Outline

1. Geometrical Optics
2. Lenses
3. Mirrors
4. Optical Systems
5. Aberrations
rays are normal to locally flat wave
rays are reflected and refracted according to Fresnel equations
phase is neglected $\Rightarrow$ incoherent sum
rays can carry polarization information
geometrical optics neglects diffraction effects: $\lambda \Rightarrow 0$
physical optics $\lambda > 0$
simplicity of geometrical optics mostly outweighs limitations
Lenses

Surface Shape of Perfect Lens

- lens material has index of refraction $n$
- $o \cdot z(r) \cdot n + z(r) f = \text{constant}$
- $n \cdot z(r) + \sqrt{r^2 + (f - z(r))^2} = \text{constant}$
- solution $z(r)$ is hyperbola with eccentricity $e = n > 1$
**Assumptions:**

1. Snell’s law for small angles of incidence ($\sin \phi \approx \phi$)
2. ray height $r$ small so that optics curvature can be neglected (plane optics, $\cos \phi \approx 1$)
3. $\tan\phi \approx \phi = r/f$

paraxial approximation reasonable for $\phi \lesssim 10$ degrees
Spherical Lenses

- surface error requirement less than $\lambda/10$
- most optical surfaces are spherical due to ease of manufacturing
- can easily join two spherical surfaces with equal radii
1.1-meter Singlet Lens of Swedish Solar Telescope

Positive/Converging Spherical Lens Parameters

- center of curvature and radii with signs: $R_1 > 0$, $R_2 < 0$
- center thickness: $d$
- positive focal length $f > 0$
Negative/Diverging Spherical Lens Parameters

- note different signs of radii: $R_1 < 0$, $R_2 > 0$
- virtual focal point
- negative focal length ($f < 0$)
**General Lens Setup**

- **Object distance** $S_1$, **object height** $h_1$
- **Image distance** $S_2$, **image height** $h_2$
- Axis through two centers of curvature is **optical axis**
- Surface point on optical axis is the **vertex**
- **Chief ray** through center maintains direction

[commons.wikimedia.org/wiki/File:Lens3.svg](commons.wikimedia.org/wiki/File:Lens3.svg)
Thin Lens Approximation

- **thick-lens equation:**

\[
\frac{1}{S_1} + \frac{1}{S_2} = (n - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} + \frac{(n - 1)d}{nR_1R_2} \right) = \frac{1}{f}
\]

- **thin-lens equation for** \(d \ll R_1R_2\):

\[
\frac{1}{S_1} + \frac{1}{S_2} = (n - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) = \frac{1}{f}
\]

Finite Imaging

- rarely image point sources, but extended object
- object and image size are proportional
- orientation of object and image are inverted
- magnification perpendicular to optical axis: \(M = h_2/h_1 = -S_2/S_1\)
Achromatic Lens

- combination of 2 lenses, different glass dispersion
- also less spherical aberration
Mirrors

Mirrors vs. Lenses

- mirrors are completely achromatic
- reflective over very large wavelength range (UV to radio)
- can be supported from the back
- can be segmented
- wavefront error is twice that of surface, lens is \((n-1)\) times surface
- only one surface to 'play' with
Spherical Mirrors

easy to manufacture
focuses light from center of curvature onto itself
focal length is half of curvature: $f = R/2$
tip-tilt misalignment does not matter
has no optical axis
does not image light from infinity correctly (spherical aberration)
Parabolic Mirrors

- want to make flat wavefront into spherical wavefront
- distance $az(r) + z(r)f = \text{const.}$
- $z(r) = \frac{r^2}{2R}$
- perfect image of objects at infinity
- has clear optical axis
8.4-meter Large Binocular Telescope Primary Mirror Nr. 2

mirrorlab.as.arizona.edu/sites/mirrorlab.as.arizona.edu/files/LBT_2_DSCN1893.jpg
Conic Sections

- circle and ellipses: cuts angle $< \text{cone angle}$
- parabola: angle $= \text{cone angle}$
- hyperbola: cut along axis
Conic Constant K

- \(r^2 - 2Rz + (1 + K)z^2 = 0 \) for \(z(r = 0) = 0\)
- \(z = \frac{r^2}{R} \frac{1}{1 + \sqrt{1 - (1 + K) \frac{r^2}{R^2}}}\)

- \(R\) radius of curvature
- \(K = -e^2\), \(e\) eccentricity
- prolate ellipsoid \((K > 0)\)
- sphere \((K = 0)\)
- oblate ellipsoid \((0 > K > -1)\)
- parabola \((K = -1)\)
- hyperbola \((K < -1)\)
- all conics are almost spherical close to origin
- analytical ray intersections
Foci of Conic Sections

- sphere has single focus
- ellipse has two foci
- parabola (ellipse with \( e = 1 \)) has one focus (and another one at infinity)
- hyperbola (\( e > 1 \)) has two focal points

en.wikipedia.org/wiki/File:Eccentricity.svg
Elliptical Mirrors

- have two foci at finite distances
- perfectly reimage one focal point into another

Hyperbolic Mirrors

- have a real focus and a virtual focus (behind mirror)
- perfectly reimage one focal point into another
combinations of several optical elements (lenses, mirrors, stops)

examples: camera “lens”, microscope, telescopes, instruments
Simple Thin-Lens Combinations

- Thin-lens combinations can be treated analytically.
- Distance > sum of focal lengths \( \Rightarrow \) real image between lenses \( \Rightarrow \)
- Apply single-lens equation successively.

