} ... \mathrm{M}_2 \mathrm{M}_1$$

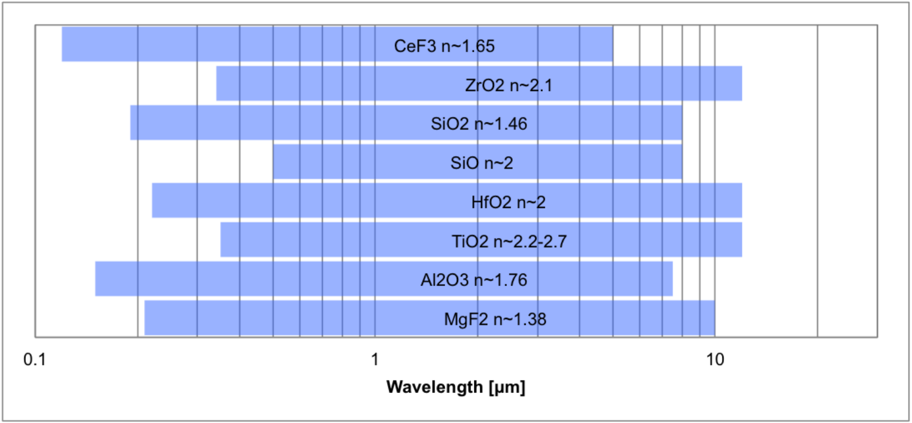
- fields in incident medium given by

$$\begin{pmatrix} E_m \\ H_m \end{pmatrix} = \mathrm{M} \begin{pmatrix} 1 \\ \eta_s \end{pmatrix}$$

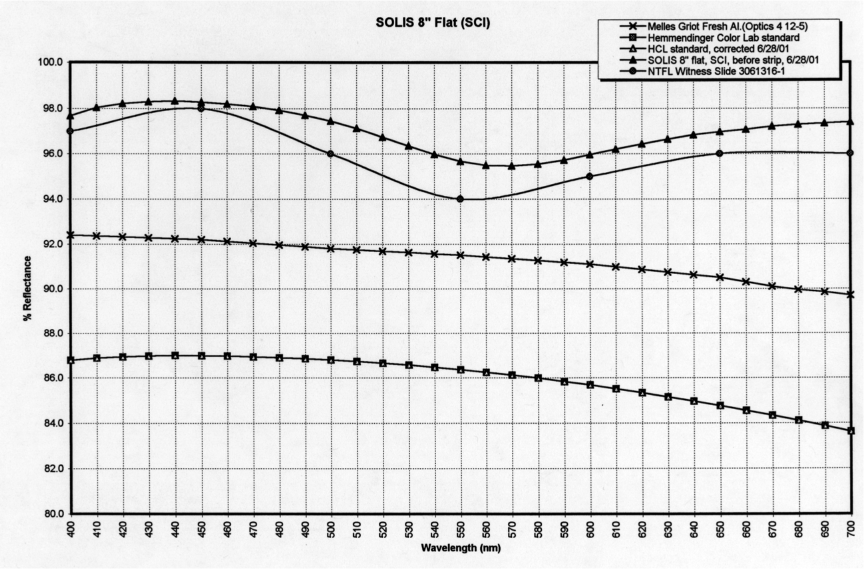
- complex reflection and transmission coefficients

$$r = \frac{\eta_m E_m - H_m}{\eta_m E_m + H_m}, \quad t = \frac{2\eta_m}{\eta_m E_m + H_m}$$

