Astronomical Observing Techniques 2018

Lecture 4: Everything You Always Wanted to Know About Optics

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Content

Waves from Maxwell Equations

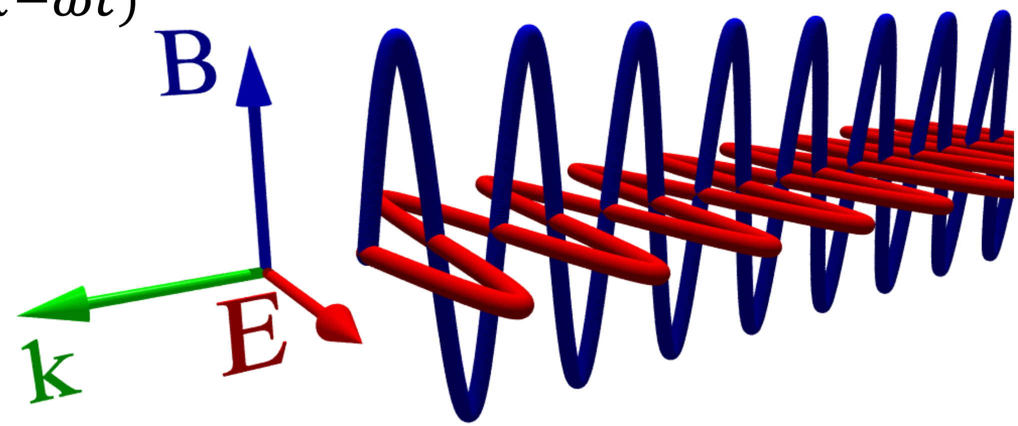
- Maxwell equations & linear material equations: differential equation for damped waves

$$\nabla^2 \vec{E} - \frac{\mu\varepsilon}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} - \frac{4\pi\mu\sigma}{c^2}\frac{\partial \vec{E}}{\partial t} = 0$$

- $\vec{E}$ electric field vector
- Material properties:
  - ε dielectric constant
  - μ magnetic permeability ($\mu = 1$ for most materials)
  - σ electrical conductivity (controls damping)
- c speed of light

Plane Waves

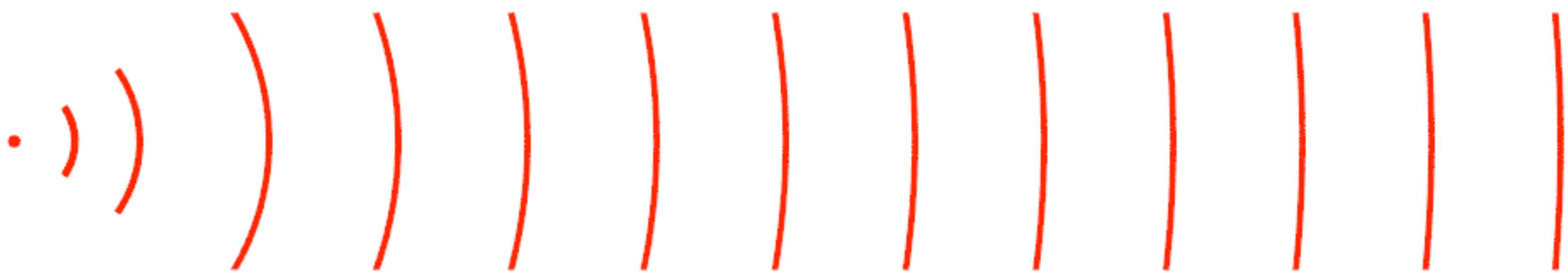
- Linear equation: sum of solutions is also solution
- Plane wave $\vec{E} = \overrightarrow{E_0} e^{i(\vec{k}\cdot\vec{x}-\omega t)}$
  - $\overrightarrow{E_0}$ complex constant vector (polarization)
  - $\vec{k}$ wave vector
  - $\vec{x}$ spatial location
  - ω angular frequency



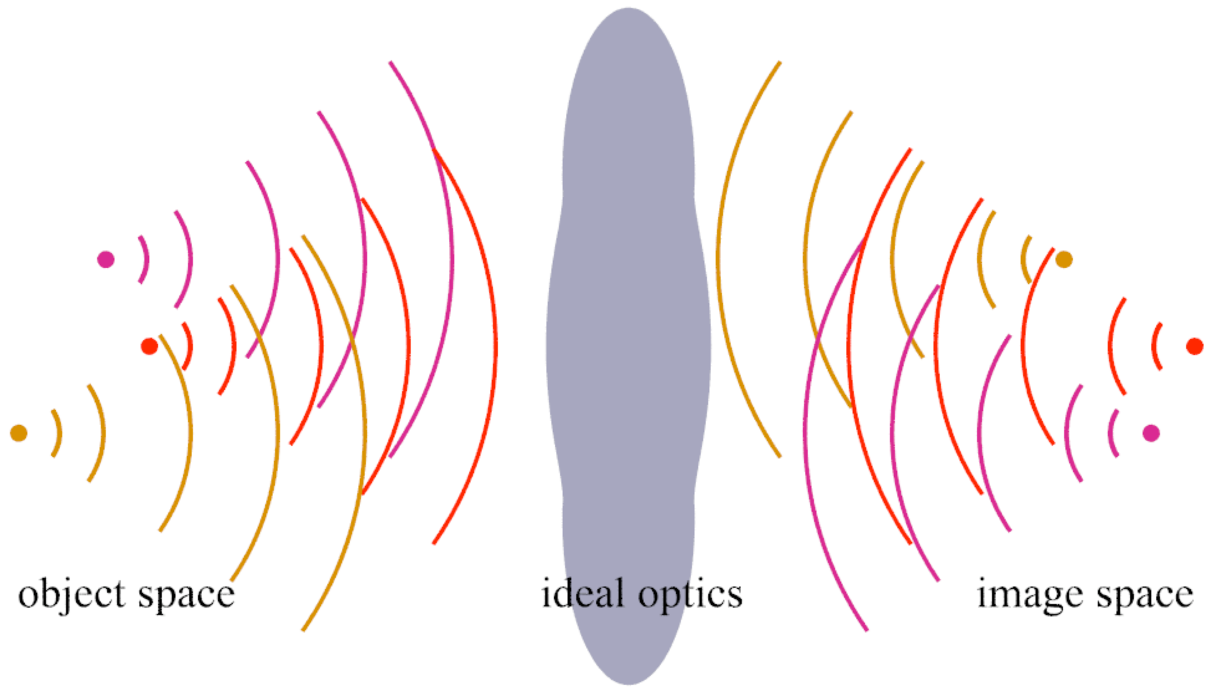
- real electric field given by real part of E
- dispersion relation: $\vec{k}\cdot\vec{k} = \frac{\omega^2}{c^2}\tilde{n}^2, \tilde{n}^2 = \mu\left(\epsilon + i\frac{4\pi\sigma}{\omega}\right)$
- complex index of refraction $\tilde{n} = n + ik$
- $\vec{E}, \vec{H}, \vec{k}$ form right-handed triplet of orthogonal vectors

Spherical and Plane Waves

- light source: collection of sources of spherical waves
- astronomical sources: almost exclusively incoherent
- lasers, masers: coherent sources
- spherical wave originating at very large distance can be approximated by plane wave



Ideal Optics

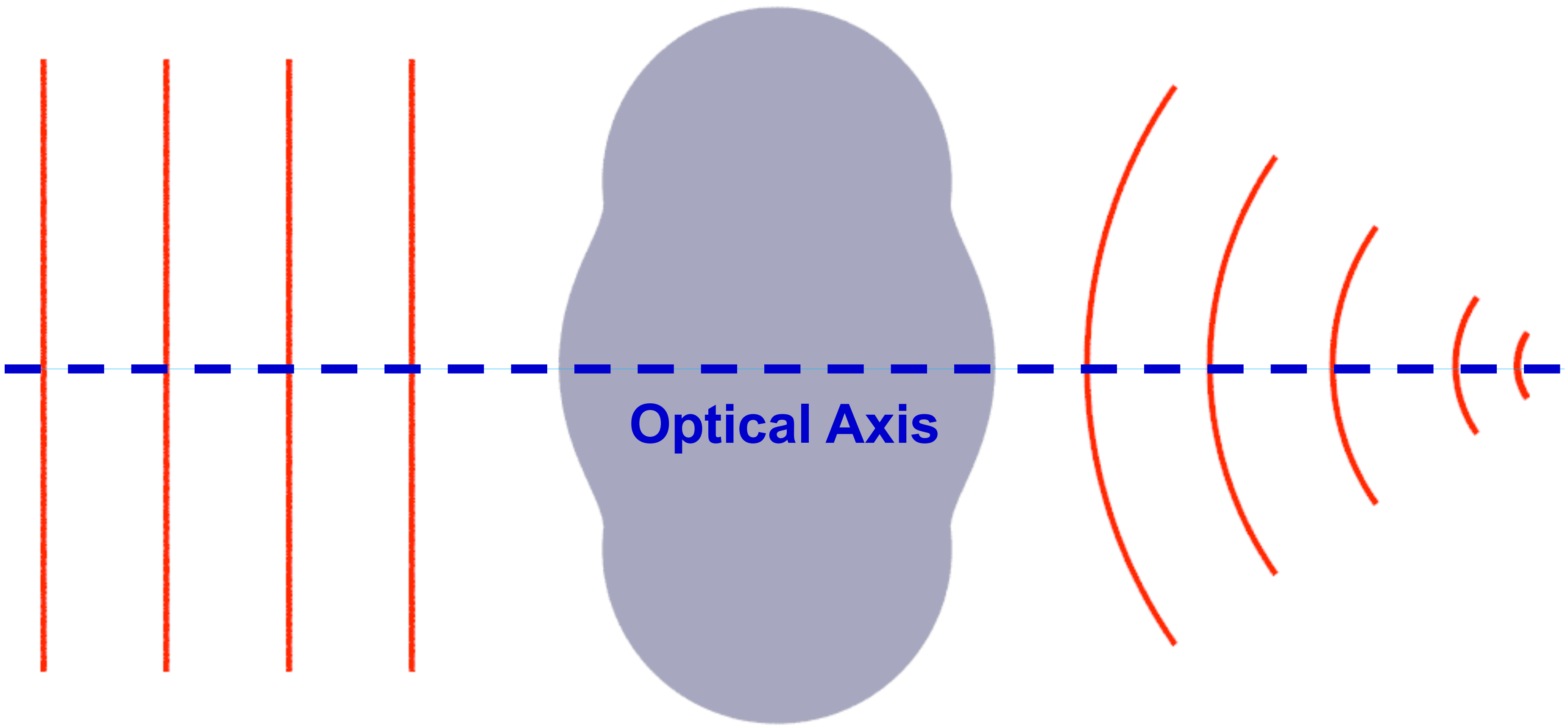


- ideal optics: spherical waves from any point in object space are
- imaged into points in image space
- corresponding points are called conjugate points
- focal point: center of converging or diverging spherical wavefront
- object space and image space are reversible

Ideal Optical System

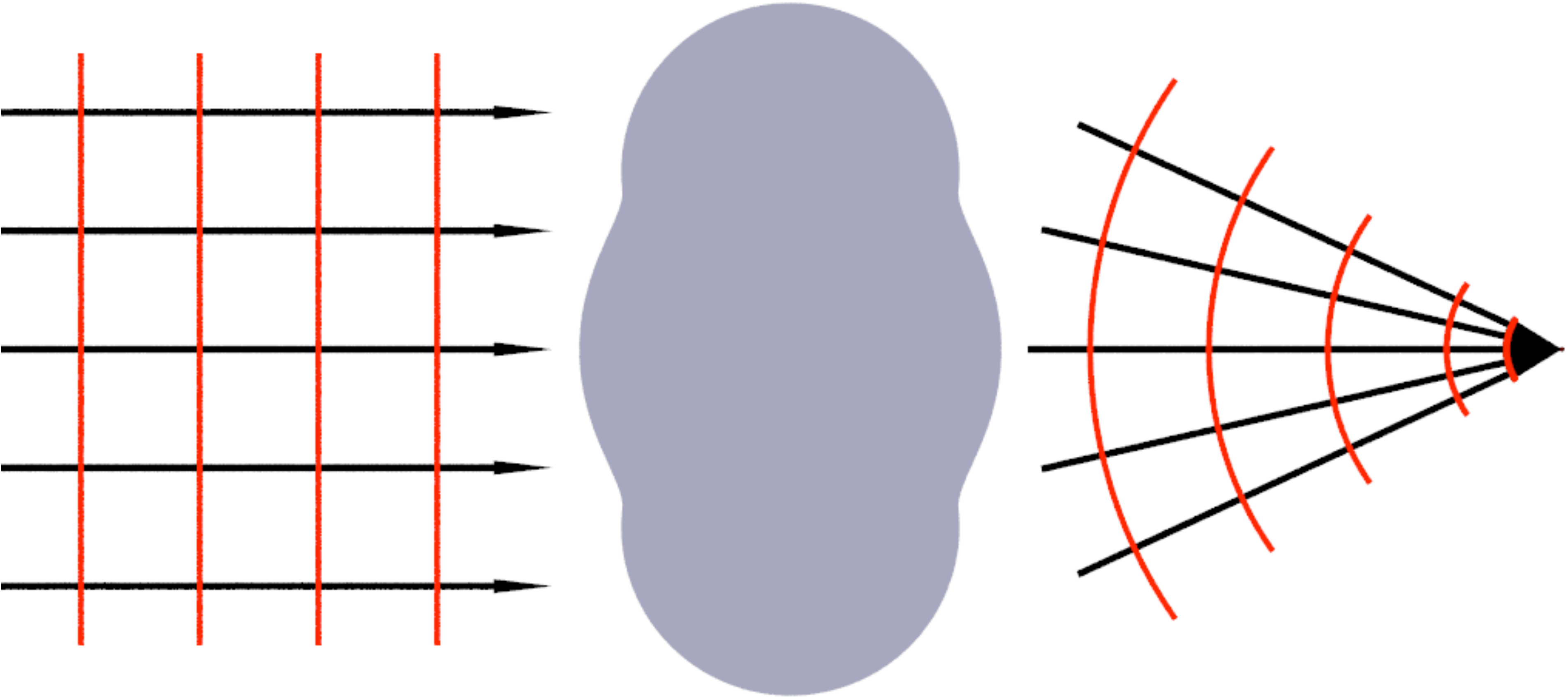
ideal optical system transforms plane wavefront into spherical, converging wavefront

Azimuthal Symmetry



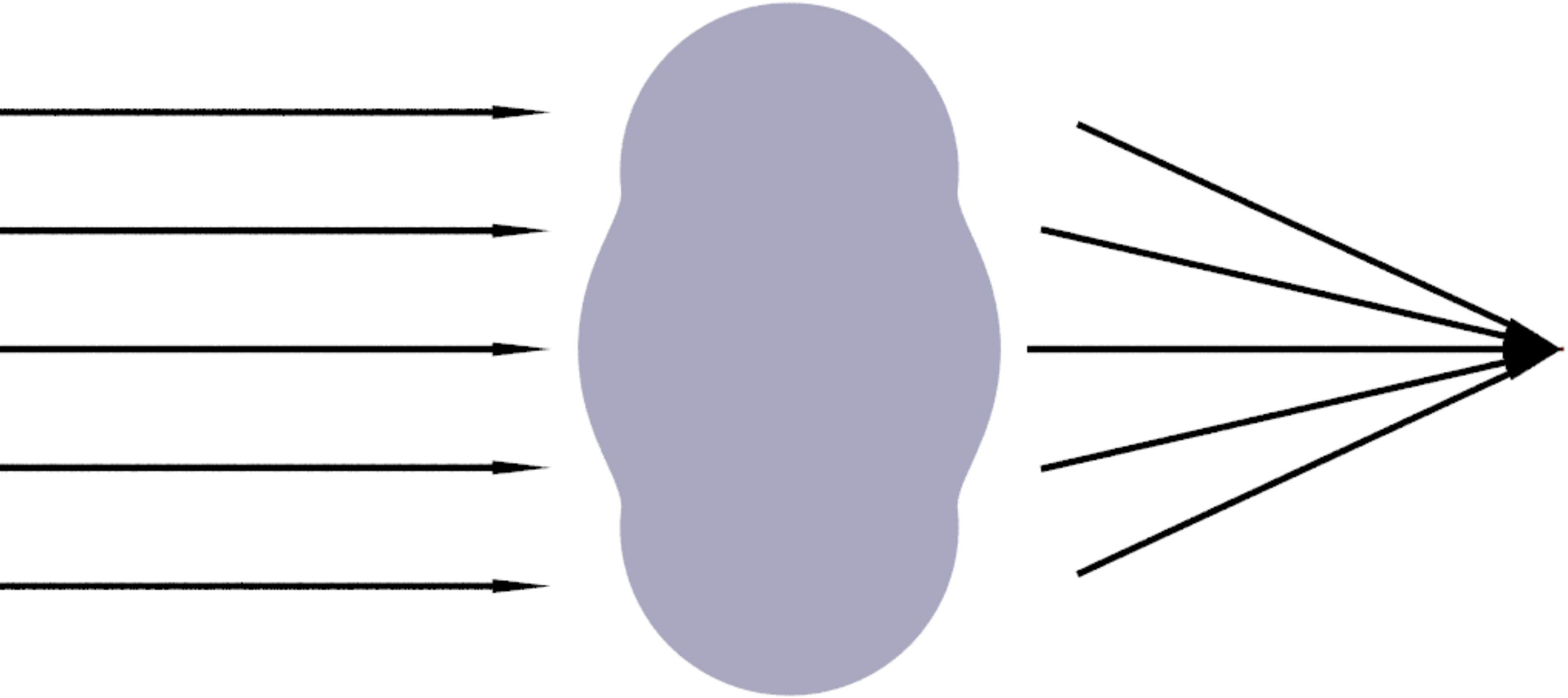
- most optical systems are azimuthally symmetric
- axis of symmetry is optical axis

