Astronomical Observing Techniques 2018

Lecture 1: Black Bodies in Space

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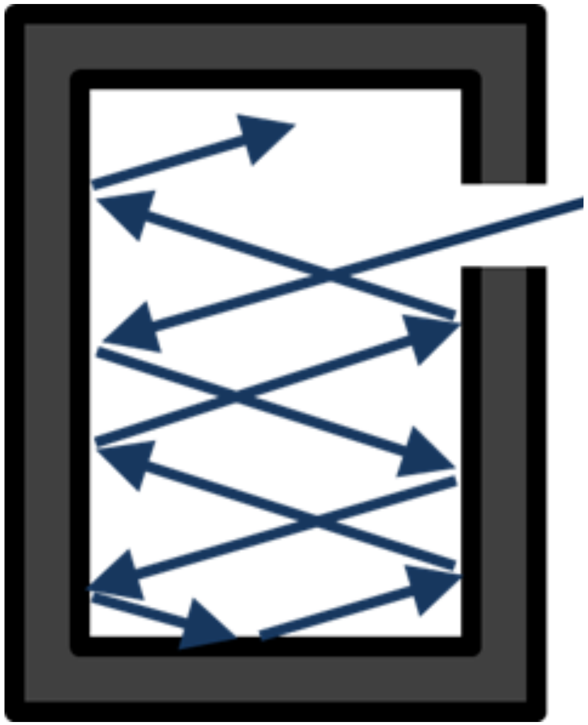
Content

Blackbody Radiation: Background

Kirchhoff (1860): black body completely absorbs all incident rays: no reflection, no transmission for all wavelengths and for all angles of incidence.



Cavity at fixed T, thermal equilibrium

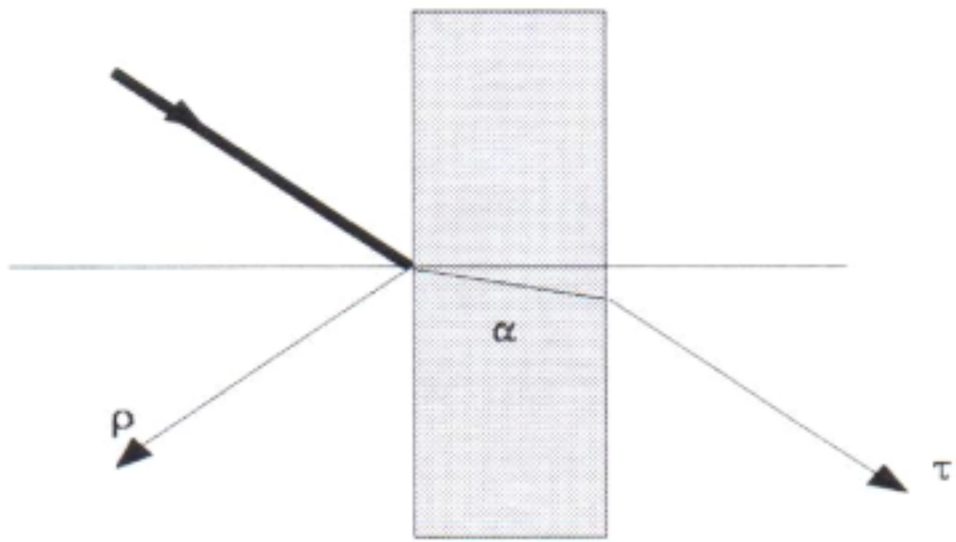
Incoming radiation is continuously absorbed and re-emitted by cavity wall

Small hole → escaping radiation will approximate black-body radiation independent of properties of cavity or hole

Blackbody Radiation

Conservation of power requires:

$$\alpha + \rho + \tau = 1$$



α = absorptivity
ρ = reflectivity
τ = transmissivity

Kirchhoff's Law

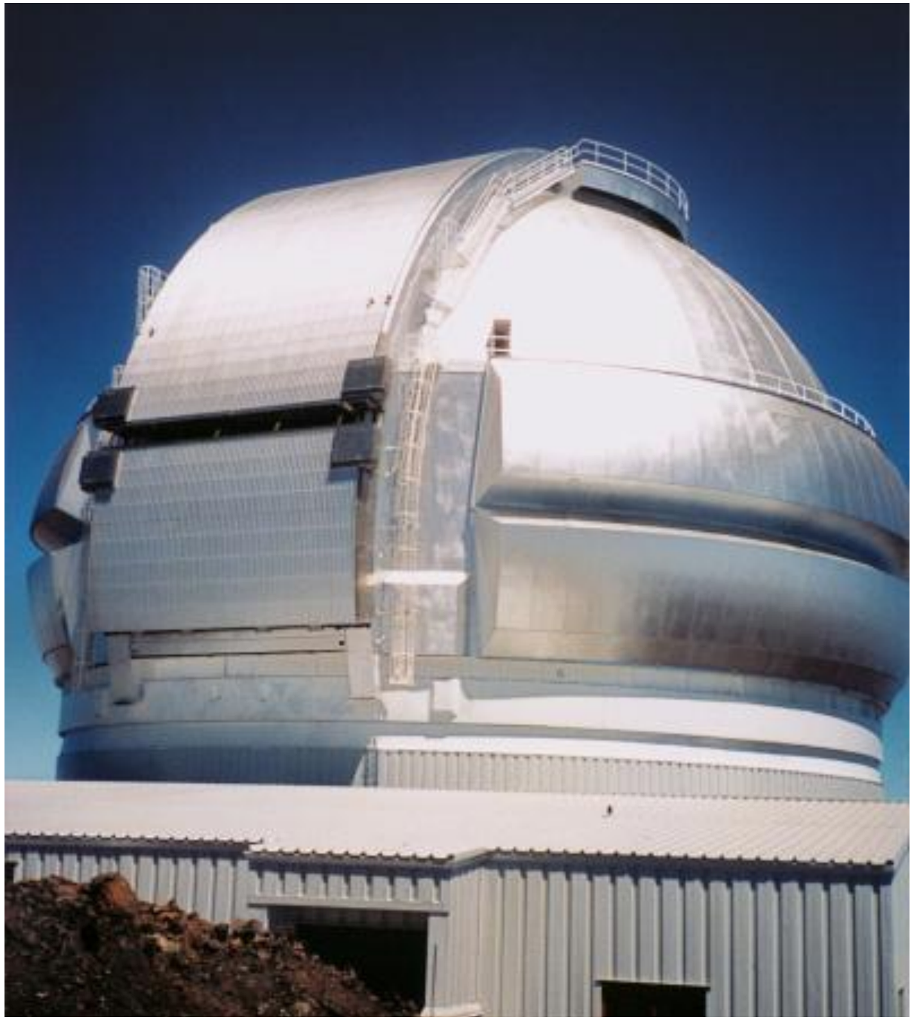
- Blackbody cavity in thermal equilibrium with completely opaque sides
- Opaque -> transmissivity $\tau = 0$
- emissivity ε = amount of emitted radiation
- Thermal equilibrium -> $\varepsilon + \rho = 1$

$$\left.\begin{array}{c} \varepsilon = 1 - \rho \\ \alpha + \rho + \tau = 1 \\ \tau = 0 \end{array}\right\} \boxed{\alpha = \varepsilon}$$