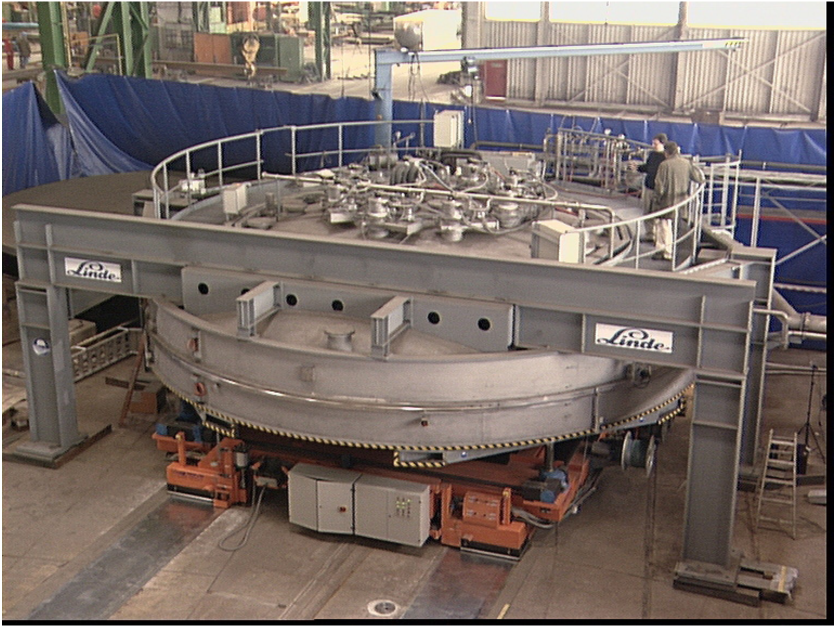
Materials and Refractive Indices



Mirror Coatings



VLT Coating Chamber



VLT Coating Chamber with Magnetogron

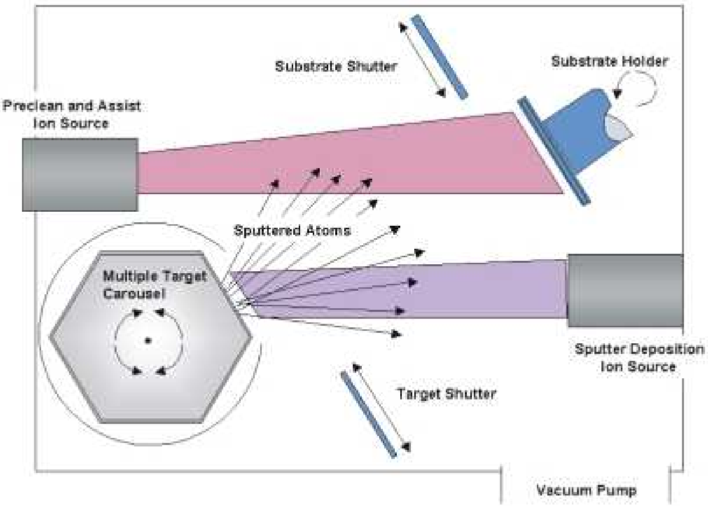
Evaporation

- evaporation of solid material through high electric current (e.g. classic Al mirror coatings)
- sputtering: plasma glow discharge ejects material from solid substance

Deposition

- uncontrolled ballistic flights, mechanical shields to homogenize beam
- ion-assisted deposition

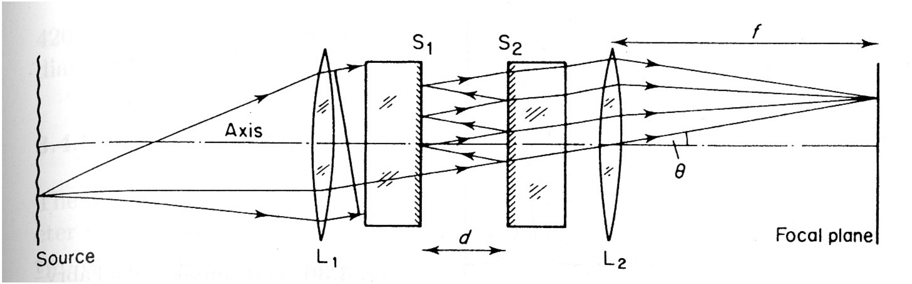
Ion-Beam Deposition



www.4waveinc.com/ibd.html

Fabry-Perot Filter

Tunable Filter



- invented by Fabry and Perot in 1899
- interference between partially transmitting plates containing medium with index of refraction n
- angle of incidence in material θ, distance d
- path difference between successive beams: $\Delta = 2nd\cos\theta$