Locally Flat Wavefronts



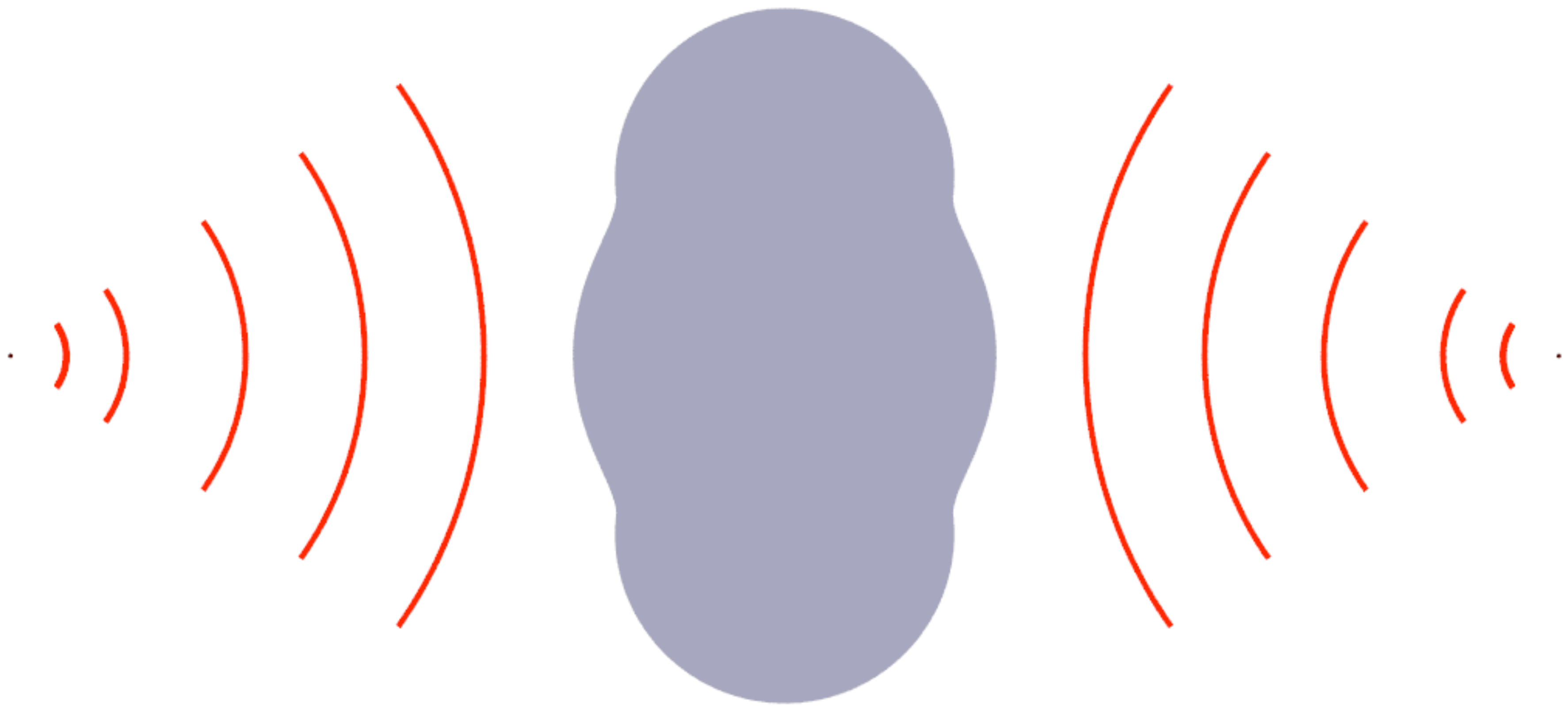
- rays normal to local wave (locations of constant phase)
- local wave around rays is assumed to be infinite, plane wave

Rays



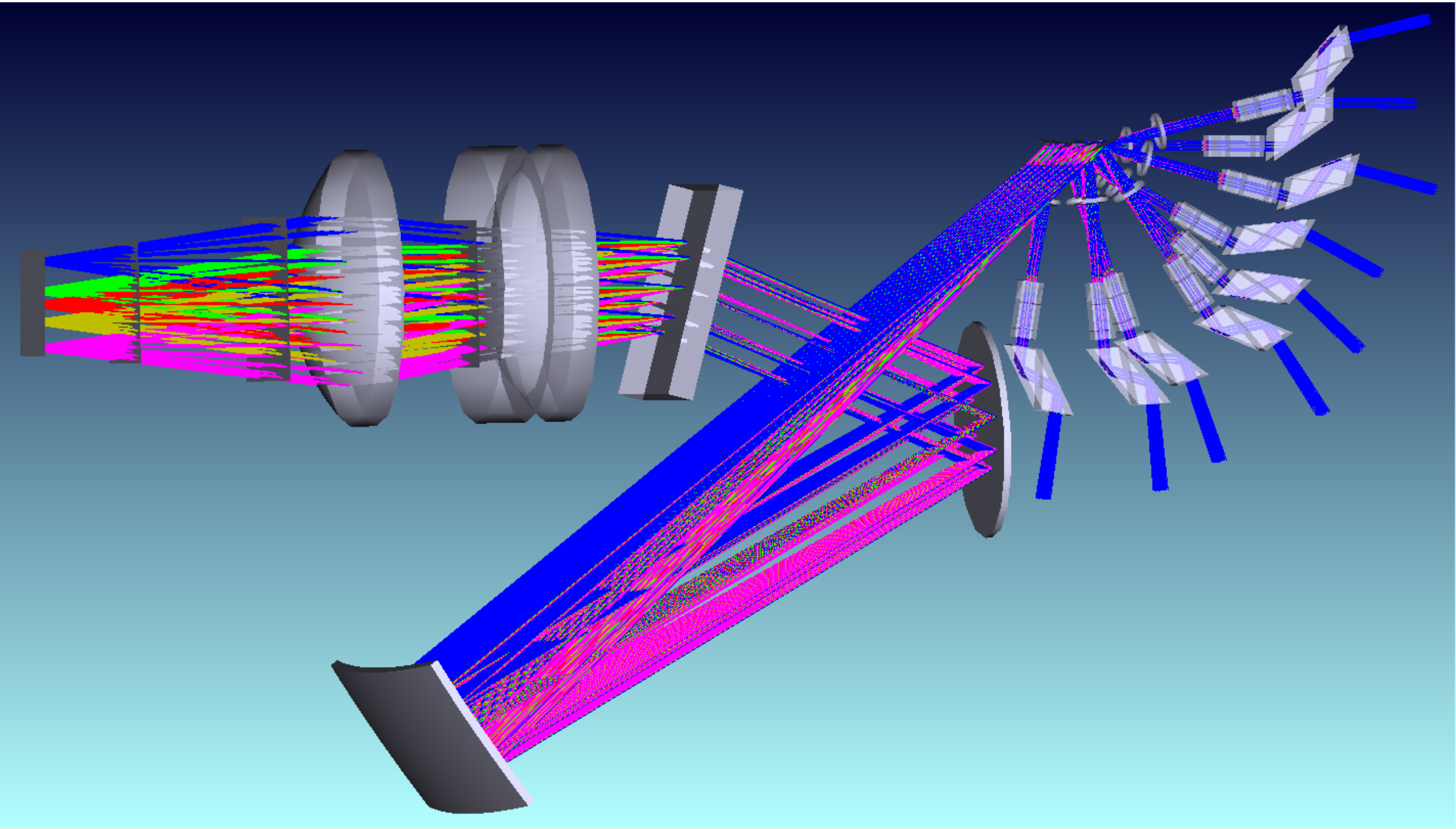
- geometrical optics works with rays only
- rays reflect and refract according to Fresnel equations
- phase is neglected (incoherent sum)

Finite Object Distance



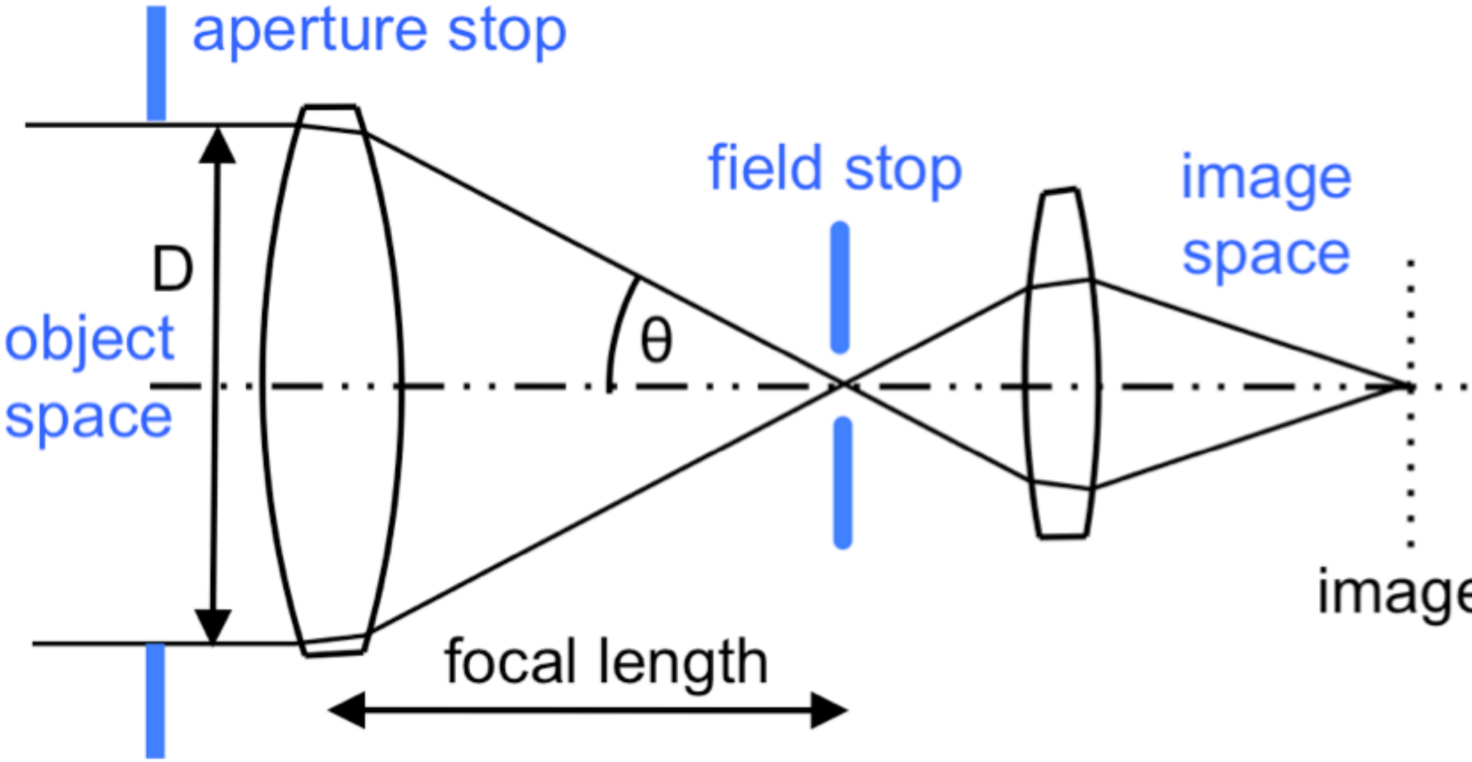
- object may also be at finite distance
- also in astronomy: reimaging within instruments and telescopes

Geometrical Optics Example: SPEX



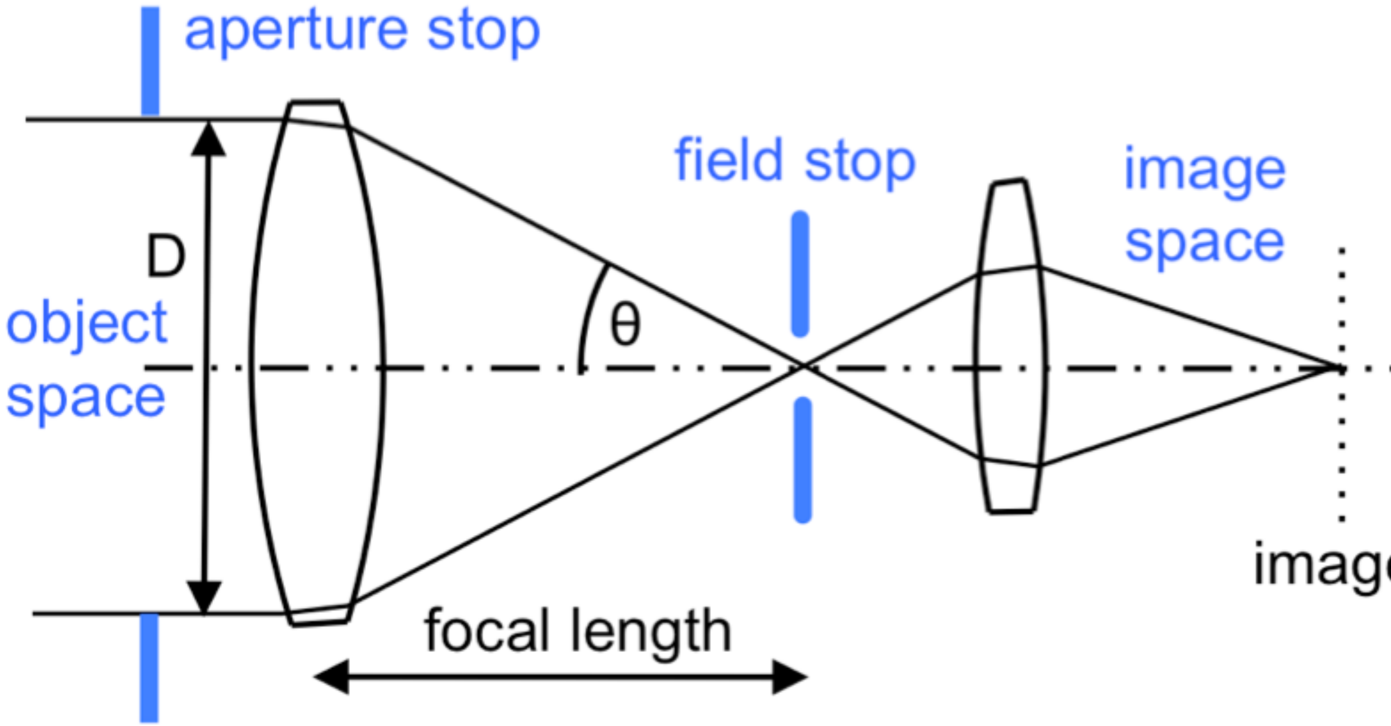
SPEX on NASA ER-2 plane

Aperture and Field Stops



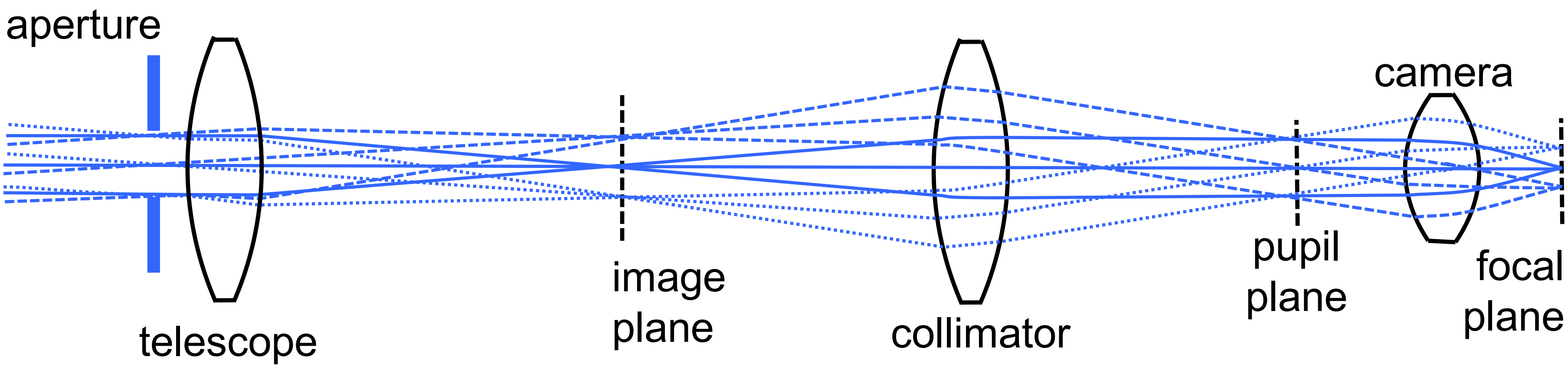
- Aperture stop: determines diameter of light cone from axial point on object.
- Field stop: determines the field of view of the system.

Images



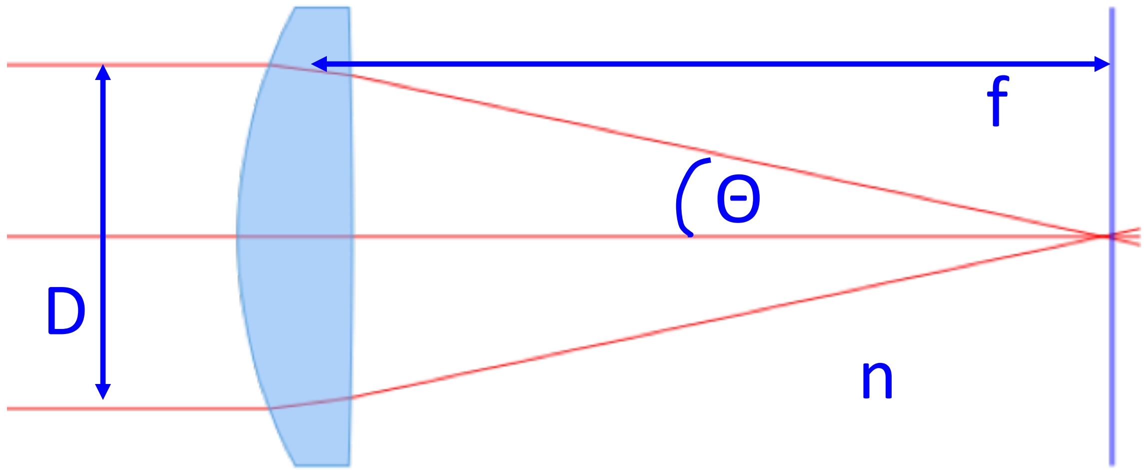
- every object point comes to focus in image plane
- light in image point comes from all pupil positions
- object information encoded in position, not angle

Pupils



- all object rays are smeared out over complete aperture
- light in one pupil point comes from different object positions
- object information is encoded in angle, not in position

Speed/F-Number/Numerical Aperture

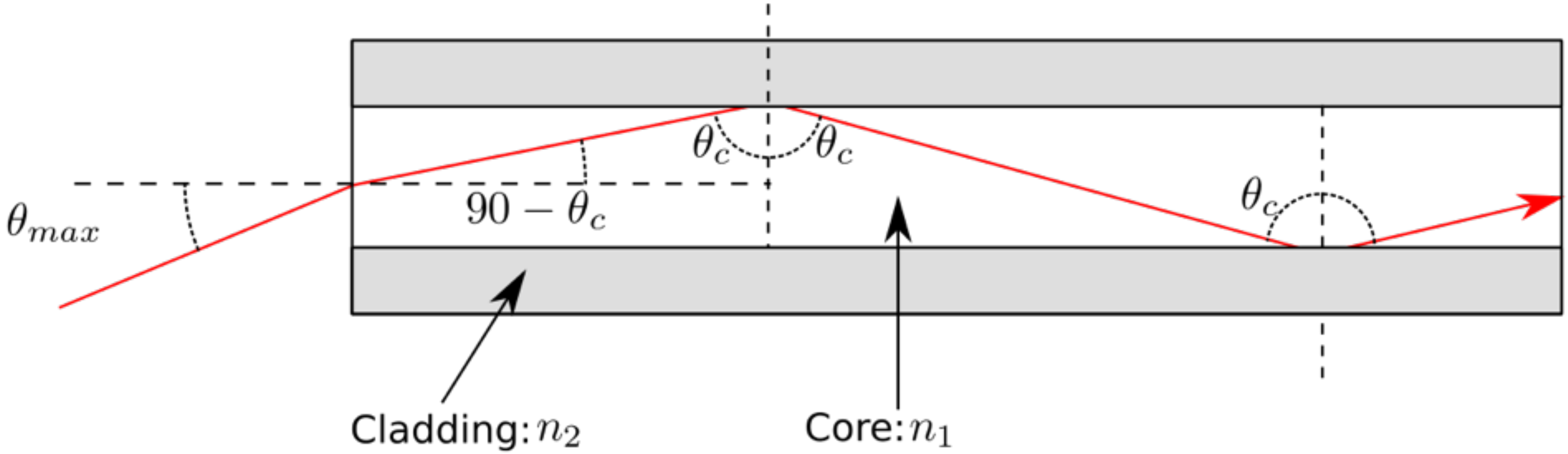


Speed of optical system described by numerical aperture (NA) or *F*-number:

$$\mathrm{NA}{=}n\cdot\sin\theta\approx n\frac{1}{2F},\qquad F=\frac{f}{D}$$

- fast optics (large NA, small F-number)
- slow optics (small NA, large F-number)
- f-ratio = 1/F, sometimes written as f/2.8

