Lecture 5: Polarization

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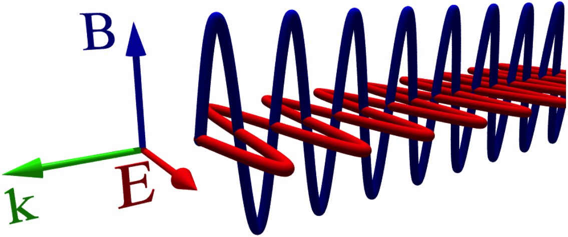
Polarized Light in the Universe

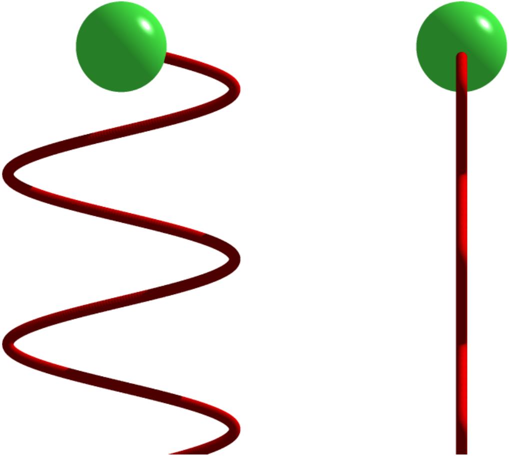


Polarization indicates *anisotropy* ⇒ not all directions are equal

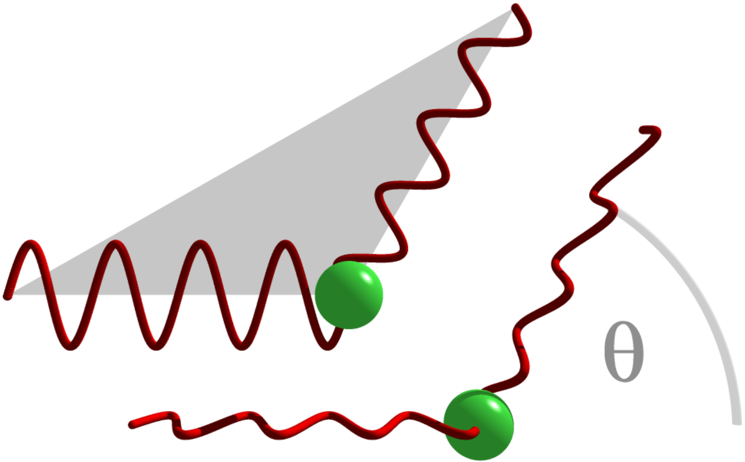
Typical anisotropies introduced by

- geometry (not everything is spherically symmetric)
- temperature gradients
- density gradients
- magnetic fields
- electrical fields

