Interferometers

ATI 2014 Lecture 12
Kenworthy and Keller

Interferometry is the combination of amplitude and phase from different telescopes

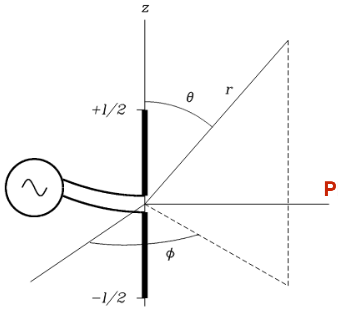
Telescope positions must be known $<< \lambda$

Lots of small telescopes are cheaper than one big telescope!

Done for radio telescopes but difficult for optical telescopes - why?

An Antenna converts EM radiation in space into electrical currents in conductors

They can be used for transmitting and receiving



A simple dipole antenna

A dipole antenna converts EM to accelerating charges appearing as an alternating electric current

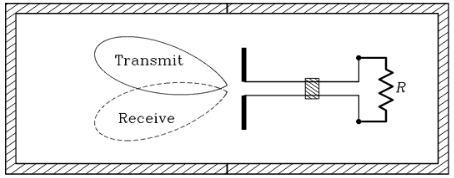
Phase of alternating current is correlated with phase of incoming EM wave!

The Antenna Theorem

The radiation pattern of an antenna is time-reversible

The transmission pattern is the same as the reception pattern!

Thermodynamic cavity temperature T
Resistor R also at temperature T



Antenna transmits power generated by the resistor R

Antenna receives power absorbed from cavity walls

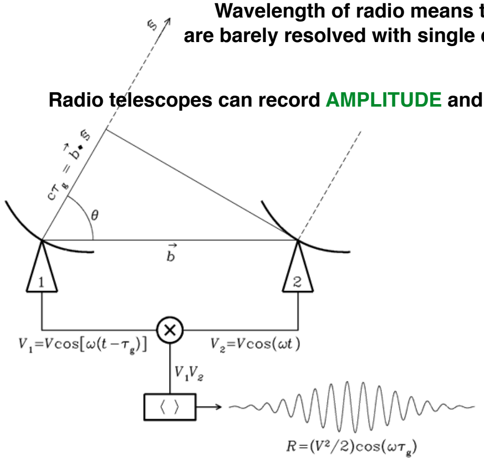
Power in equals power out otherwise resistor temperature changes from T.

The power received and transmitted by the antenna must be the same, otherwise the cavity wall in directions where the transmitted power was greater than the received power would rise in temperature and the cavity wall in directions of lower transmitted/received power ratio would cool, leading to a violation of the second law of thermodynamics.

Interferometers developed at radio wavelengths

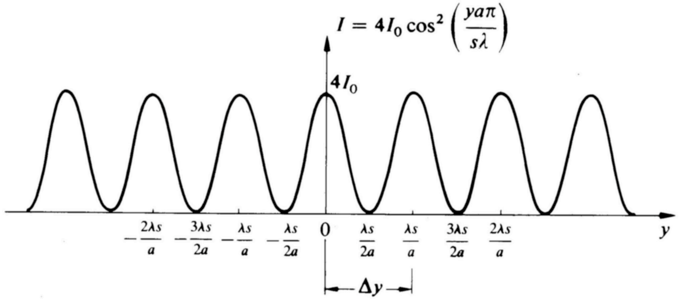
Wavelength of radio means that galaxies
are barely resolved with single dish telescopes

Radio telescopes can record AMPLITUDE and PHASE of incoming radiation



Interference

Two point interferometer fringes from a monochromatic point source

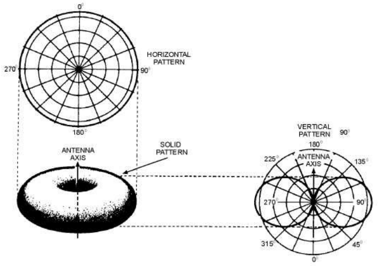


λ	Wavelength of observation
s	Distance to the observing plane
y	Coordinate perpendicular to fringes
a	Interferometer element spacing

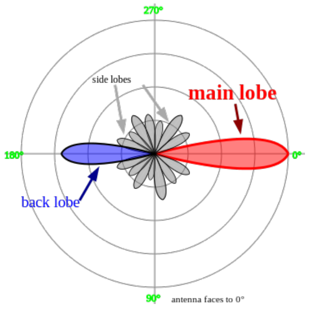
The radiation pattern for an antenna is exactly equivalent to the PSF for a telescope