Blackbody absorbs all radiation: $\alpha=\varepsilon=1$

Kirchhoff's law applies to perfect black body at all wavelengths

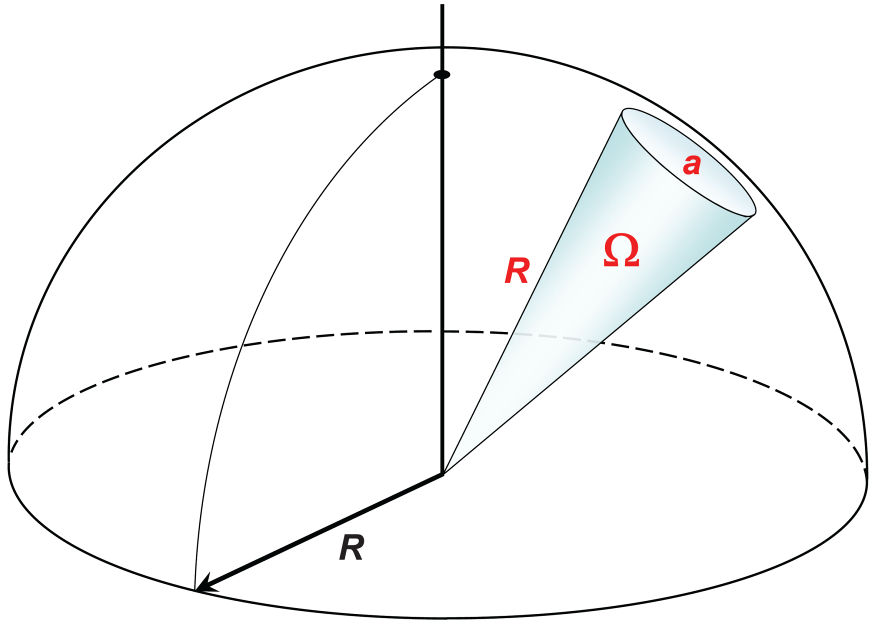
Blackbody Radiation: Telescope Domes



Credit NOAO/AURA/NSF: www.noao.edu/image_gallery/telescopes.html

Solid Angle steradian

- Solid Angle Ω: 2D angle in 3D space
- Measures how large the object appears to an observer
- Solid angle is expressed in a dimensionless unit called a steradian (sr)
- Full sphere is 4π sr



Planck Curve: Equation

Specific intensity I_ν of blackbody given by Planck's law:

$$I_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{\exp\left(\frac{h\nu}{kT}\right) - 1}$$ in units of [W m^{-2} sr^{-1} Hz^{-1}]

In wavelength units:

$$I_\lambda(T) = \frac{2hc^2}{\lambda^5} \frac{1}{\exp\left(\frac{hc}{\lambda kT}\right) - 1}$$ in units of [W m^{-3} sr^{-1}]

Conversion between frequency ⇔ wavelength units:

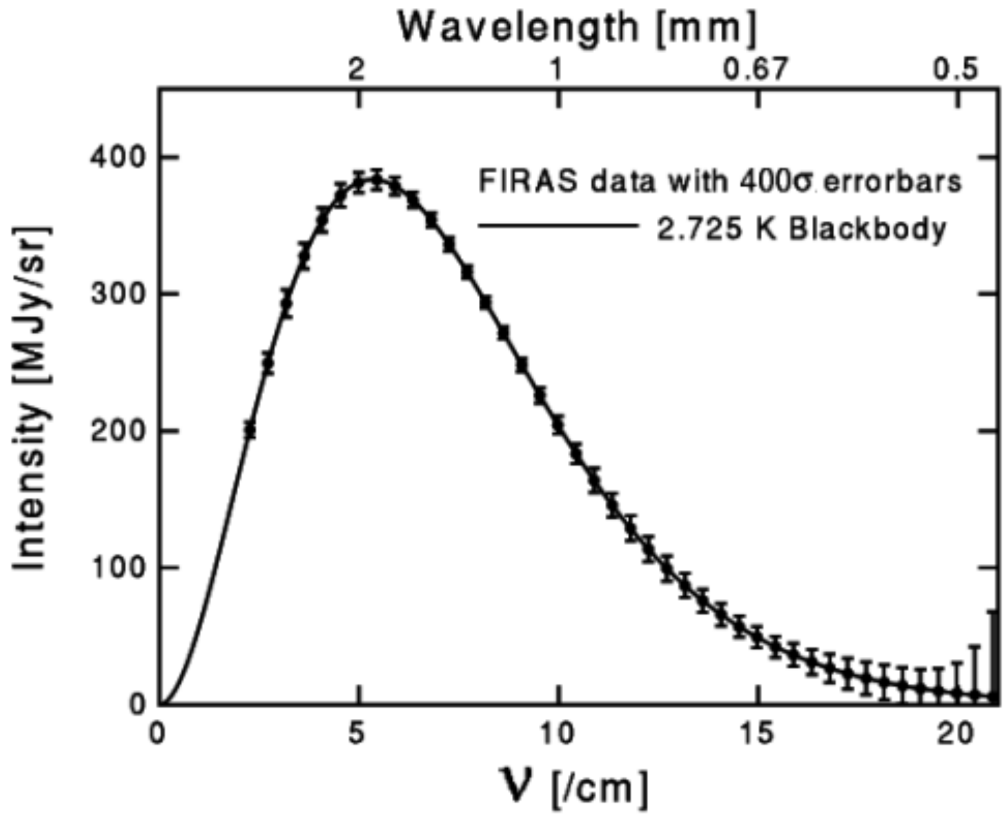
$$dv = \frac{c}{\lambda^2} d\lambda \quad \text{or} \quad d\lambda = \frac{c}{\nu^2} d\nu$$

Blackbody Radiation: Black Body

- Black body (BB) is idealized object that absorbs all EM radiation
- Cold (T~0K) BBs are black (no emitted or reflected light)
- At T > 0 K BBs absorb and re-emit characteristic EM spectrum

Many astronomical sources emit close to a black body.