Scattering Polarization 1

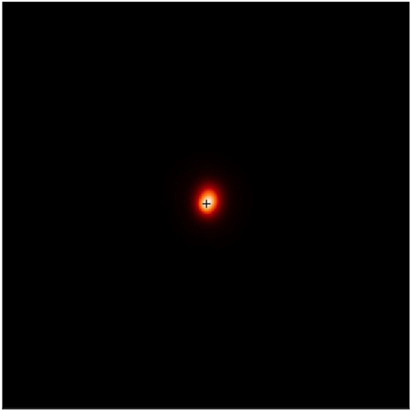


Scattering Polarization 2

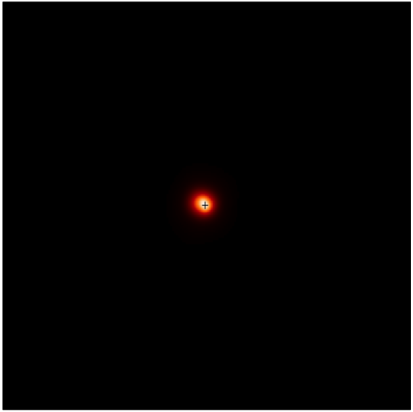


The Power of Polarized Light Measurements

T Tauri in intensity

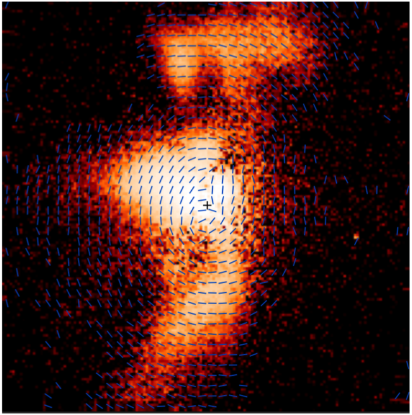


MWC147 in intensity

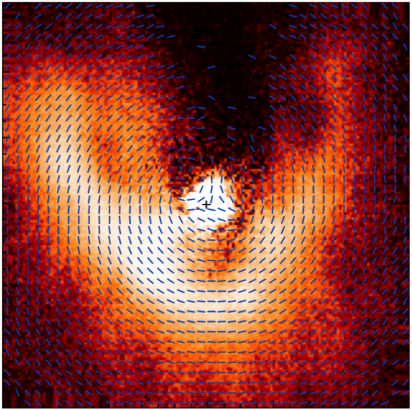


The Power of Polarimetry

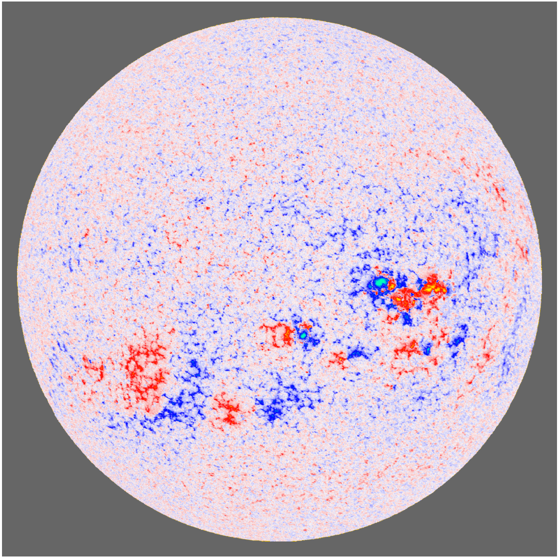
T Tauri in Linear Polarization



MWC147 in Linear Polarization



Solar Magnetic Field Maps from Longitudinal Zeeman Effect



Summary of Polarization Origin

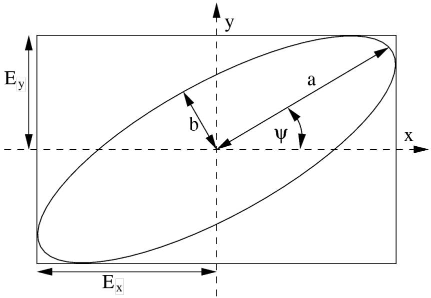
- Plane Vector Wave ansatz $\vec{E} = \vec{E}_0 e^{i\left(\vec{k}\cdot\vec{x}-\omega t\right)}$
- spatially, temporally constant vector $\vec{E}_0$ lays in plane perpendicular to propagation direction $\vec{k}$
- represent $\vec{E}_0$ in 2-D basis, unit vectors $\vec{e}_x$ and $\vec{e}_y$, both perpendicular to $\vec{k}$

  $$\vec{E}_0 = E_x\vec{e}_x + E_y\vec{e}_y.$$

  E_x, E_y: arbitrary complex scalars
- damped plane-wave solution with given ω, $\vec{k}$ has 4 degrees of freedom (two complex scalars)
- additional property is called *polarization*
- many ways to represent these four quantities
- if E_x and E_y have identical phases, $\vec{E}$ oscillates in fixed plane
- sum of plane waves is also a solution

