Lecture 3: Physical Optics

Content

1. Motivation
2. Interference
3. Coherence
4. Two-Element Interferometer
5. Fraunhofer and Fresnel Diffraction
6. Transfer Functions
7. Van Cittert-Zernike Theorem

3 Ways to Understanding Optics

1. **Geometrical Optics:** Light can be described as rays
2. **Physical Optics:** Light can be described as waves
3. **Quantum Optics:** Light can be described as discrete particles

Very Large Telescope Interferometer (VLTI)

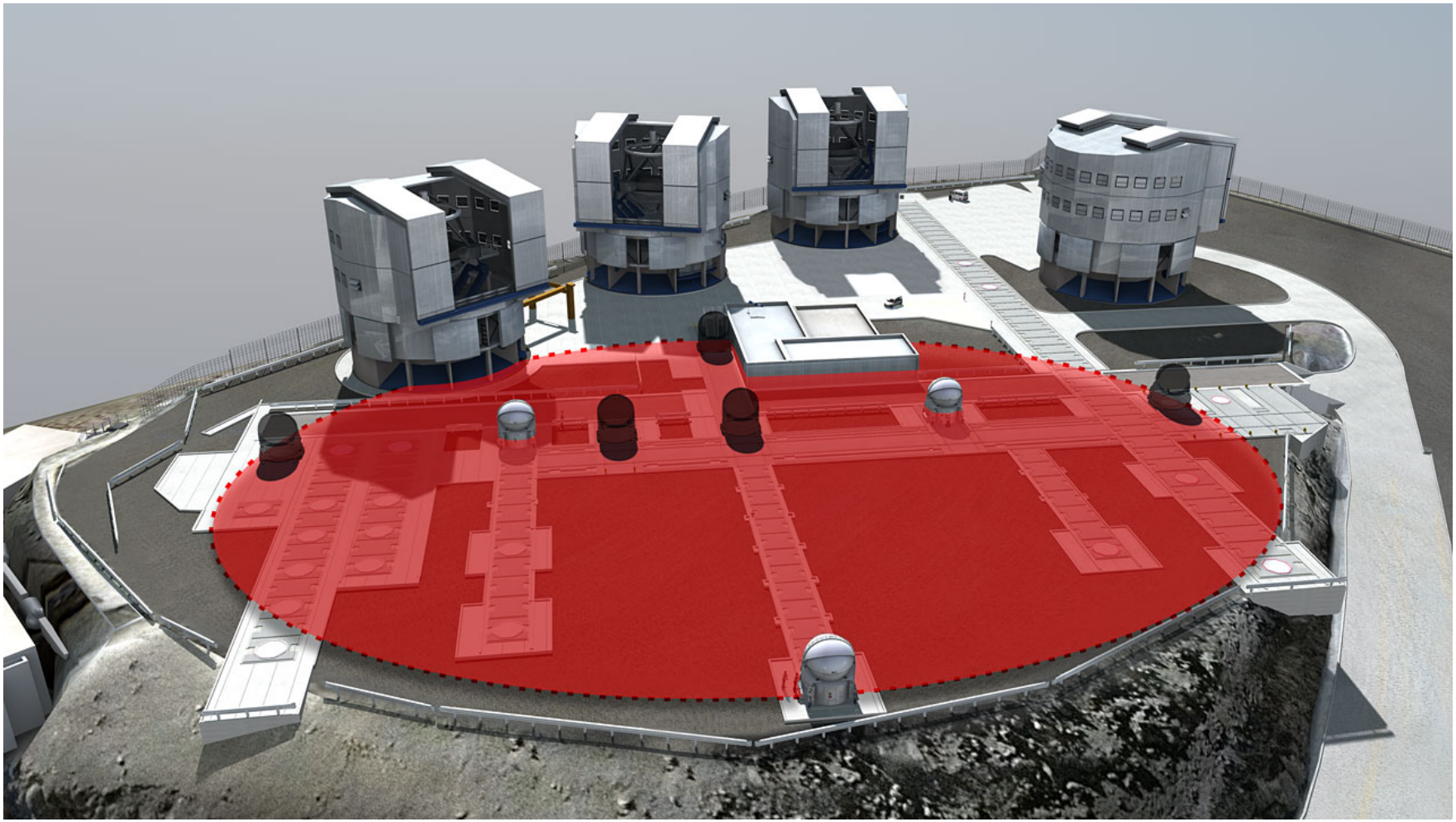


Image credit: ESO

T Leporis: a Mira-like Star with the VLTI in the NIR

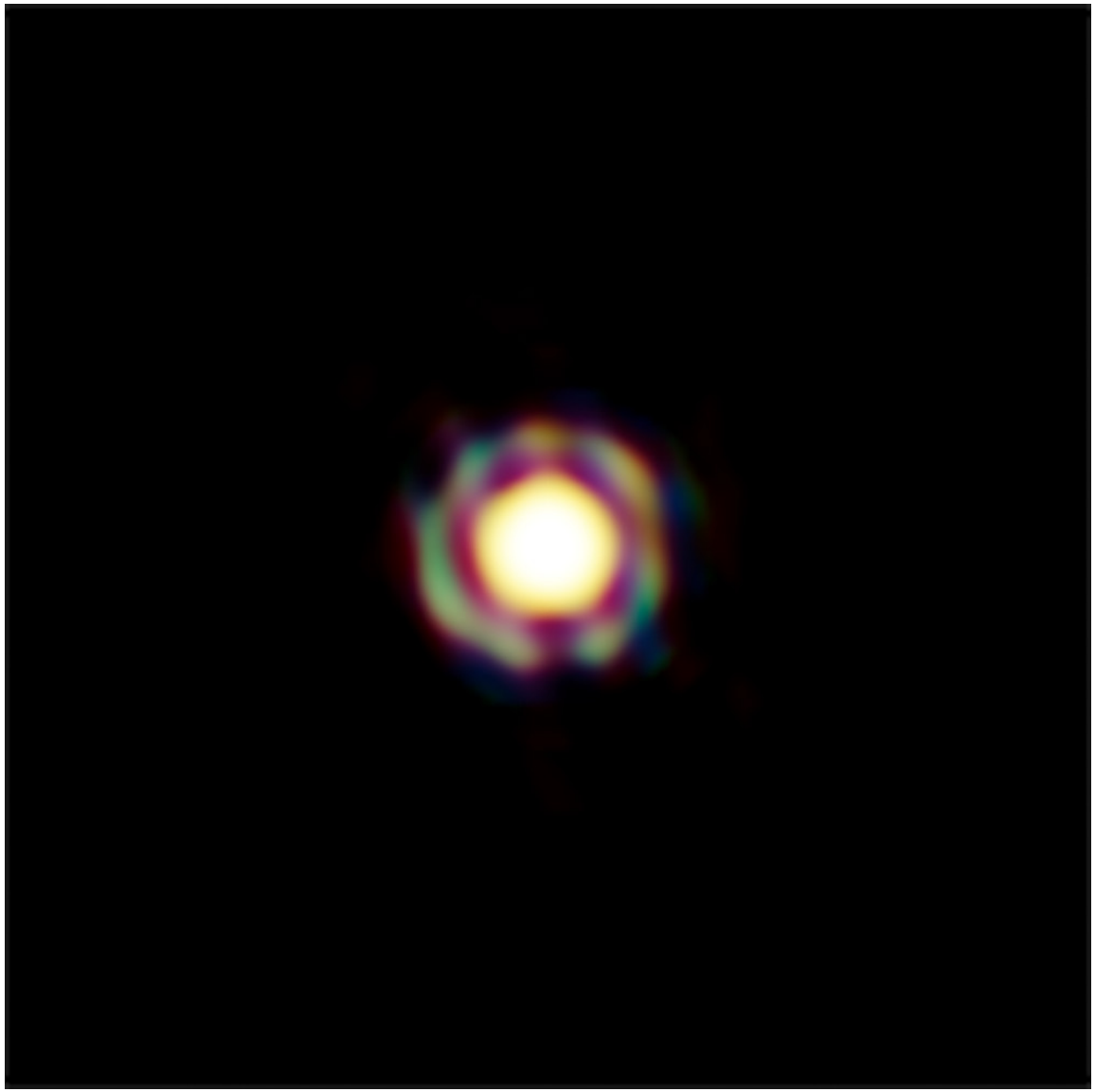
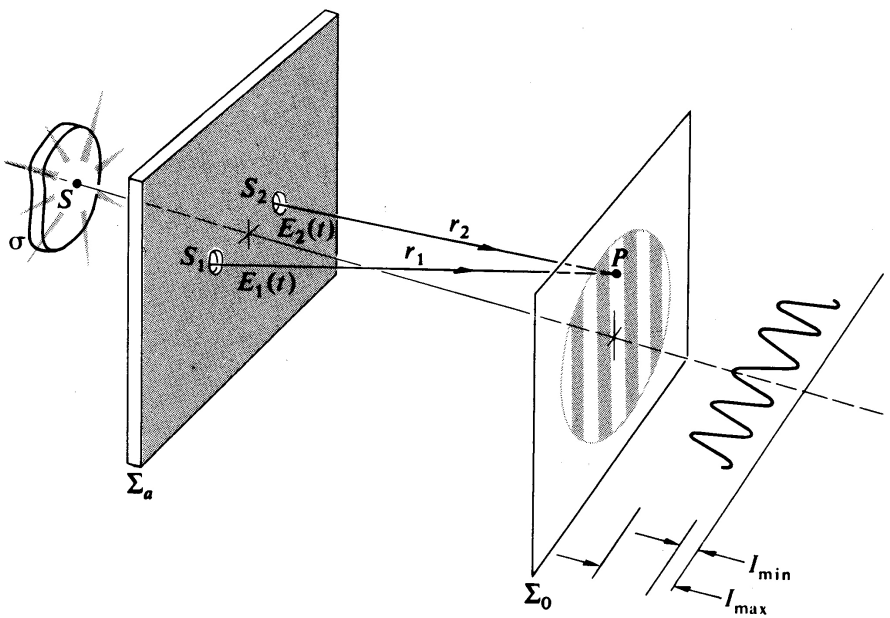


Image credit: ESO/J.-B. Le Bouquin et al.