Dipole has transmission over 1 steradian

Radio telescope parabolic dishes
give FWHM beam widths of few degrees



http://electriciantraining.tpub.com/14182/css/14182_186.htm



http://en.wikipedia.org/wiki/Radiation_pattern#mediaviewer/File:Sidelobes_en.svg

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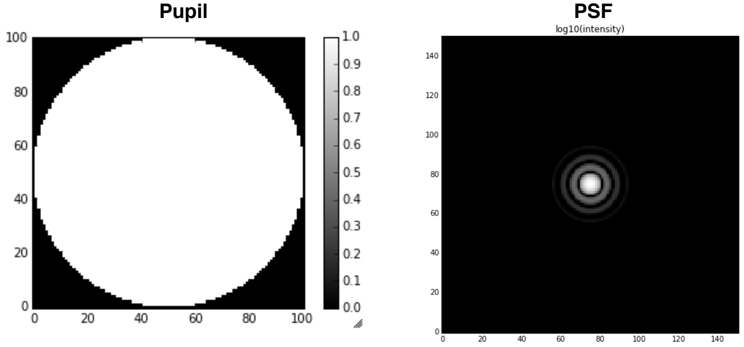
Reminder: single dish angular resolution

Single filled dish telescopes have angular resolutions of $\sim \frac{\lambda}{D}$

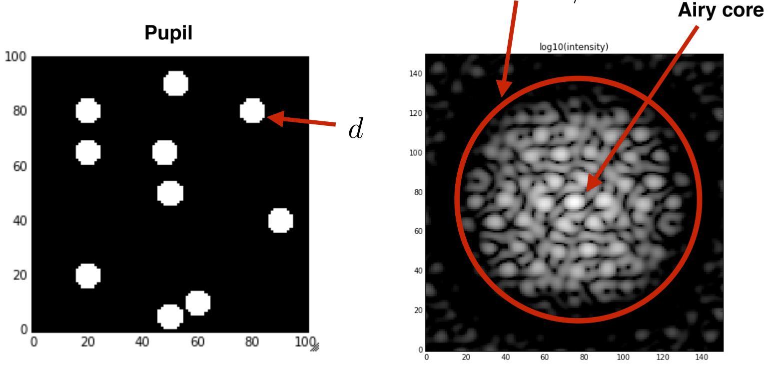
Imaging at optical on 10m telescopes $\sim 10\,\mathrm{mas}$



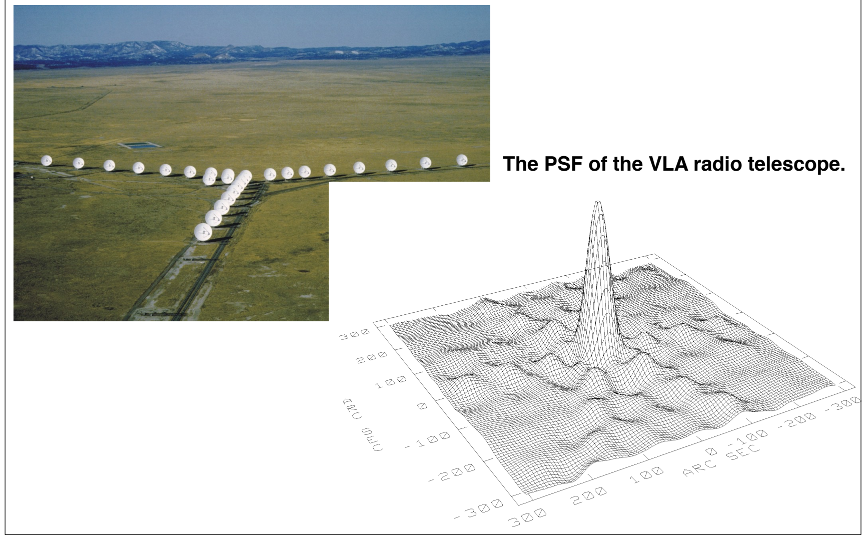
Higher angular resolution with multiple telescopes

A sparsely filled aperture of diameter D has the same angular resolution but total flux is lower and PSF has power removed from the Airy core

Individual apertures of diameter d lead to first minimum of λ/d

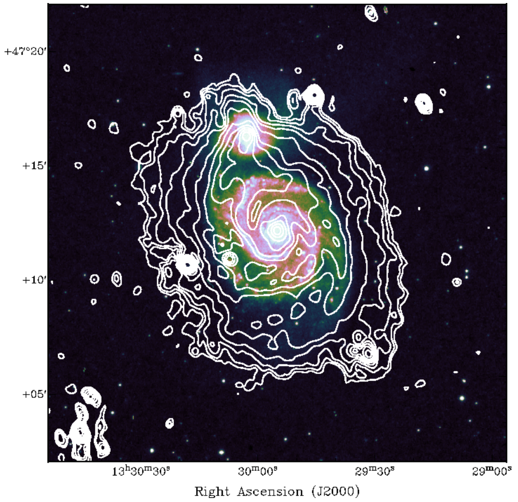


VLA point spread function



The PSF of the VLA radio telescope.

