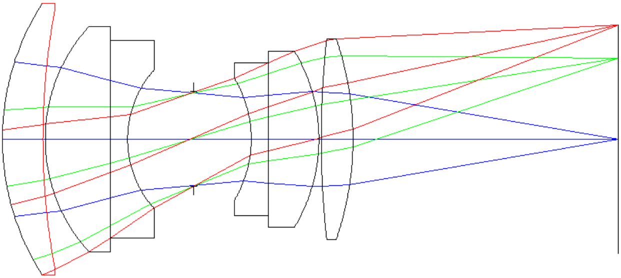
Lecture 4: Geometrical Optics 2

Outline

1. Optical Systems
2. Images and Pupils
3. Rays
4. Wavefronts
5. Aberrations

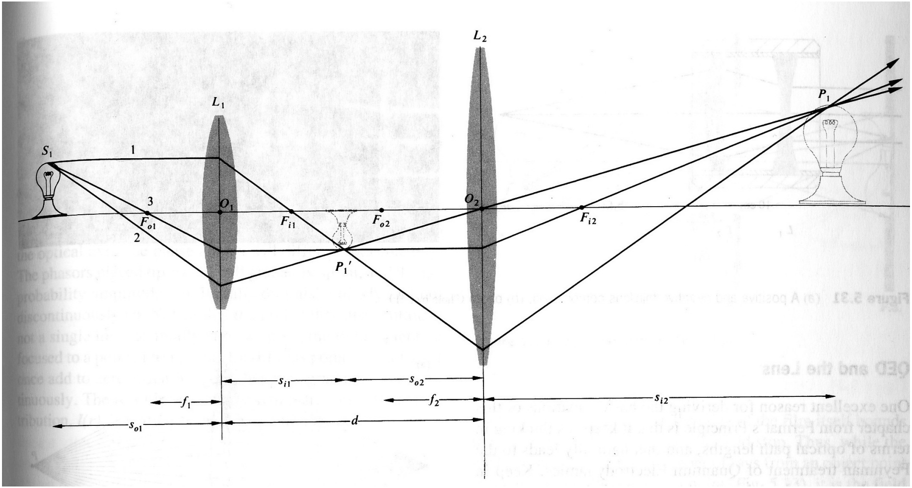
Optical Systems

Overview



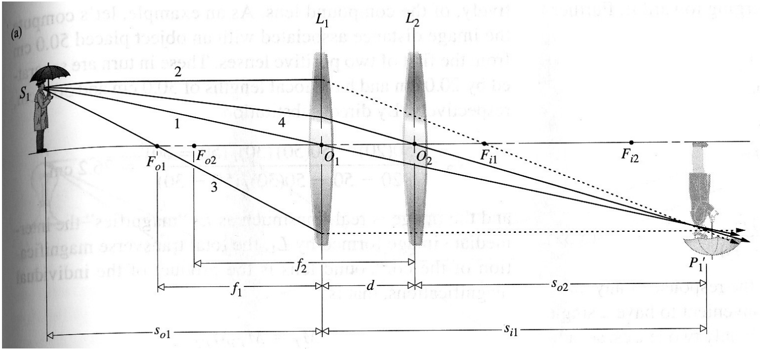
- combinations of several optical elements (lenses, mirrors, stops)
- examples: camera “lens”, microscope, telescopes, instruments
- thin-lens combinations can be treated analytically
- effective focal length: $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$

Simple Thin-Lens Combinations



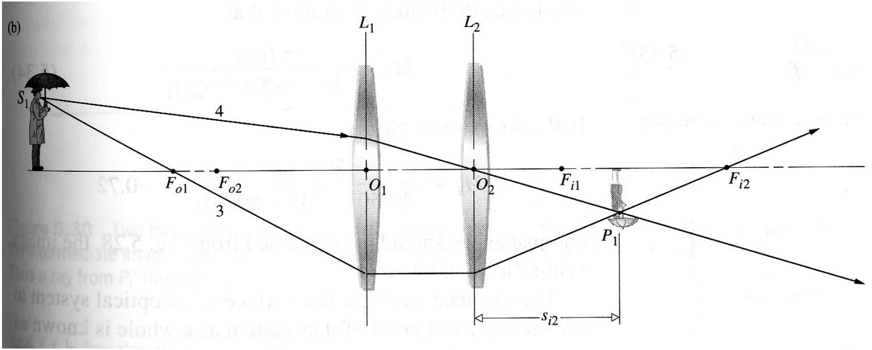
- distance $>$ sum of focal lengths $\Rightarrow$ real image between lenses
- apply single-lens equation successively

Thin-Lens Combinations 1



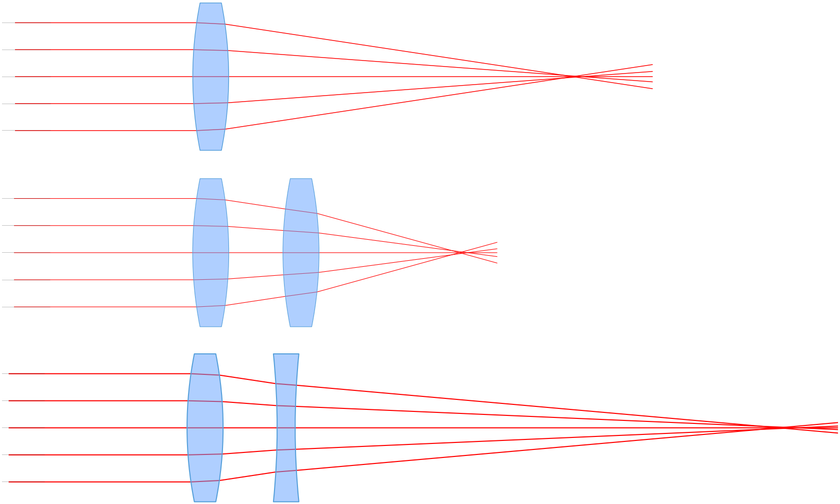
- construct image formed by lens 1 using rays 2 and 3
- ray 2 passes through focal point F_{i1}
- ray 3 passes through focal point F_{o1}
- ray 4 passes backwards through center of lens 2