Young's Double Slit Experiment

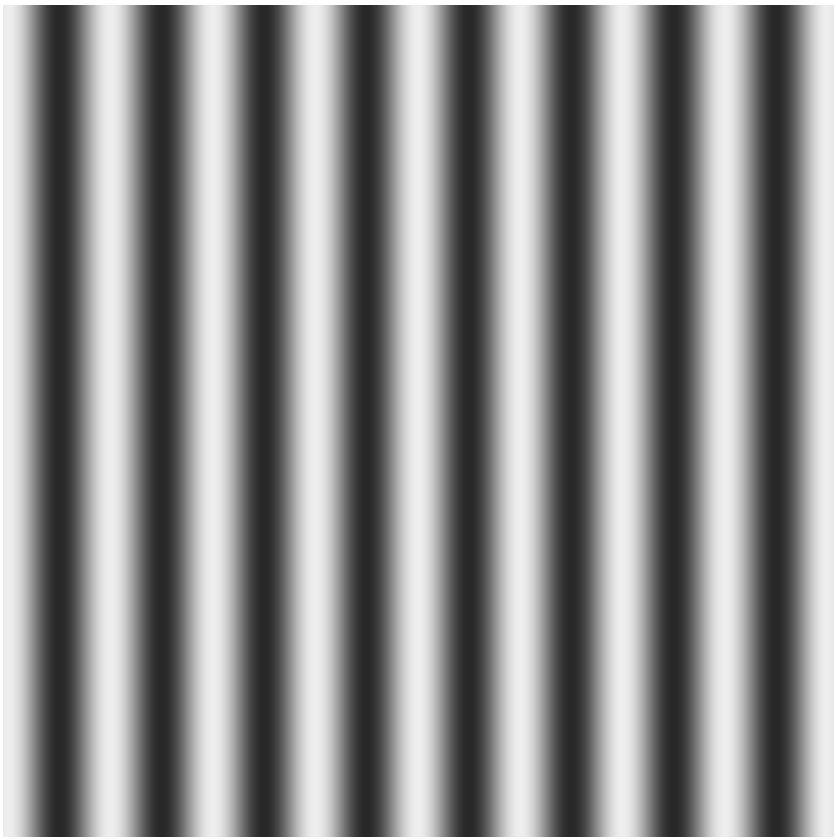


- monochromatic wave
- infinitely small (pin)holes
- source S generates fields $\tilde{E}(\vec{r}_1, t) \equiv \tilde{E}_1(t)$ at S_1 and $\tilde{E}(\vec{r}_2, t) \equiv \tilde{E}_2(t)$ at S_2
- two spherical waves from pinholes interfere on screen
- electrical field at P (without propagators i/λ)

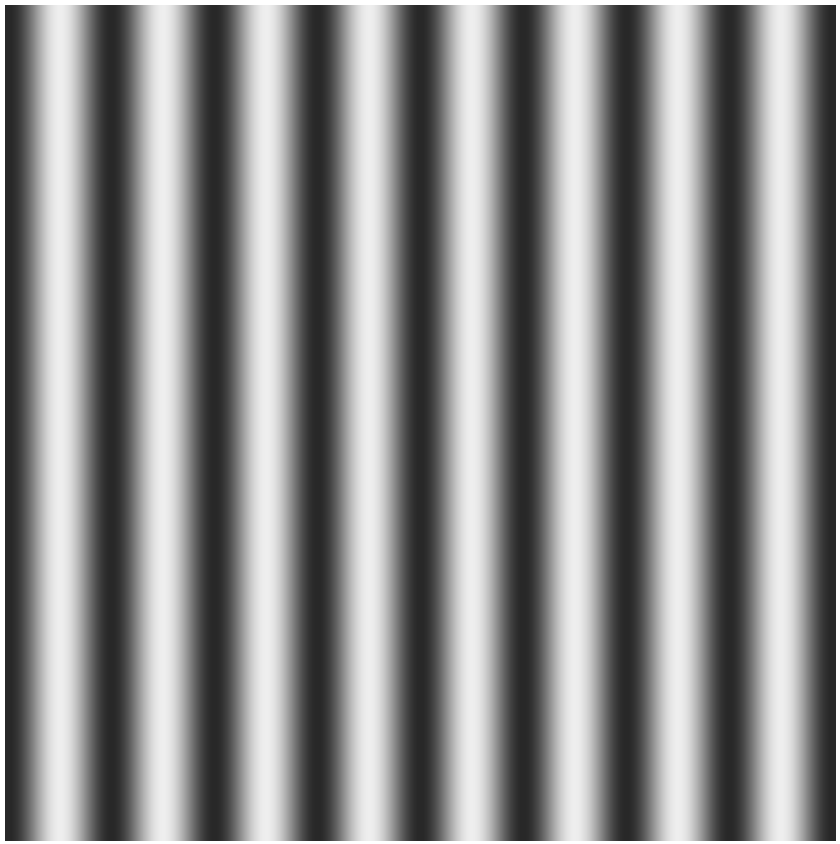
$$\tilde{E}_P(t) = \tilde{E}_1(t - t_1) + \tilde{E}_2(t - t_2)$$

- $t_1 = r_1/c$, $t_2 = r_2/c$
- r_1, r_2: path lengths from S_1, S_2 to P

no tilt



tilt by 0.5 λ/d

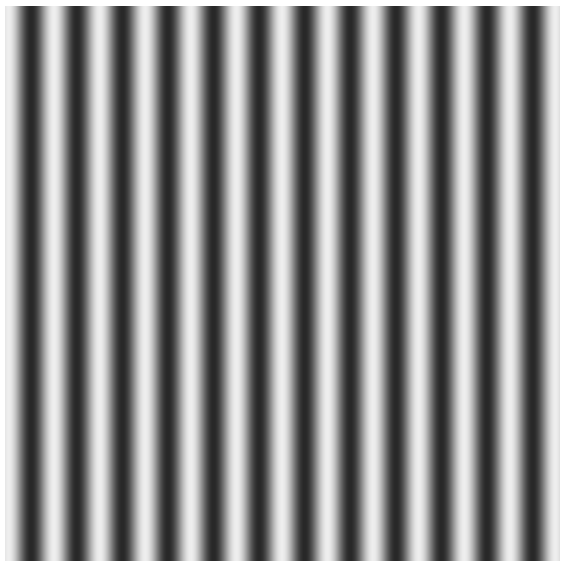


Change in Angle of Incoming Wave

- phase of fringe pattern changes, but not fringe spacing
- tilt of λ/d produces identical fringe pattern

long wavelength

short wavelength

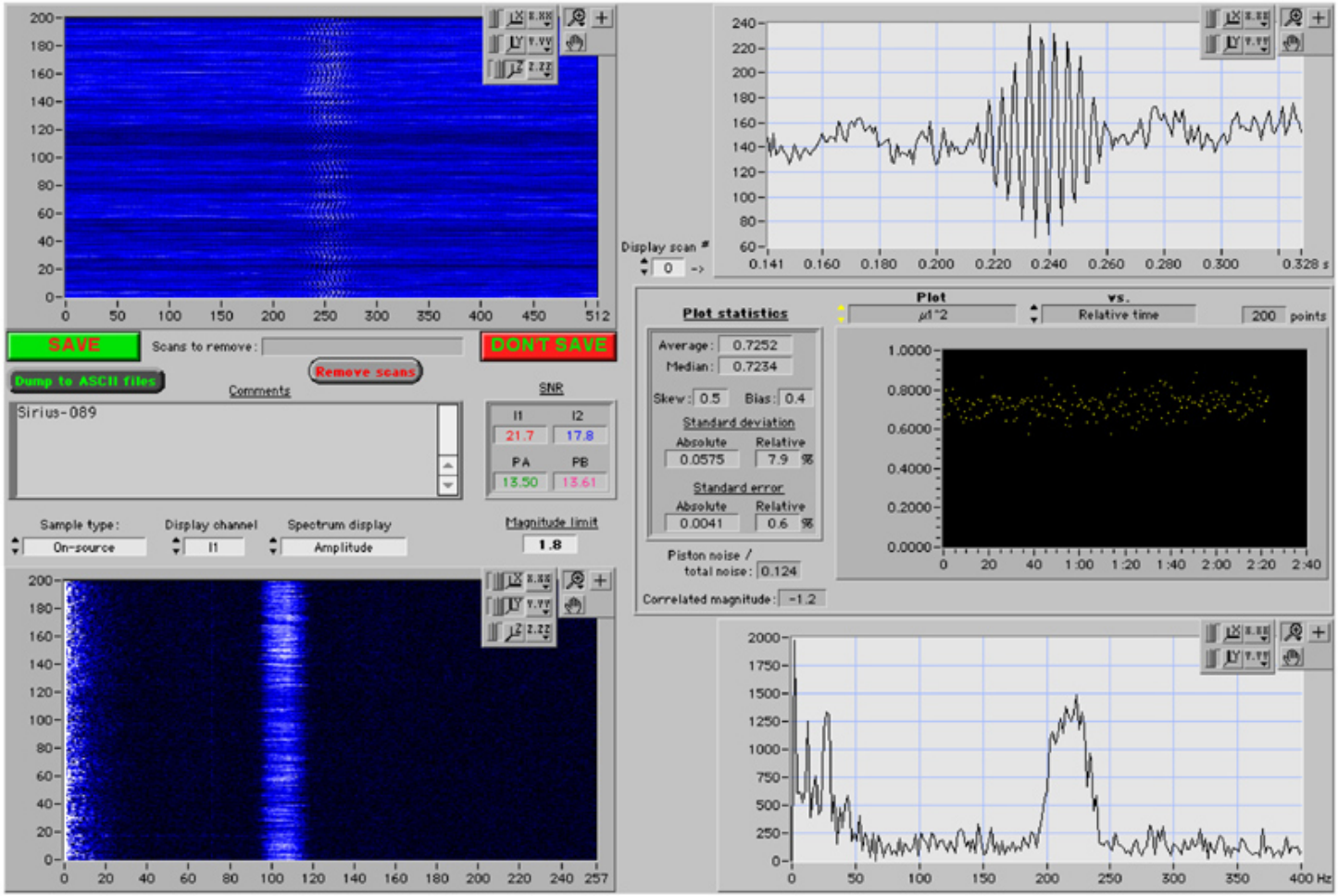


wavelength average

Change in Wavelength

- fringe spacing changes, central fringe broadens
- integral over 0.8 to 1.2 of central wavelength
- intergral over wavelength makes fringe envelope