Polarization Ellipse

Polarization Ellipse



Polarization

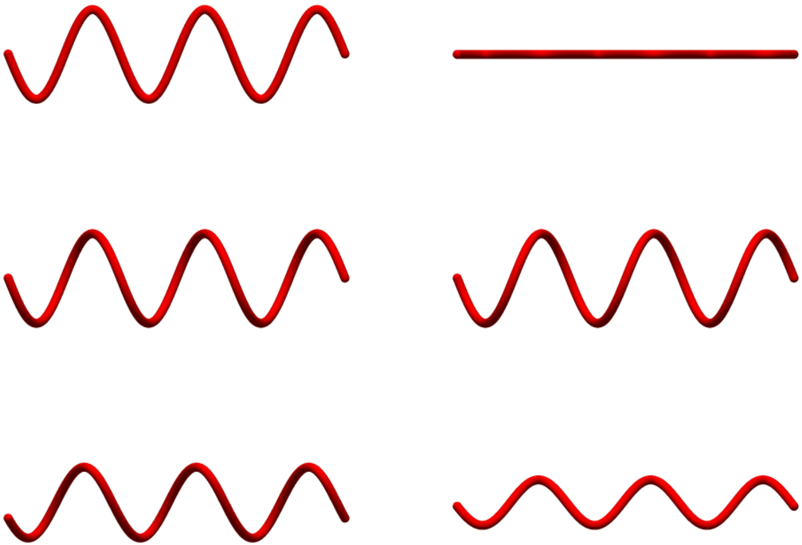
$$\vec{E}(t) = \vec{E}_0 e^{i\left(\vec{k}\cdot\vec{x}-\omega t\right)}$$

$$\vec{E}_0 = |E_x| e^{i\delta_x} \vec{e}_x + |E_y| e^{i\delta_y} \vec{e}_y$$

- wave vector in z-direction
- $\vec{e}_x$, $\vec{e}_y$: unit vectors in x, y
- $|E_x|$, $|E_y|$: (real) amplitudes
- $\delta_{x,y}$: (real) phases

Polarization Description

- 2 complex scalars not the most useful description
- at given $\vec{x}$, time evolution of $\vec{E}$ described by *polarization ellipse*
- ellipse described by axes a, b, orientation ψ



Jones Formalism

Jones Vectors

$$\vec{E}_0 = E_x \vec{e}_x + E_y \vec{e}_y$$

- beam in z-direction
- $\vec{e}_x$, $\vec{e}_y$ unit vectors in x, y-direction
- complex scalars $E_{x,y}$
- Jones vector

$$\vec{e} = \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

- phase difference between E_x, E_y multiple of π, electric field vector oscillates in a fixed plane $\Rightarrow$ *linear polarization*
- phase difference $\pm\frac{\pi}{2} \Rightarrow$ *circular polarization*

Summing and Measuring Jones Vectors

$$\vec{E}_0 = E_x \vec{e}_x + E_y \vec{e}_y, \quad \vec{e} = \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

- Maxwell's equations linear $\Rightarrow$ sum of two solutions again a solution
- Jones vector of sum of two waves $=$ sum of Jones vectors of individual waves if wave vectors $\vec{k}$ the same
- waves must have the same wave vector and direction of propagation
- addition of Jones vectors: *coherent* superposition of waves
- elements of Jones vectors are not observed directly
- observables always depend on products of elements of Jones vectors, i.e. intensity $I = \vec{e} \cdot \vec{e}^* = e_x e_x^* + e_y e_y^*$

Jones matrices

- influence of medium on polarization described by 2×2 complex *Jones matrix* J

$$\vec{e}' = \mathrm{J}\vec{e} = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix} \vec{e}$$

- assumes that medium not affected by polarization state
- different media 1 to N in order of wave direction $\Rightarrow$ combined influence described by

$$\mathrm{J} = \mathrm{J}_N \mathrm{J}_{N-1} \cdots \mathrm{J}_2 \mathrm{J}_1$$

- order of matrices in product is crucial
- Jones calculus describes coherent superposition of polarized light

Linear Polarization

- horizontal: $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
- vertical: $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$
- 45°: $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Circular Polarization

- left: $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}$
- right: $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix}$

Notes on Jones Formalism

- Jones formalism operates on amplitudes, not intensities
- coherent superposition important for coherent light (lasers, interference effects)
- Jones formalism describes 100% polarized light