Thin-Lens Combinations 2



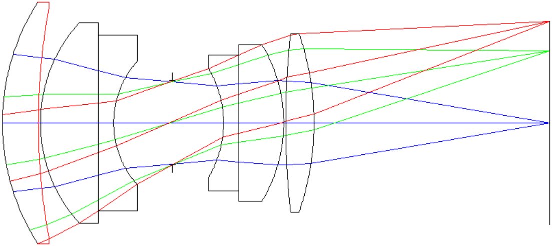
- adding lens 2 does not refract ray 4
- ray 3 is refracted to image focus F_{i2}
- intersection of rays 3 and 4 determine image location
- lens 2 adds convergence or divergence

Second Lens Adds Convergence or Divergence



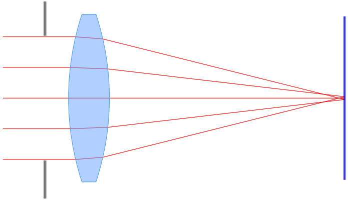
F-number and Numerical Aperture

Aperture

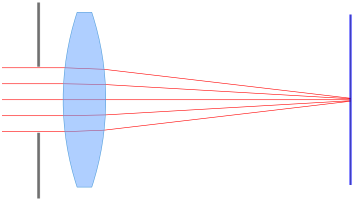


- all optical systems have a place where ’aperture’ is limited
- main mirror of telescopes
- aperture stop in photographic lenses
- aperture typically has a maximum diameter
- aperture size is important for diffraction effects

F-number



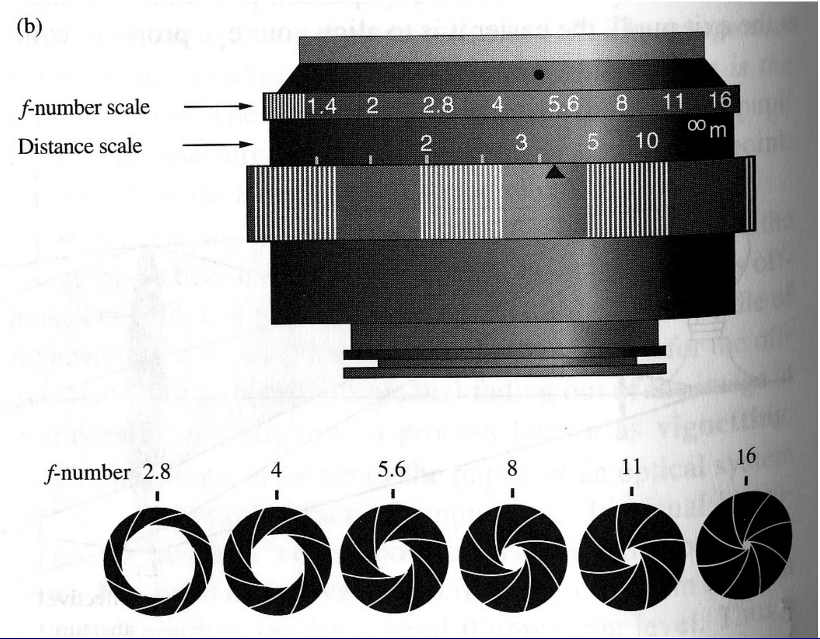
f/2:



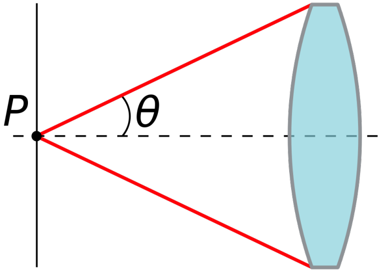
f/4:

- describes the light-gathering ability of the lens
- f-number given by $F = f/D$
- also called focal ratio or f-ratio, written as: f/F
- the bigger F, the better the paraxial approximation works
- fast system for $F < 2$, slow system for $F > 2$

F-number on Camera Lens



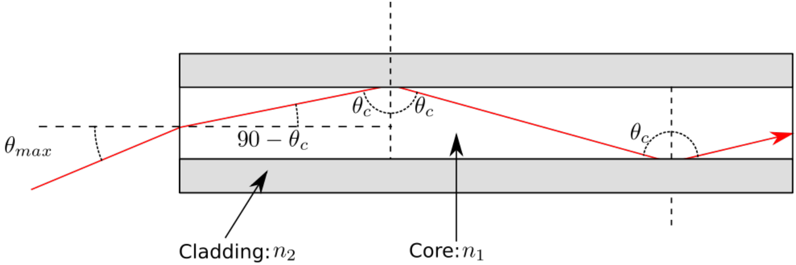
Numerical Aperture



en.wikipedia.org/wiki/File:Numerical_aperture.svg

- numerical aperture (NA): $n \sin\theta$
- n index of refraction of working medium
- θ half-angle of maximum cone of light that can enter or exit lens
- important for microscope objectives (n often not 1)