First Fringes from VLT Interferometer



'First Fringes' from Sirius with VLTI

ESO PR Photo 10a/01 (18 March 2001)

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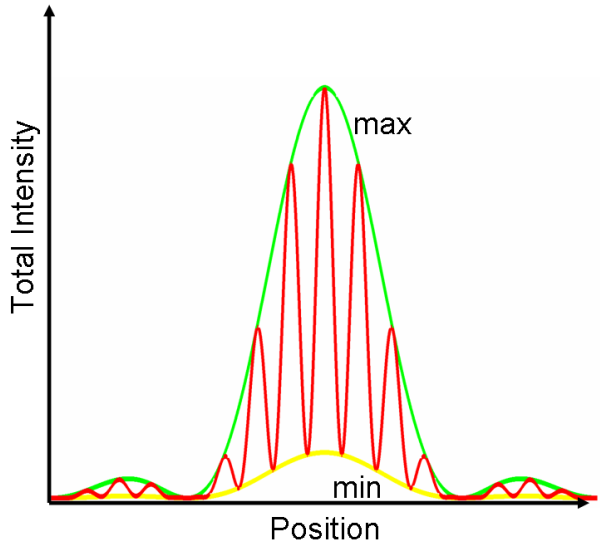
Visibility



en.wikipedia.org/wiki/Interferometric_visibility

- "quality" of fringes described by **Visibility function**

$$V = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

- $I_{\max}$, $I_{\min}$ are maximum and adjacent minimum in fringe pattern
- first introduced by Michelson

Mutual Coherence

- total field in point P (neglecting propagators)

$$\tilde{E}_P(t) = \tilde{E}_1(t - t_1) + \tilde{E}_2(t - t_2)$$

- irradiance at P, averaged over time (expectation operator **E**)

$$I = \mathbf{E}|\tilde{E}_P(t)|^2 = \mathbf{E}\left\{\tilde{E}_P(t)\tilde{E}_P^*(t)\right\}$$

- writing out all the terms

$$I = \mathbf{E}\left\{\tilde{E}_1(t - t_1)\tilde{E}_1^*(t - t_1)\right\} + \mathbf{E}\left\{\tilde{E}_2(t - t_2)\tilde{E}_2^*(t - t_2)\right\}$$
$$+\mathbf{E}\left\{\tilde{E}_1(t - t_1)\tilde{E}_2^*(t - t_2)\right\} + \mathbf{E}\left\{\tilde{E}_1^*(t - t_1)\tilde{E}_2(t - t_2)\right\}$$

Mutual Coherence (continued)

- as before

$$I = \mathbf{E}\left\{\tilde{E}_1(t-t_1)\tilde{E}_1^*(t-t_1)\right\} + \mathbf{E}\left\{\tilde{E}_2(t-t_2)\tilde{E}_2^*(t-t_2)\right\}$$
$$+\mathbf{E}\left\{\tilde{E}_1(t-t_1)\tilde{E}_2^*(t-t_2)\right\} + \mathbf{E}\left\{\tilde{E}_1^*(t-t_1)\tilde{E}_2(t-t_2)\right\}$$

- *stationary* wave field, time average independent of absolute time

$$I_{S_1} = \mathbf{E}\left\{\tilde{E}_1(t)\tilde{E}_1^*(t)\right\}, \; I_{S_2} = \mathbf{E}\left\{\tilde{E}_2(t)\tilde{E}_2^*(t)\right\}$$

- irradiance at P is now

$$I = I_{S_1} + I_{S_2}$$
$$+\mathbf{E}\left\{\tilde{E}_1(t-t_1)\tilde{E}_2^*(t-t_2)\right\} + \mathbf{E}\left\{\tilde{E}_1^*(t-t_1)\tilde{E}_2(t-t_2)\right\}$$

Mutual Coherence (continued)

- as before

$$I = I_{S_1} + I_{S_2}$$
$$+\mathbf{E}\left\{\tilde{E}_1(t-t_1)\tilde{E}_2^*(t-t_2)\right\} + \mathbf{E}\left\{\tilde{E}_1^*(t-t_1)\tilde{E}_2(t-t_2)\right\}$$

- time difference $\tau = t_2 - t_1 \Rightarrow$ last two terms become

$$\mathbf{E}\left\{\tilde{E}_1(t+\tau)\tilde{E}_2^*(t)\right\} + \mathbf{E}\left\{\tilde{E}_1^*(t+\tau)\tilde{E}_2(t)\right\}$$

- equivalent to

$$2\ Re\left[\mathbf{E}\left\{\tilde{E}_1(t+\tau)\tilde{E}_2^*(t)\right\}\right]$$

- cross-term becomes $2Re\left[\mathbf{E}\left\{\tilde{E}_1(t+\tau)\tilde{E}_2^*(t)\right\}\right]$

Mutual Coherence (continued)

- irradiance at P

$$I = I_{S_1} + I_{S_2} + 2Re\left[\mathbf{E}\left\{\tilde{E}_1(t+\tau)\tilde{E}_2^*(t)\right\}\right]$$

- **mutual coherence function** of wave field at S_1 and S_2

$$\tilde{\Gamma}_{12}(\tau) = \mathbf{E}\left\{\tilde{E}_1(t+\tau)\tilde{E}_2^*(t)\right\}$$

- therefore $I = I_{S_1} + I_{S_2} + 2\ Re\,\tilde{\Gamma}_{12}(\tau)$
- $I_1 = I_{S_1}$, $I_2 = I_{S_2}$: irradiances at P from single aperture

$$I = I_1 + I_2 + 2\ Re\,\tilde{\Gamma}_{12}(\tau)$$

Self-Coherence

- $S_1 = S_2 \Rightarrow$ mutual coherence function = autocorrelation

$$\tilde{\Gamma}_{11}(\tau) = \tilde{R}_1(\tau) = \mathbf{E}\left\{\tilde{E}_1(t+\tau)\tilde{E}_1^*(t)\right\}$$
$$\tilde{\Gamma}_{22}(\tau) = \tilde{R}_2(\tau) = \mathbf{E}\left\{\tilde{E}_2(t+\tau)\tilde{E}_2^*(t)\right\}$$

- autocorrelation functions are also called *self-coherence functions*
- for $\tau = 0$

$$I_{S_1} = \mathbf{E}\left\{\tilde{E}_1(t)\tilde{E}_1^*(t)\right\} = \Gamma_{11}(0) = \mathbf{E}\left\{|\tilde{E}_1(t)|^2\right\}$$
$$I_{S_2} = \mathbf{E}\left\{\tilde{E}_2(t)\tilde{E}_2^*(t)\right\} = \Gamma_{22}(0) = \mathbf{E}\left\{|\tilde{E}_2(t)|^2\right\}$$

- autocorrelation function with zero lag ($\tau = 0$) represent (average) irradiance (power) of wave field at S_1, S_2

Complex Degree of Coherence

- normalized mutual coherence defines the **complex degree of coherence**

$$\tilde{\gamma}_{12}(\tau) \equiv \frac{\tilde{\Gamma}_{12}(\tau)}{\sqrt{\Gamma_{11}(0)\Gamma_{22}(0)}} = \frac{\mathbf{E}\left\{\tilde{E}_1(t+\tau)\tilde{E}_2^*(t)\right\}}{\sqrt{\mathbf{E}\left\{|\tilde{E}_1(t)|^2\right\}\mathbf{E}\left\{|\tilde{E}_2(t)|^2\right\}}}$$

- irradiance in point P as *general interference law for a partially coherent radiation field*

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\, Re\, \tilde{\gamma}_{12}(\tau)$$

Spatial and Temporal Coherence

- complex degree of coherence

$$\tilde{\gamma}_{12}(\tau) \equiv \frac{\tilde{\Gamma}_{12}(\tau)}{\sqrt{\Gamma_{11}(0)\Gamma_{22}(0)}} = \frac{\mathbf{E}\left\{\tilde{E}_1(t+\tau)\tilde{E}_2^*(t)\right\}}{\sqrt{\mathbf{E}\left\{|\tilde{E}_1(t)|^2\right\}\mathbf{E}\left\{|\tilde{E}_2(t)|^2\right\}}}$$

- measures both
  - *spatial coherence* at S_1 and S_2
  - *temporal coherence* through time lag τ
- $\tilde{\gamma}_{12}(\tau)$ is a complex variable and can be written as:

$$\tilde{\gamma}_{12}(\tau) = |\tilde{\gamma}_{12}(\tau)|e^{i\psi_{12}(\tau)}$$

- $0 \leq |\tilde{\gamma}_{12}(\tau)| \leq 1$
- phase angle $\psi_{12}(\tau)$ relates to
  - phase angle between fields at S_1 and S_2
  - phase angle difference in P resulting in time lag τ