Image of M51 with LOFAR



$$\lambda = 2\,\mathrm{m}$$

$$\theta_{diff} = 20''$$

$$D \sim 20\,\mathrm{km}$$

LOFAR array coherently combines antennae scattered over many kilometres

Mulcahy 2014 A&A

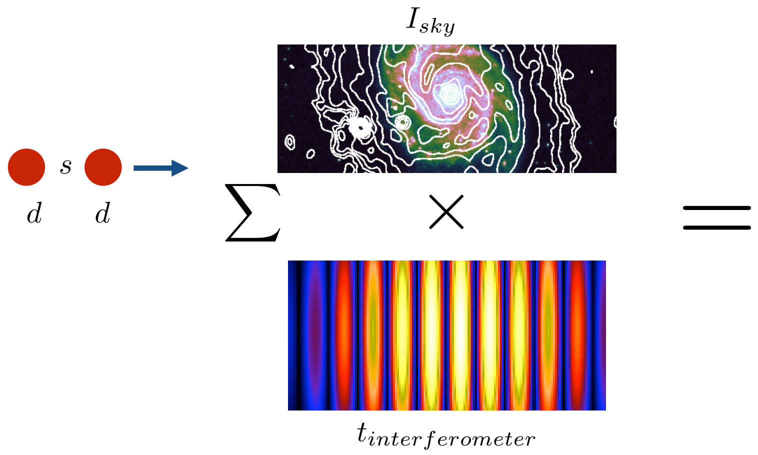
Image of M51 with LOFAR

$$\lambda = 2\,\mathrm{m}$$

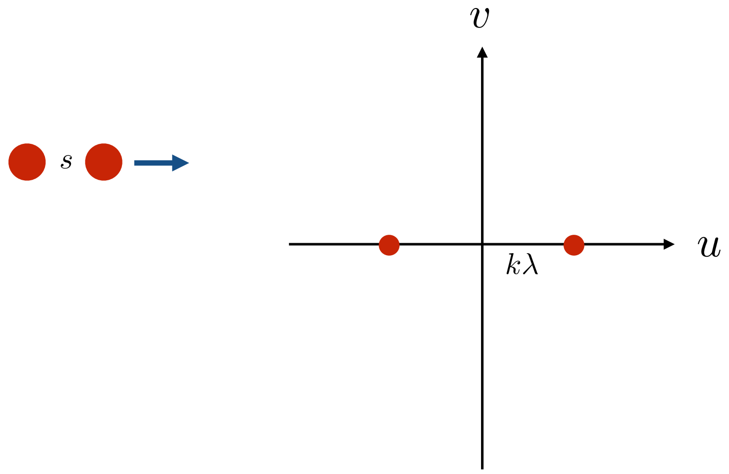
LOFAR Superterp, Exloo, Netherlands

LOFAR array consists of thousands of radio antennae over large baselines

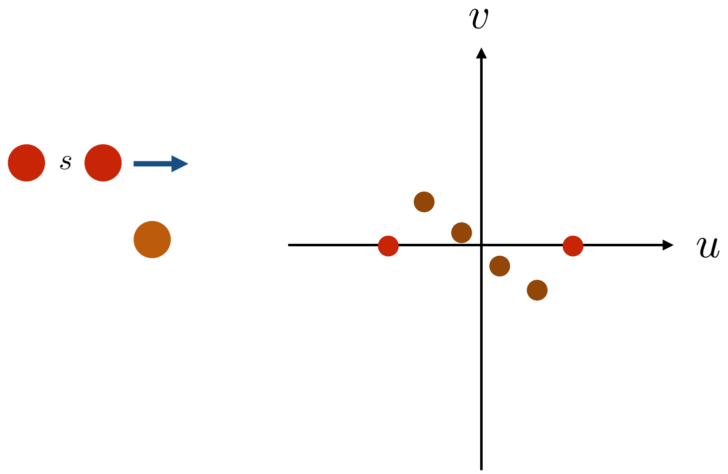
One pair of telescopes makes a set of fringes projected on the sky



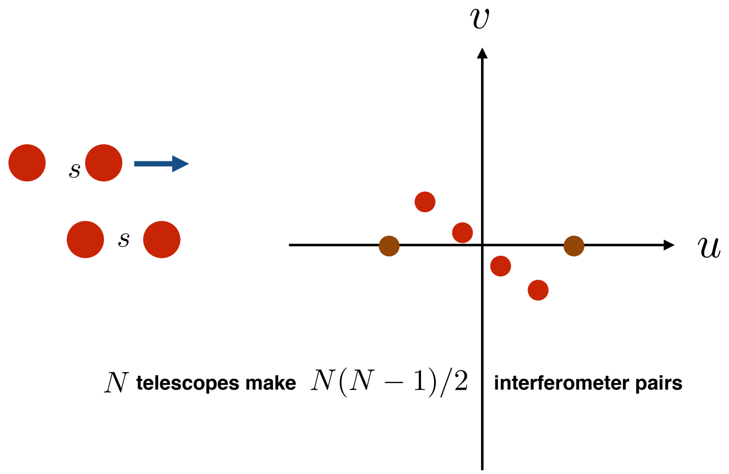
One pair of telescopes samples one point in the $u - v$ spatial frequency plane