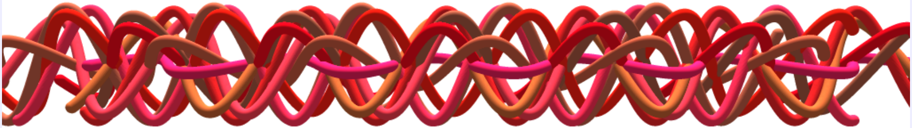
Stokes Vector

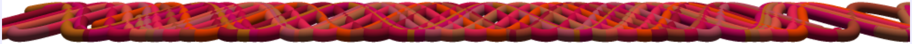
- formalism to describe polarization of quasi-monochromatic light
- directly related to measurable intensities
- Stokes vector fulfills these requirements

$$\vec{I} = \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} E_x E_x^* + E_y E_y^* \\ E_x E_x^* - E_y E_y^* \\ E_x E_y^* + E_y E_x^* \\ i\left(E_x E_y^* - E_y E_x^*\right) \end{pmatrix} = \begin{pmatrix} |E_x|^2 + |E_y|^2 \\ |E_x|^2 - |E_y|^2 \\ 2|E_x||E_y|\cos\delta \\ 2|E_x||E_y|\sin\delta \end{pmatrix}$$

Jones vector elements $E_{x,y}$, real amplitudes $|E_{x,y}|$, phase difference $\delta = \delta_y - \delta_x$

- $I^2 \geq Q^2 + U^2 + V^2$
- can describe unpolarized ($Q = U = V = 0$) light





Stokes Vector Interpretation

$$\vec{I} = \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} \text{intensity} \\ \text{linear } 0^\circ - \text{linear } 90^\circ \\ \text{linear } 45^\circ - \text{linear } 135^\circ \\ \text{circular left} - \text{right} \end{pmatrix}$$

- *degree of polarization*

$$P = \frac{\sqrt{Q^2 + U^2 + V^2}}{I}$$

  1 for fully polarized light, 0 for unpolarized light
- summing of Stokes vectors $=$ *incoherent* adding of quasi-monochromatic light waves

Linear Polarization

- horizontal: $\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$
- vertical: $\begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix}$
- 45°: $\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$

Circular Polarization

- left: $\begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$
- right: $\begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix}$

Mueller Matrices

- 4×4 real Mueller matrices describe (linear) transformation between Stokes vectors when passing through or reflecting from media

$$\vec{I}' = \mathrm{M}\vec{I}\,,$$

$$\mathrm{M} = \begin{pmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{pmatrix}$$

- N optical elements, combined Mueller matrix is

$$\mathrm{M}' = \mathrm{M}_N \mathrm{M}_{N-1} \cdots \mathrm{M}_2 \mathrm{M}_1$$

Rotating Mueller Matrices

- optical element with Mueller matrix M
- Mueller matrix of the same element rotated by θ around the beam given by

$$\mathrm{M}(\theta) = \mathrm{R}(-\theta)\mathrm{M}\mathrm{R}(\theta)$$

with

$$\mathrm{R}\left(\theta\right) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\theta & \sin 2\theta & 0 \\ 0 & -\sin 2\theta & \cos 2\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Polarizers

- polarizer: optical element that produces polarized light from unpolarized input light
- linear, circular, or in general elliptical polarizer, depending on type of transmitted polarization
- linear polarizers by far the most common
- large variety of polarizers

Jones Matrix for Linear Polarizers

- Jones matrix for linear polarizer: $\mathrm{J}_p = \begin{pmatrix} p_x & 0 \\ 0 & p_y \end{pmatrix}$
- $0 \leq p_x \leq 1$ and $0 \leq p_y \leq 1$, real: transmission factors for x, y-components of electric field: $E'_x = p_x E_x, \quad E'_y = p_y E_y$
- $p_x = 1,\ p_y = 0$: linear polarizer in $+Q$ direction
- $p_x = 0,\ p_y = 1$: linear polarizer in $-Q$ direction
- $p_x = p_y$: neutral density filter