Numerical Aperture in Fiber



https://commons.wikimedia.org/wiki/File:Optic_fibre-numerical_aperture_diagram.svg

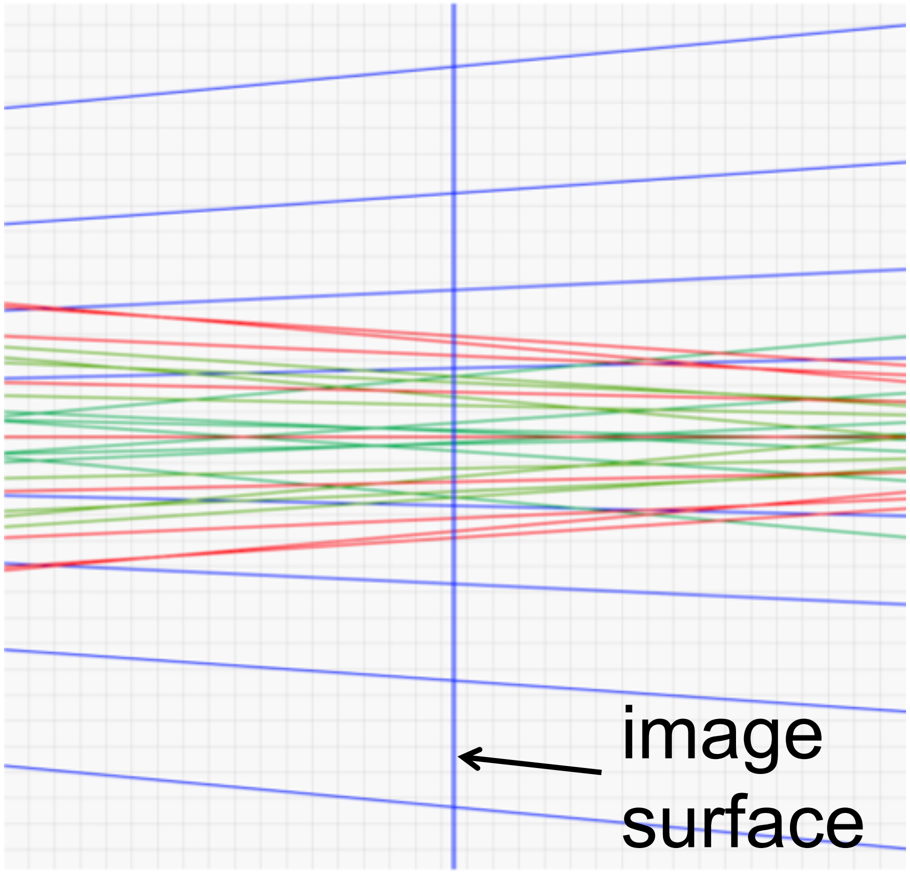
- total internal reflection for $\theta < \theta_{\max}$
- $n \cdot \sin\theta_{\max} = \sqrt{n_1^2 - n_2^2}$ =NA
- maximum acceptance angle expressed as numerical aperture

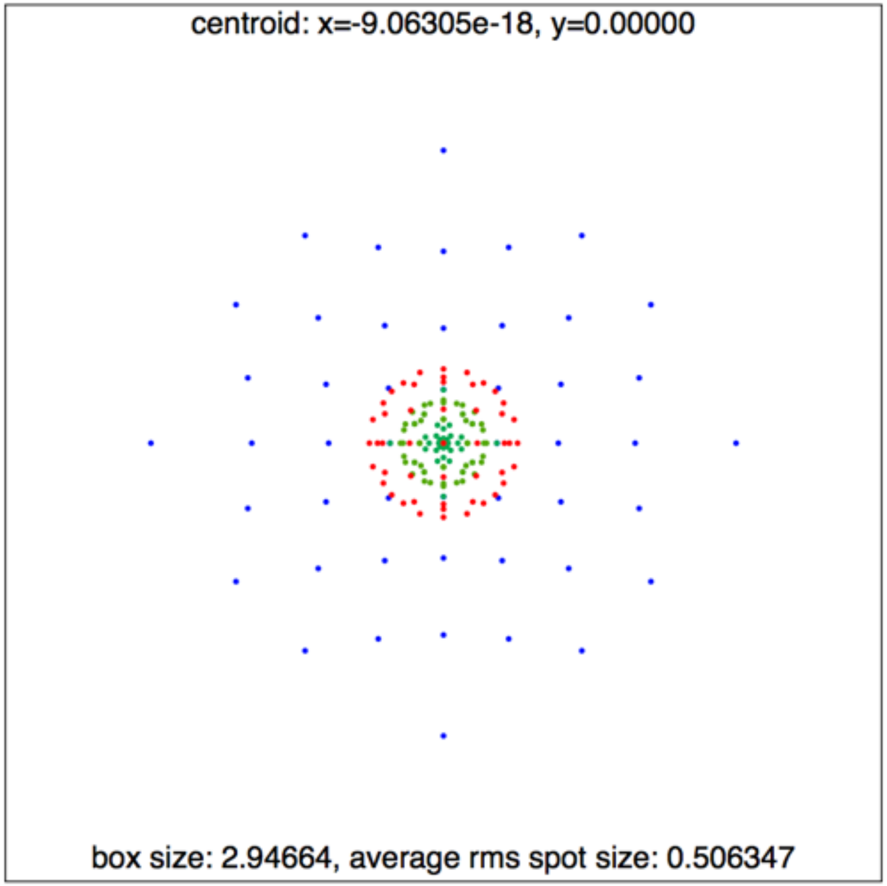
Aberrations

Aberrations are departures of the performance of an optical system from the ideal optical system.

1. On-axis aberrations: aberrations that can be seen everywhere in the image, also on the optical axis (center of the image)
2. Off-axis aberrations: aberrations that are absent on the optical axis (center of the image)
   a) Aberrations that degrade the image
   b) Aberrations that alter the image position

Spot Diagram

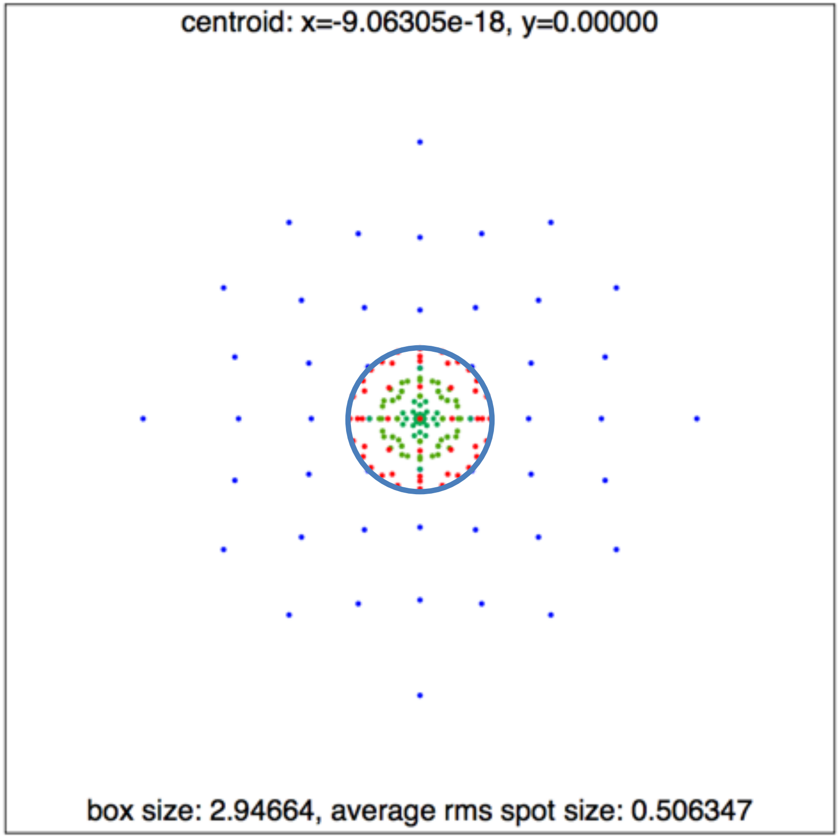




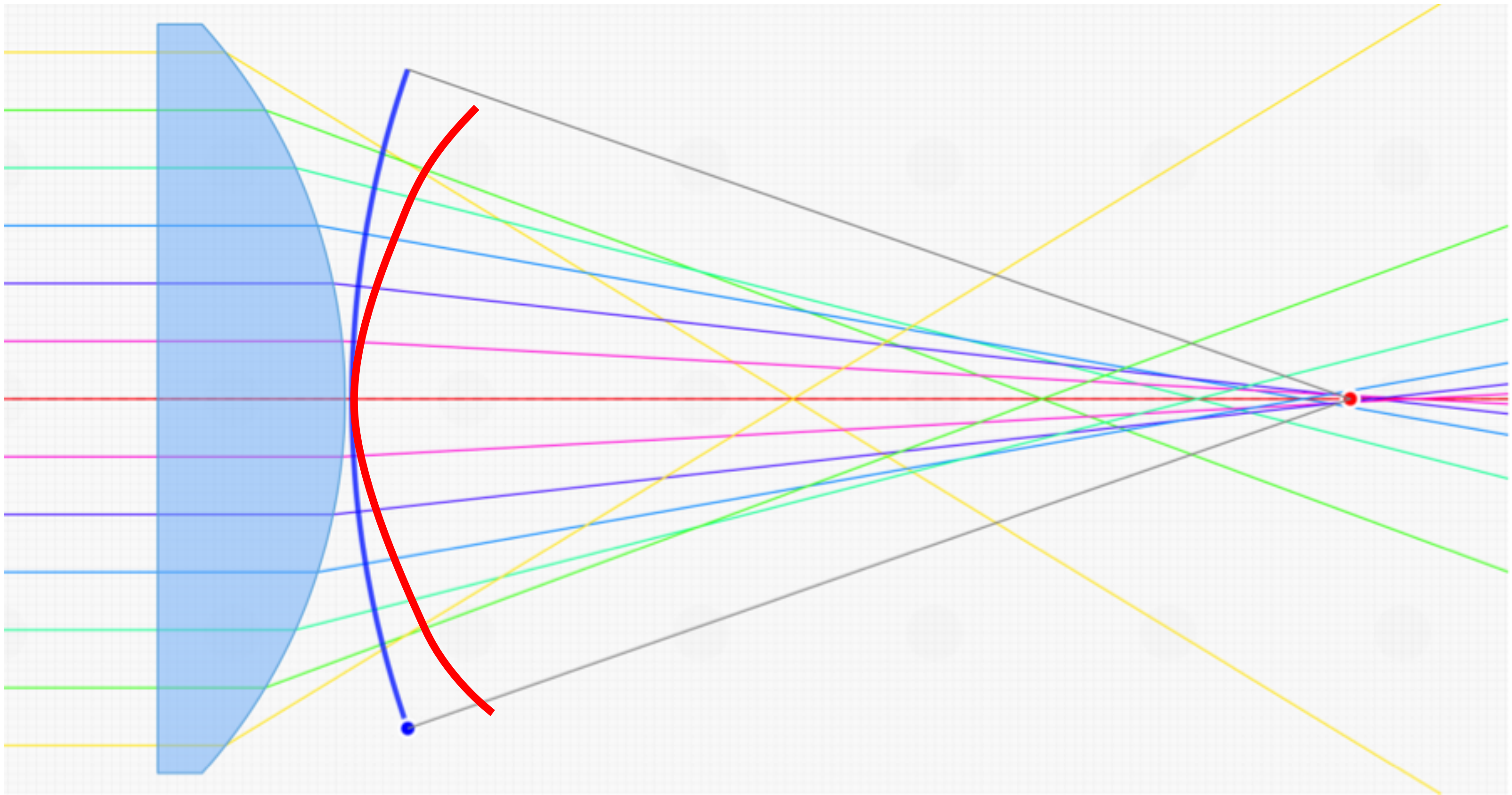
intersection of rays with image surface

RMS Spot Radius

- calculate rms radius of all spots from the perfect center
- provides a rough measure of image quality
- optics are virtually perfect if rms spot radius $\leqq \lambda/2D$

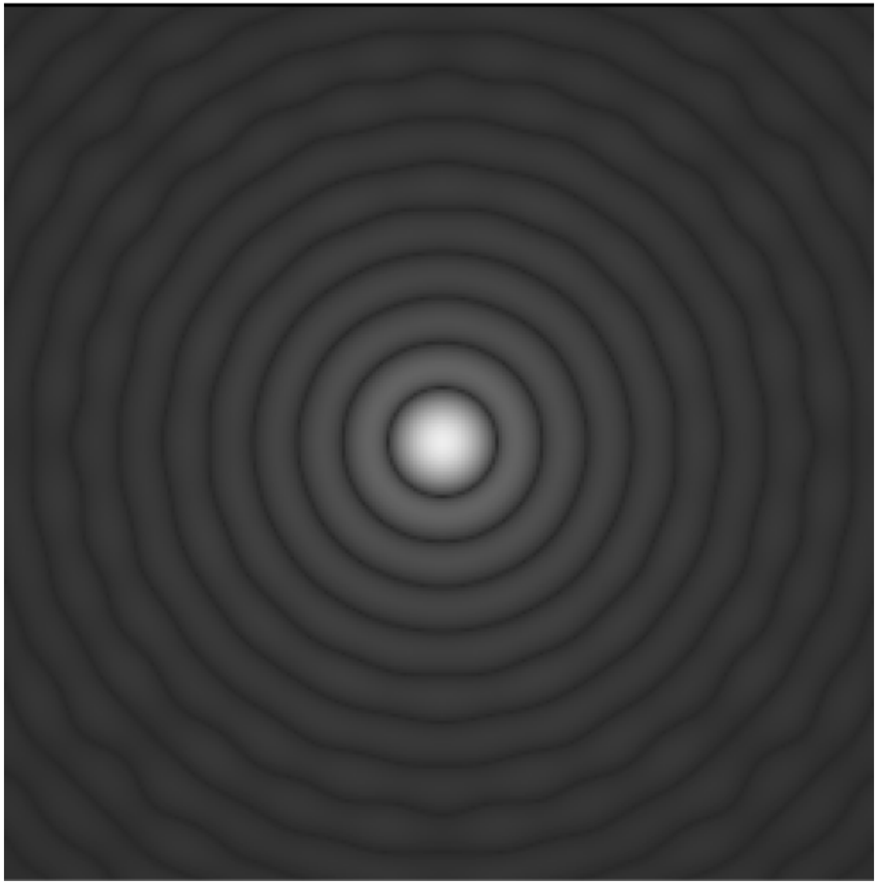


Wavefront Error

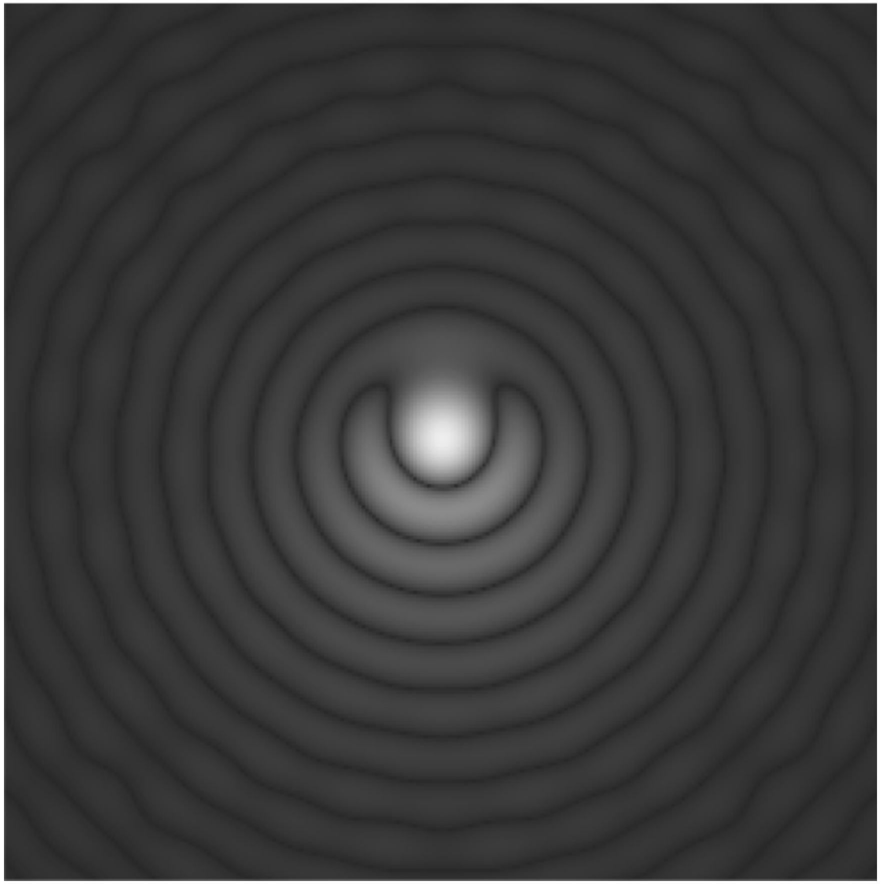


- deviation of surface that is normal to rays and spherical reference surface (distance between red & blue line)
- often shown as grayscale image or 3D surface