Fringes in Fiber-fed High-Resolution Spectrograph

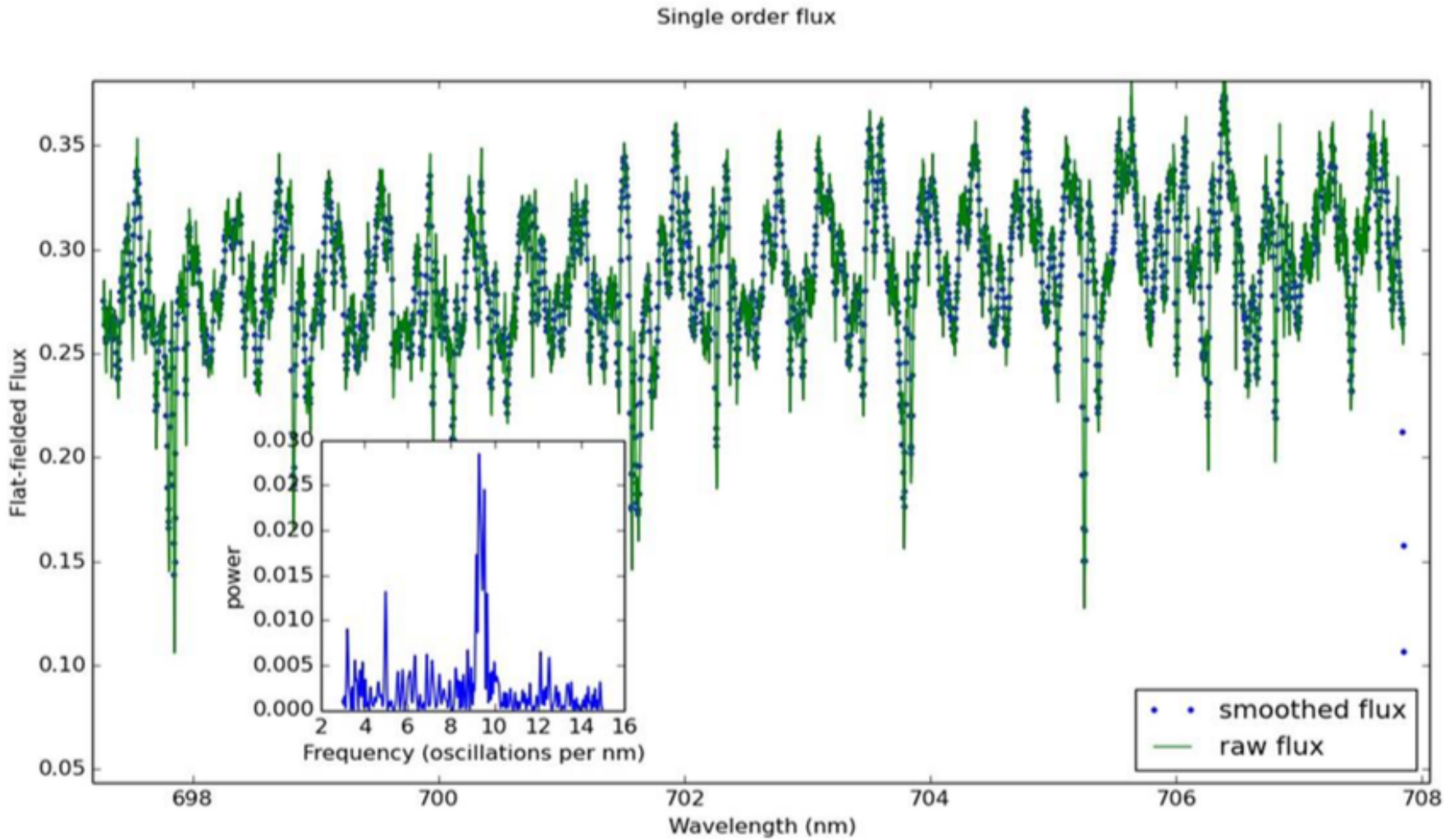
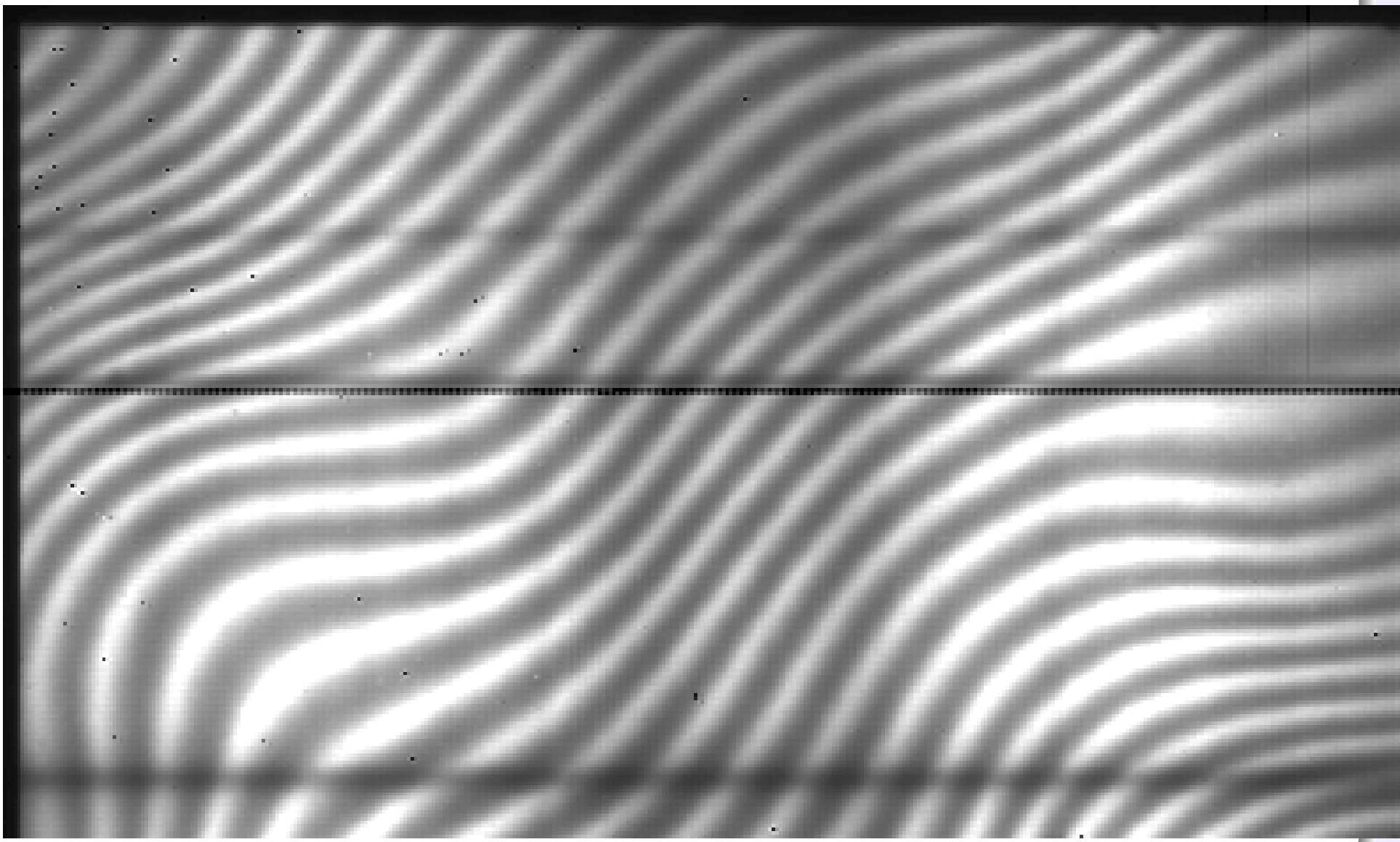


Figure 7 from Rains et al. 2016, SPIE 9908, 990876

Detector Fringes in HyViSi Detector



Coherence of Quasi-Monochromatic Light

- quasi-monochromatic light, mean wavelength $\overline{\lambda}$, frequency $\overline{\nu}$, phase difference ϕ due to optical path difference:

$$\phi \;=\; \frac{2\pi}{\overline{\lambda}}(r_2 - r_1) \;=\; \frac{2\pi}{\overline{\lambda}} c(t_2 - t_1) \;=\; 2\pi\overline{\nu}\tau$$

- with phase angle $\alpha_{12}(\tau)$ between fields at pinholes S_1, S_2

$$\psi_{12}(\tau) = \alpha_{12}(\tau) - \phi$$

- and

$$Re\,\tilde{\gamma}_{12}(\tau) \;=\; |\tilde{\gamma}_{12}(\tau)|\cos\left[\alpha_{12}(\tau) - \phi\right]$$

- intensity in P becomes

$$I \;=\; I_1 \;+\; I_2 \;+\; 2\sqrt{I_1 I_2}\,|\tilde{\gamma}_{12}(\tau)|\cos\left[\alpha_{12}(\tau) - \phi\right]$$

Visibility of Quasi-Monochromatic, Partially Coherent Light

- intensity in P

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\,|\tilde{\gamma}_{12}(\tau)|\cos\left[\alpha_{12}(\tau) - \phi\right]$$

- maximum, minimum I for $\cos(...) = \pm 1$
- visibility V at position P

$$V = \frac{2\sqrt{I_1}\sqrt{I_2}}{I_1 + I_2}|\tilde{\gamma}_{12}(\tau)|$$

- for $I_1 = I_2 = I_0$

$$\begin{aligned} I &= 2I_0\left\{1 + |\tilde{\gamma}_{12}(\tau)|\cos\left[\alpha_{12}(\tau) - \phi\right]\right\} \\ V &= |\tilde{\gamma}_{12}(\tau)| \end{aligned}$$