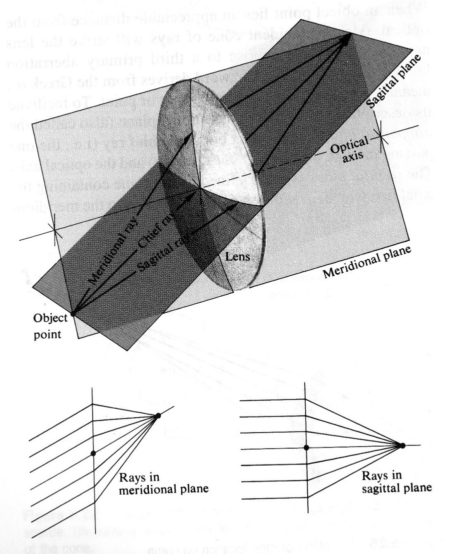
Numerical Aperture in Fibers



en.wikipedia.org/wiki/File:OF-na.svg

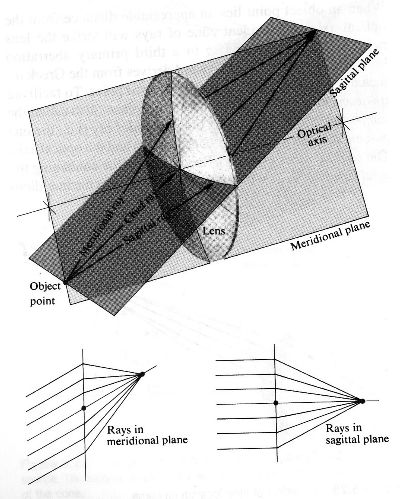
- acceptance cone of the fiber determined by materials
- $NA = n \sin\theta = \sqrt{n_1^2 - n_2^2}$
- n index of refraction of working medium

Ray Definitions



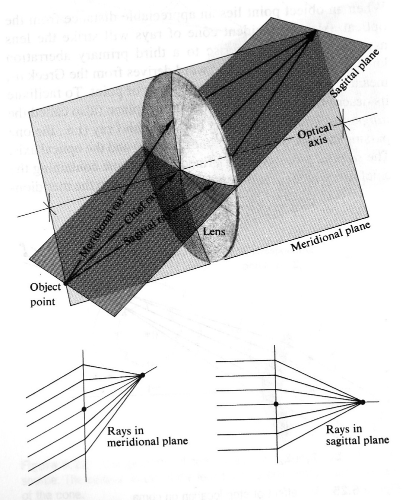
Planes and Rays

- meridional plane defined by optical axis and chief ray going through center of optical system
- sagittal plane is perpendicular to it



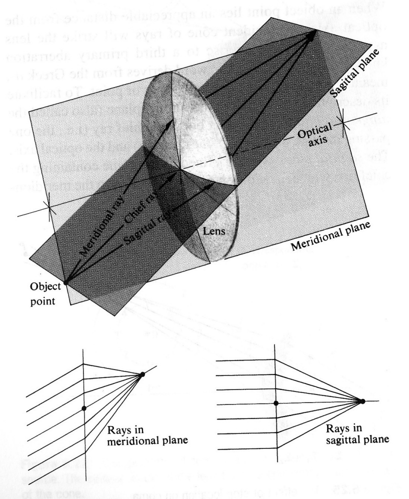
Meridional (or Tangential) Ray

- confined to plane containing optical axis and object point from which ray originates



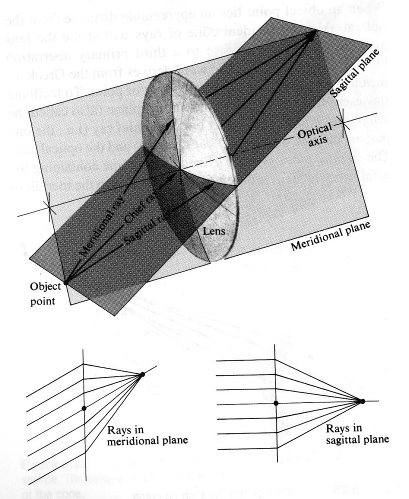
Chief (or Principal) Ray

- goes through center of aperture
- meridional ray that starts at edge of object, and passes through center of aperture stop
- crosses optical axis at locations of pupils
- chief rays are equivalent to the rays in pinhole camera
- distance between chief ray and optical axis at an image location defines size of image



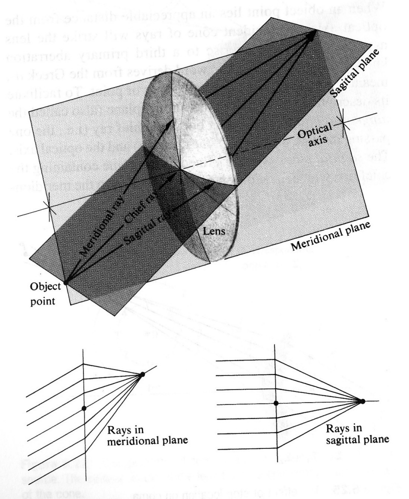
Skew Ray

- does not propagate in plane that contains both object point and optical axis
- does not cross optical axis anywhere, and not parallel to it



