Outline

1. Optical Systems
2. Images and Pupils
3. Rays
4. Wavefronts
5. Aberrations
combinations of several optical elements (lenses, mirrors, stops)
examples: camera “lens”, microscope, telescopes, instruments
thin-lens combinations can be treated analytically
effective focal length: $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$
Simple Thin-Lens Combinations

- distance > sum of focal lengths ⇒ real image between lenses
- apply single-lens equation successively
Thin-Lens Combinations 1

- Construct image formed by lens 1 using rays 2 and 3
- Ray 2 passes through focal point $F_{i1}$
- Ray 3 passes through focal point $F_{o1}$
- Ray 4 passes backwards through center of lens 2
adding lens 2 does not refract ray 4
ray 3 is refracted to image focus $F_{i2}$
intersection of rays 3 and 4 determine image location
lens 2 adds convergence or divergence
Second Lens Adds Convergence or Divergence
all optical systems have a place where 'aperture' is limited
main mirror of telescopes
aperture stop in photographic lenses
aperture typically has a maximum diameter
aperture size is important for diffraction effects
F-number

-describes the light-gathering ability of the lens
-f-number given by $F = \frac{f}{D}$
-also called focal ratio or f-ratio, written as: $\frac{f}{F}$
-the bigger $F$, the better the paraxial approximation works
-fast system for $F < 2$, slow system for $F > 2$
F-number on Camera Lens

(b)

- **f-number scale**
- **Distance scale**

$\text{f-number scale}$

- 1.4
- 2
- 2.8
- 4
- 5.6
- 8
- 11
- 16

$\text{Distance scale}$

- 2
- 3
- 5
- 10
- $\infty m$

$\text{f-number}$ 2.8

4

5.6

8

11

16
**Numerical Aperture**

- numerical aperture (NA): $n \sin \theta$
- $n$ index of refraction of working medium
- $\theta$ half-angle of maximum cone of light that can enter or exit lens
- important for microscope objectives ($n$ often not 1)
Numerical Aperture in Fibers

- acceptance cone of the fiber determined by materials

\[ NA = n \sin \theta = \sqrt{n_1^2 - n_2^2} \]

\[ n \] index of refraction of working medium
Ray Definitions

Planes and Rays

- meridional plane defined by optical axis and chief ray going through center of optical system
- sagittal plane is perpendicular to it
Meridional (or Tangential) Ray

- confined to plane containing optical axis and object point from which ray originates
Chief (or Principal) Ray

- goes through center of aperture
- meridional ray that starts at edge of object, and passes through center of aperture stop
- crosses optical axis at locations of pupils
- chief rays are equivalent to the rays in pinhole camera
- distance between chief ray and optical axis at an image location defines size of image
Skew Ray

- does not propagate in plane that contains both object point and optical axis
- does not cross optical axis anywhere, and not parallel to it
Marginal Ray

- is meridional ray that starts at point where object crosses optical axis and touches edge of aperture stop
- useful because it crosses optical axis again at locations where image is formed
- distance of marginal ray from optical axis at entrance and exit pupils defines their sizes
Sagittal (or Transverse) Ray

- comes from off-axis object point, propagates in plane perpendicular to meridional plane
- intersects the pupil along a line that is perpendicular to meridional plane
- chief ray is both sagittal and meridional
- all other sagittal rays are skew rays
Paraxial Ray

- makes a small angle to the optical axis of the system
- lies close to the axis throughout the system
- can be modeled reasonably well by using the paraxial approximation.
Images and Pupils

- **image**
  - every object point comes to a focus in an image plane
  - light in one image point comes from pupil positions
  - object information is encoded in position, not in angle

- **pupil**
  - all object rays are smeared out over complete aperture
  - light in one pupil point comes from different object positions
  - object information is encoded in angle, not in position
Aperture and Field Stops

- Aperture stop limits the amount of light reaching the image.
- Aperture stop determines light-gathering ability of optical system.
- Field stop limits the image size or angle.
Entrance and Exit Pupils

- pupil is an image of the aperture stop
- entrance pupil: image of the aperture stop as seen from a point on the optical axis and on the object through optical elements preceding the aperture stop
- exit pupil: image of the aperture stop as seen from a point on the optical axis and in the image through optical elements after the aperture stop
Entrance and Exit Pupils

Entrance pupil

Exit pupil

Chief ray

$E_{xp}$

$E_{np}$

A.S.