Interpretation of Visibility

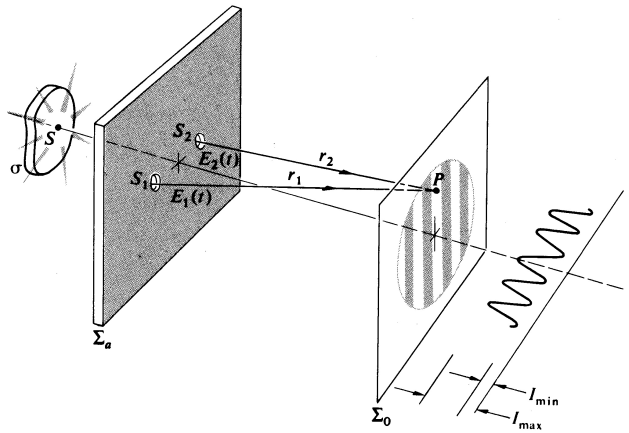
- for $I_1 = I_2 = I_0$

$$\begin{aligned} I &= 2I_0 \{1 + |\tilde{\gamma}_{12}(\tau)| \cos [\alpha_{12}(\tau) - \phi]\} \\ V &= |\tilde{\gamma}_{12}(\tau)| \end{aligned}$$

- *modulus of complex degree of coherence = visibility of fringes*
- modulus can therefore be measured
- shift in location of central fringe (no optical path length difference, $\phi = 0$) is measure of $\alpha_{12}(\tau)$
- measurements of visibility and fringe position yield amplitude and phase of complex degree of coherence

Two-Element Interferometer

Fringe Pattern



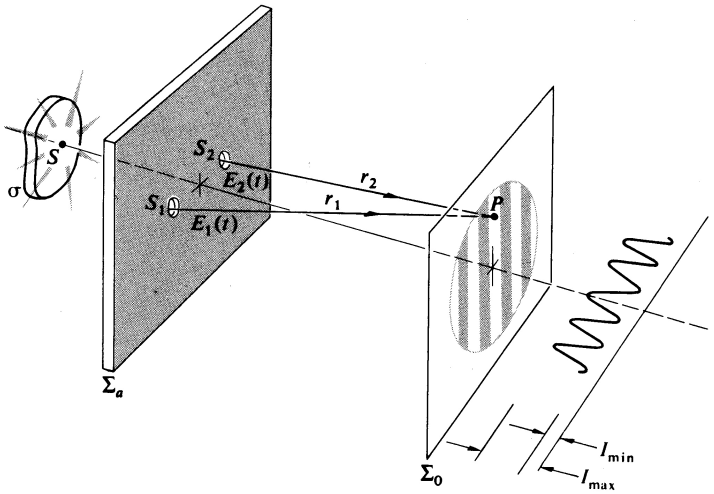
- for $I_1 = I_2 = I_0$

$$I = 2I_0 \{1 + |\tilde{\gamma}_{12}(\tau)| \cos [\alpha_{12}(\tau) - \phi]\} \quad V = |\tilde{\gamma}_{12}(\tau)|$$

- source S on central axis, fully coherent waves from two holes

$$I = 2I_0(1 + \cos\phi) = 4I_0 \cos^2 \frac{\phi}{2}$$

Fringe Pattern (continued)



$$I = 4I_0 \cos^2 \frac{\phi}{2}$$

$$\phi = \frac{2\pi}{\lambda}(r_2 - r_1) = 2\pi\bar{\nu}\tau$$

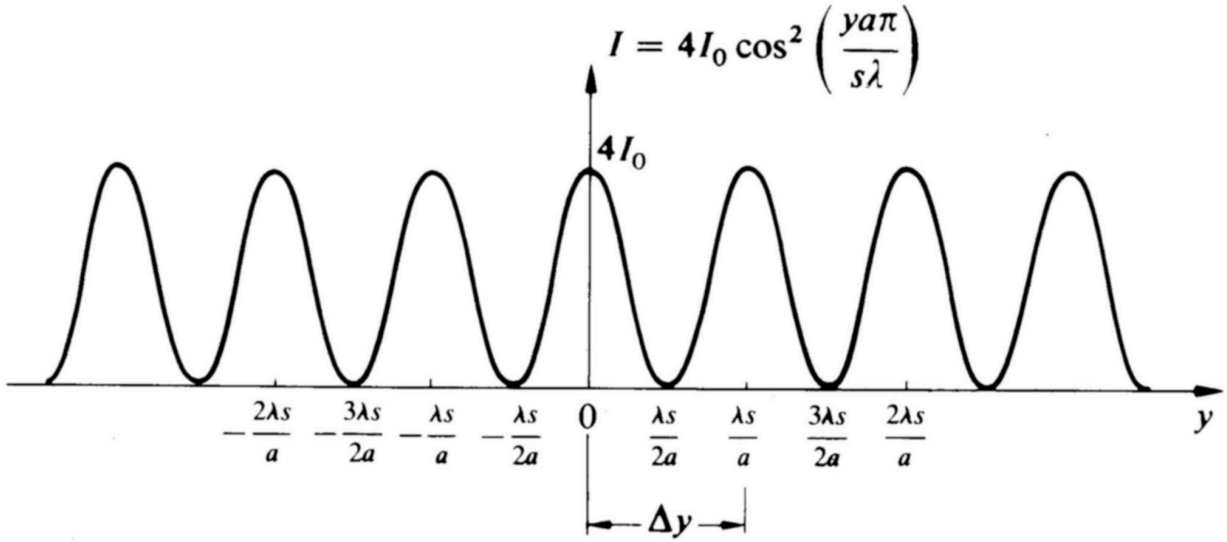
- distance a between pinholes
- distance s to observation plane Σ_O, $s \gg a$
- path difference $(r_2 - r_1)$ in equation for ϕ in good approximation

$$r_2 - r_1 = a\theta = \frac{a}{s} y$$

- and therefore

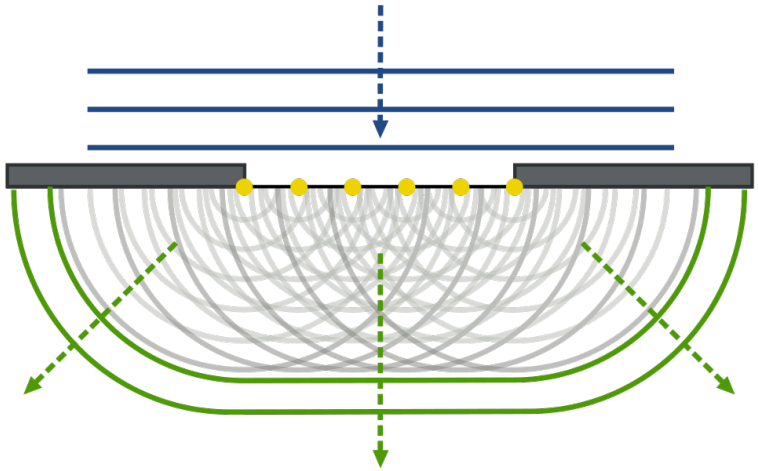
$$I = 4I_0 \cos^2 \frac{\pi a y}{s\bar{\lambda}}$$

Interference Fringes from Monochromatic Point Source



- irradiance as a function of the y-coordinate of the fringes in observation plane Σ_O
- irradiance vs. distance distribution is Point-Spread Function (PSF) of ideal two-element interferometer

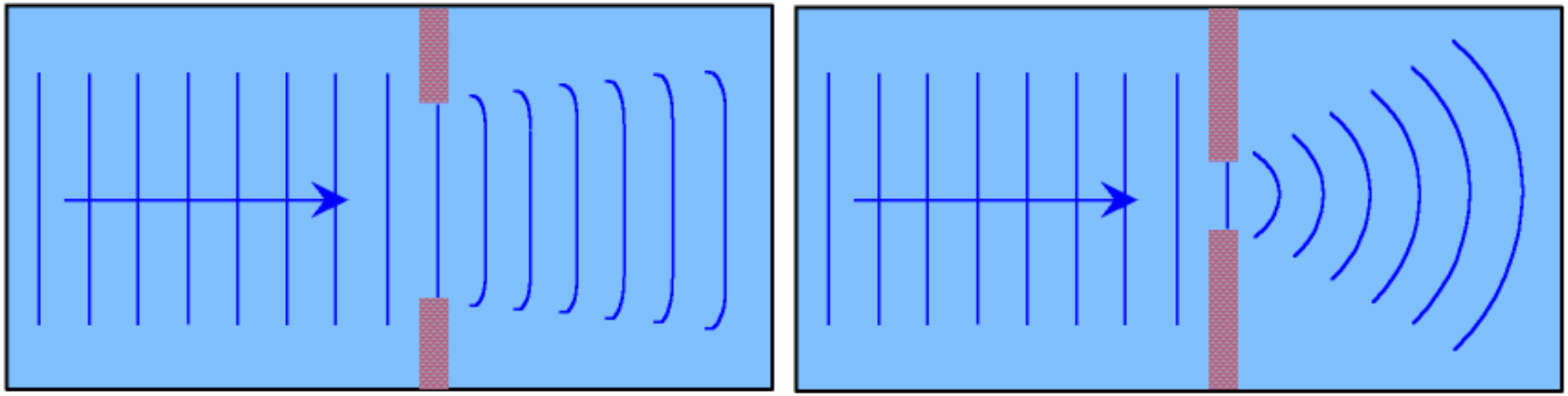
Huygens-Fresnel Principle



en.wikipedia.org/wiki/File:Refraction_on_an_aperture_-_Huygens-Fresnel_principle.svg