Example: COBE measurement of the cosmic microwave background



Planck Curve: Emission, Power & Temperature

Total radiated power per unit surface proportional to fourth power of temperature *T*:

$$\int_{\Omega}\int_{\nu} I_{\nu}(T)\, d\nu\, d\Omega = M = \sigma T^4$$

σ = 5.67·10^{-8} W m^{-2} K^{-4} (Stefan-Boltzmann constant)

Assuming BB radiation, astronomers often specify the emission from objects via their effective temperature.

Planck Curve: Approximations

Planck:

$$I_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{\exp\left(\frac{h\nu}{kT}\right) - 1}$$

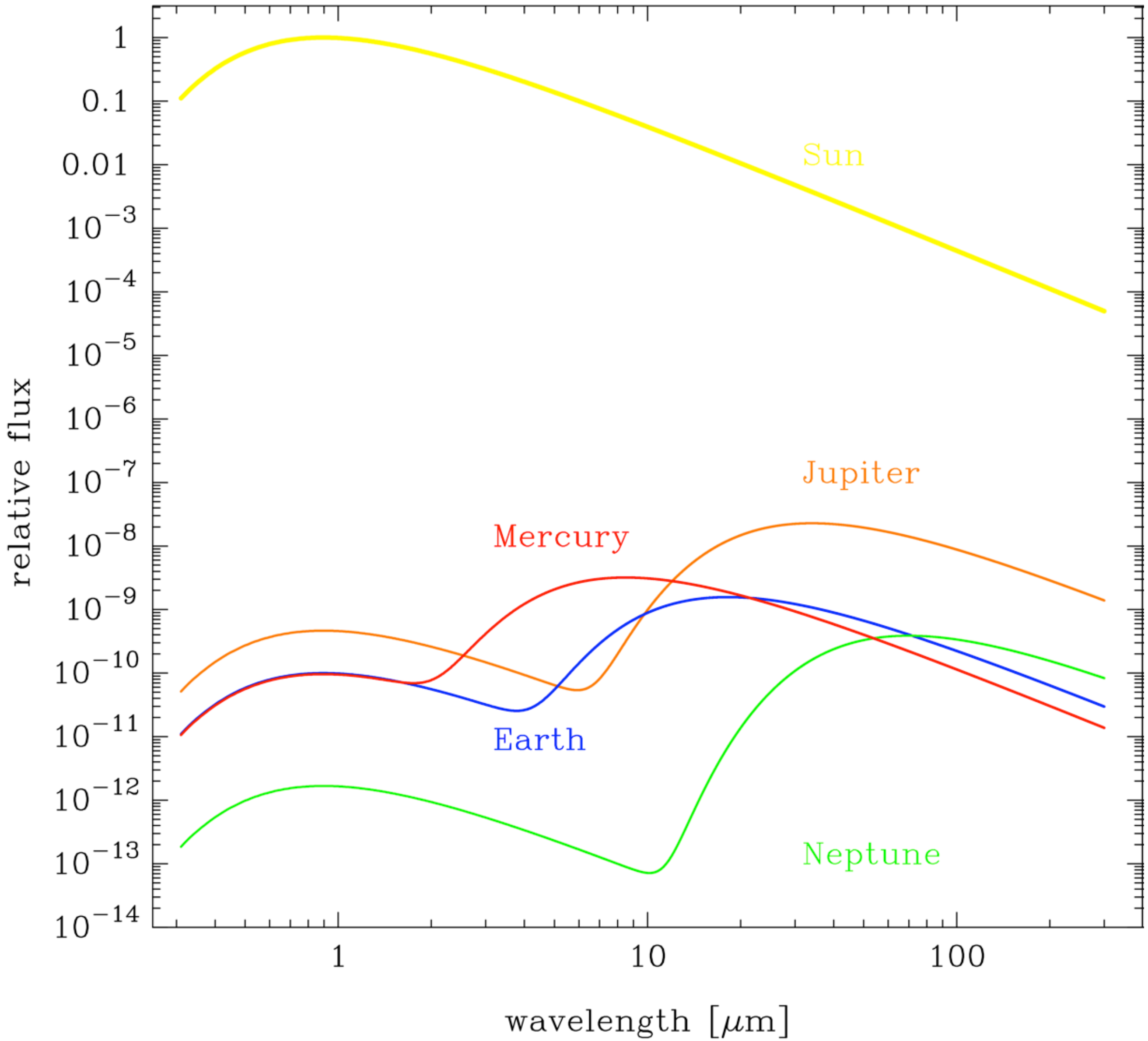
High frequencies (h*ν* >> kT) ➔ Wien approximation:

$$I_\nu(T) = \frac{2h\nu^3}{c^2} \exp\left(-\frac{h\nu}{kT}\right)$$

Low frequencies (hv << kT) ➔ Rayleigh-Jeans approximation:

$$I_\nu(T) \approx \frac{2\nu^2}{c^2} kT = \frac{2kT}{\lambda^2}$$

Planck Curve: Planet Radiation

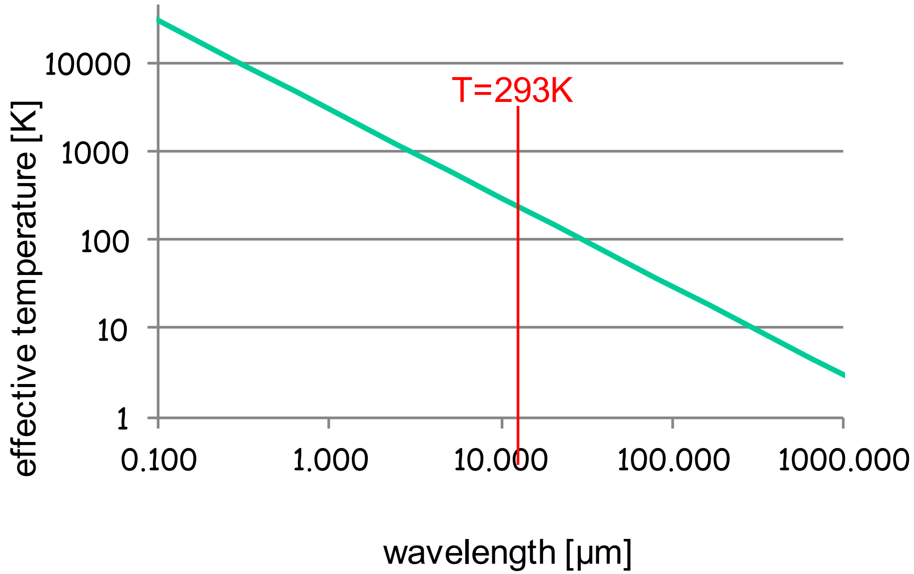


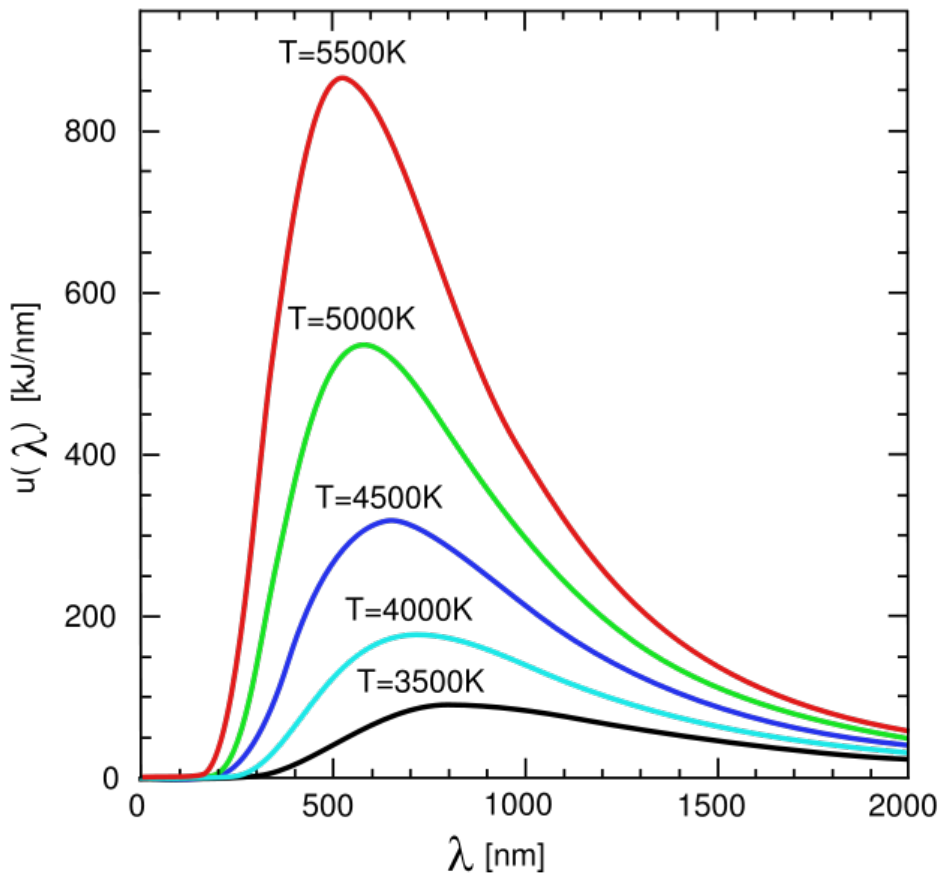
Effective Temperature: Wien's Law

Temperature corresponding to maximum specific intensity given by Wien's displacement law:

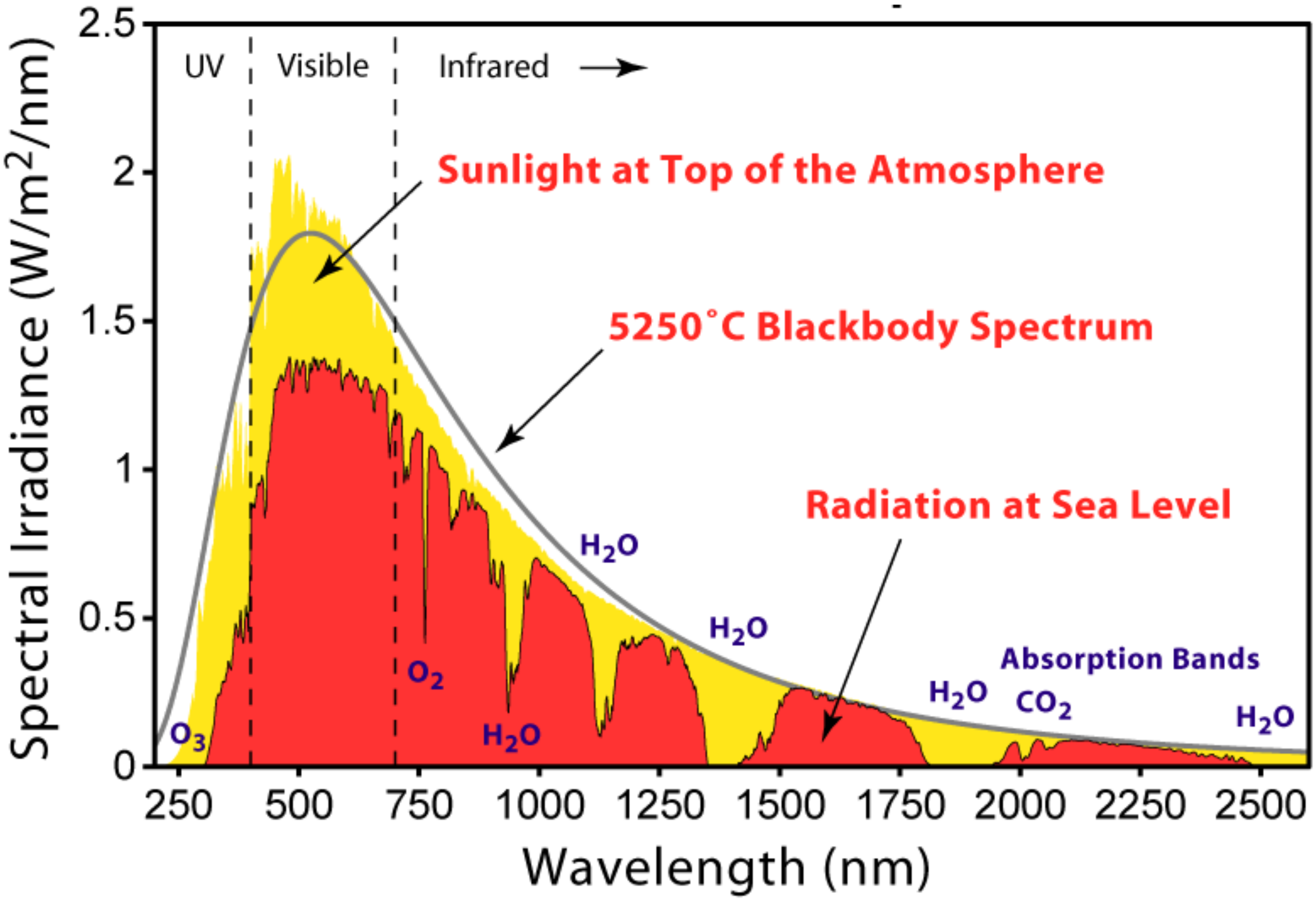
$$\frac{c}{\nu_{\max}} T = 5.096 \cdot 10^{-3} \text{ mK} \quad \text{or} \quad \lambda_{\max} T = 2.98 \cdot 10^{-3} \text{ mK}$$