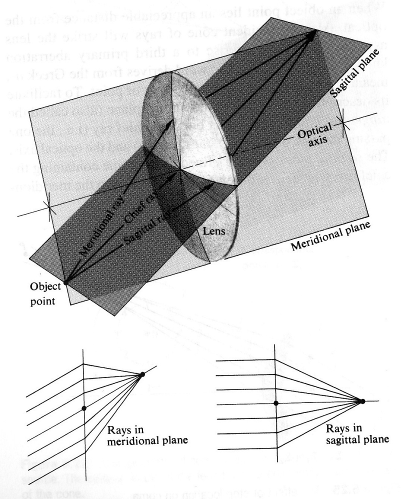
Marginal Ray

- is meridional ray that starts at point where object crosses optical axis and touches edge of aperture stop
- useful because it crosses optical axis again at locations where image is formed
- distance of marginal ray from optical axis at entrance and exit pupils defines their sizes



Sagittal (or Transverse) Ray

- comes from off-axis object point, propagates in plane perpendicular to meridional plane
- intersects the pupil along a line that is perpendicular to meridional plane
- chief ray is both sagittal and meridional
- all other sagittal rays are skew rays

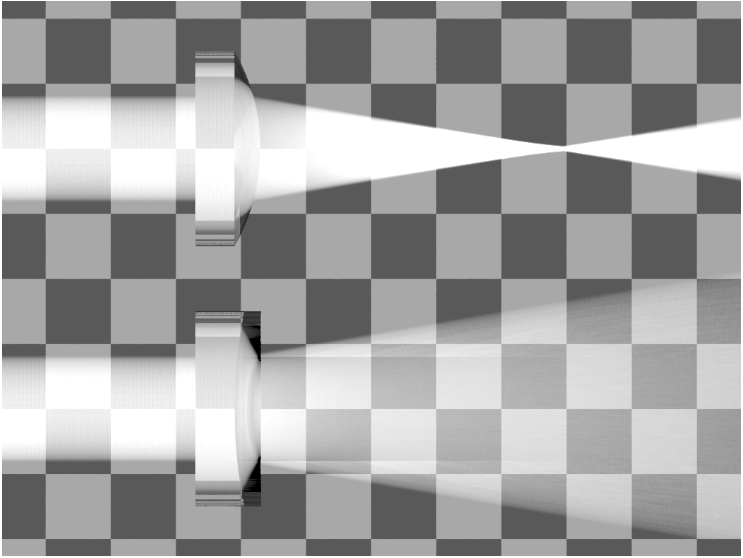


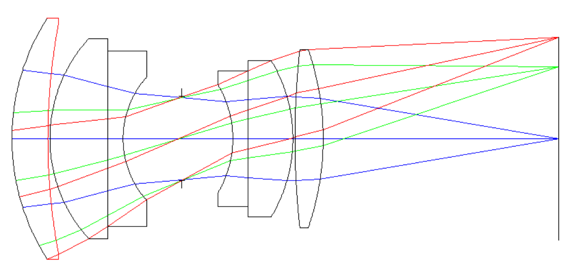
Paraxial Ray

- makes a small angle to the optical axis of the system
- lies close to the axis throughout the system
- can be modeled reasonably well by using the paraxial approximation.

Images and Pupils

Converging, Diverging and Collimated beams

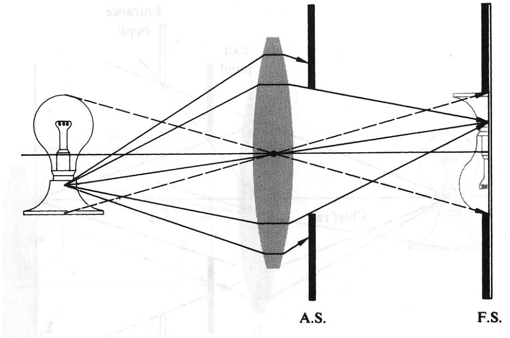




Images and Pupils

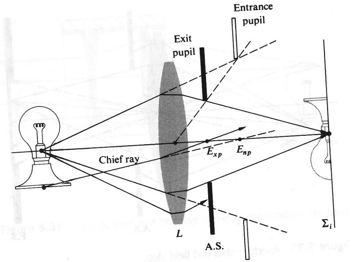
- image
  - every object point comes to a focus in an image plane
  - light in one image point comes from pupil positions
  - object information is encoded in position, not in angle
- pupil
  - all object rays are smeared out over complete aperture
  - light in one pupil point comes from different object positions
  - object information is encoded in angle, not in position

Aperture and Field Stops



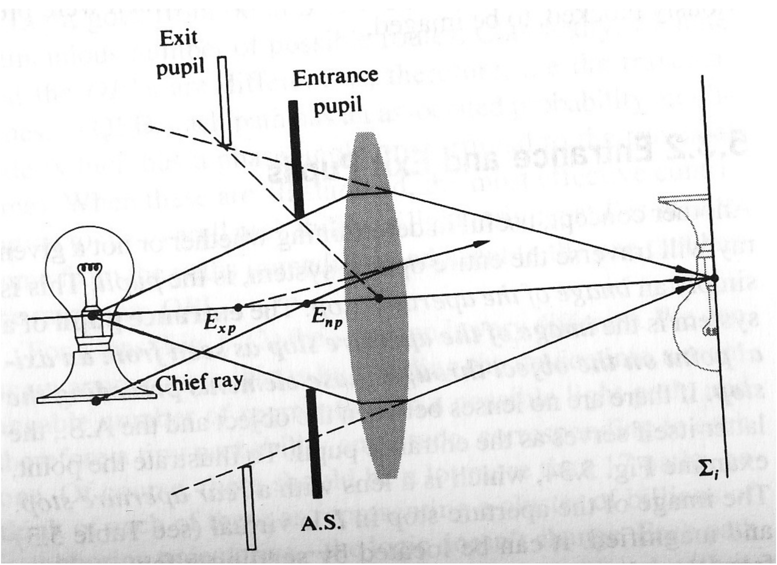
- aperture stop limits the amount of light reaching the image
- aperture stop determines light-gathering ability of optical system
- field stop limits the image size or angle