Aberrated Point-Spread Function



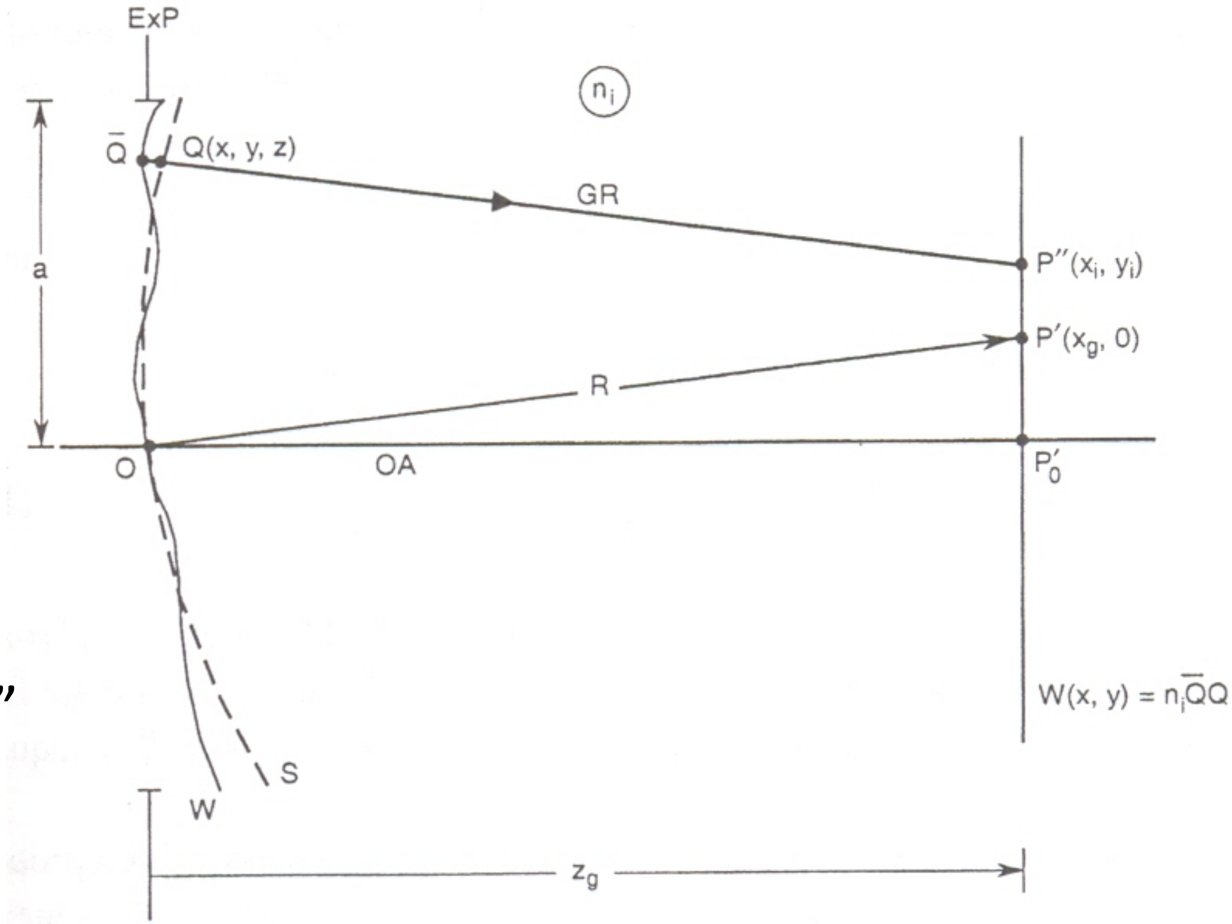
perfect



aberrated

Wave and Ray Aberrations

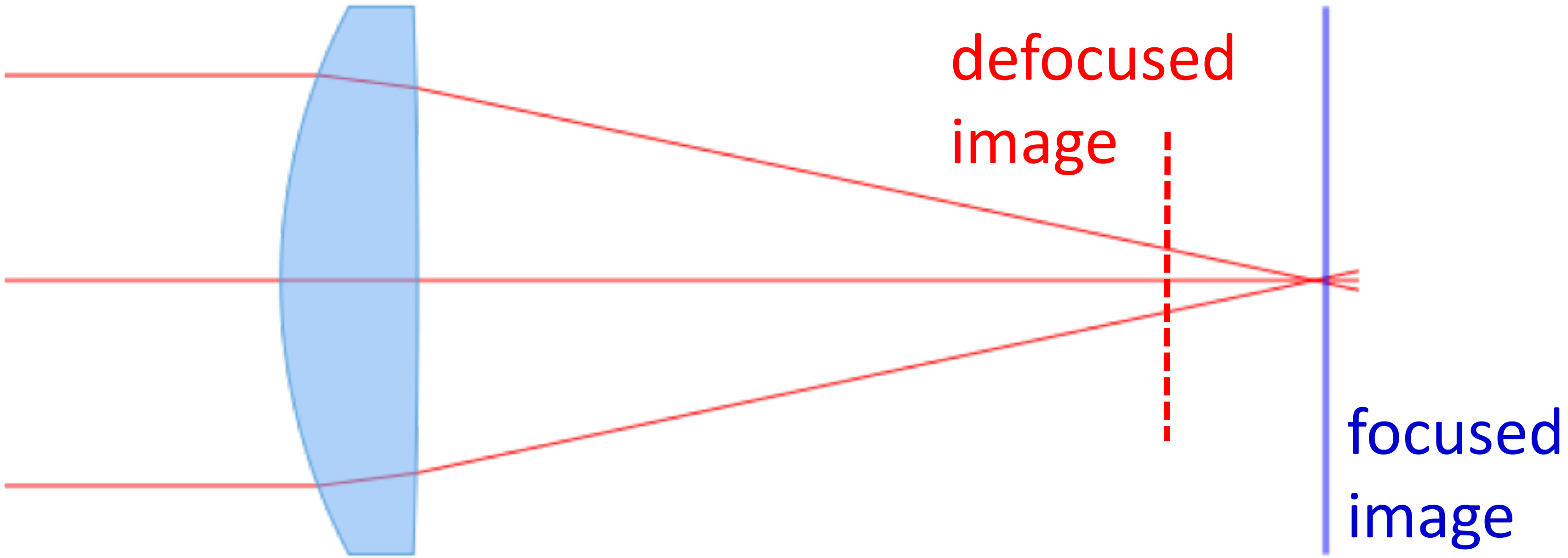
- Reference sphere S with radius R for off-axis point P' and aberrated wavefront W
- "Aberrated" ray from object intersects image plane at P"
- Ray aberration is P'P"
- Wave aberration is $n_i \cdot Q\bar{Q}$



Small FOV, radially symmetric wavefront W(r)

$$r_i = \frac{R}{n_i}\frac{\partial W(r)}{\partial r}$$

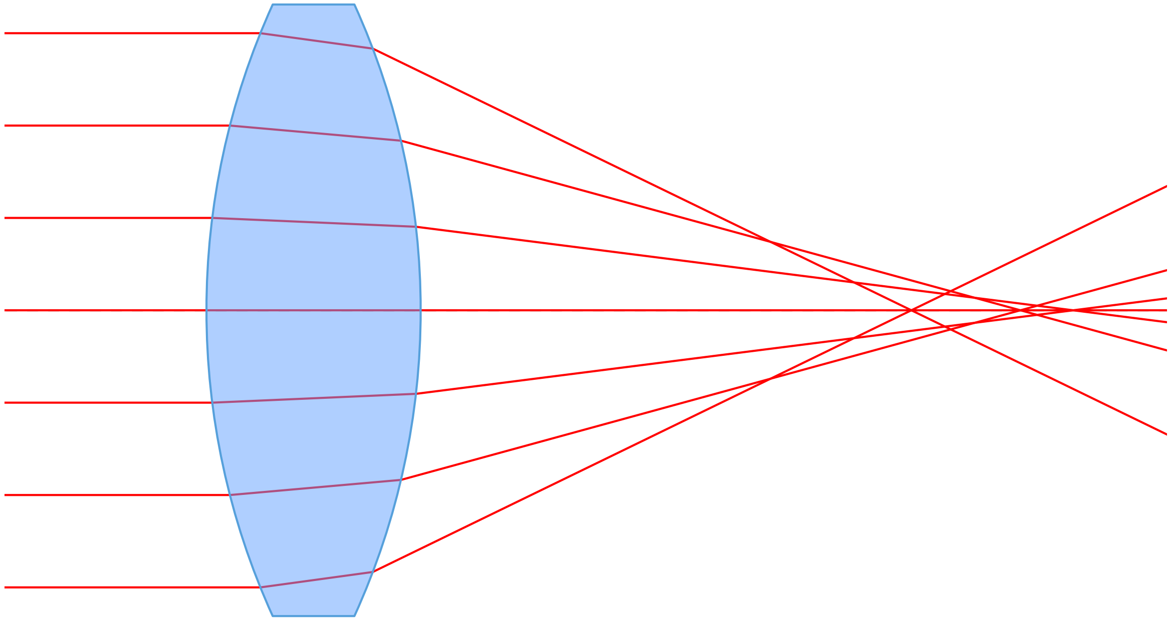
Defocus (Out of Focus)

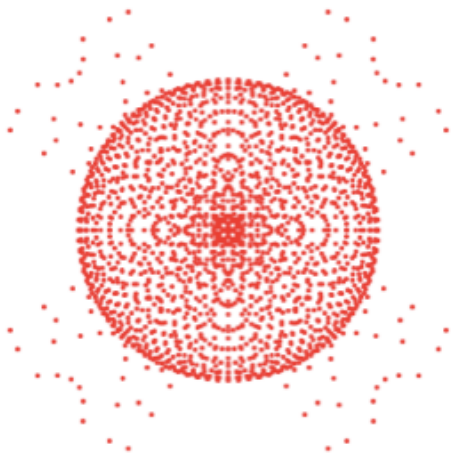


Depth of focus: $\delta = 2\lambda F^2 = \frac{\lambda}{2}\left(\frac{1}{\mathrm{NA}}\right)^2$

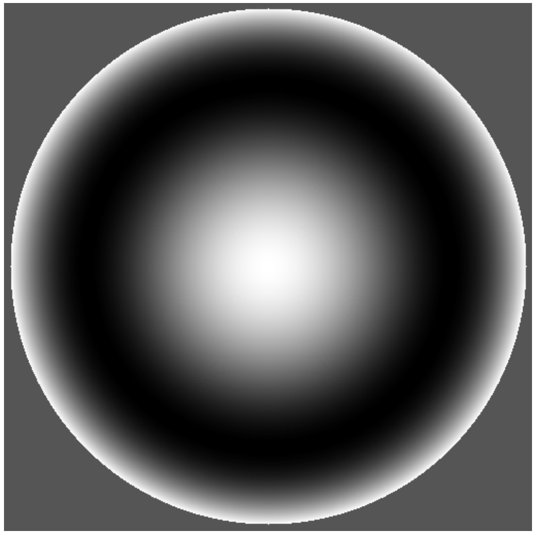
Usually refers to optical path difference of $\lambda/4$.

Spherical Aberration





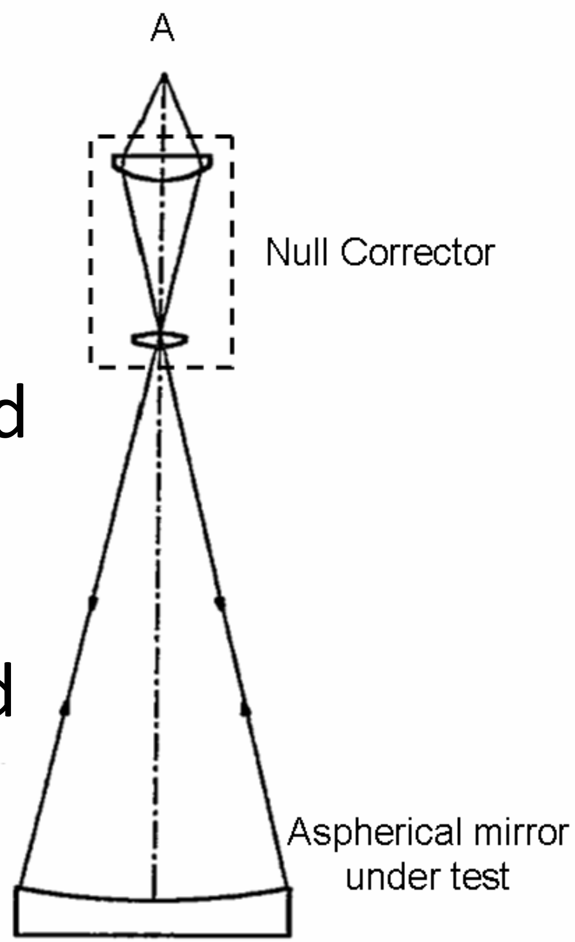
Rays further from the optical axis have a different focal point than rays closer to the optical axis.



Spherical Aberration: Hubble Trouble

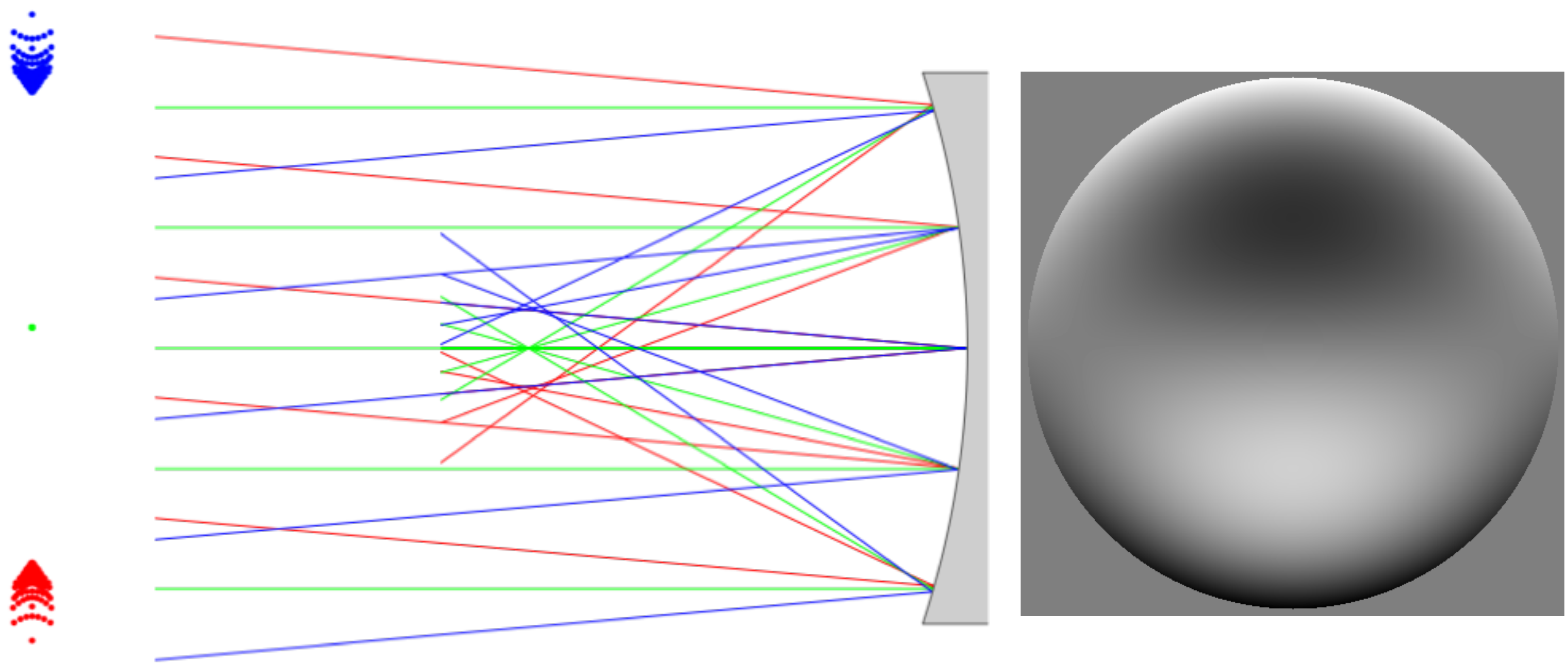
HST Primary Mirror Aberration

- Null corrector cancels non-spherical portion of aspheric mirror shape. Viewed from point A, combination looks precisely spherical
- Null corrector had one lens misplaced by 1.3 mm
- Manufacturer analyzed surface with other null correctors, which indicated the problem, but ignored results because they were believed to be less accurate



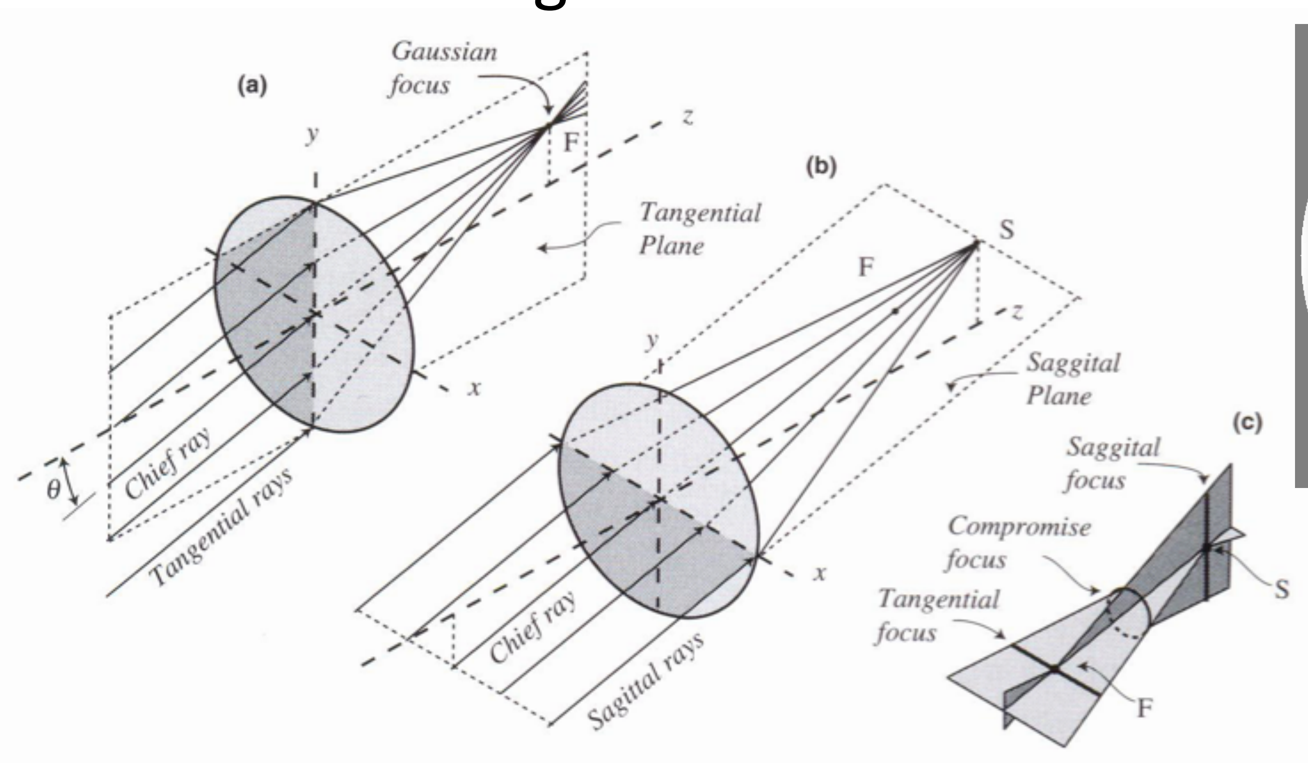
Coma

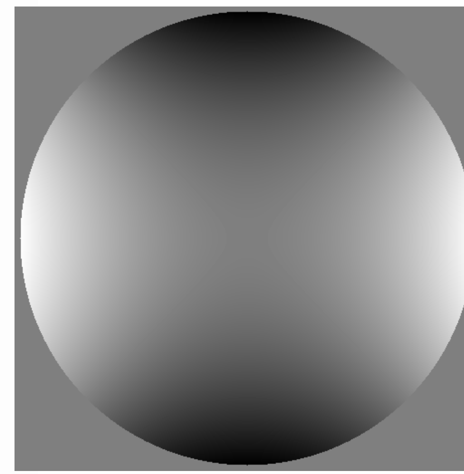
Variation of magnification across entrance pupil. Point sources will show a come-like tail. Coma is an inherent property of telescopes using parabolic mirrors