Cooler BBs have peak emission (effective temperatures) at longer wavelengths and at lower intensities:





Effective Temperature: Solar Spectrum



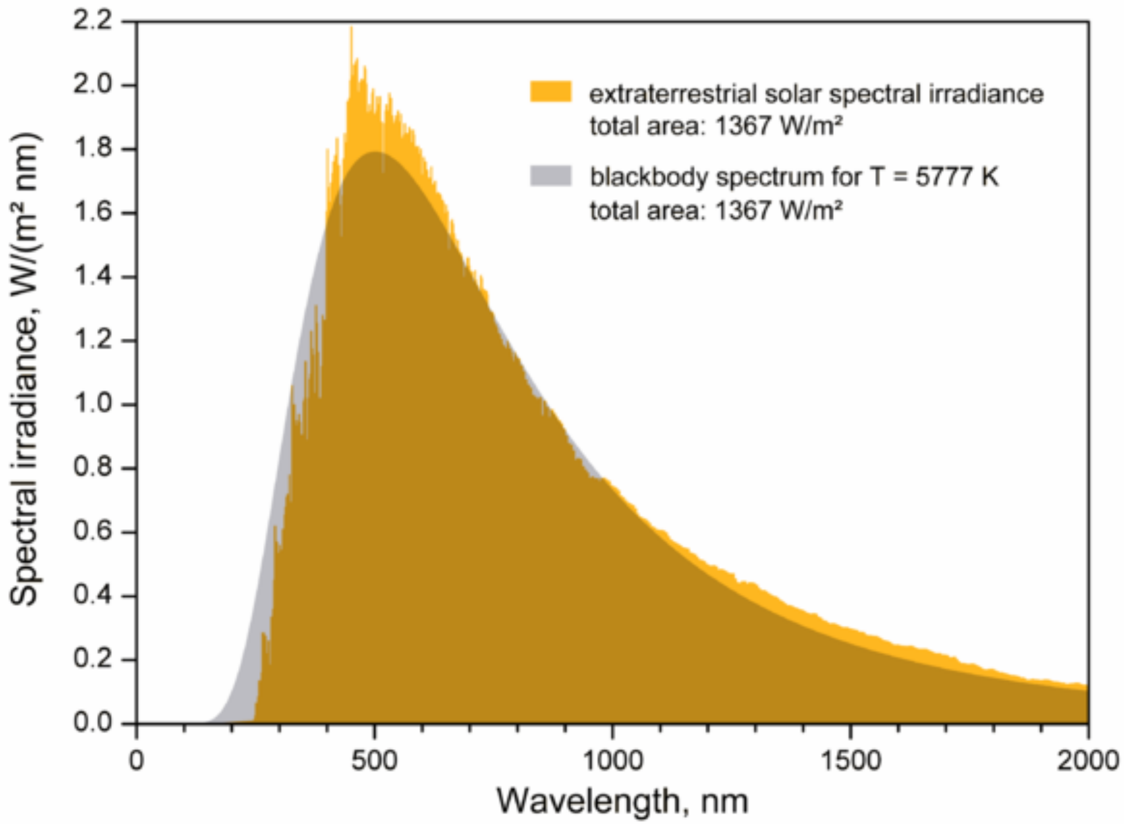
http://en.wikipedia.org/wiki/Sunlight#mediaviewer/File:Solar_Spectrum.png

Brightness Temperature: Grey Bodies

Many emitters close to but not perfect black bodies.
With wavelength-dependent emissivity ε<1:

$$I_\lambda(T) = \varepsilon(\lambda) \cdot \frac{2hc^2}{\lambda^5} \frac{1}{\exp\left(\frac{hc}{\lambda kT}\right) - 1}$$

Example: the Sun
(like many stars)



Brightness Temperature: Definition

Brightness temperature: temperature of a perfect black body that reproduces the observed intensity of a grey body object at frequency *ν.*

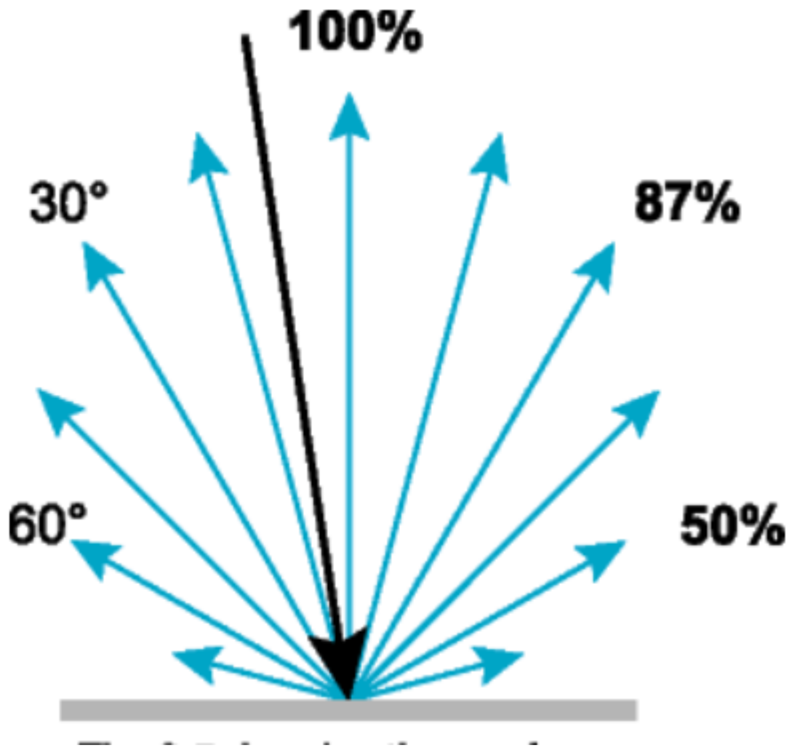
For low frequencies ($h\nu << kT$):

$$T_b = \varepsilon(\nu)\cdot T \overset{\text{Rayleigh-}}{\underset{\text{Jeans}}{=}} \varepsilon(\nu)\cdot\frac{c^2}{2k\nu^2}I_\nu$$

Only for perfect BBs is T_b the same for all frequencies.