Entrance and Exit Pupils

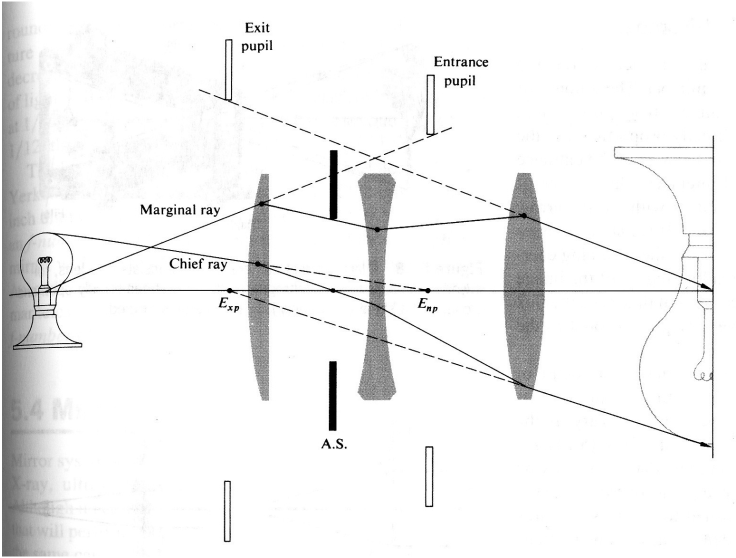


- pupil is an image of the aperture stop
- entrance pupil: image of the aperture stop as seen from a point on the optical axis and on the object through optical elements preceeding the aperture stop
- exit pupil: image of the aperture stop as seen from a point on the optical axis and in the image through optical elements after the aperture stop

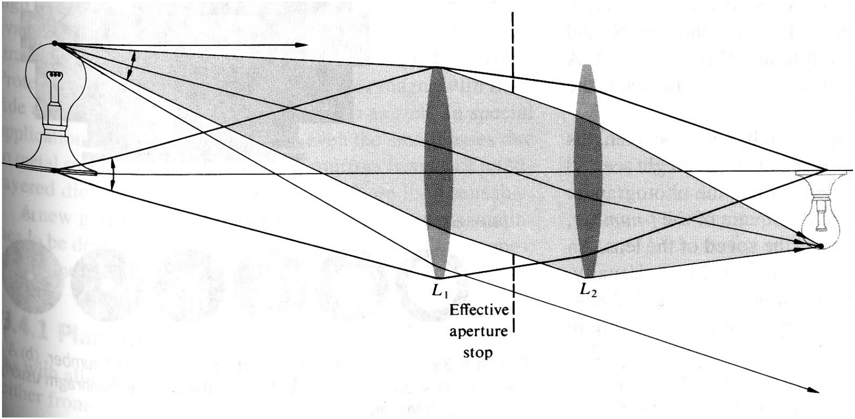
Entrance and Exit Pupils



Entrance and Exit Pupils

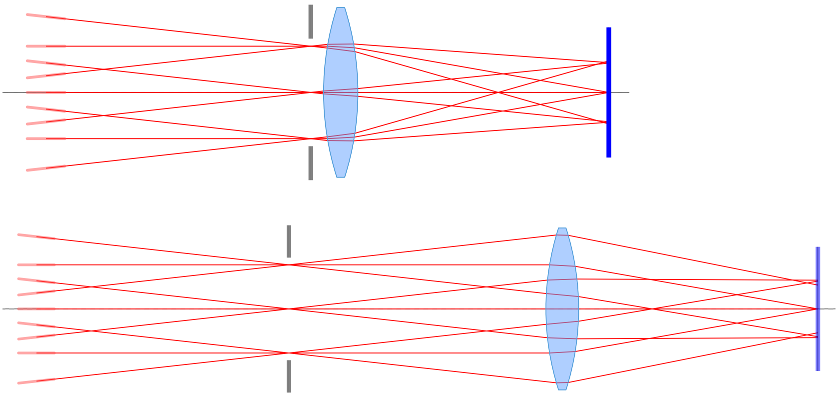


Vignetting



- effective aperture stop depends on position in object
- image fades toward its edges

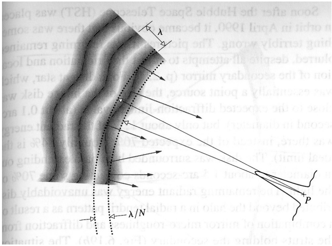
Telecentric Arrangement



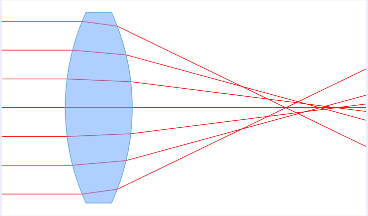
- as seen from image, pupil is at infininity
- easy: lens is its focal length away from pupil (image)
- magnification does not change with focus positions
- ray cones for all image points have the same orientation