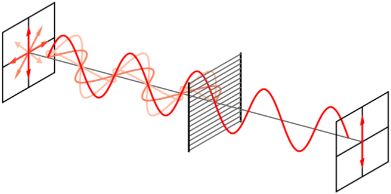
Mueller Matrix for Linear Polarizers

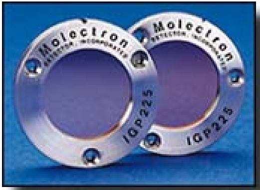
$$\mathsf{M}_{\mathrm{horizontal}} = \frac{1}{2}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \mathsf{M}_{\mathrm{vertical}} = \frac{1}{2}\begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Mueller Matrix for Ideal Linear Polarizer at Angle θ

$$\mathsf{M}_{\mathrm{pol}}(\theta) = \frac{1}{2}\begin{pmatrix} 1 & \cos 2\theta & \sin 2\theta & 0 \\ \cos 2\theta & \cos^2 2\theta & \sin 2\theta \cos 2\theta & 0 \\ \sin 2\theta & \sin 2\theta \cos 2\theta & \sin^2 2\theta & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

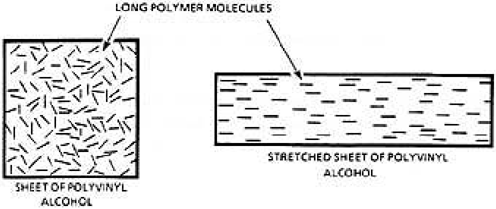
Wire Grid Polarizers





- parallel conducting wires, spacing $d \lesssim \lambda$ act as polarizer
- electric field parallel to wires induces electrical currents in wires
- induced electrical current reflects polarization parallel to wires
- polarization perpendicular to wires is transmitted
- rule of thumb:
  - $d < \lambda/2 \Rightarrow$ strong polarization
  - $d \gg \lambda \Rightarrow$ high transmission of both polarization states (weak polarization)

Polaroid-type Polarizers



- developed by Edwin Land in 1938 ⇒ Polaroid
- sheet polarizers: stretched polyvynil alcohol (PVA) sheet, laminated to sheet of cellulose acetate butyrate, treated with iodine
- PVA-iodine complex analogous to short, conducting wire
- cheap, can be manufactured in large sizes

Crystal-Based Polarizers

- crystals make highest-quality polarizers
- precise arrangement of atom/molecules and anisotropy of index of refraction separate incoming beam into two beams with precisely orthogonal linear polarization states
- work well over large wavelength range
- many different configurations
- calcite most often used in crystal-based polarizers because of large birefringence, low absorption in visible
- many other suitable crystals including quartz

Indices of Refraction of Crystal

- in anisotropic material: dielectric constant is a tensor
- Maxwell equations imply symmetric dielectric tensor

$$\epsilon = \epsilon^T = \begin{pmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{12} & \epsilon_{22} & \epsilon_{23} \\ \epsilon_{13} & \epsilon_{23} & \epsilon_{33} \end{pmatrix}$$

- symmetric tensor of rank 2 $\Rightarrow$ Cartesian coordinate system exists where tensor is diagonal
- 3 *principal indices of refraction* in coordinate system spanned by principal axes

$$\vec{D} = \begin{pmatrix} n_x^2 & 0 & 0 \\ 0 & n_y^2 & 0 \\ 0 & 0 & n_z^2 \end{pmatrix} \vec{E}$$

Uniaxial Materials

- anisotropic materials: $n_x \neq n_y \neq n_z$
- *uniaxial materials*: $n_x = n_y \neq n_z$
- *optic axis* is axis that has different index of refraction, also called *c* or *crystallographic axis*
- *ordinary index*: $n_o = n_x = n_y$
- *extraordinary index*: $n_e = n_z$
- *fast axis*: axis with smallest index
- rotation of coordinate system around *z* has no effect
- most materials used in polarimetry are (almost) uniaxial, e.g. calcite