Fabry Perot continued

- path difference between successive beams: $\Delta = 2nd\cos\theta$
- phase difference: $\delta = 2\pi\Delta/\lambda = 4\pi nd\cos\theta/\lambda$
- incoming wave: $e^{i\omega t}$
- intensity transmission at surface: T
- intensity reflectivity at surface: R
- outgoing wave is the *coherent sum* of all beams

$$Ae^{i\omega t} = Te^{i\omega t} + TRe^{i(\omega t+\delta)} + TR^2e^{i(\omega t+2\delta)} + ...$$

- write this as

$$A = T(1 + Re^{i\delta} + R^2e^{i2\delta} + ... = \frac{T}{1 - Re^{i\delta}}$$

Fabry Perot continued

- emerging amplitude

$$A = \frac{T}{1 - Re^{i\delta}}$$

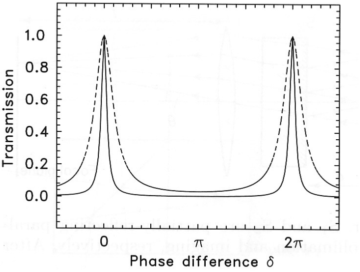
- emerging intensity is therefore

$$I = AA^* = \frac{T^2}{1 - 2R\cos\delta + R^2} = \frac{T^2}{(1-R)^2 + 4R\sin^2\frac{\delta}{2}}$$

- with $I_{\max} = T^2/(1-R)^2$

$$I = I_{\max}\frac{1}{1 + \frac{4R}{(1-R)^2}\sin^2\frac{\delta}{2}}$$

Fabry Perot Properties



$$I = I_{\max} \frac{1}{1 + \frac{4R}{(1-R)^2} \sin^2 \frac{\delta}{2}}$$

- transmission is periodic
- distance between transmission peaks, *free spectral range*

$$\mathrm{FSR} = \frac{\lambda_0^2}{2nd \cos\theta}$$

- Full-Width at Half Maximum (FWHM) $\Delta\lambda$

$$\Delta\lambda = \mathrm{FSR}/F$$

- *finesse F*

$$F = \frac{\pi\sqrt{R}}{1-R}$$

Anti-Reflection Coatings

Reflection Off Uncoated Substrate

- reflectivity of bare substrate:

$$R = \left(\frac{n_m - n_s}{n_m + n_s}\right)^2$$

  - fused silica at 600 nm: $n_s = 1.46 \Rightarrow R = 0.035$
  - extra-dense flint glass at 600 nm: $n_s = 1.75 \Rightarrow R = 0.074$
  - silicon at 600 nm: $n_s = 3.96 \Rightarrow R = 0.6$
- loss in transparency
- ghost reflections

Analytical treatment of single layer

- assume single-layer coating
- determine optimum thickness and index of material
- amplitude reflection from coating/air interface