$\Sigma_i$
Entrance and Exit Pupils

- Exit pupil
- Entrance pupil
- Marginal ray
- Chief ray
- $E_{xp}$
- $E_{np}$
- A.S.
Vignetting

- effective aperture stop depends on position in object
- image fades toward its edges
Telecentric Arrangement

- As seen from image, pupil is at infinity.
- Easy: lens is its focal length away from pupil (image).
- Magnification does not change with focus positions.
- Ray cones for all image points have the same orientation.
Aberrations

Spot Diagrams and Wavefronts

- Plane of least confusion is the location where the image of a point source has the smallest diameter.
- Spot diagram: shows the ray locations in the plane of least confusion.
- Spot diagrams are closely connected with wavefronts.
- Aberrations are deviations from spherical wavefronts.
Spherical Aberrations

- different focal lengths of paraxial and marginal rays
- longitudinal spherical aberration along optical axis
- transverse (or lateral) spherical aberration in image plane
- much more pronounced for short focal ratios
Minimizing Spherical Aberrations
Spherical Aberration of Spherical Lens

- foci from paraxial beams are further away than marginal rays
- spot diagram shows central area with fainter disk around it
Spherical Aberration Spots and Waves

- spot diagram shows central area with fainter disk around it
- wavefront has peak and turned-up edges
Aspheric Lens

- conic constant $K = -1 - \sqrt{n}$ makes perfect lens
- difficult to manufacture
- but possible these days
Coma

- typically seen for object points away from optical axis
- leads to 'tails' on stars
Positive Coma
Coma Spots and Waves

- parabolic mirror with perfect on-axis performance
- spots and wavefront for off-axis image points
- wavefront is tilted in inner part
Astigmatism

- Image of a point forms focal lines at the sagittal and tangential foci in between an elliptical shape.
Tilted Glass Plate in Converging Beam

- astigmatism and spherical aberration
- note beam shift
- tilted plates: beam shifters, filters, beamsplitters
Astigmatism Spots and Waves

- focus in two orthogonal directions, but not in both at the same time
- difference of two parabolae with different curvatures
- wavefront has saddle shape
Field Curvature

- field (Petzval) curvature: image lies on curved surface
- problems with flat detectors (e.g. CCDs)
- solution: field flattening lens close to focus
Distortion

- Image is sharp but geometrically distorted
  - (a) object
  - (b) positive (or pincushion) distortion
  - (c) negative (or barrel) distortion
Aperture Stop Creates Distortion

(a) Orthoscopic
(b) Barrel
(c) Pin-cushion
### Seidel Aberrations

- Ludwig von Seidel (1857)
- Taylor expansion of $\sin \phi$
  \[
  \sin \phi = \phi - \frac{\phi^3}{3!} + \frac{\phi^5}{5!} - \ldots
  \]
- paraxial: first-order optics
- Seidel optics: third-order optics
- Seidel aberrations: spherical, astigmatism, coma, field curvature, distortion
**Zernike Polynomials**

<table>
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<th>Tip</th>
<th>Tilt</th>
<th>Focus</th>
<th>Astigmatism (45 deg)</th>
<th>Astigmatism 0 deg</th>
<th>Coma (0 deg)</th>
<th>Coma (90 deg)</th>
<th>Trefoil (0 deg)</th>
<th>Trefoil (30 deg)</th>
<th>Third-order spherical</th>
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- low orders equal Seidel aberrations
- form orthonormal basis on unit circle

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Lecture 4: Geometrical Optics 2