Wave Propagation in Uniaxial Media

- two solutions to wave equation with orthogonal linear polarizations
- ordinary ray propagates as in isotropic medium with index n_o
- extraordinary ray sees direction-dependent index of refraction

$$n_2(\theta) = \frac{n_o n_e}{\sqrt{n_o^2 \sin^2\theta + n_e^2 \cos^2\theta}}$$

n_2 direction-dependent index of refraction of the extraordinary ray
n_o ordinary index of refraction
n_e extraordinary index of refraction
θ angle between extraordinary wave vector and optic axis

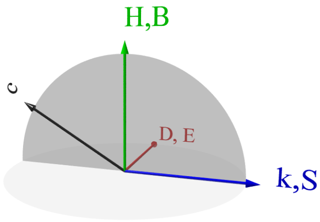
- for $\theta = 0$ $n_2 = n_o$, for $\theta = 90^\circ$ $n_2 = n_e$

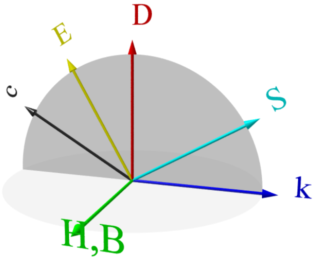
Energy Propagation in Uniaxial Media

- ordinary ray propagates along wave vector $\vec{k}$ with electric field perpendicular to c-axis
- extraordinary ray and wave vector make *dispersion angle* α

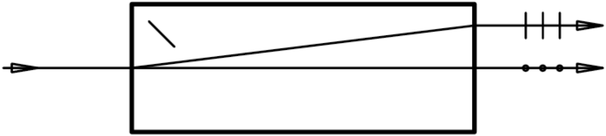
$$\tan\alpha = \frac{(n_e^2 - n_o^2)\tan\theta}{n_e^2 + n_o^2\tan^2\theta}$$

- dispersion angle $\alpha = 0$ for $\theta = 0$ or $\theta = 90^\circ$
- extraordinary electric field in plane of wave vector and optic axis



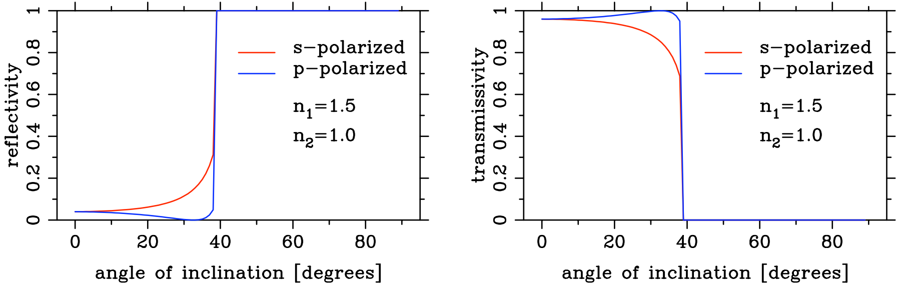


Crystal-Based Polarizing Beamsplitter



- one linear polarization goes straight through as in isotropic material (*ordinary ray*)
- perpendicular linear polarization propagates at an angle (*extraordinary ray*)
- different optical path lengths
- crystal aberrations

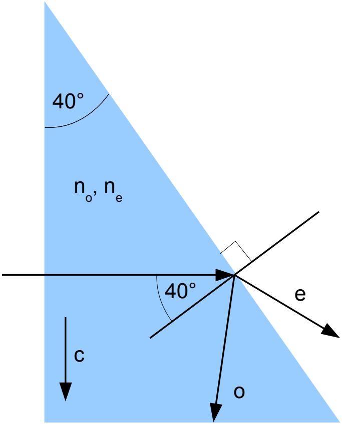
Total Internal Reflection (TIR)