- every unobstructed point of a wavefront at a given moment in time serves as a source of spherical, secondary waves with the same frequency as the primary wave
- the amplitude of the optical field at any point beyond is the coherent superposition of all these secondary, spherical waves

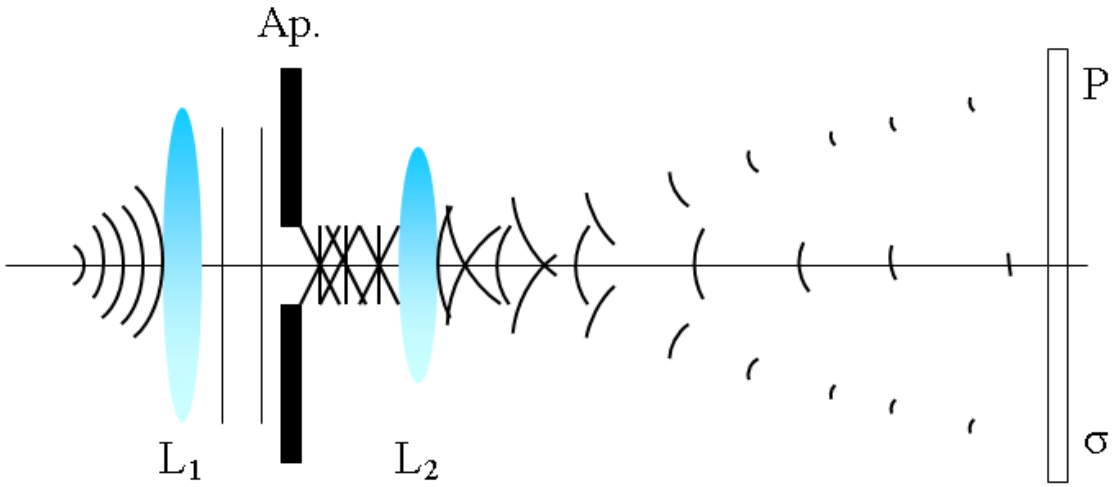
Diffraction



www.smkbud4.edu.my/Data/sites/vschool/phy/wave/diffraction.htm

- if obstructing structures are small compared to the wavelength, waves will spread out $\Rightarrow$ diffraction
- really need to solve wave equation with boundary constraints $\Rightarrow$ riguorous solution for only a few special cases
- various numerical ways to solve such problems (e.g. Rigorous Coupled Wave Analysis)
- Huygens-Fresnel is useful for most applications

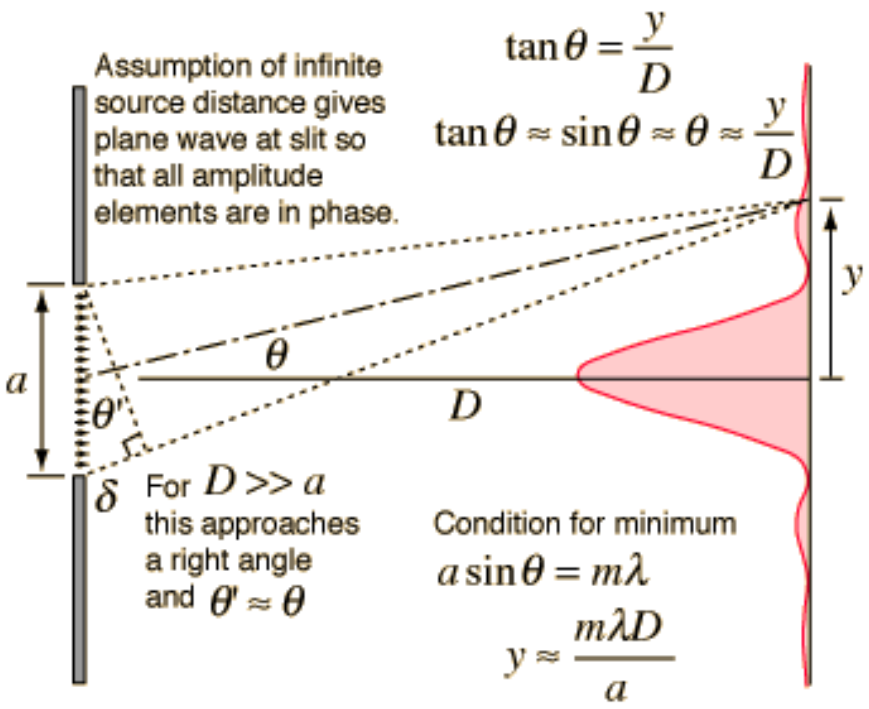
Fraunhofer and Fresnel Diffraction



en.wikipedia.org/wiki/File:Fraunhofer_diffraction_pattern_image.PNG

- wave shape changes as it moves away from obstruction
- *Fresnel* (or near-field) diffraction close to obstruction
- *Fraunhofer* (or far-field) diffraction far away from obstruction
- rule of thumb: Fraunhofer diffraction for $R > a^2/\lambda$
  - a greatest width of obscuration
  - λ wavelength
  - R greater of distance between source/detector and obscuration

Slit Diffraction



hyperphysics.phy-astr.gsu.edu/hbase/phyopt/sinslit.html

- with E_0 the strength of each slit segment i at point P is
  $E_i(P) = \frac{E_L}{r_i}\sin(k\omega - kr_i)\Delta y_i$
  - i segment index $(1 - M)$
  - E_L source strength per unit length
  - r_i distance between segment and point P
  - Δy_i small segment of slit
  - D length of slit

Fraunhofer Diffraction at Single Slit

- integrate along slit

$$E = E_L \int_{-D/2}^{D/2} \frac{\sin \omega t - kr}{r} dy$$

- express r as a function of y:

$$r = R - y \sin\theta + \frac{y^2}{2R} \cos^2\theta + ...$$

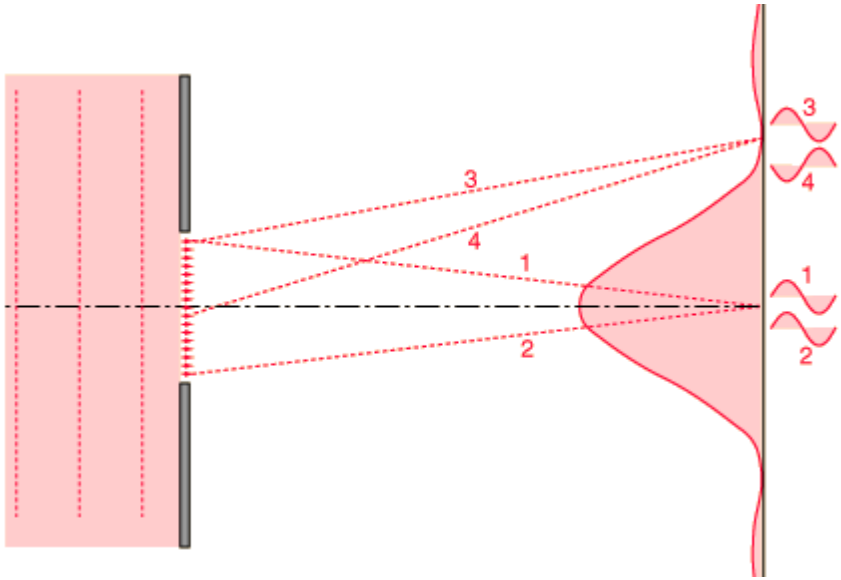
R distance between center of slit and point P

- substituting, integrating and squaring for intensity:

$$I(\theta) = I(0) \left(\frac{\sin\beta}{\beta} \right)^2$$

- $\beta = (kD/2) \sin\theta$

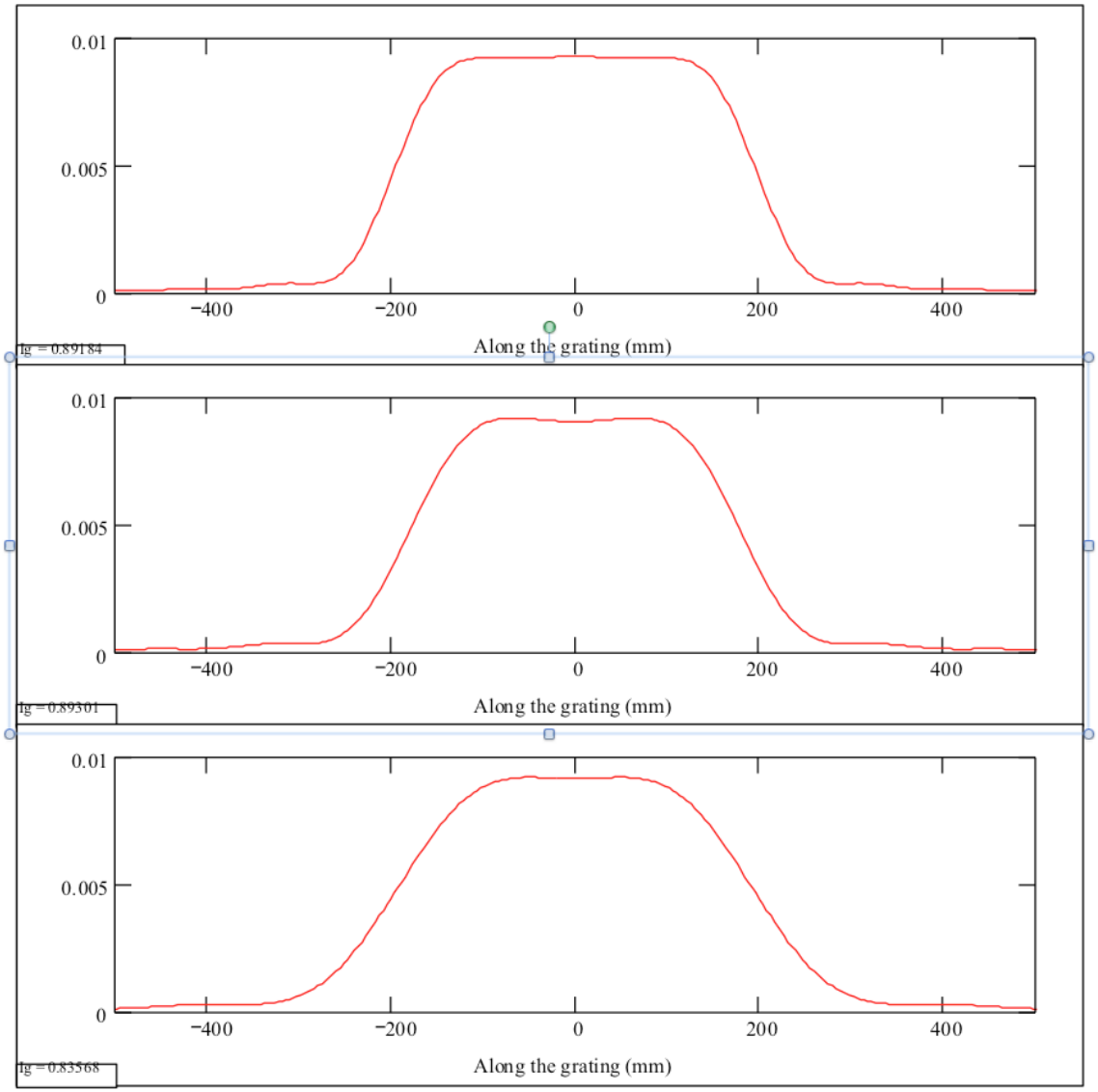
Interpretation of Single Slit Diffraction