Adding one more telescope adds many more points in the $u - v$ plane

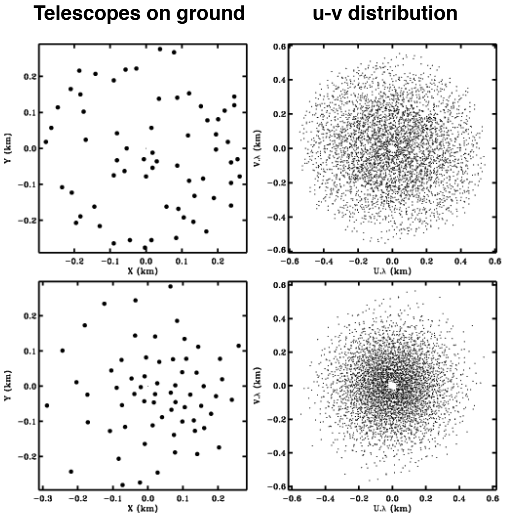


...but if you duplicate a baseline, you are wasting your resources



N telescopes make $N(N-1)/2$ interferometer pairs

You put your telescopes in a non-redundant pattern so that you sample the uv plane equally



Optical Transfer Function of 2 aperture interferometer

$$OTF = 2\left(\frac{\lambda}{R}\right)^2 \left[\delta(\vec{\zeta}) + \frac{1}{2}\delta\left(\vec{\zeta} - \vec{s}/\lambda\right) + \frac{1}{2}\delta\left(\vec{\zeta} + \vec{s}/\lambda\right)\right]$$

- pair of pinholes transmits three spatial frequencies
  - DC-component $\delta(\vec{0})$
  - two high frequencies related to length of baseline vector $\vec{s}$ at $\pm\vec{s}/\lambda$
- 3 spatial frequencies represent three-point sampling of the **uv-plane** in 2-d spatial frequency space
- complete sampling of uv-plane provides sufficient information to completely reconstruct original brightness distribution

PSF and OTF (Optical Transfer Function)

We know that the image of a point source gives the Point Source Function (PSF):

$$PSF = |FT(A)|^2$$

where A is the aperture function (==pupil shape) and FT is the Fourier transform.

The image of any general object i is then the convolution of the object o with the PSF:

$$i = o * s$$

The Optical Transfer Function (OTF) is the FT of the PSF:

$$I = O \times S$$

OTF is then the auto-correlation of the aperture function A

Point Spread Function of 2 aperture interferometer

- PSF is Fourier Transform of OTF

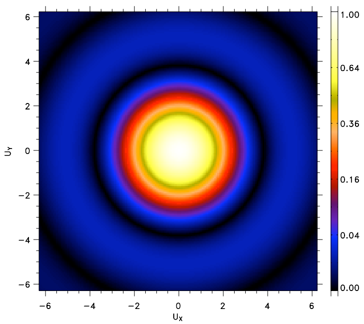
$$\delta(\vec{\zeta}) \quad \Leftrightarrow \quad 1$$
$$\delta\left(\vec{\zeta} - \vec{s}/\lambda\right) \quad \Leftrightarrow \quad e^{i2\pi\vec{\theta}\cdot\vec{s}/\lambda}$$
$$\delta\left(\vec{\zeta} + \vec{s}/\lambda\right) \quad \Leftrightarrow \quad e^{-i2\pi\vec{\theta}\cdot\vec{s}/\lambda}$$

- Point-Spread Function of 2-element interferometer

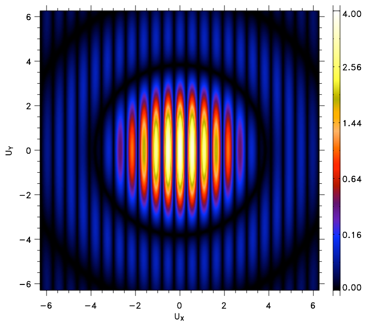
$$\left(\frac{\lambda}{R}\right)^2 \left[2(1 + \cos 2\pi\vec{\theta}\cdot\vec{s}/\lambda)\right] = 4\left(\frac{\lambda}{R}\right)^2 \cos^2 \pi\vec{\theta}\cdot\vec{s}/\lambda$$

- $\vec{\theta}$: 2-d angular coordinate vector
- attenuation factor $(\lambda/R)^2$ from spherical expansion