Lambert: Cosine Law

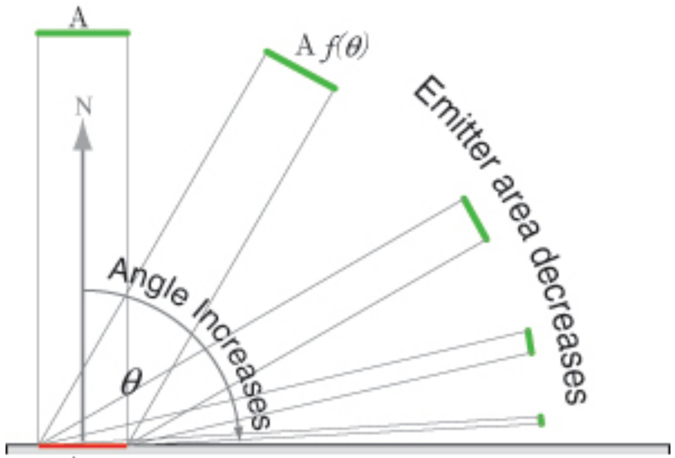
Lambert's cosine law: radiant intensity from an ideal, diffusively reflecting surface is directly proportional to the cosine of the angle θ between the surface normal and the observer.



Johann Heinrich Lambert
(1728 – 1777)

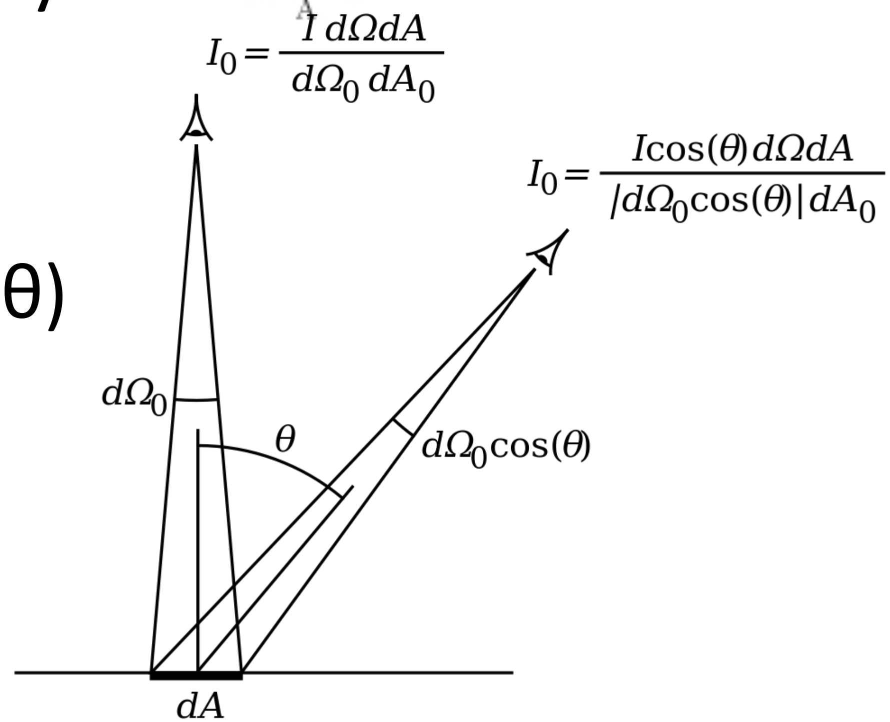
Lambert: Lambertian Emitters

Radiance of Lambertian emitters is independent of direction θ of observation (i.e., isotropic).



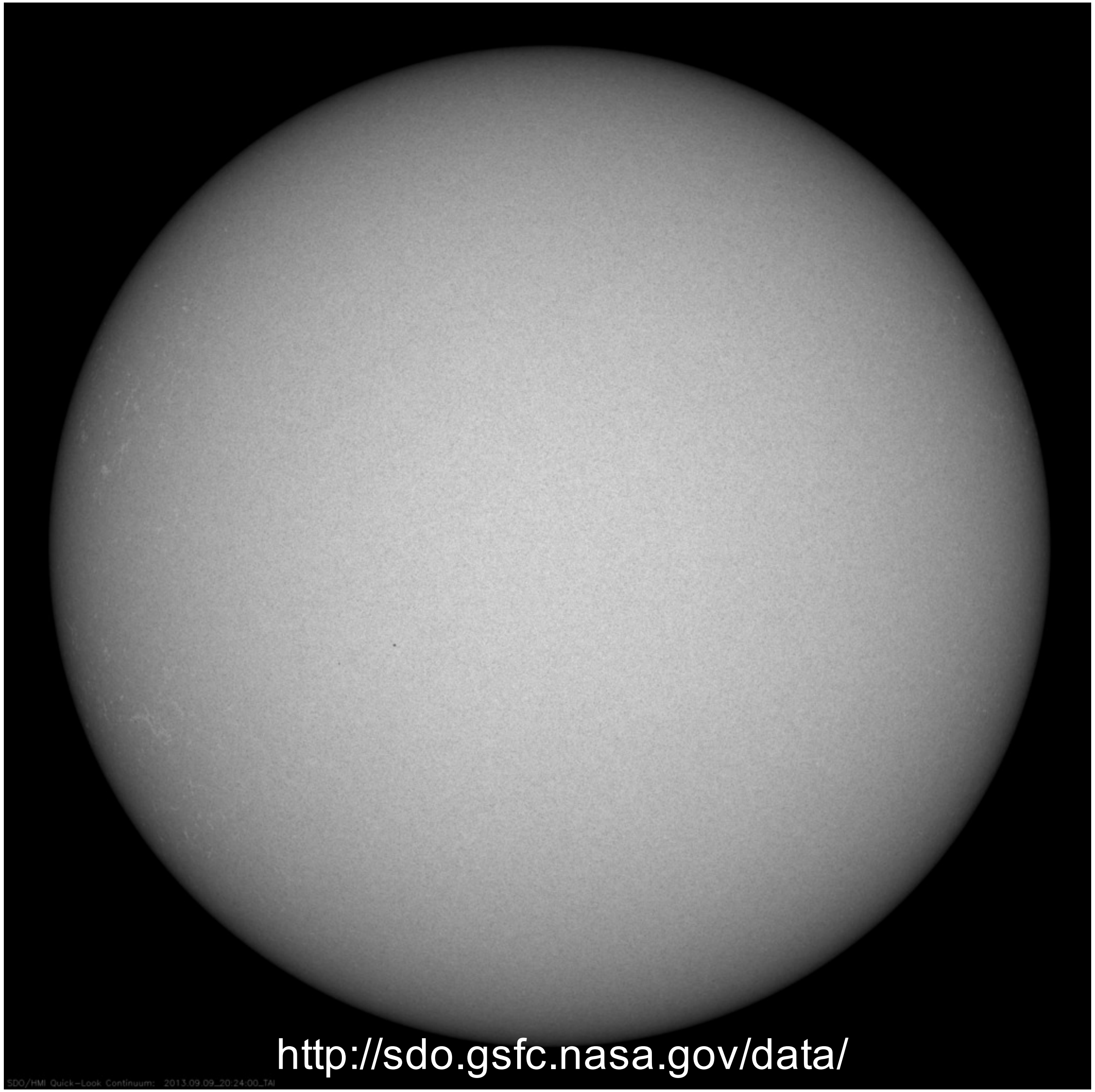
Two effects that cancel each other:

1. Lambert's cosine law → radiant intensity and *dΩ* are reduced by cos(θ)
2. Emitting surface area *dA* for a given *dΩ* is increased by $\cos^{-1}(\theta)$



Perfect black bodies are Lambertian emitters!

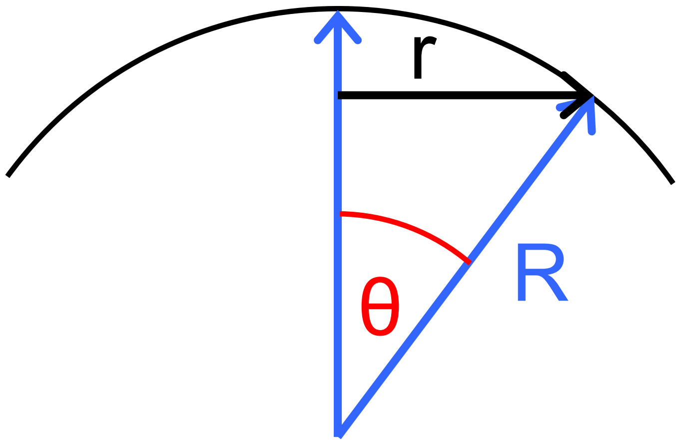
Lambert: The Sun a Lambertian Emitter?



Flux and Intensity

- Energy flux *F* of star = π × intensity *I* averaged over disk
- Stellar disk average in polar coordinates *r,φ*

$$\bar{I} = \frac{1}{\pi R^2} \int_0^{2\pi} \int_0^{R} I(r) r\, dr\, d\varphi$$



- Substitute *r* with *R*sinθ, μ=cosθ

$$\bar{I} = 2 \int_0^{\pi/2} I(\theta) \sin\theta \cos\theta d\theta = 2\int_0^1 I \mu\, d\mu$$

- Flux integrated over hemisphere

$$F = \int_0^{2\pi} \int_0^{\pi/2} I(\theta) \cos\theta \sin\theta d\theta d\phi = 2\pi \int_0^1 I \mu\, d\mu$$

Summary of Radiometric Quantities

Name	Symbol	Unit	Definition	Equation
Spectral radiance *or* specific intensity	$L_\nu \; , \; I_\nu$	$W\,m^{-2}\,Hz^{-1}\,sr^{-1}$	Power leaving unit projected surface area into unit solid angle and unit Δν	
Spectral radiance *or* specific intensity	$L_\lambda \; , \; I_\lambda$	$W\,m^{-3}\,sr^{-1}$	Power leaving unit projected surface area into unit solid angle and unit Δλ	
Radiance *or* Intensity	$L \; , \; I$	$W\,m^{-2}\,sr^{-1}$	Spectral radiance integrated over spectral bandwidth	$L = \int L_\nu \, d\nu$
Radiant exitance	M	$W\,m^{-2}$	Total power emitted per unit surface area	$M = \int L(\theta)\, d\Omega$
Flux *or* luminosity	$\Phi \; , \; L$	W	Total power emitted by a source of surface area A	$\Phi = \int M \, dA$
Spectral irradiance *or* flux density	L_ν, F_ν, I_ν	$W\,m^{-2}\,Hz^{-1}$ *	Power received at a unit surface element per unit Δν	
Spectral irradiance *or* flux density	$L_\lambda, F_\lambda, I_\lambda$	$W\,m^{-3}$ *	Power received at a unit surface element per unit Δλ	
Irradiance	E	$W\,m^{-2}$	Power received at a unit surface element	$E = \frac{\int M\, dA}{4\pi r^2}$

*10^{-26} W m^{-2} Hz^{-1} = 10^{-23} erg $s^{-1}cm^{-2}$ Hz^{-1} is called 1 Jansky

Magnitudes: Origin

Origins in Greek classification of stars according to their visual brightness.
Brightest stars were m = 1, faintest detected with bare eye were m = 6.

Formalized by Pogson (1856): 1^{st} mag ~ 100 × 6^{th} mag

Magnitude	Example	#stars brighter
-27	Sun	
-13	Full moon	
-5	Venus	
0	Vega	4
2	Polaris	48
3.4	Andromeda	250
6	Limit of naked eye	4800
10	Limit of good binoculars	
14	Pluto	
27	Visible light limit of 8m telescopes	

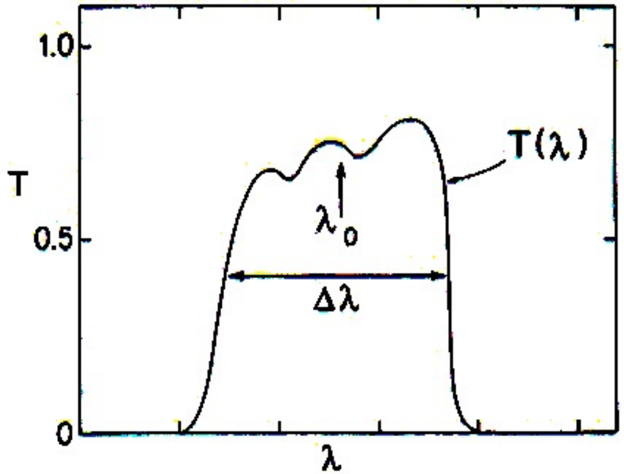
Magnitudes: Apparent Magnitude

Apparent magnitude is *relative* measure of monochromatic flux density F_λ of a source:

$$m_\lambda - M_0 = -2.5 \cdot \log\left(\frac{F_\lambda}{F_0}\right)$$

M_0 defines reference point (usually magnitude zero).