Focal length of general thin lens combination:

- Focal lengths: \( f_1, f_2 \)
- Distance between thin lenses: \( d_{12} \)
- Effective focal length:
  \[ \frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d_{12}}{f_1 f_2} \]
Example Thin-Lens Combinations

[Diagram showing various thin-lens combinations with ray paths]
all optical systems have a place where 'aperture' is limited
main mirror of telescopes
aperture stop in photographic lenses
aperture typically has a maximum diameter
aperture size is important for diffraction effects
F-number

describes the light-gathering ability of the optical system
f-number given by $F = \frac{f}{D}$
also called focal ratio or f-ratio, written as: $\frac{f}{F}$
the bigger $F$, the better the paraxial approximation works
fast system for $F < 2$, slow system for $F > 2$
Numerical Aperture in Fibers

- acceptance cone of the fiber determined by materials
- $NA = n \sin \theta = \sqrt{n_1^2 - n_2^2}$
- $n$ index of refraction of external medium

$n_1$ core index
$n_2$ cladding index
$\theta_c$ critical angle
$\theta_{\text{max}}$ maximum acceptance angle

en.wikipedia.org/wiki/File:OF-na.svg
Telecentric Arrangement

- as seen from image, pupil is at infinity
- easy: lens is its focal length away from pupil (image)
- magnification does not change with focus positions
- ray cones for all image points have the same orientation
Aberrations

Spot Diagrams and Wavefronts

- plane of least confusion is location where image of point source has smallest diameter
- spot diagram: shows ray locations in plane of least confusion
- spot diagrams are closely connected with wavefronts
- aberrations are deviations from spherical wavefront
Spherical Aberration of Spherical Lens

- different focal lengths of paraxial and marginal rays
- longitudinal spherical aberration along optical axis
- transverse (or lateral) spherical aberration in image plane
- much more pronounced for short focal ratios
- foci from paraxial beams are further away than marginal rays
- spot diagram shows central area with fainter disk around it
Minimizing Spherical Aberrations
Spherical Aberration Spots and Waves

- spot diagram shows central area with fainter disk around it
- wavefront has peak and turned-up edges
Aspheric Lens

- conic constant $K = -1 - \sqrt{n}$ makes perfect lens
- difficult to manufacture
- but possible these days
Coma

- typically seen for object points away from optical axis
- leads to 'tails' on stars
Coma Spots and Waves

- parabolic mirror with perfect on-axis performance
- spots and wavefront for off-axis image points
- wavefront is tilted in inner part
- **Astigmatism due to Tilted Glass Plate in Converging Beam**

  - Astigmatism: focus in two orthogonal directions, but not in both at the same time
  - Tilted glass-plate: astigmatism, spherical aberration, beam shift
  - Tilted plates: beam shifters, filters, beamsplitters
  - Difference of two parabolae with different curvatures
Field Curvature

- field (Petzval) curvature: image lies on curved surface
- curvature comes from lens thickness variation across aperture
- problems with flat detectors (e.g. CCDs)
- potential solution: field flattening lens close to focus
Petzval Field Flattening

- Petzval curvature only depends on index of refraction ($n_i$) and focal length ($f_i$) of lenses
- Petzval curvature is independent of lens position!
- Petzval curvature vanishes for $\sum_i \frac{1}{n_i f_i} = 0$
- Field flattener also makes image much more telecentric
Distortion

- image is sharp but geometrically distorted
- (a) object
- (b) positive (or pincushion) distortion
- (c) negative (or barrel) distortion
Vignetting

- effective aperture stop depends on position in object
- image fades toward its edges
- only influences transmission, not image sharpness
Aberration Descriptions

Seidel Aberrations

- introduced by Ludwig von Seidel (1857)
- Taylor expansion of $\sin \phi = \phi - \frac{\phi^3}{3!} + \frac{\phi^5}{5!} - \ldots$
- paraxial approximation: first-order optics, $\sin \phi \sim \phi$
- Seidel aberrations: third-order optics, $\sin \phi \sim \phi - \frac{\phi^3}{3!}$
- describes spherical, astigmatism, coma, field curvature, distortion

Zernike Polynomials

- orthonormal basis on unit circle
- lowest orders correspond to Seidel aberrations
- $Z_n^m (r, \phi) = R_n^m (r) \cos m\phi$, $Z_{n}^{-m} (r, \phi) = R_n^m (r) \sin m\phi$
- $n \geq m \geq 0$
- $R_n^m (r) = \sum_{k=0}^{\frac{n-m}{2}} (-1)^k \binom{n-k}{k} \left( \frac{n-2k}{n-m-k} \right) r^{n-2k}$
Zernike Polynomials

tip
tilt
focus
astigmatism
astigmatism
com a 0°
coma 90°
trefoil 0°
trefoil 30°
third-order spherical