Two element interferometer

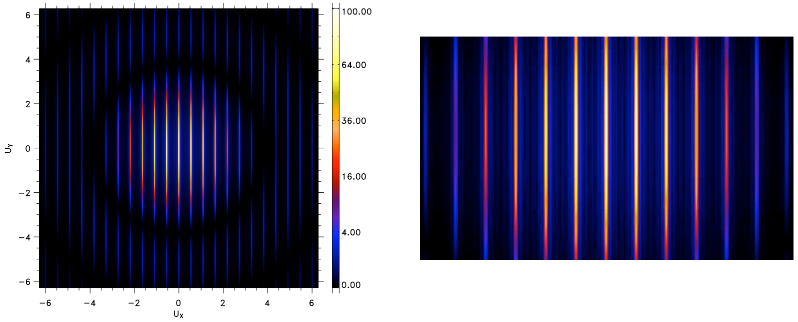


A single dish PSF on the sky



Two dishes on the sky diameter d = 25m separated by s=144m

Ten element interferometer



10 elements makes the fringes narrower

Equally spaced array of telescopes

- scalar function due to circular symmetry

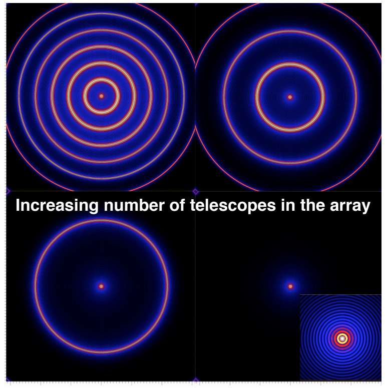
$$PSF_{ERAS} = \left(\frac{\lambda}{R}\right)^2 \left[\frac{1}{4}\pi\,(d/\lambda)^2\right]^2 \left[\frac{2J_1(u)}{u}\right]^2 \frac{\sin^2 N(u\triangle L/D)}{\sin^2(u\triangle L/D)}$$

  with $u = \pi\theta D/\lambda$ and θ, the radially symmetric, diffraction angle
- central peak: similar to Airy function with spatial resolution

$$\triangle\theta = \frac{\lambda}{2L_{max}}\,radians$$

  with $2L_{max}$ the maximum diameter of the array in the YZ-plane
- concentric grating lobes: angular distances of annuli from central peak follow from the location of principal maxima given by modulation term $\sin^2 N(u\triangle L/D)/\sin^2(u\triangle L/D)$

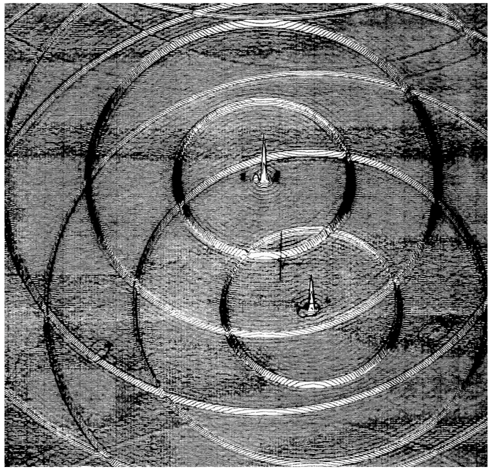
Rotating this array of telescopes on the sky causes symmetric PSF



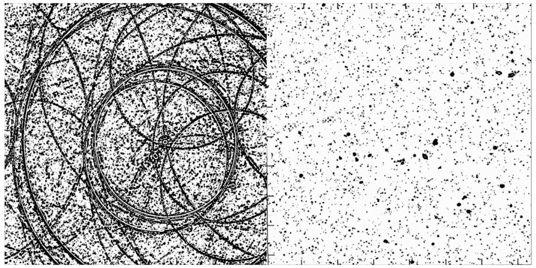
For an N element array with separation these angular positions are:

$$\theta_{grating} = \frac{\lambda}{\Delta L}, 2\frac{\lambda}{\Delta L}, ..., (N-1)\frac{\lambda}{\Delta L}$$

Reconstruction of a pair of point sources seen by an array of telescopes



Fit the PSF of the array and remove it, but it is a tricky inverse problem….



undersampling of uv-plane, grating lobes within field of view

decrease distance between antennas 9 and A during second half of rotation for 36 meter increment coverage

four half rotations in 48 hours can increase coverage to 18 meter increments ⇒ complete uv coverage

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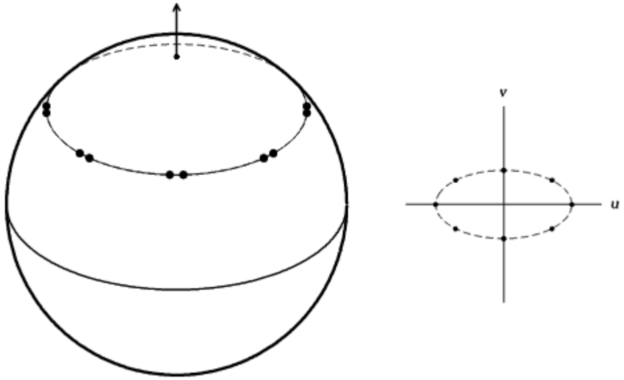
Earth Rotation fills in uv plane