hyperphysics.phy-astr.gsu.edu/hbase/phyopt/sinslitd.html

- assume infinite distance from aperture for source and observation plane
- equivalent to plane waves coming from aperture into different directions
- first minimum when phase delay at edge is exactly one wave

Diffraction Effects in High-Resolution Spectrographs



λ=630.2 nm, grating angle=65.09 degree

λ=854.2 nm, grating angle=62.745 degree

λ=1083 nm, grating angle=65.377 degree

SOLIS VSM Slit Diffraction on Grating

Arbitrary Diffracting Aperture

- from before forgetting common phase term and $1/R$ amplitude drop-off

$$E(Y,Z) = \int\int_{\text{aperture}} A(y,z) e^{ik(Yy+Zz)/R} dS$$

- complex aperture function $A(y,z)$ describing non-uniform absorption and phase delays
- finite aperture $\Rightarrow$ change integration boundaries to infinity
- with $k_y = kY/R$ and $k_z = kZ/R$ we obtain

$$E(k_y,k_z) = \int\int_{\text{aperture}} A(y,z) e^{i(k_y y+k_z z)} dy\, dz$$

- field distribution in Fraunhofer diffraction pattern is Fourier transform of field distribution across aperture

Transfer Functions

Introduction

- linear black box system
- measure response to delta function input (transfer function)
- express output as convolution between input signal and transfer function

Point-Spread Function

- intensity is modulus squared of field distribution $\Rightarrow$ point-spread function
- image of a point source: *Point Spread Function (PSF)*
- image of arbitrary object is a convolution of object with PSF
  $i = o * s$
  - i observed image
  - o true object, constant in time
  - s point spread function
  - $*$ convolution

Optical Transfer Function

- after Fourier transformation:

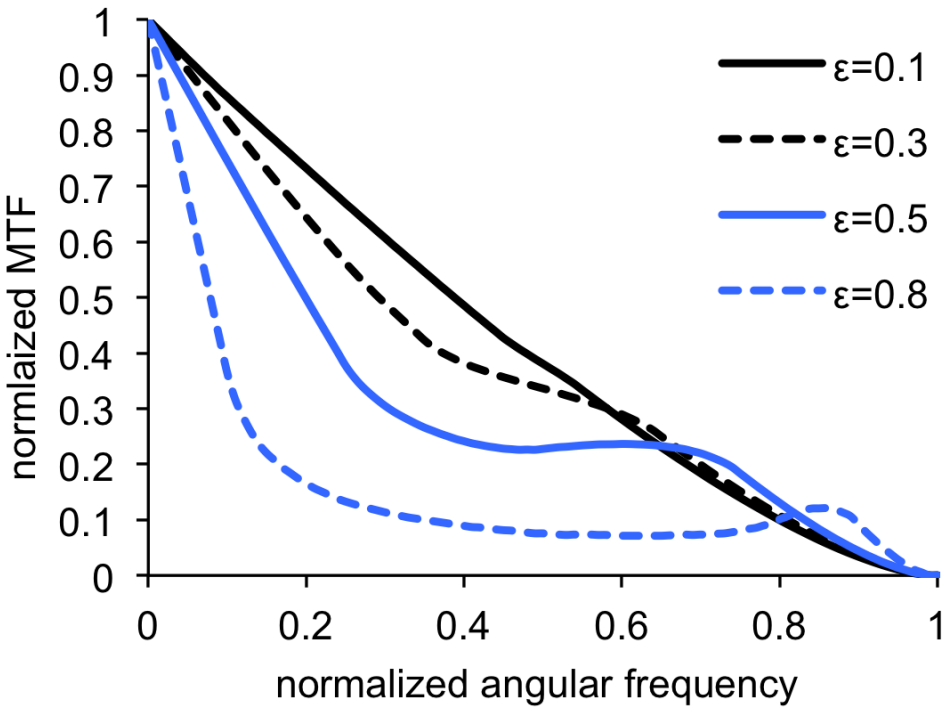
$$I = O \cdot S$$

  - Fourier transformed
  - I Fourier transform of image
  - O Fourier transform of object
  - S Optical Transfer Function (OTF)
- OTF is Fourier transform of PSF and vice versa

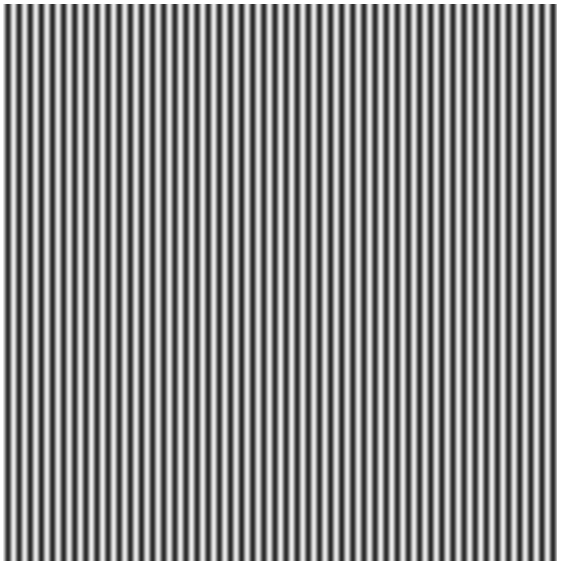
Modulation Transfer Function (MTF)

- is the absolute value of the optical transfer function
- describes the amplitude reduction of a sinusoidal source
- is the autocorrelation of the aperture function A
- $\mathrm{OTF} = \mathrm{FT}^{-1}(\mathrm{PSF}) = \mathrm{FT}^{-1}(|\mathrm{FT}(A)|^2) = \mathrm{FT}^{-1}(\mathrm{FT}(A)\cdot\mathrm{FT}(A)^*) = A * A$

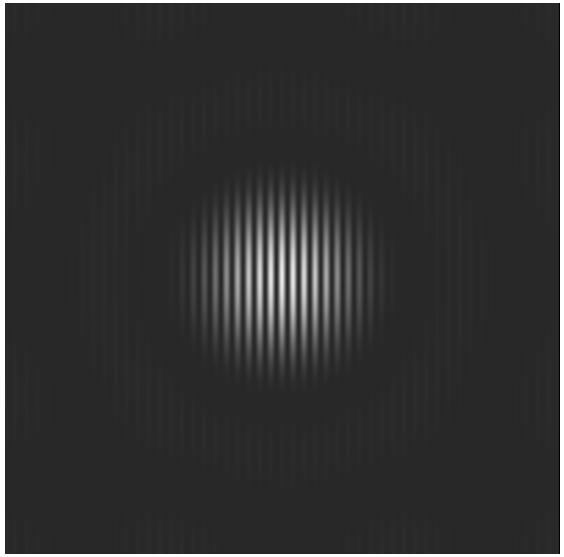
MTF of Circular Telescope with Central Obscuration ϵ