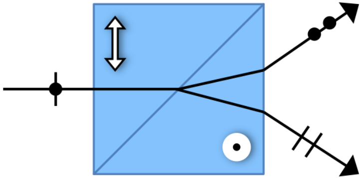
- Snell's law: $\sin\theta_t = \frac{n_1}{n_2}\sin\theta_i$
- high-index to lower index medium (e.g. glass to air): $n_1/n_2 > 1$
- right-hand side > 1 for $\sin\theta_i > \frac{n_2}{n_1}$
- all light is reflected in high-index medium ⇒ *total internal reflection*
- transmitted wave has complex phase angle ⇒ damped wave along interface

Total Internal Reflection (TIR) in Crystals

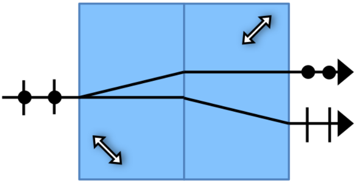
- $n_o \neq n_e \Rightarrow$ one beam can be totally reflected while other is transmitted
- principal of most crystal polarizers
- calcite at 632.8 nm: $n_o = 1.6558$, $n_e = 1.4852$
- requirement for total reflection $\frac{n_2}{\sin}\theta > 1$
- entrance: extraordinary ray not refracted, two rays propagate according to indices n_o,n_e
- exit: rays (and wave vectors) at 40° to surface
- for $\theta = 40^\circ$ extraordinary ray is transmitted, ordinary ray undergoes TIR



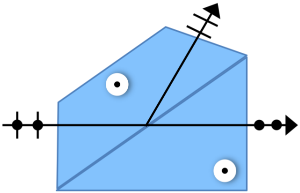
Wollaston Prism



Savart Plate

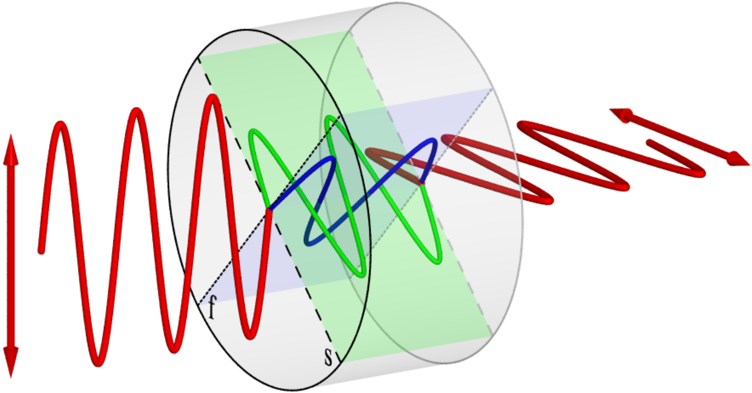


Foster Prism



Linear Retarders or Wave Plates

- uniaxial crystal, optic axis parallel to surface ($\theta = 90^\circ$)
- fast axis (f) has lowest index, slow axis (s) has highest index
- example: halfwave retarder



Phase Delay between Ordinary and Extraordinary Rays

- ordinary and extraordinary wave propagate in same direction
- ordinary ray propagates with speed $\frac{c}{n_o}$
- extraordinary beam propagates at different speed $\frac{c}{n_e}$
- $\vec{E}_o$, $\vec{E}_e$ perpendicular to each other $\Rightarrow$ plane wave with arbitrary polarization can be (coherently) decomposed into components parallel to $\vec{E}_o$ and $\vec{E}_e$
- 2 components will travel at different speeds
- (coherently) superposing 2 components after distance $d \Rightarrow$ phase difference between 2 components $\frac{\omega}{c}(n_e - n_o)d$ radians
- phase difference $\Rightarrow$ change in polarization state
- basis for constructing linear retarders

Retarder Properties

- does not change intensity or degree of polarization
- characterized by two (not identical, not trivial) Stokes vectors of incoming light that are not changed by retarder ⇒ *eigenvectors* of retarder
- depending on polarization described by eigenvectors, retarder is
  - *linear retarder*
  - *circular retarder*
  - *elliptical retarder*
- linear, circular retarders are special cases of elliptical retarders
- circular retarders sometimes called *rotators* since they rotate the orientation of linearly polarized light
- linear retarders by far the most common type of retarder
- retardation depends strongly on wavelength
- achromatic retarders: combinations of different materials or the same materials with different fast axis directions