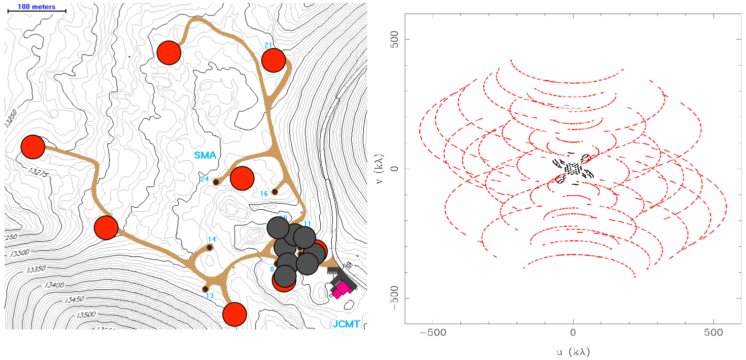
Pair of telescopes in East-West line appear to rotate about each other once every 24 hours when see by a celestial object



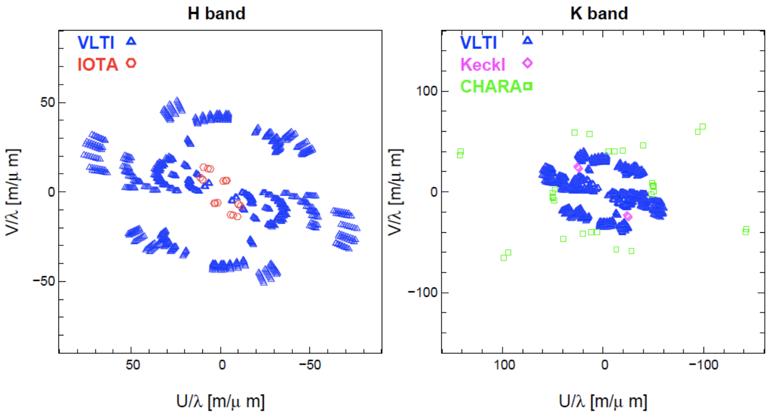
The v axis of the ellipse is smaller by cosine (declination)

Earth Rotation fills in uv plane

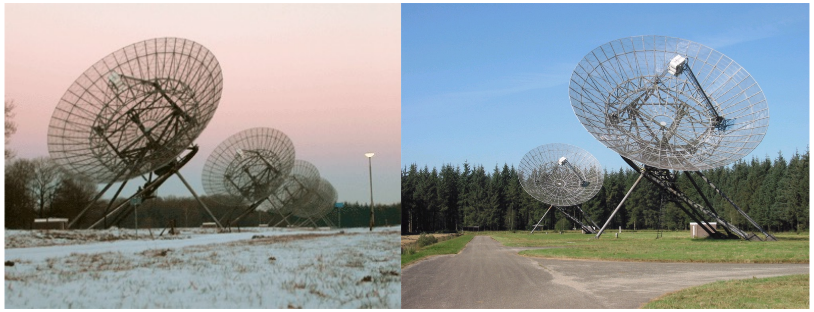
2 configurations of 8 SMA telescopes



Rotation of Earth during observations 'fills in' the uv plane



Westerbork Synthesis Radio Telescope (WSRT)



- after 12 hours, 38 concentric semi-circles with radii ranging from $L_{min} = 72$ meters to $L_{max} = 2736$ meters in increments of $\triangle L = 72$ meters
- correlators integrate over 10 s, sampling of semi-circles every 1/24 degrees

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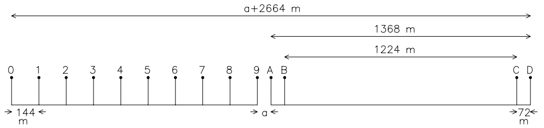
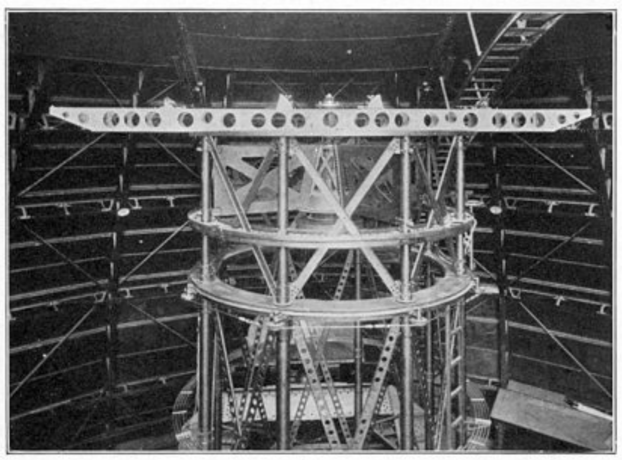
Westerbork Synthesis Radio Telescope (WSRT)



- 14 parabolic antennae, diameters $D = 25$ m
- lined up along East-West direction over ≈ 2750 m
- 10 antennae have fixed mutual distance of 144 m
- 4 antennae can be moved collectively with respect to fixed array
- 14 antennae comprise 40 simultaneously operating interferometers
- array is rotated in plane containing Westerbork perpendicular to Earth's rotation axis
- limited to sources near the North polar axis
- standard distance *a* between 9 and A equals 72 meters