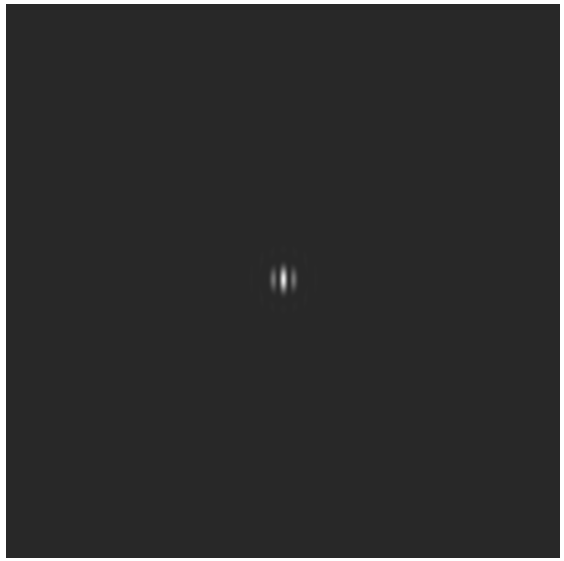
2 pinholes



2 small holes



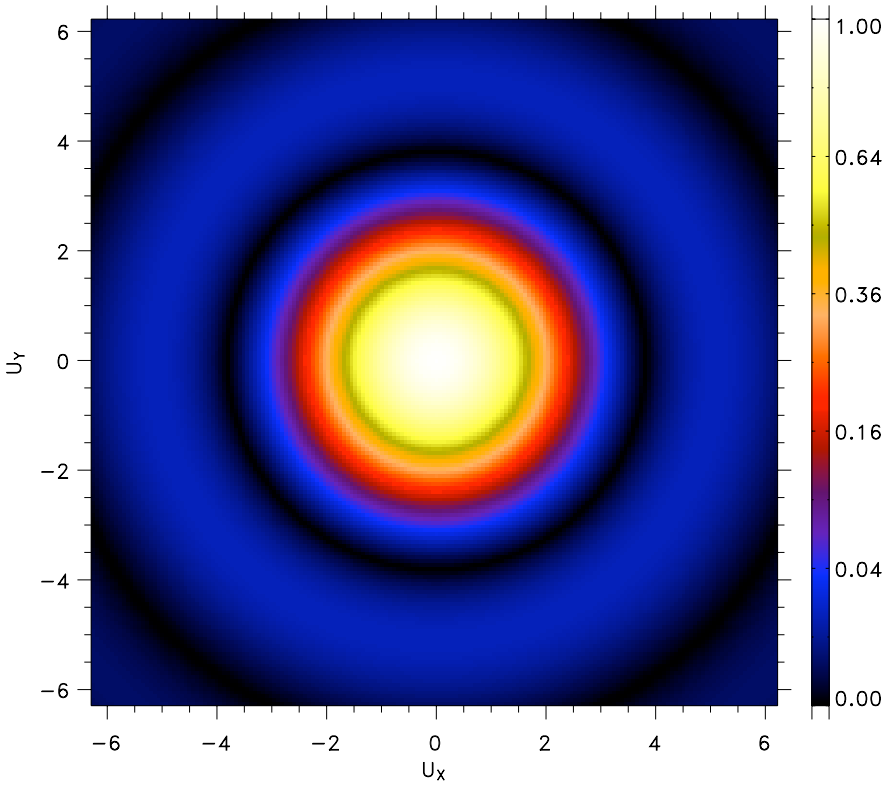
2 large holes



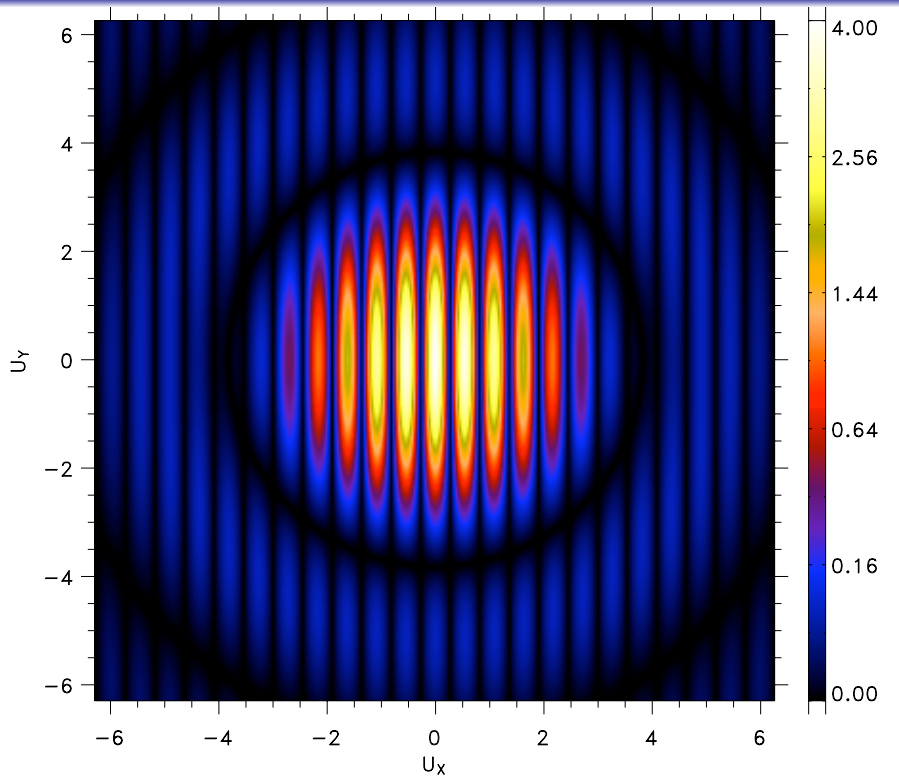
Finite Hole Diameter

- fringe spacing only depends on separation of holes and wavelength
- the smaller the hole, the larger the 'illuminated' area
- fringe envelope is Airy pattern (diffraction pattern of a single hole)

2-d Brightness Distribution



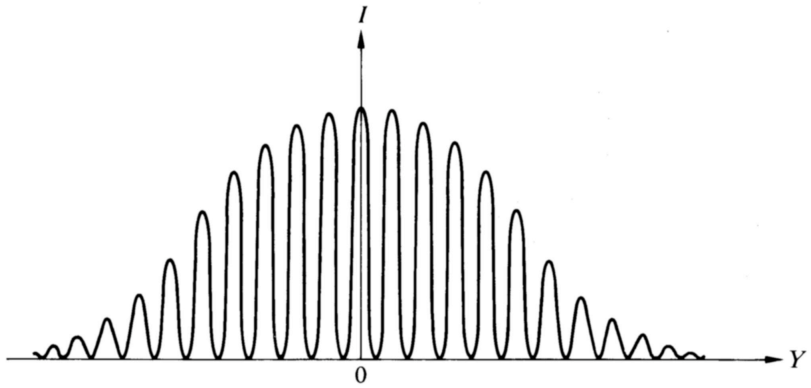
PSF of single circular aperture



PSF of two-element interferometer, aperture diameter $d = 25$ m, length of baseline vector $|\vec{s}| = 144$ m

- double beam interference fringes showing modulation effect of diffraction by aperture of a single pinhole

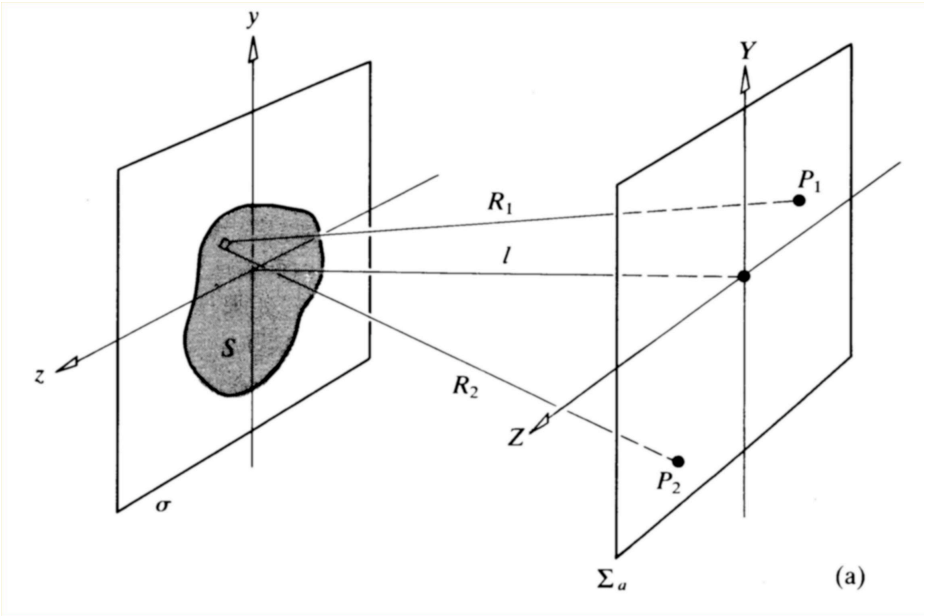
Modulation Effect of Aperture



- typical one-dimensional cross-section through central part of interferogram
- visibilities are equal to one, because $I_{min} = 0$
- $|\tilde{\gamma}_{12}(\tau)| = 1$ for all values of τ and any pair of spatial points, if and only if the radiation field is *strictly monochromatic*

Van Cittert-Zernike Theorem

The Problem



- relates brightness distribution of extended source and phase correlation between two points in radiation field
- extended source S incoherent, quasi-monochromatic
- positions P_1 and P_2 in observers plane Σ

$$\tilde{E}_1(t)\tilde{E}_2^*(t) = \mathbf{E}\{\tilde{E}_1(t)\tilde{E}_2^*(t)\} = \tilde{\Gamma}_{12}(0)$$

The Solution

- $I(\vec{\Omega})$ is intensity distribution of extended source as function of unit direction vector $\vec{\Omega}$ as seen from observation plane Σ
- $\tilde{\Gamma}(\vec{r})$ is coherence function in Σ-plane
- vector $\vec{r}$ represents arbitrary baseline
- van Cittert-Zernike theorem

$$\tilde{\Gamma}(\vec{r}) = \int\int_{\text{source}} I(\vec{\Omega}) e^{\frac{2\pi i \vec{\Omega}.\vec{r}}{\lambda}} d\vec{\Omega}$$

$$I(\vec{\Omega}) = \lambda^{-2} \int\int_{\Sigma\text{-plane}} \tilde{\Gamma}(\vec{r}) e^{-\frac{2\pi i \vec{\Omega}.\vec{r}}{\lambda}} d\vec{r}$$

- $\tilde{\Gamma}(\vec{r})$ and $I(\vec{\Omega})$ are linked through Fourier transform, except for scaling with wavelength λ
- "true" Fourier transform with *conjugate variables* $\vec{\Omega}$, $\vec{r}/\lambda$,
- Fourier pair: $I(\vec{\Omega}) \quad \Leftrightarrow \quad \tilde{\Gamma}(\vec{r}/\lambda)$