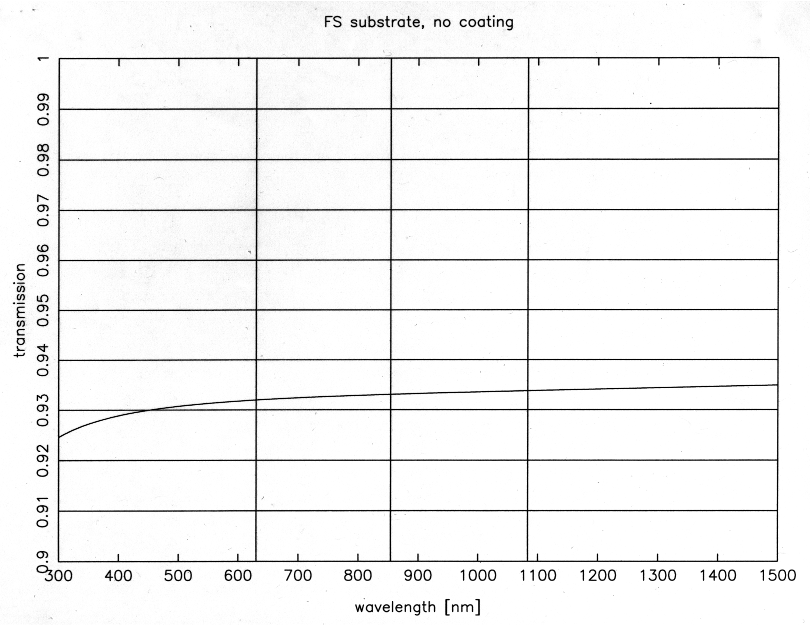
$$A_1 = \frac{n_1 - 1}{n_1 + 1}$$

- amplitude reflection from coating/substrate interface

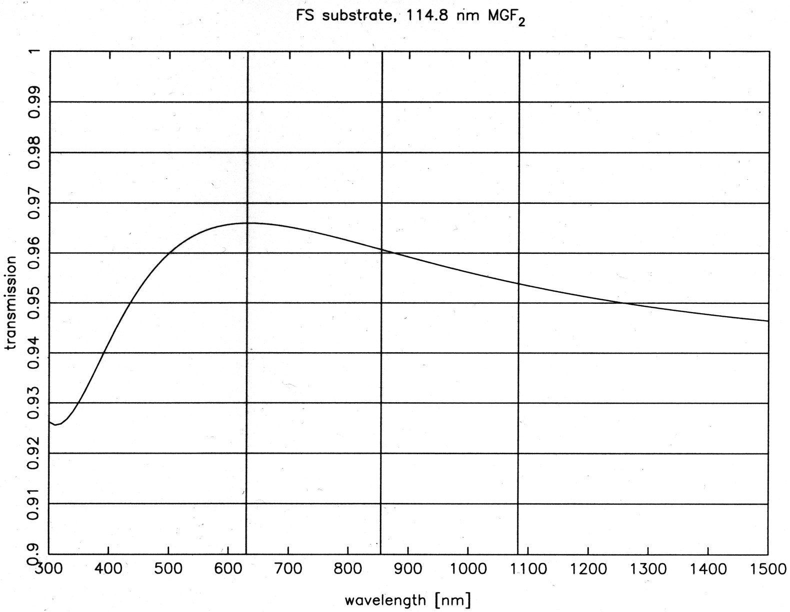
$$A_1 = \frac{n_s - n_1}{n_s + n_1}$$

- amplitudes subtract for 180 degree phase difference $\Rightarrow$ coating should have optical path length $\lambda/4$ thick
- best cancellation for $n_1 = \sqrt{n_s}$