Aberrations

Spot Diagrams and Wavefronts



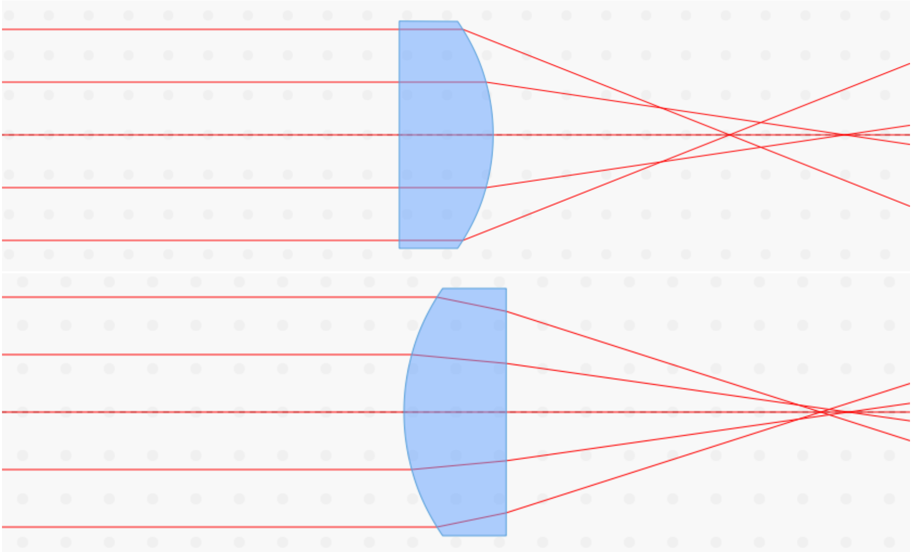
- plane of least confusion is location where image of point source has smallest diameter
- spot diagram: shows ray locations in plane of least confusion
- spot diagrams are closely connected with wavefronts
- aberrations are deviations from spherical wavefront



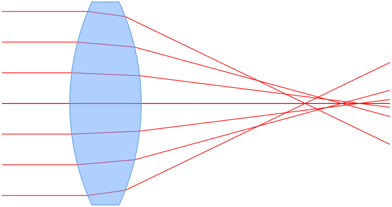
Spherical Aberrations

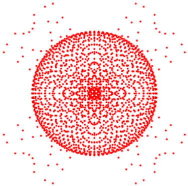
- different focal lengths of paraxial and marginal rays
- longitudinal spherical aberration along optical axis
- transverse (or lateral) spherical aberration in image plane
- much more pronounced for short focal ratios

Minimizing Spherical Aberrations



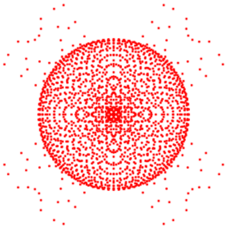
Spherical Aberration of Spherical Lens

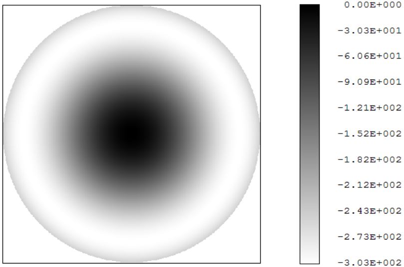




- foci from paraxial beams are further away than marginal rays
- spot diagram shows central area with fainter disk around it

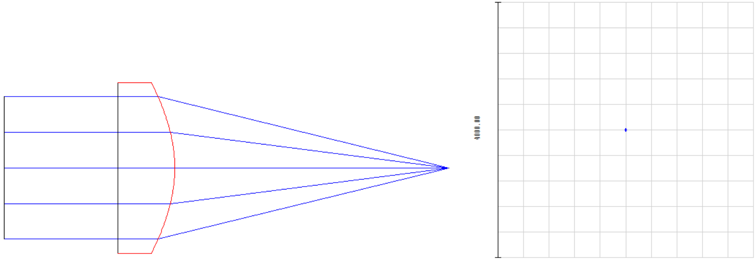
Spherical Aberration Spots and Waves





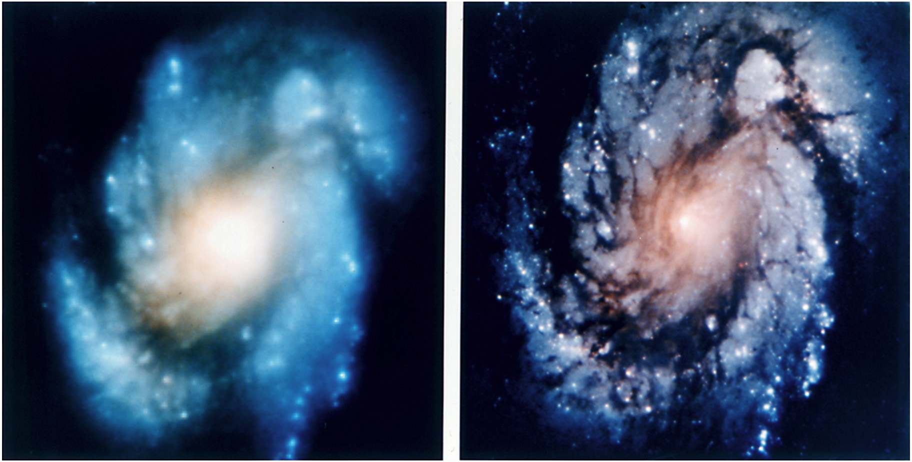
- spot diagram shows central area with fainter disk around it
- wavefront has peak and turned-up edges