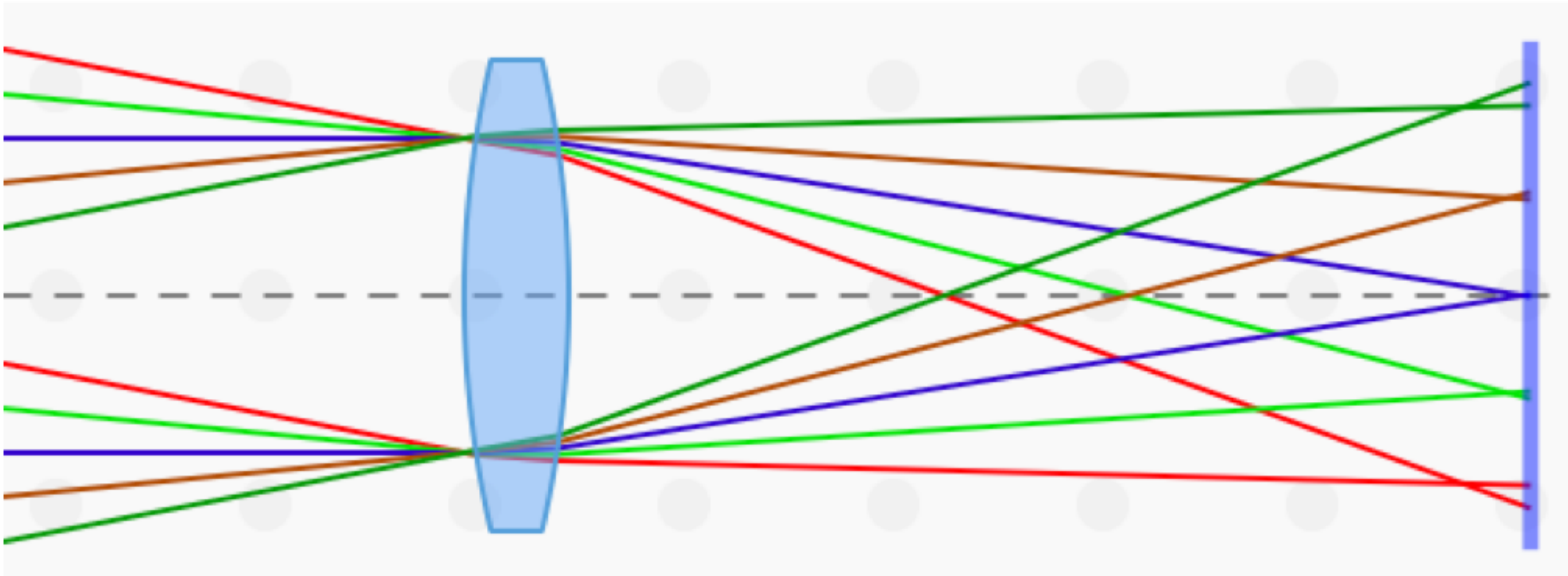
Astigmatism

From off-axis point A lens does not appear symmetrical but shortened in plane of incidence (tangential plane).
Emergent wave will have a smaller radius of curvature for tangential plane than for plane normal to it (sagittal plane) and form an image closer to the lens.





Field Curvature

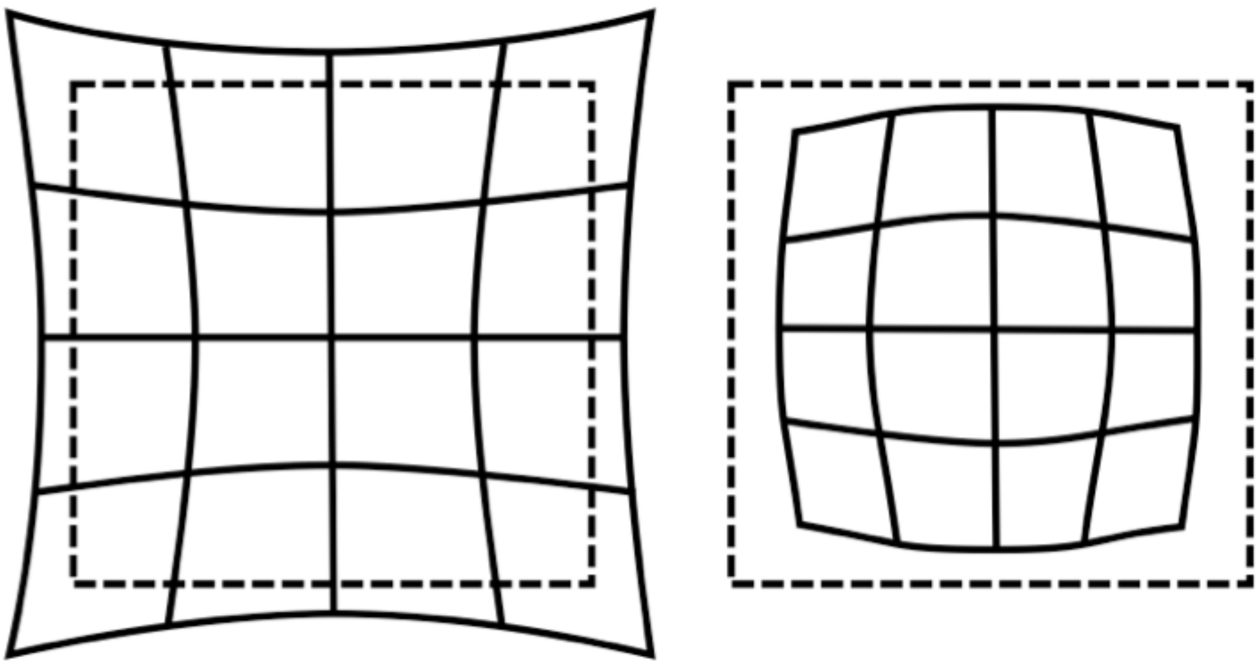


Only objects close to optical axis will be in focus on flat image plane. Off-axis objects will have different focal points.

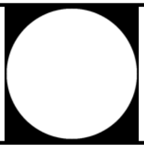
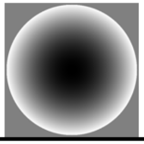
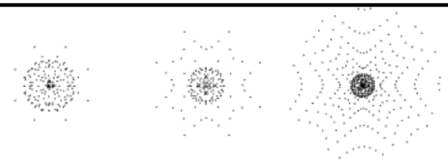
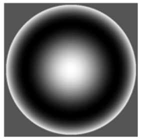
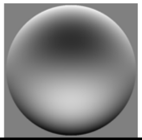
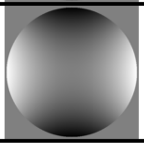
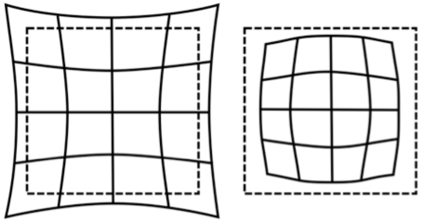
Distortion

Straight line on sky becomes curved line in focal plane because magnification depends on distance to optical axis.

1. Outer parts have larger magnification → pincushion
2. Outer parts have smaller magnification → barrel

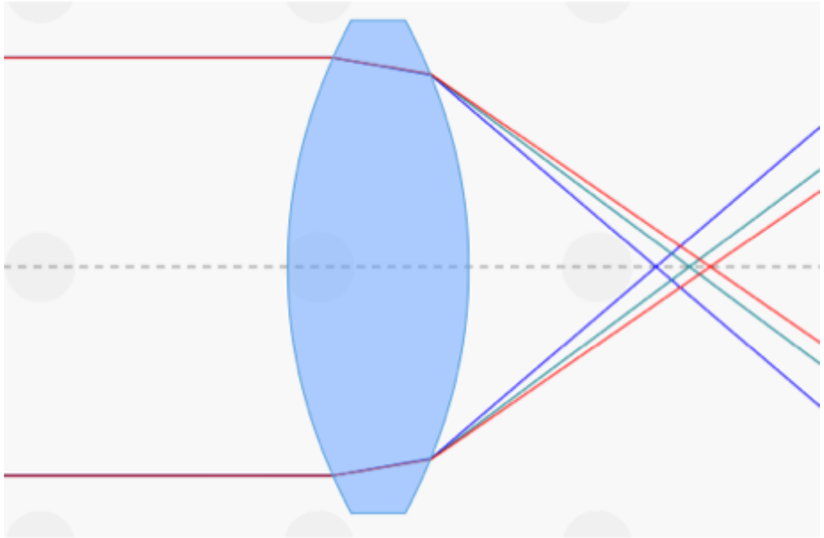


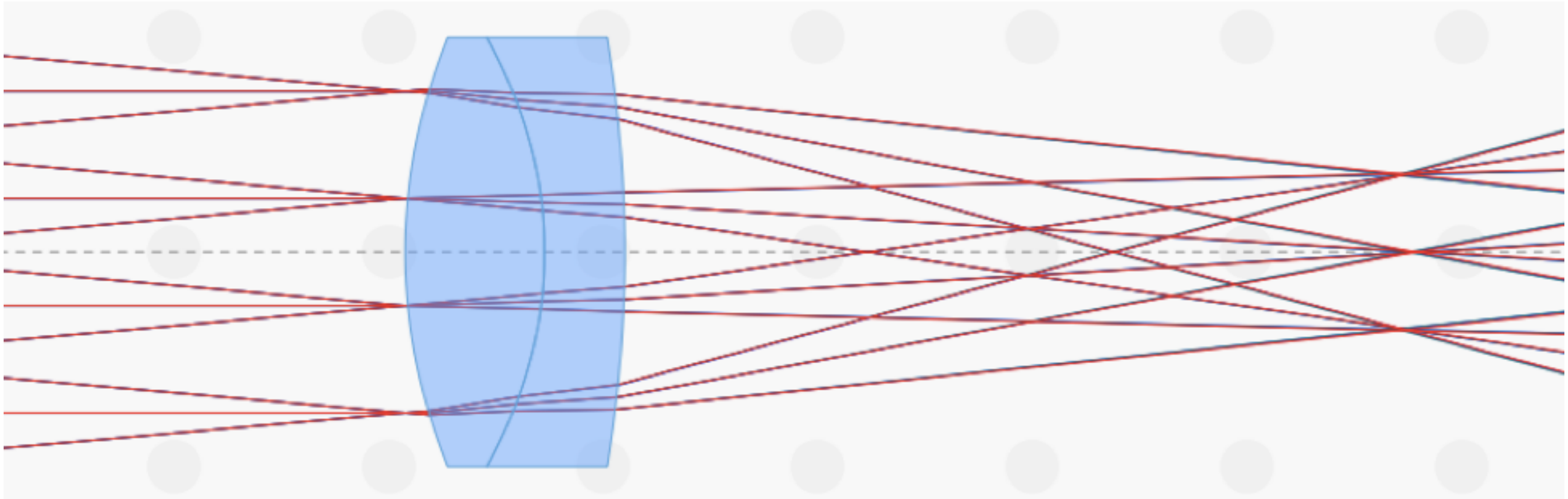
Aberrations Summary

aberration	spot diagram / image	wavefront	scaling	
perfect			-	-
focus			$1/F^2$	-
spherical			$1/F^3$	-
coma			$1/F^2$	y
astigmatism			$1/F^2$	y^2
field curvature			$1/F^2$	y^2
distortion			-	y^3

Chromatic Aberration

Refractive index variation with wavelength n(λ) results in focal length of lens f(λ) to depend on wavelength; different wavelengths have different foci

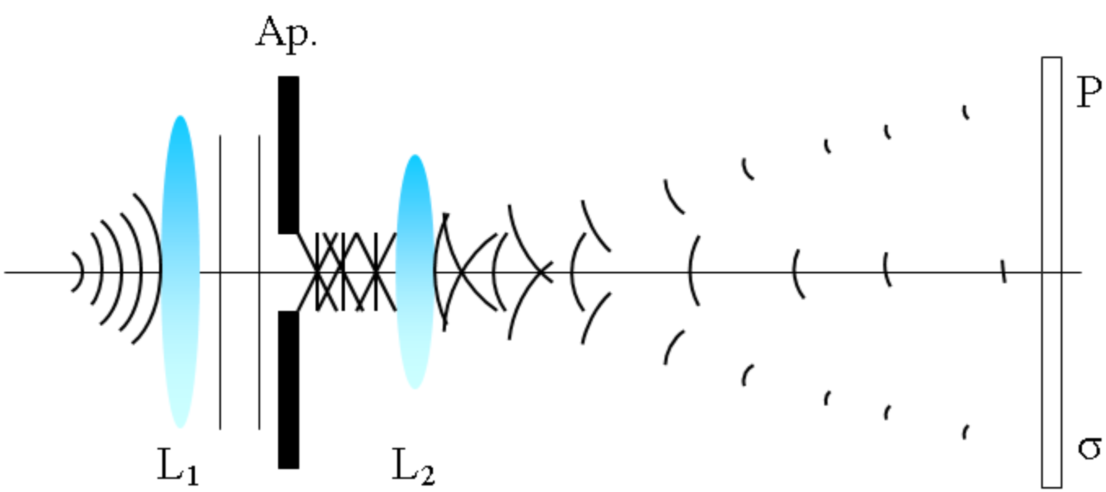




Fresnel and Fraunhofer Diffraction

Fresnel diffraction = near-field diffraction: When a wave passes through an aperture and diffracts in the near field it causes the observed diffraction pattern to differ in size and shape for different distances.

For Fraunhofer diffraction at infinity (far-field) the wave becomes planar.



Fresnel: $F = \frac{r^2}{d \cdot \lambda} \geq 1$

Fraunhofer: $F = \frac{r^2}{d \cdot \lambda} << 1$

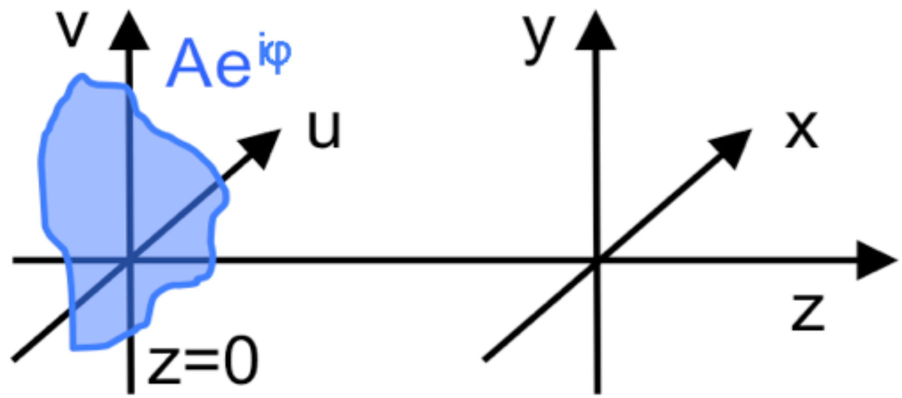
F = Fresnel number

r = aperture size

d = distance to screen

Fraunhofer Diffraction

- Electric field in image plane is Fourier transform
  of electric field in aperture for



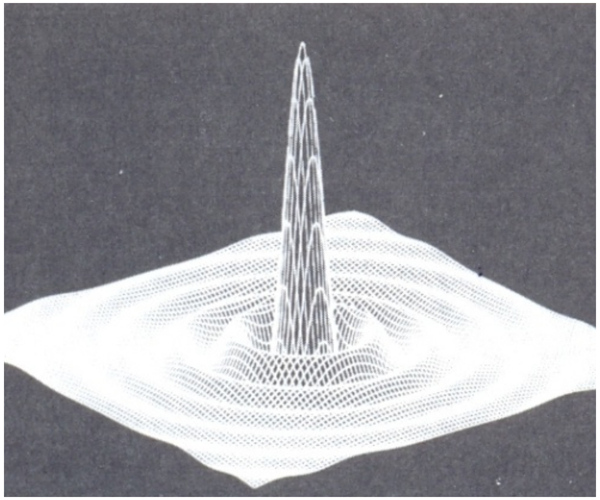
$$E(x,y,z) = \iint A(u,v)\, e^{i\varphi(u,v)}\, e^{-i\frac{2\pi}{\lambda z}(xu+yv)}\, du\, dv$$

- Complex aperture function: $Ae^{i\phi}$

Point Spread Function

Circular pupil illuminated by point source: PSF is Airy function:

$$I_1(\theta) = \left(\frac{2J_1(2\pi\, r_0\theta/\lambda)}{2\pi\, r_0\theta/\lambda} \right)^2$$



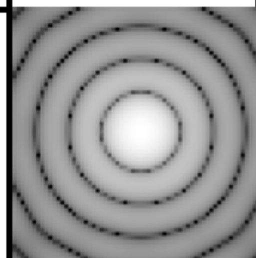
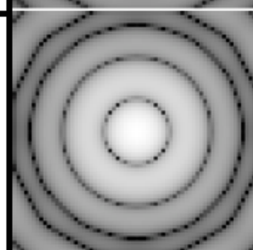
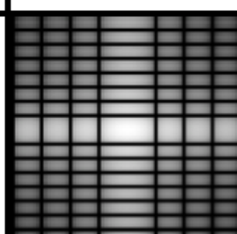
First dark ring (minimum) at:

$$r_1 = 1.22\lambda F \qquad \text{or} \qquad \alpha_1 = \frac{r_1}{f} = 1.22\frac{\lambda}{D}$$

PSF often characterized by Full Width at Half Maximum (FWHM) in angular units. Airy function: FWHM=1.028 λ/D

Nyquist-Shannon sampling theorem requires sampling at least every 0.5 FWHM. Airy function: at least every 0.5λ/D

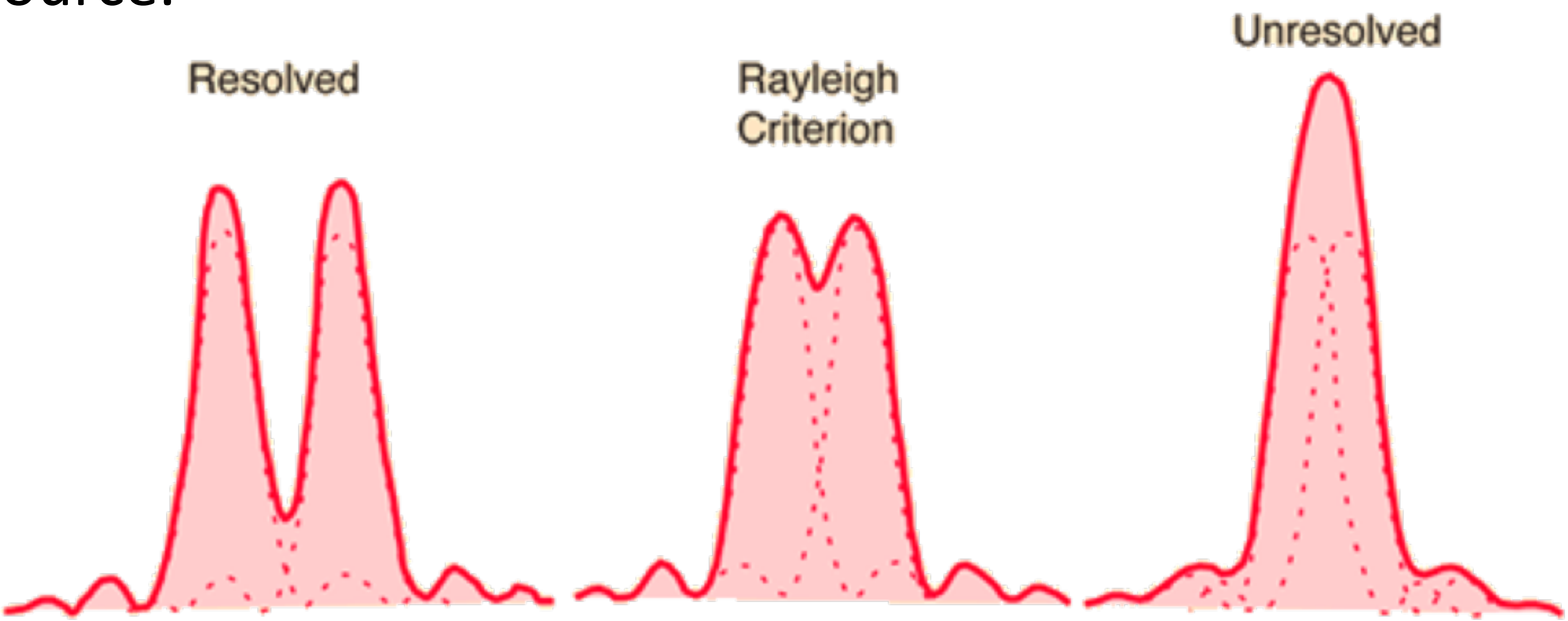
Point Spread Functions

Aperture	PSF	PSF equation
round, diameter d_x		$\left(\frac{2J_1(x)}{x}\right)^2$
obscured round, diameter d_x, obscuration ratio ε		$\frac{1}{\left(1-\varepsilon^2\right)^2}\left(\frac{2J_1(x)}{x}-\frac{2J_1(\varepsilon x)}{x}\right)^2$
rectangle, sides $d_{x,y}$		$\left(\frac{\sin x}{x}\right)^2\left(\frac{\sin y}{y}\right)^2$

Angular Resolution: Rayleigh Criterion

Two sources can be resolved if the peak of the second source is no closer than the 1st dark Airy ring of the first source.