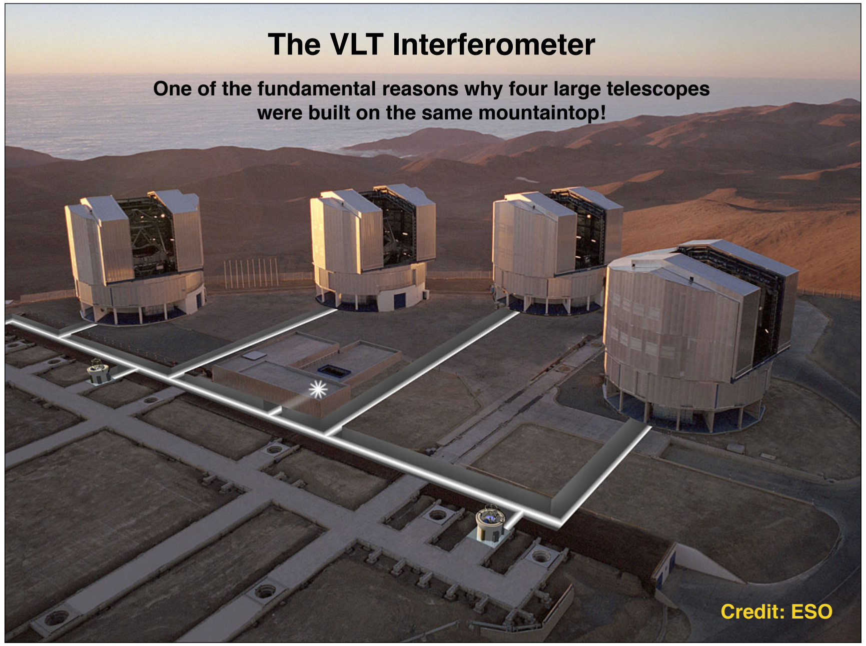
Michelson's Interferometer

1908 on the 200 inch telescope at Palomar

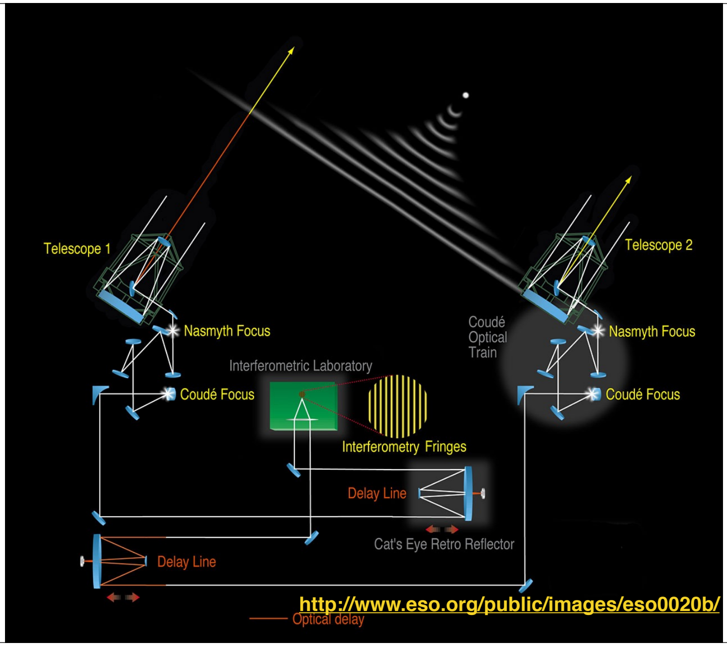


The VLT Interferometer

One of the fundamental reasons why four large telescopes were built on the same mountaintop!



Credit: ESO



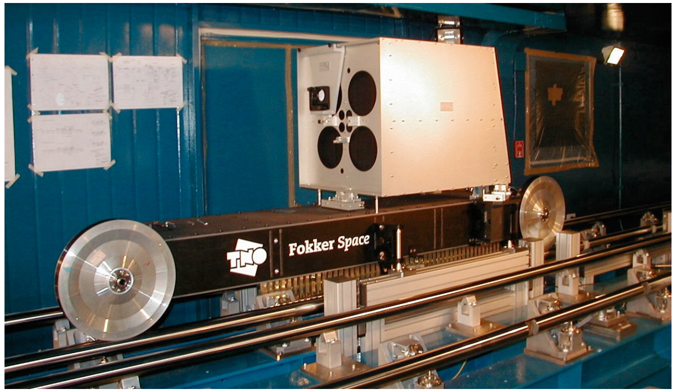
http://www.eso.org/public/images/eso0020b/