Uncoated Fused Silica

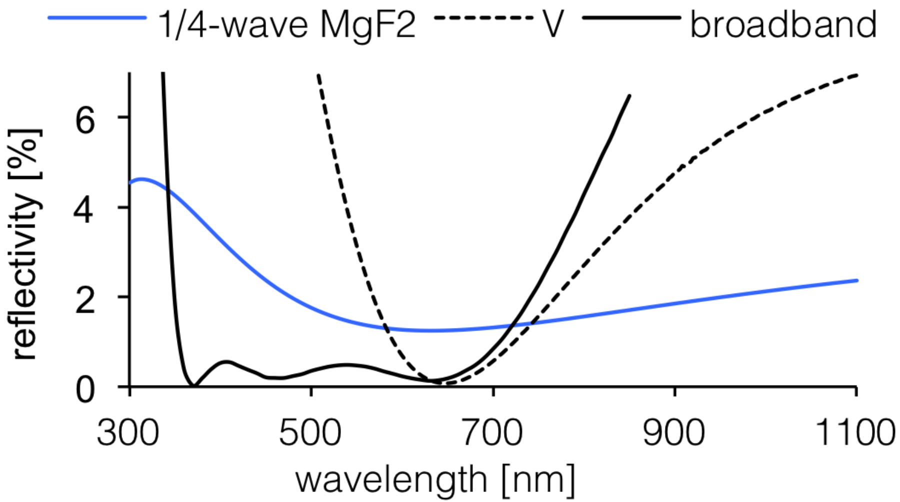


MgF_2 Coating on Fused Silica



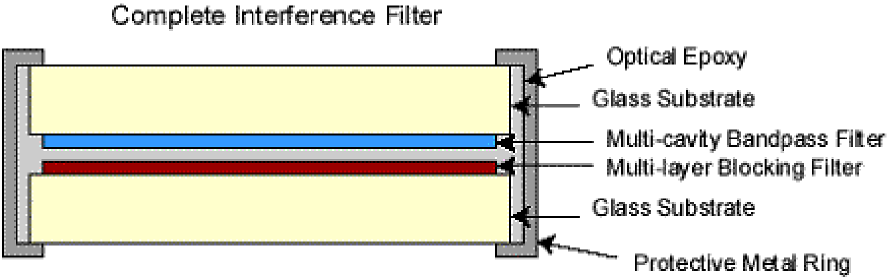
Multi-Layer Coatings on Fused Silica

AR Coating Types



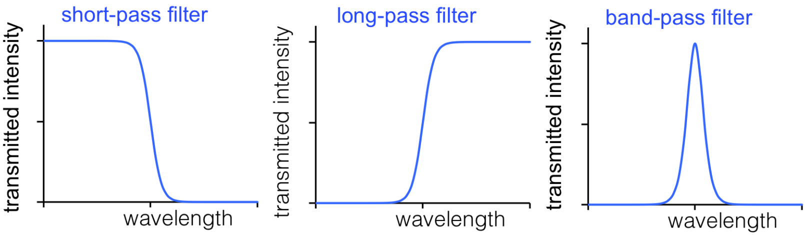
Interference Filters

Overview

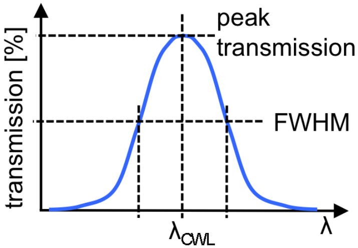


- many layers can achieve many things
- band-pass, long-pass, short-pass, dichroic filters
- colored glass substrates often used in addition to coatings
- sensitivie to angle of incidence
- evaporated coatings are very temperature sensitive

Types of Interference Filters

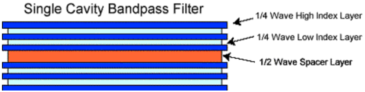


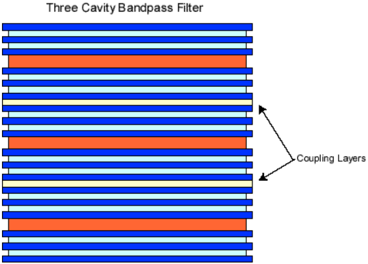
Bandpass Interference Filter Terminology



- **Bandpass:** range of wavelengths passed by filter
- **Blocking:** degree of attenuation outside of passband
- **Center Wavelength (CWL):** wavelength at midpoint of passband FWHM
- **Number of cavities:** Fabry-Perot-like thin-film arrangement
- **Full-width Half-Maximum (FWHM):** width of bandpass at one-half of peak transmission
- **Peak Transmission in %:** maximum transmission in passband

Cavities





www.fluorescence.com/tutorial/int-filt.htm

- basic design element: Fabry-Perot cavity
- cavity: $\lambda/2$ spacer and reflective multi-layer coatings on either side
- stacks of cavities provide much better performance

Multiple Cavity Performance