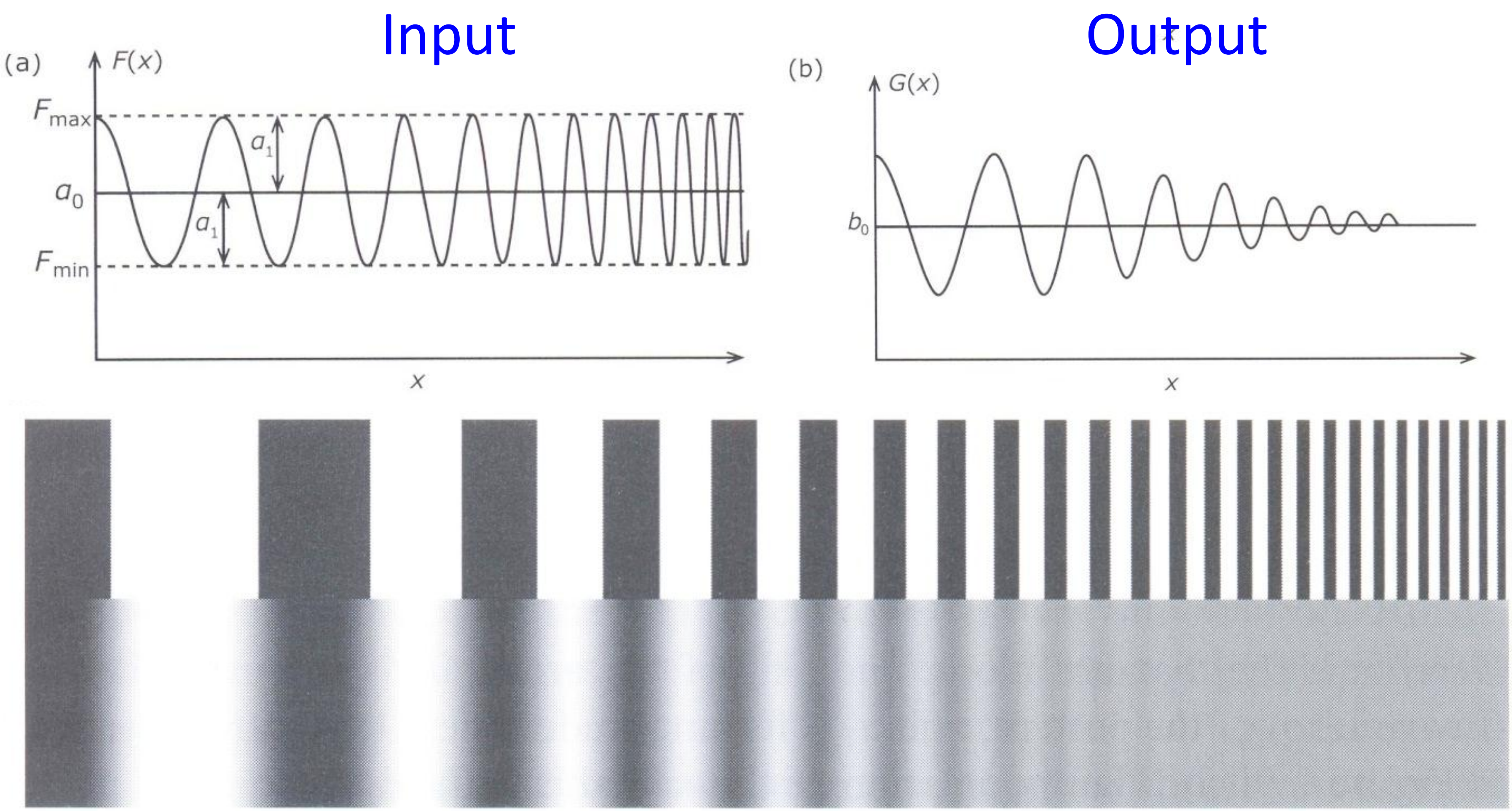
$$\sin\Theta = 1.22\frac{\lambda}{D}$$



http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/Raylei.html

Modulation Transfer Function (MTF)

Contrast of image of a sinusoidal gray-scale pattern

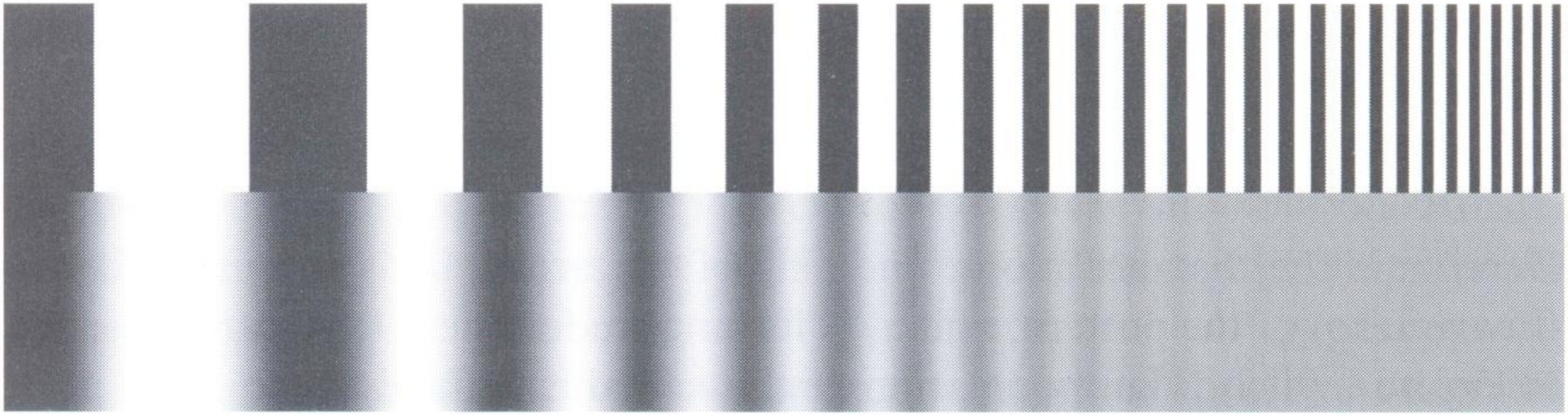


Modulation Transfer Function (MTF)

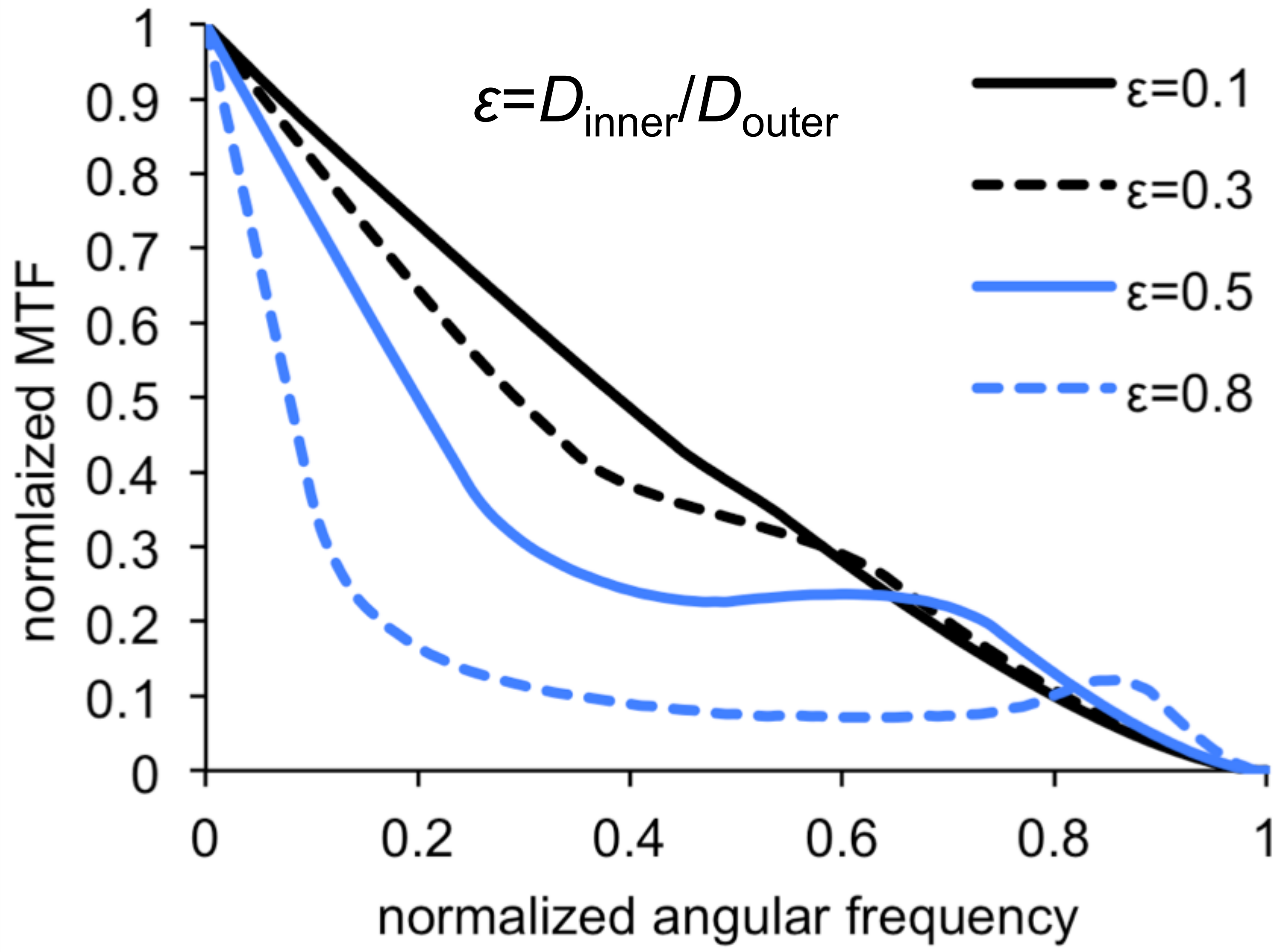
Contrast of image of a sinusoidal gray-scale pattern

$$MTF(f) = \frac{C(f)}{C_0}, \qquad \text{where } C = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

MTF(*f*) = |Fourier Transform(PSF)|



MTF of Obscured Telescope



Optical Transfer Function (OTF)

- Optical Transfer Function (OTF) is Fourier transform of PSF
- OTF is auto-correlation of complex aperture function

$$\mathrm{OTF} = \mathrm{FT}(\mathrm{PSF}) = \mathrm{FT}(|\mathrm{FT}(Ae^{i\varphi})|^2) = Ae^{i\varphi} \otimes Ae^{i\varphi}$$

- FT: Fourier transform
- *: complex conjugate
- $\otimes$ correlation operator
- MTF is absolute value of OTF
- MTF of optical system often approximated by product of MTFs of parts: MTF(telescope)*MTF(instrument)

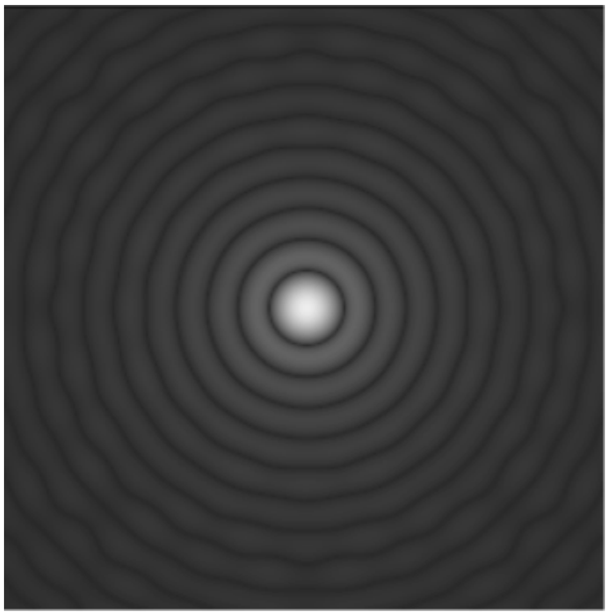
Strehl Ratio

- Strehl ratio is convenient measure of optical quality
- Strehl ratio (SR) is ratio of observed *peak intensity* of PSF compared to theoretical maximum peak intensity of point source seen with perfect imaging system
- With wave number k=2π/λ, RMS wavefront error ω

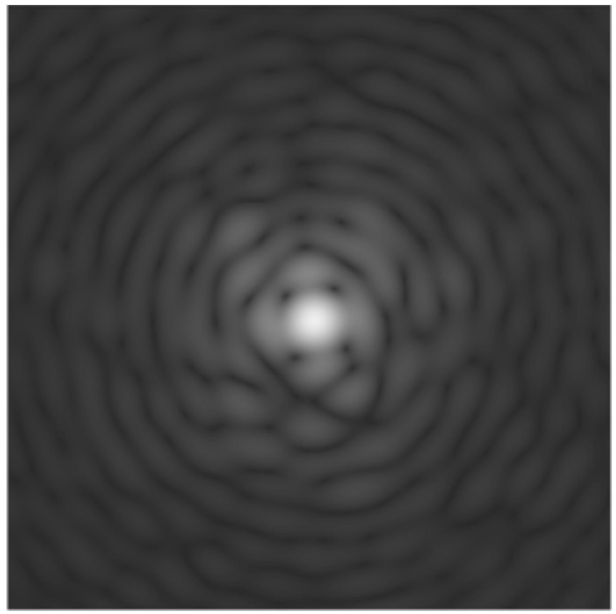
$$SR = e^{-k^2\omega^2} \approx 1 - k^2\omega^2$$

- <u>Examples:</u>
- SR > 80% considered diffraction-limited → WFE ~ λ/14
- typical adaptive optics system delivers SR ~ 10-80%
- seeing-limited PSF on 8m telescope has a SR~0.1-0.01%

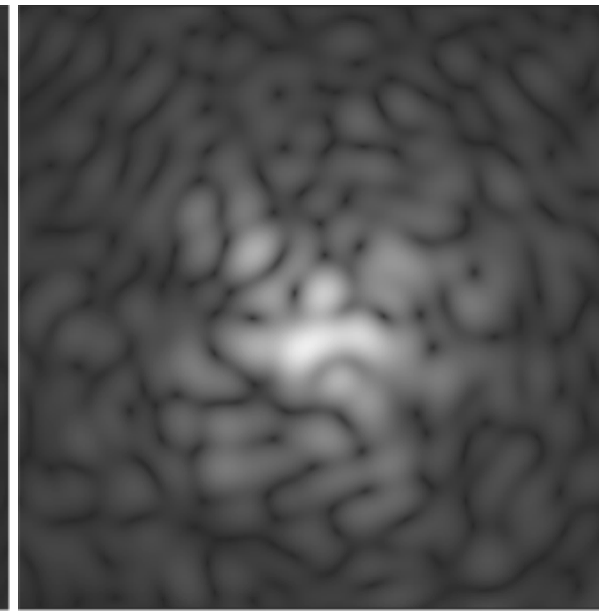
Images of Strehl Ratios



S=1.0



S=0.73



S=0.18

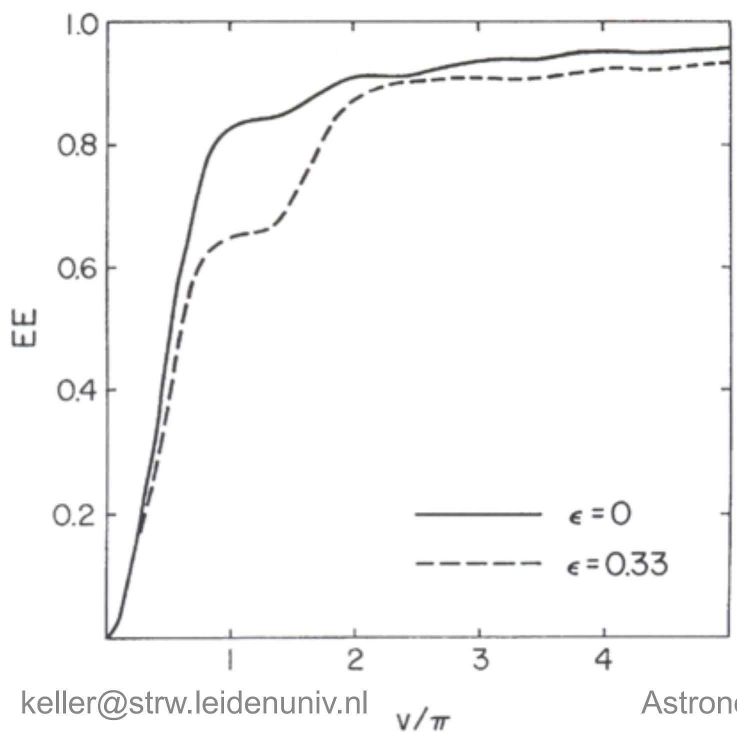
Encircled Energy

Fraction of total intensity (=energy) within a certain radius given by encircled energy (EE):

$$EE(r) = 1 - J_0^2\left(\frac{\pi r}{\lambda F}\right) - J_1^2\left(\frac{\pi r}{\lambda F}\right)$$

F is the f/# number

EE depends strongly on the central obscuration ε of the telescope:

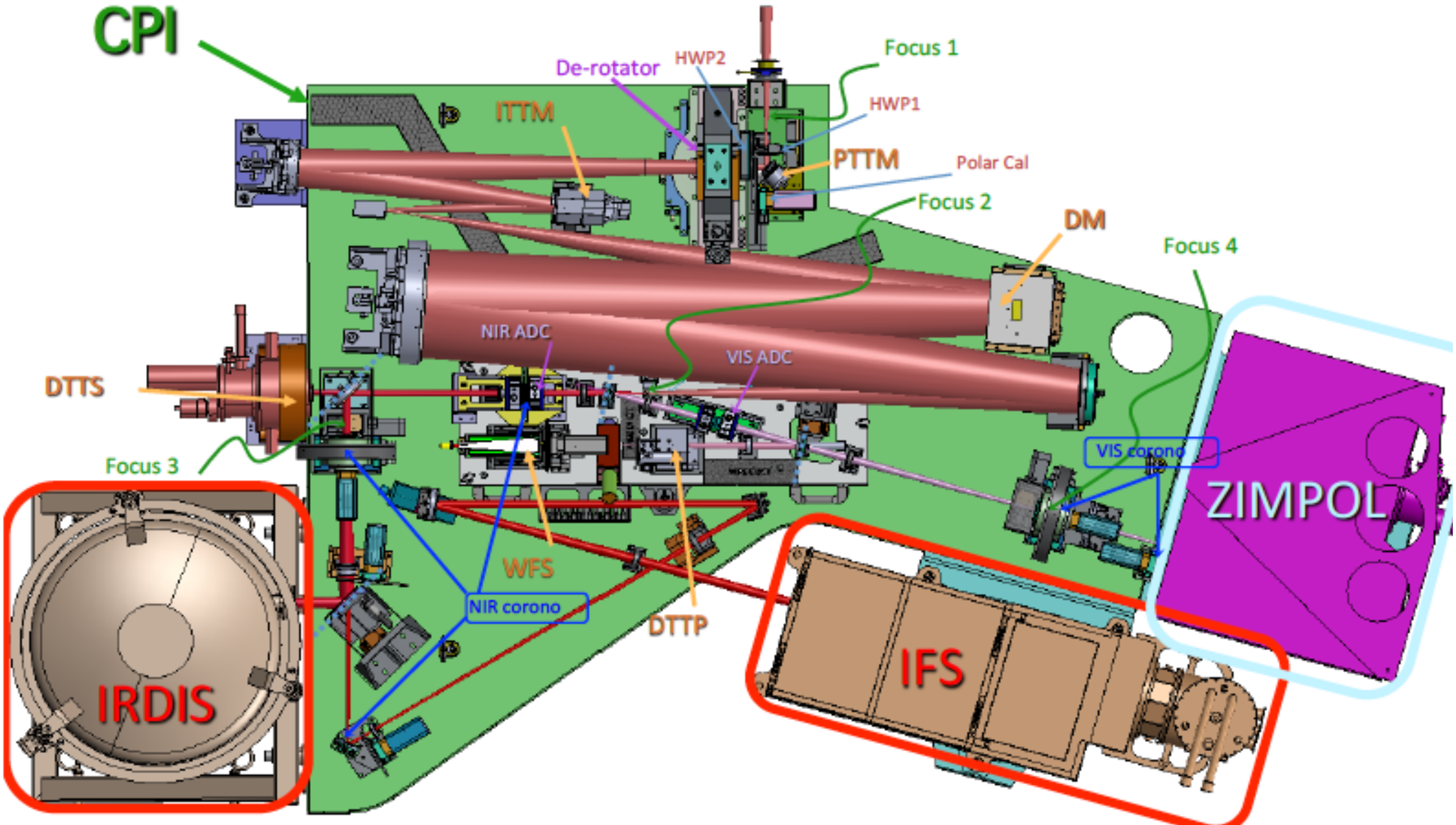


Encircled Energy Fraction within Airy Dark Rings[a]

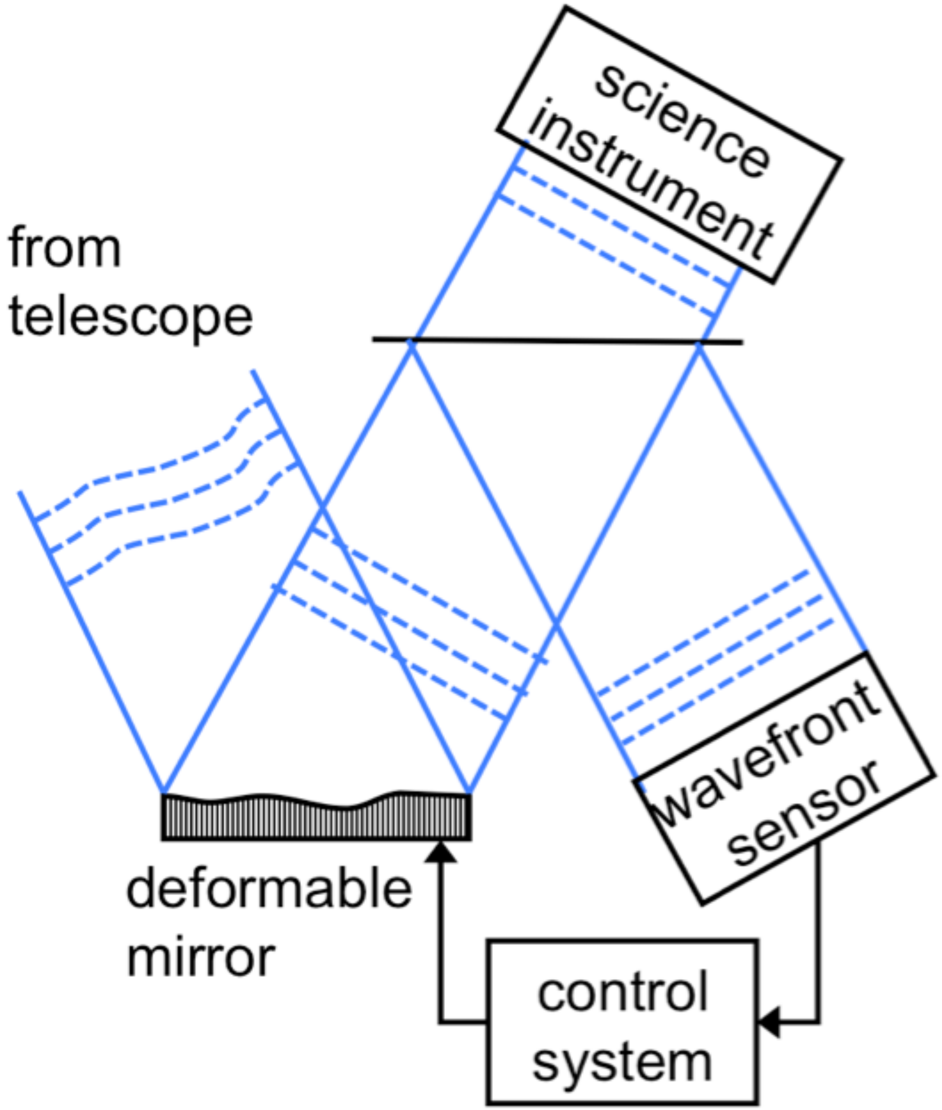
ε	EE_1	EE_2	EE_3
0.00	0.838	0.910	0.938
0.10	0.818	0.906	0.925
0.20	0.764	0.900	0.908
0.33	0.654	0.898	0.904
0.40	0.584	0.885	0.903
0.50	0.479	0.829	0.901
0.60	0.372	0.717	0.873

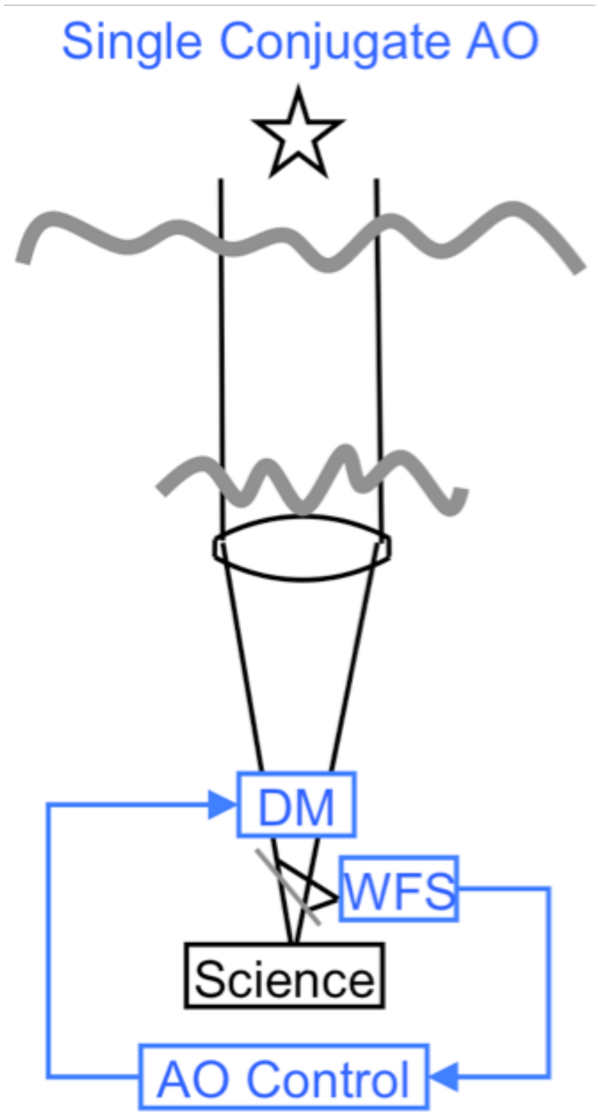
[a] Subscript on EE is number of dark ring starting at innermost ring.

High-Contrast Imaging: SPHERE at VLT

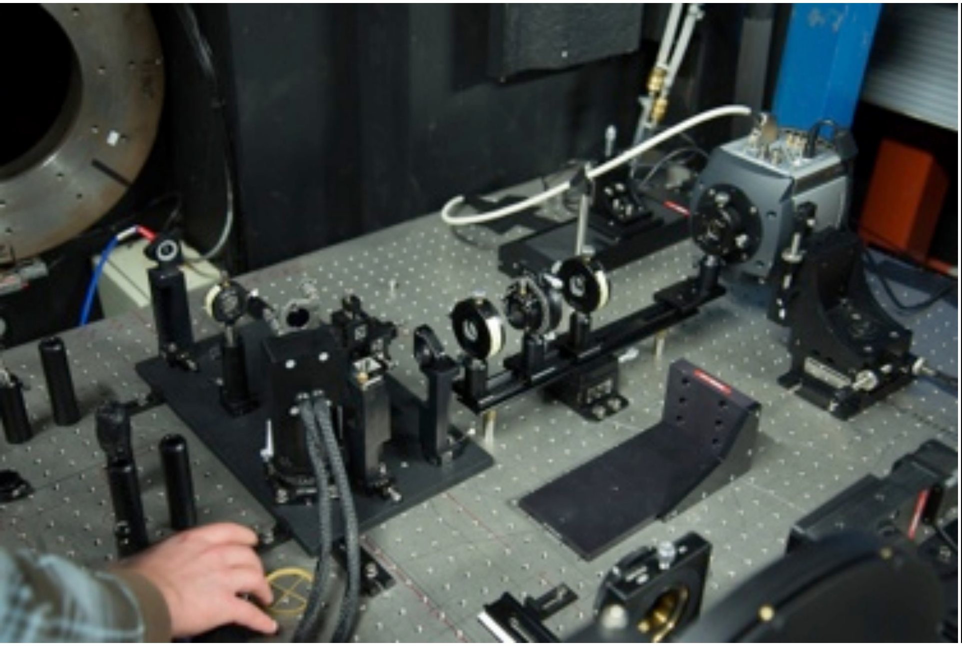


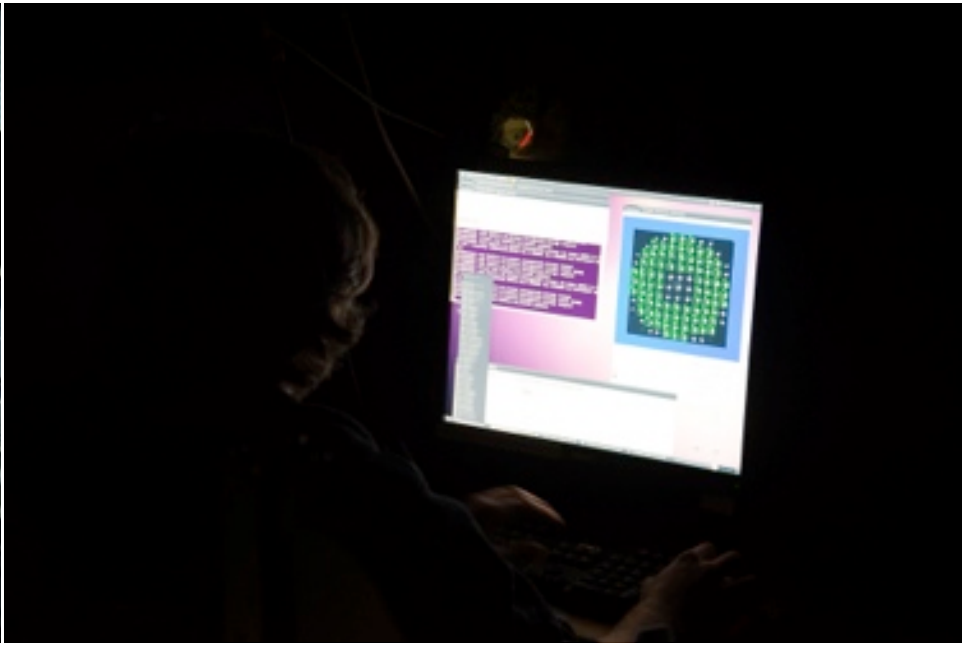
Adaptive Optics (more in lecture 13)