Aspheric Lens

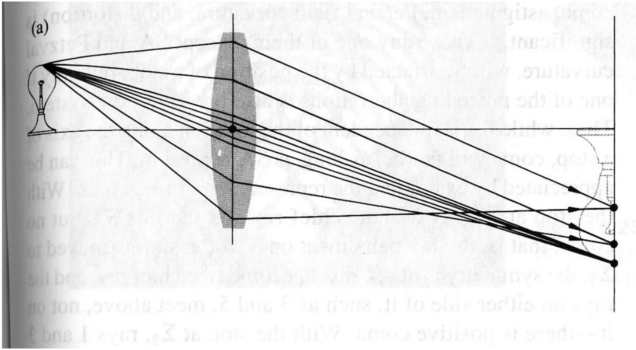


- conic constant $K = -1 - \sqrt{n}$ makes perfect lens
- difficult to manufacture
- but possible these days

HST

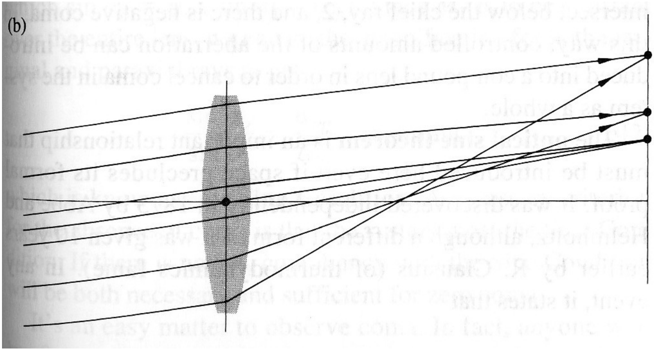


Coma

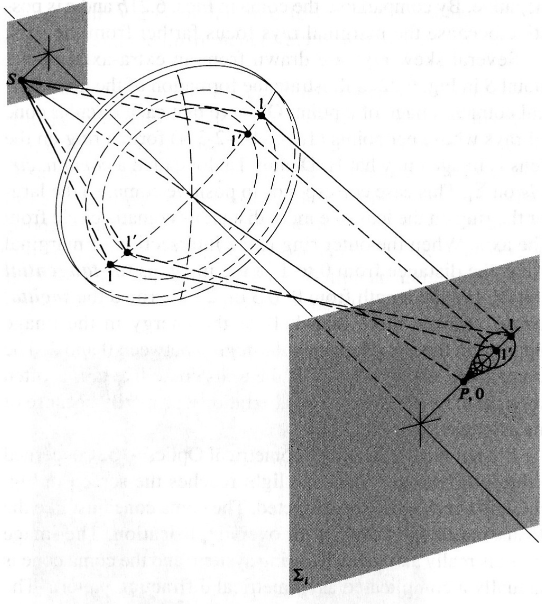


- typically seen for object points away from optical axis
- leads to 'tails' on stars

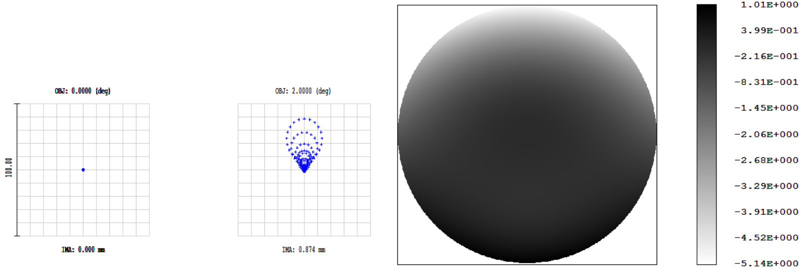
Positive Coma



Coma

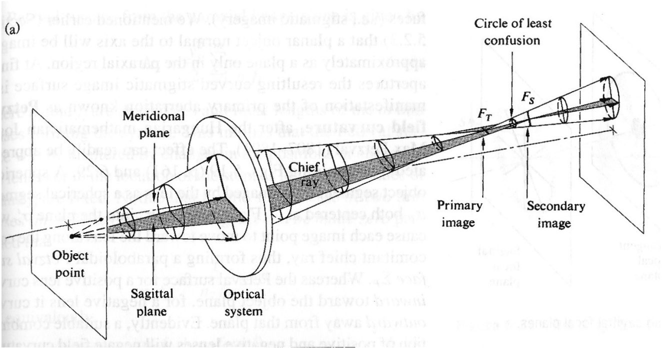


Coma Spots and Waves



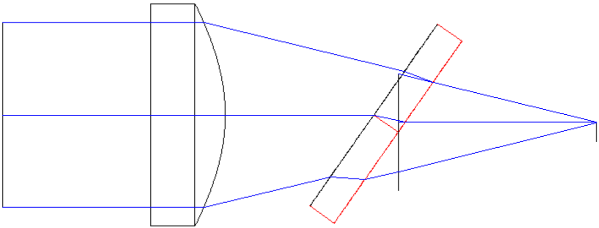
- parabolic mirror with perfect on-axis performance
- spots and wavefront for off-axis image points
- wavefront is tilted in inner part

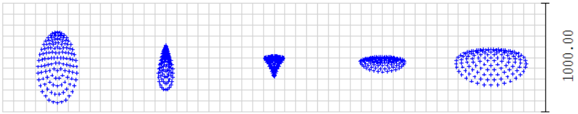
Astigmatism



- image of a point forms focal lines at the sagittal and tangental foci
- in between an elliptical shape