Jones Matrix for Linear Retarders

- linear retarder with fast axis at 0° characterized by Jones matrix

$$\mathrm{J}_r(\delta) = \begin{pmatrix} e^{i\delta} & 0 \\ 0 & 1 \end{pmatrix}, \quad \mathrm{J}_r(\delta) = \begin{pmatrix} e^{i\frac{\delta}{2}} & 0 \\ 0 & e^{-i\frac{\delta}{2}} \end{pmatrix}$$

- δ: phase shift between two linear polarization components (in radians)
- absolute phase does not matter

Mueller Matrix for Linear Retarder

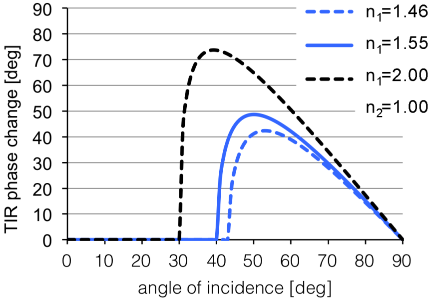
$$\mathrm{M}_r = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\delta & -\sin\delta \\ 0 & 0 & \sin\delta & \cos\delta \end{pmatrix}$$

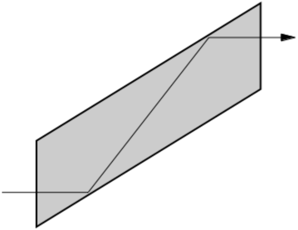
Phase Change on Total Internal Reflection (TIR)

- TIR induces phase change that depends on polarization
- complex ratios: $r_{s,p} = |r_{s,p}| e^{i\delta_{s,p}}$
- phase change $\delta = \delta_s - \delta_p$

$$\tan\frac{\delta}{2} = \frac{\cos\theta_i \sqrt{\sin^2\theta_i - \left(\frac{n_2}{n_1}\right)^2}}{\sin^2\theta_i}$$

- relation valid between critical angle and grazing incidence
- at critical angle and grazing incidence $\delta = 0$

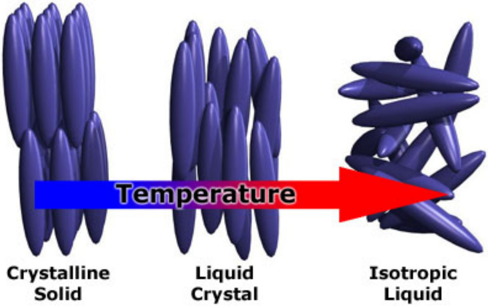




Variable Retarders

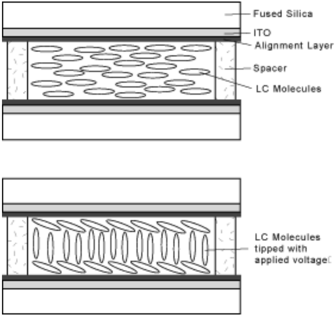
- sensitive polarimeters requires retarders whose properties (retardance, fast axis orientation) can be varied quickly (*modulated*)
- retardance changes (change of birefringence):
  - liquid crystals
  - Faraday, Kerr, Pockels cells
  - piezo-elastic modulators (PEM)
- fast axis orientation changes (change of *c*-axis direction):
  - rotating fixed retarder
  - ferro-electric liquid crystals (FLC)

Liquid Crystals



- liquid crystals: fluids with elongated molecules
- at high temperatures: liquid crystal is isotropic
- at lower temperature: molecules become ordered in orientation and sometimes also space in one or more dimensions
- liquid crystals can line up parallel or perpendicular to external electrical field

Liquid Crystal Retarders



- dielectric constant anisotropy often large $\Rightarrow$ very responsive to changes in applied electric field
- birefringence δn can be very large (larger than typical crystal birefringence)
- liquid crystal layer only a few μm thick
- birefringence shows strong temperature dependence