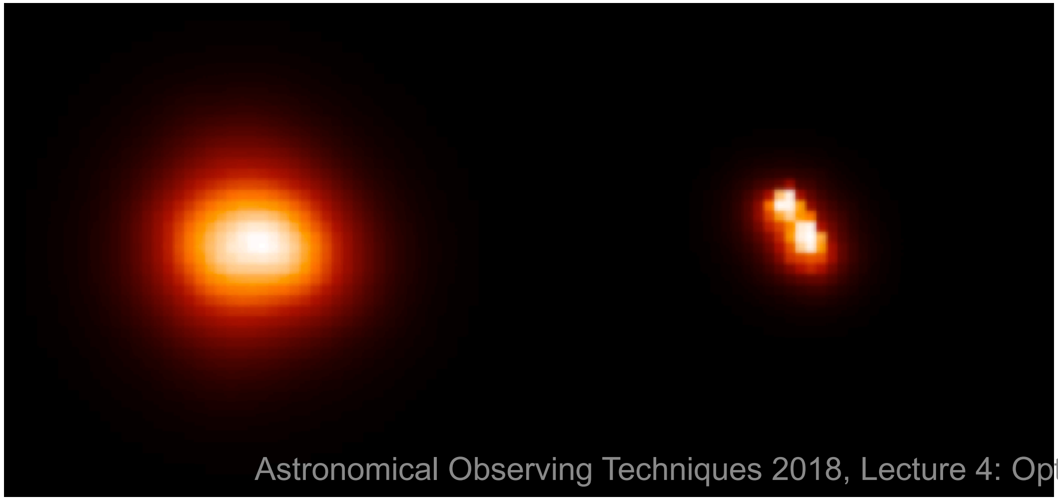




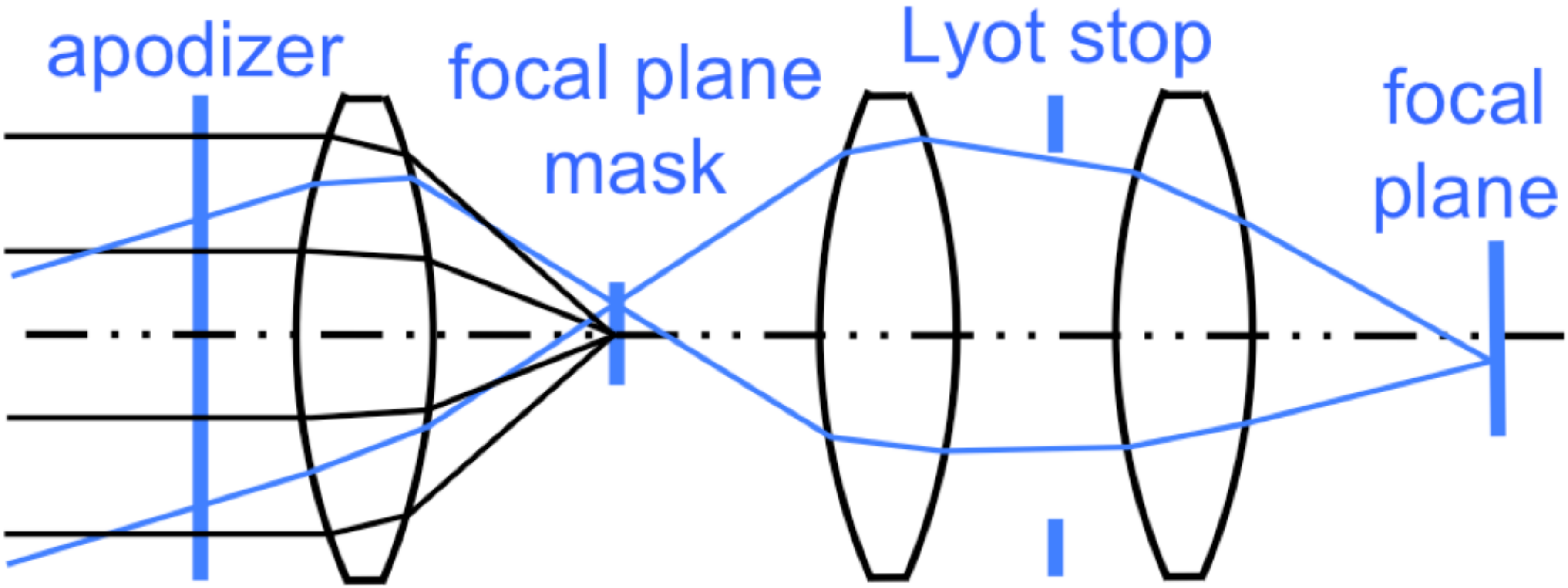
Student-Built Leiden AO





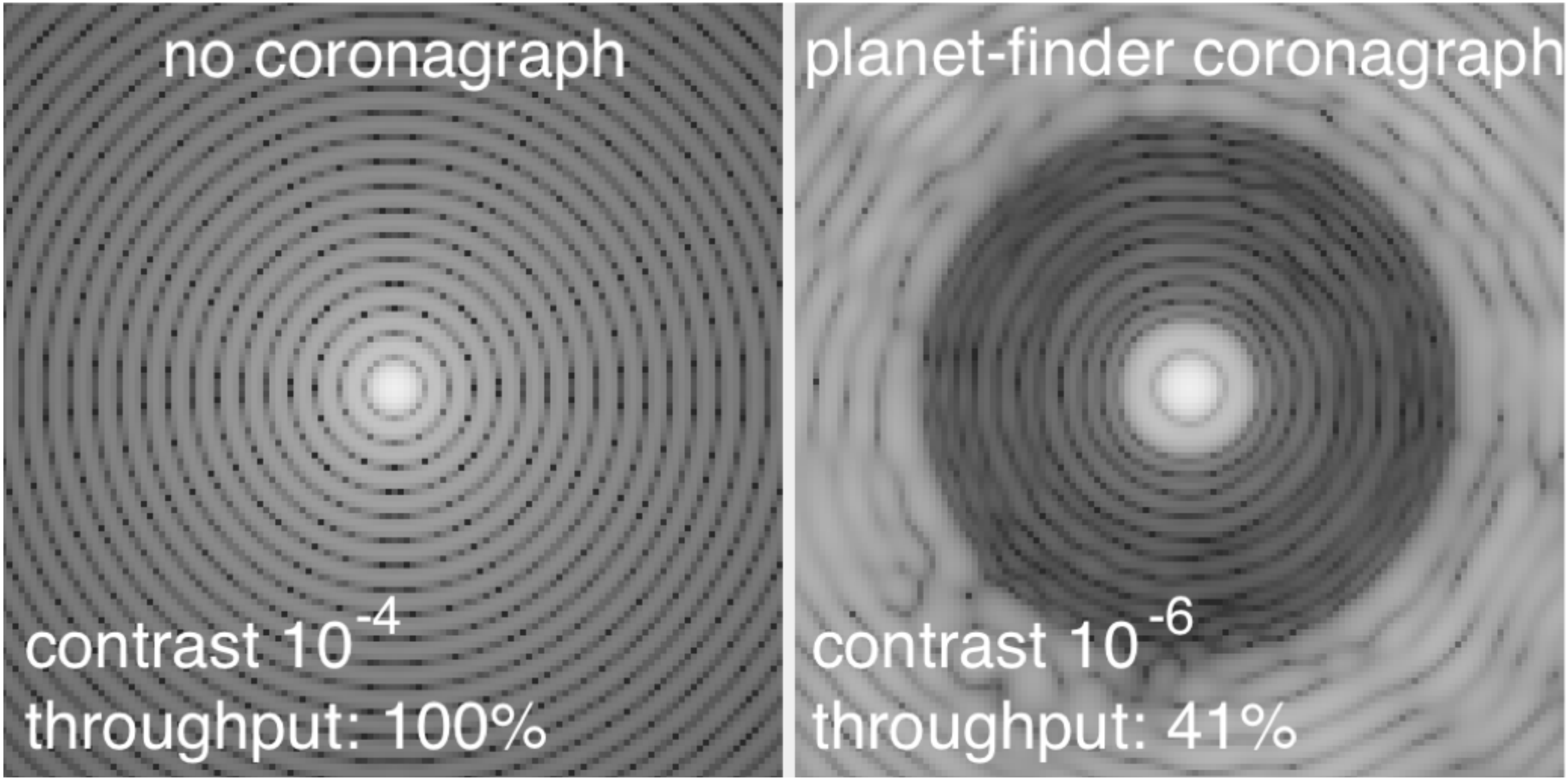


Coronagraph

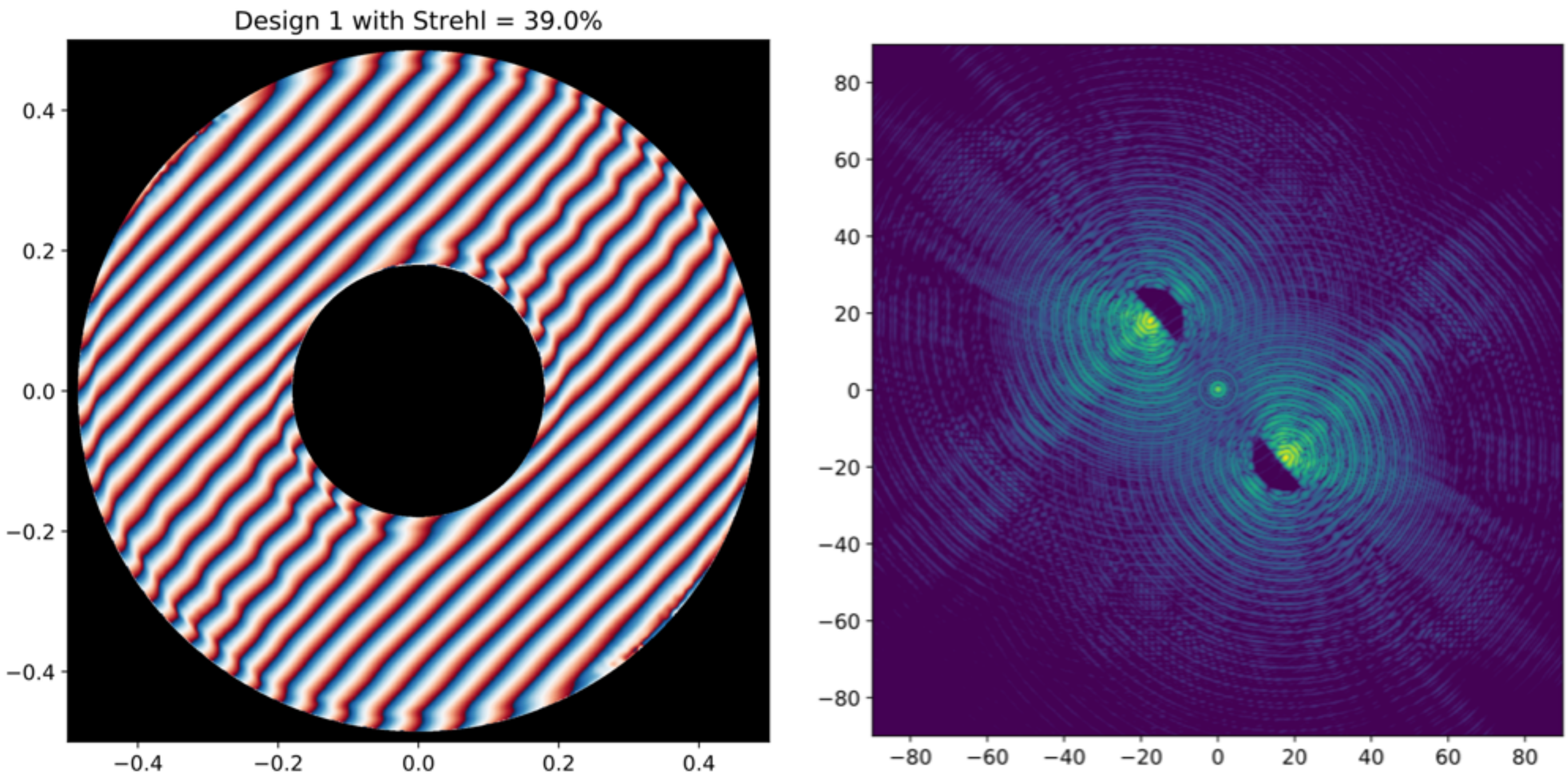


- goal: minimize diffracted light close to star
- can introduce optics in pupil and/or focal planes

Apodizing Pupil Phase Coronagraph

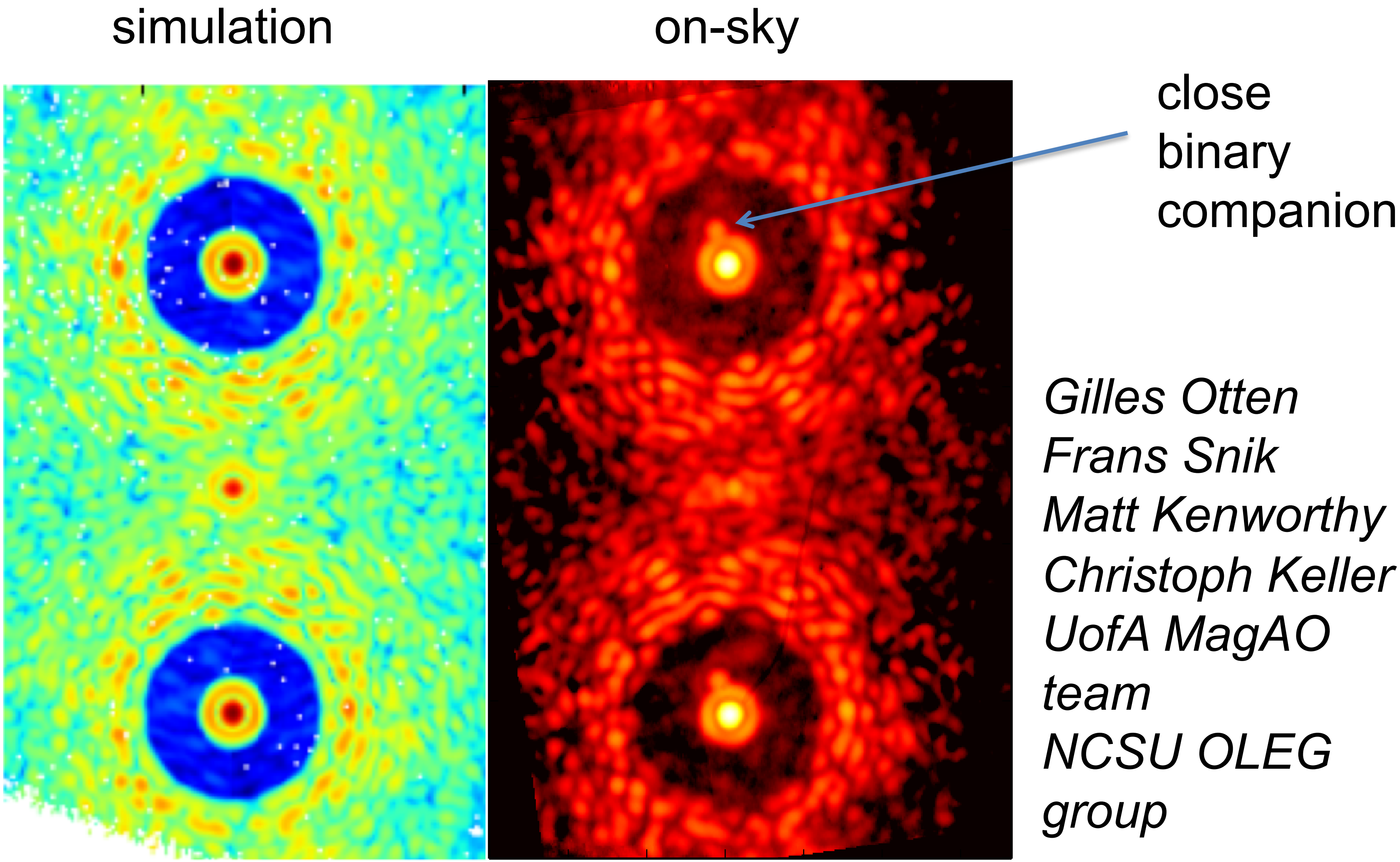


Apodizing Phase Plate



courtesy David Doelman, Leiden Observatory

360° vAPP on-sky at MagAO



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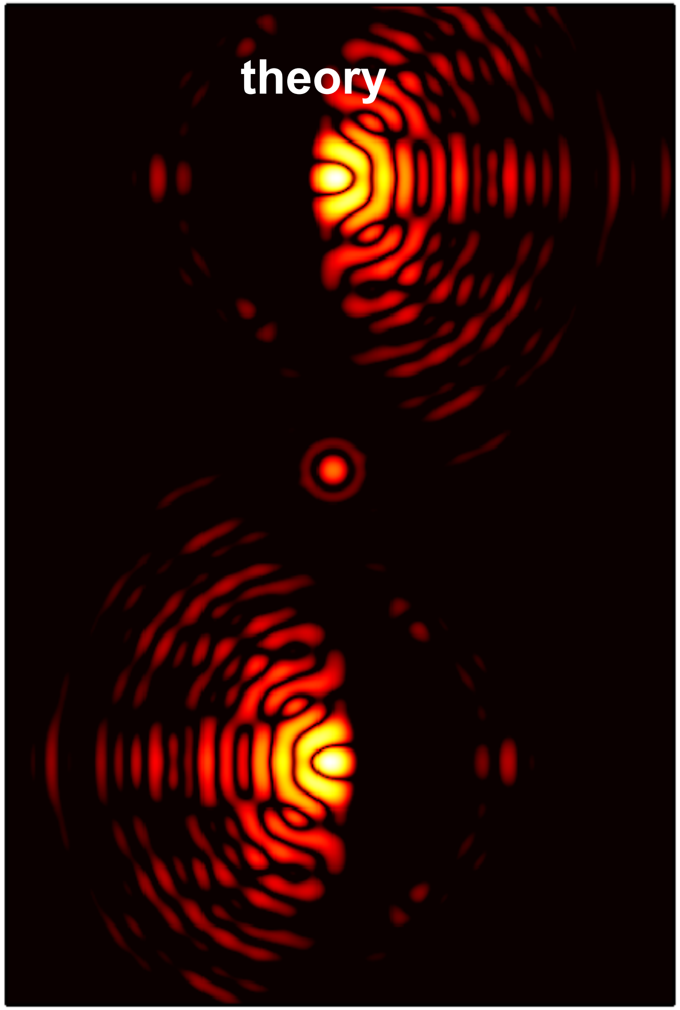

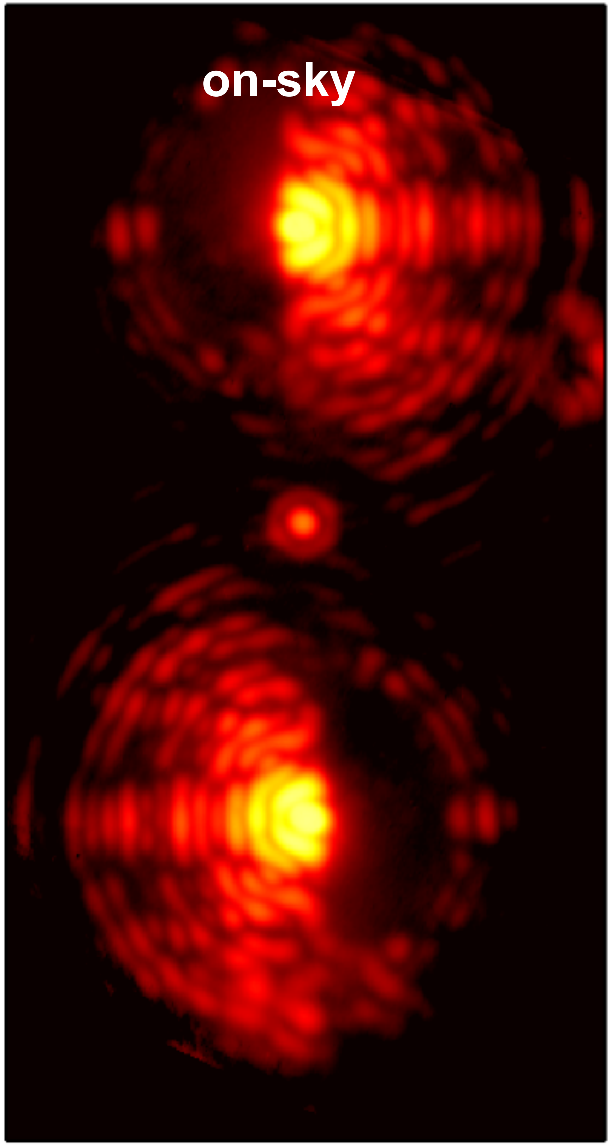

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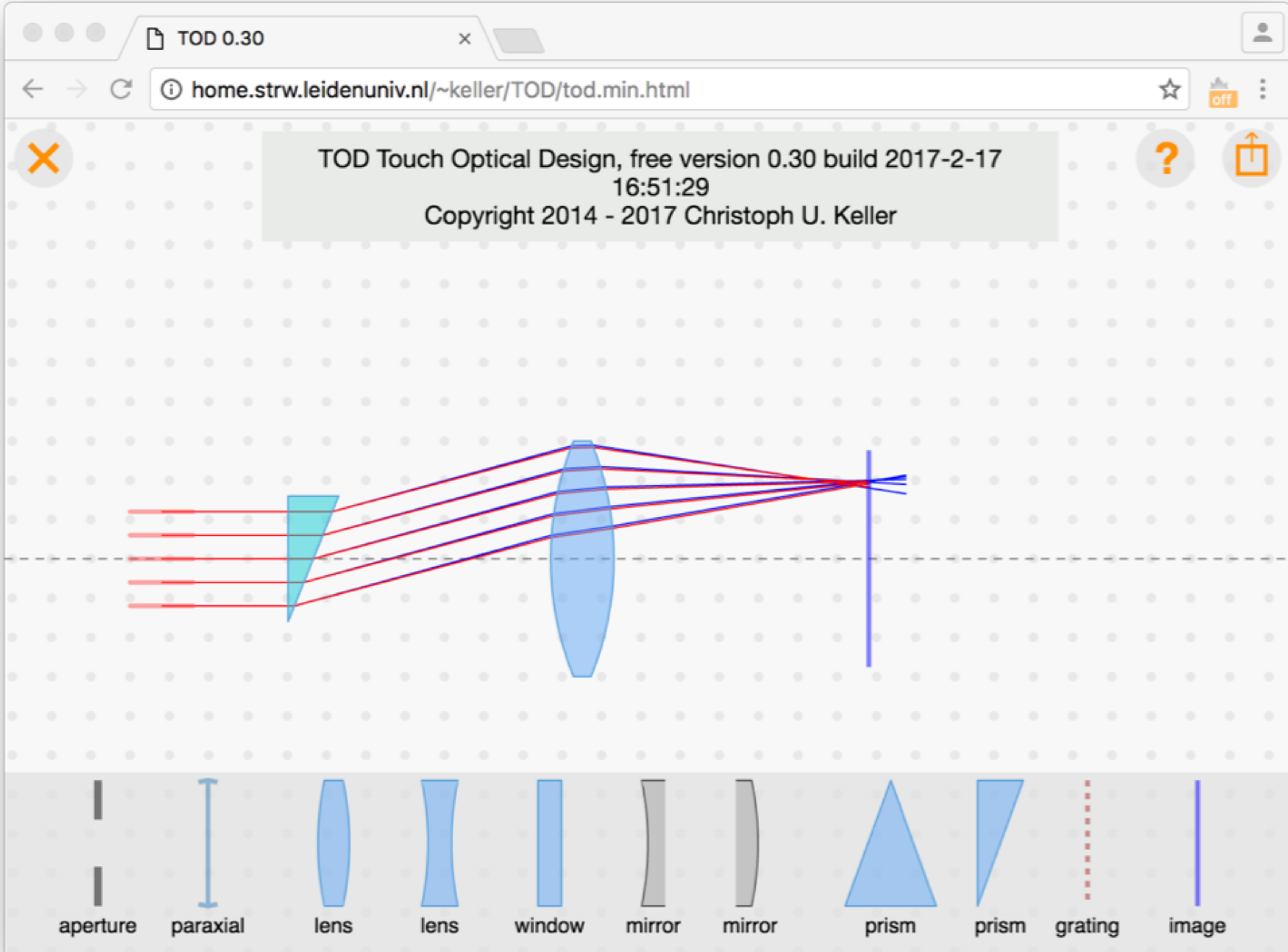
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