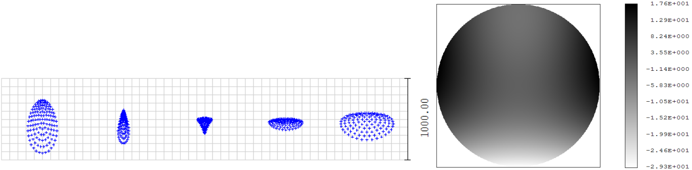
Tilted Glass Plate in Converging Beam





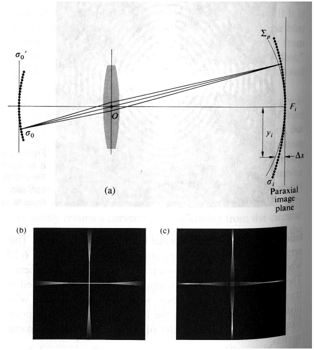
- astigmatism and spherical aberration
- note beam shift
- tilted plates: beam shifters, filters, beamsplitters

Astigmatism Spots and Waves



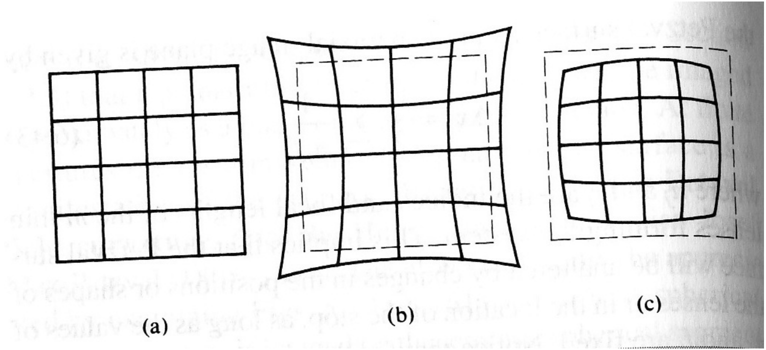
- focus in two orthogonal directions, but not in both at the same time
- difference of two parabolae with different curvatures
- wavefront has saddle shape

Field Curvature



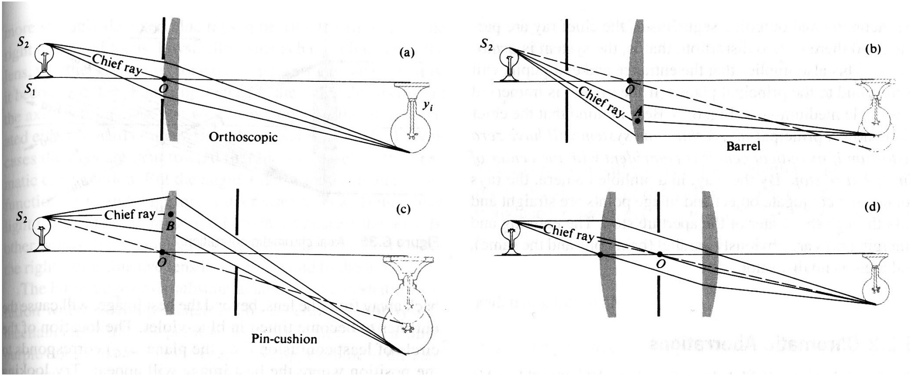
- field (Petzval) curvature: image lies on curved surface
- problems with flat detectors (e.g. CCDs)
- solution: field flattening lens close to focus

Distortion



- image is sharp but geometrically distorted
- (a) object
- (b) positive (or pincushion) distortion
- (c) negative (or barrel) distortion

Aperture Stop Creates Distortion



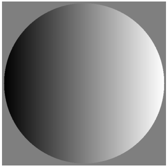
Aberration Descriptions

Seidel Aberrations

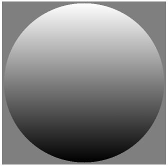
- Ludwig von Seidel (1857)
- Taylor expansion of $\sin\phi$
- $\sin\phi = \phi - \frac{\phi^3}{3!} + \frac{\phi^5}{5!} - ...$
- paraxial: first-order optics
- Seidel optics: third-order optics
- Seidel aberrations: spherical, astigmatism, coma, field curvature, distortion

Zernike Polynomials

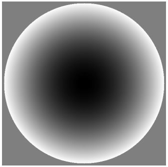
tip



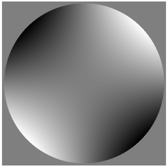
tilt



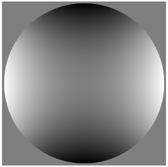
focus



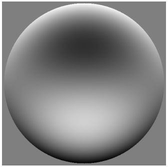
astigmatism (45 deg)



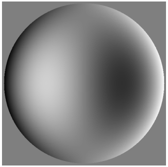
astigmatism 0 deg



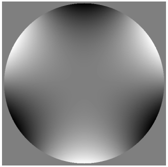
coma (0 deg)



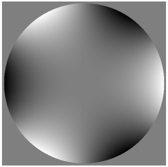
coma (90 deg)



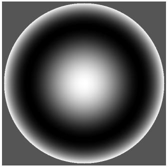
trefoil (0 deg)



trefoil (30 deg)



third-order spherical



- low orders equal Seidel aberrations
- form orthonormal basis on unit circle