Delay lines under the mountain

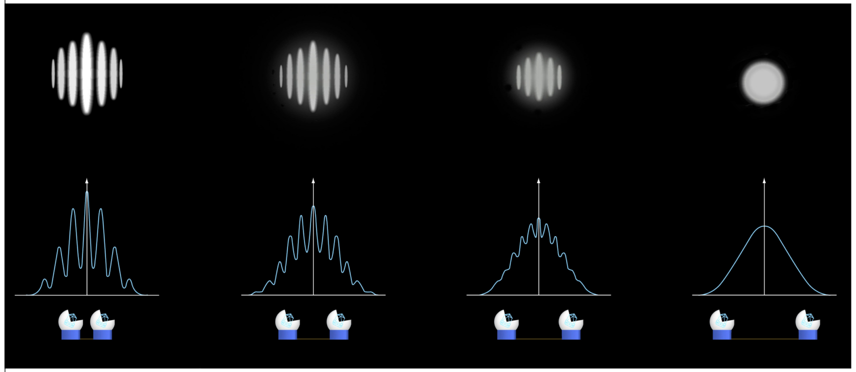
Compensate for optical path difference between telescopes with a beam compensation room

Continuous beam path compensation

Movable carts need to position retroreflectors with an accuracy of 10 nanometers over a length of 60 metres



Fringes disappear when diameter of star is equal to the fringe separation in the interferometer



Original experiment by Michelson determined diameter of Betelgeuse

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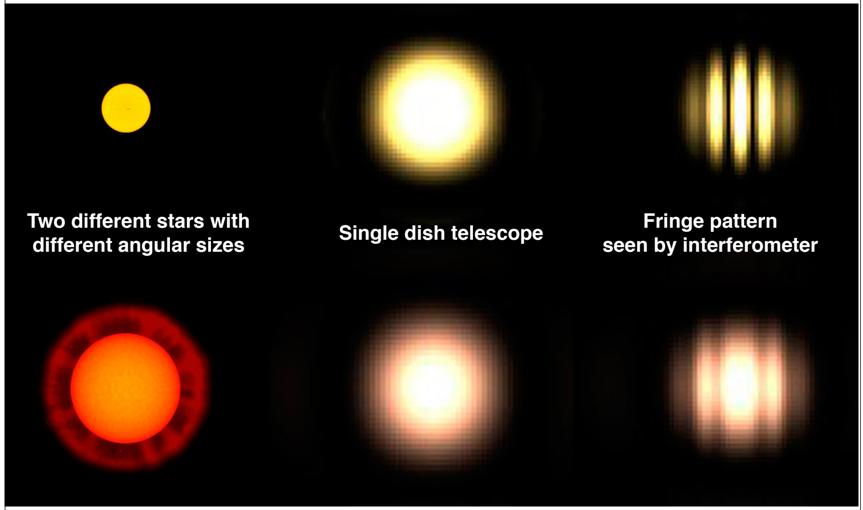
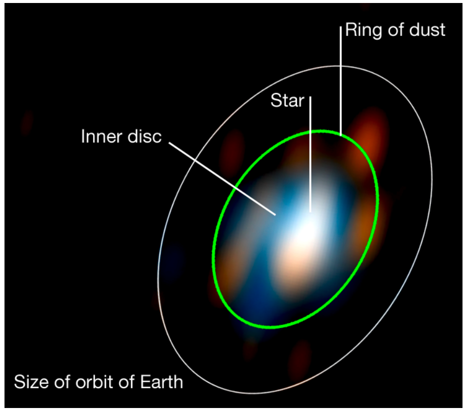
Stellar diameter measurements due to fringe visibility



VLTI results in resolved images of giant stars



Herbig Ae star HD 163296

Field of view of 25 mas

H and K band data

Combination of VLTI, IOTA, Keck I and CHARA interferometers

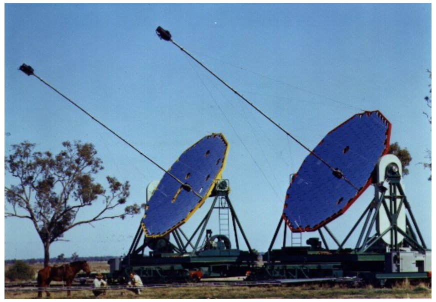
Renard et al. 2010

Intensity Interferometry

Correlation with intensity as seen by two telescopes

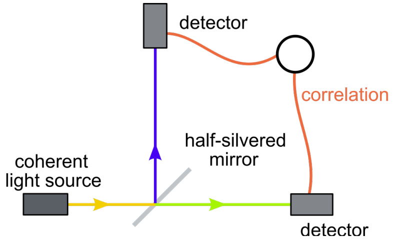
Hanbury-Brown and Twiss Effect

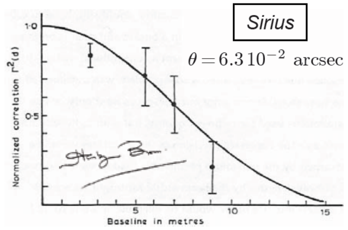
Nairobi Interferometer 1956



Light buckets - do not need to be high optical quality

Measuring the diameter of hot nearby stars





looking at plane waves with two detectors and phase difference ϕ

$$I_1 = E_1^2 \sin^2 \omega t \qquad I_2 = E_1^2 \sin^2(\omega t + \phi)$$

intensity correlation

$$< I_1 I_2 >= E^4/4 \left(1 + \cos^2 2\phi\right)$$