In practice, measurements through transmission filter $T(\lambda)$ that defines bandwidth:



$$m_\lambda - M = -2.5 \log \int_0^\infty T(\lambda) F_\lambda d\lambda + 2.5 \log \int_0^\infty T(\lambda) d\lambda$$

Magnitudes: Absolute Magnitude

- absolute magnitude = apparent magnitude of source if it were at distance D = 10 parsecs

$$M = m + 5 - 5\log D$$

- $m_\odot \approx -27$, $M_\odot = 4.83$ at visible wavelengths
- $M_{\text{Milky Way}} = -20.5 \rightarrow M_\odot - M_{\text{Milky Way}} = 25.3$
- Luminosity $L_{\text{Milky Way}} = 1.4\times10^{10}\ L_\odot$

Magnitudes: Bolometric Magnitude

Bolometric magnitude is luminosity expressed in magnitude units = integral of monochromatic flux over all wavelengths:

$$M_{bol} = -2.5 \cdot \log \frac{\int_0^\infty F(\lambda) d\lambda}{F_{bol}} \qquad ; F_{bol} = 2.52 \cdot 10^{-8} \frac{\mathrm{W}}{\mathrm{m}^2}$$

If source radiates isotropically:

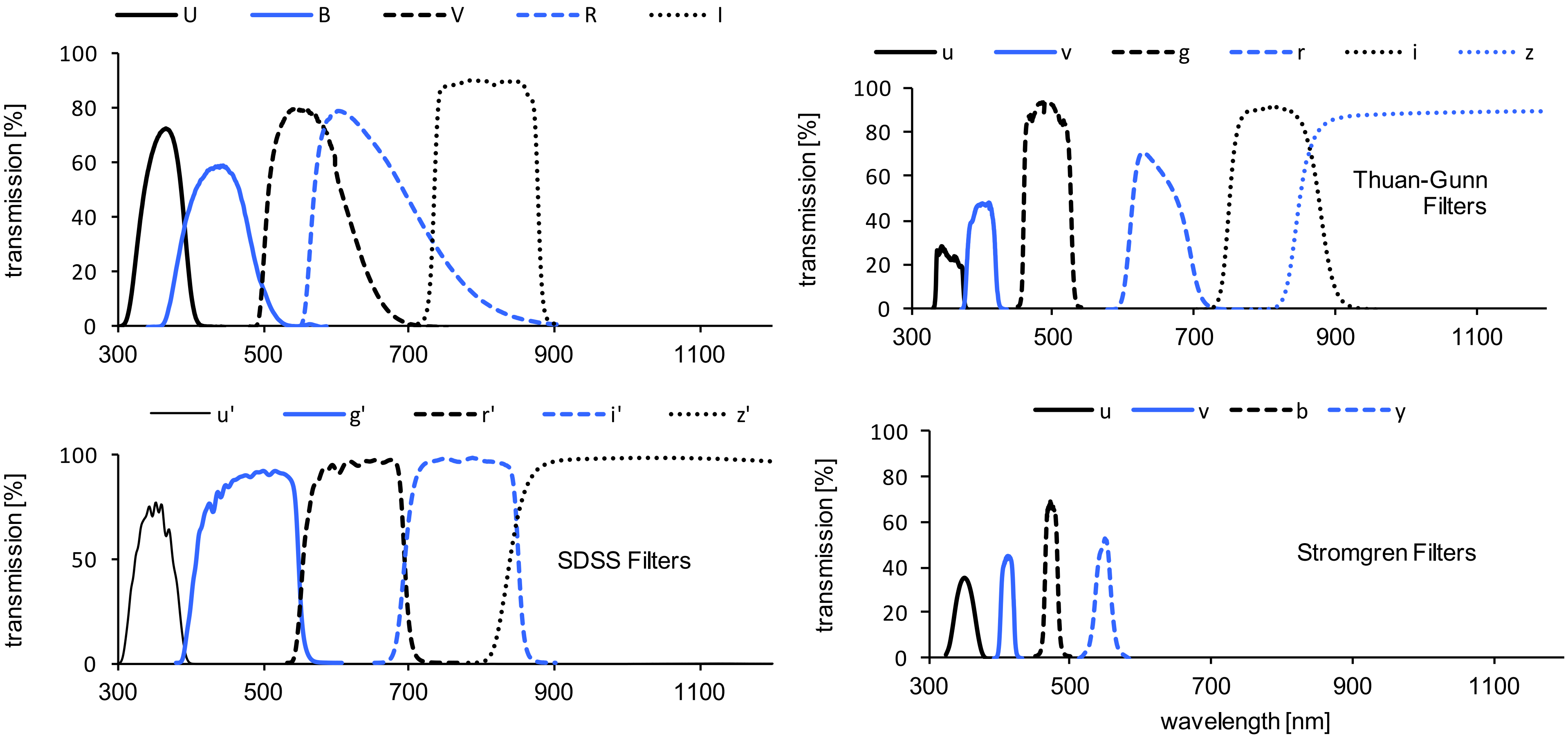
$$M_{bol} = -0.25 + 5 \cdot \log D - 2.5 \cdot \log \frac{L}{L_\Theta} \qquad ; L_\Theta = 3.827 \cdot 10^{26}\ \mathrm{W}$$

Bolometric magnitude can also be derived from visual magnitude plus a bolometric correction BC: $M_{bol} = M_V + BC$

BC is large for stars that have a peak emission very different from the Sun's.

Photometric Systems

Filters usually matched to atmospheric transmission
→ different observatories = different filters
→ many photometric systems:



Photometric Systems: AB and STMAG

For given flux density F_ν, AB magnitude is defined as:

$$m(AB) = -2.5 \cdot \log F_\nu - 48.60$$

- object with constant flux per unit frequency interval has zero color
- zero point defined to match zero points of Johnson V-band
- used by SDSS and GALEX
- F_ν in units of [erg s^{-1} cm^2 Hz^{-1}]

STMAG system defined such that object with constant flux per unit wavelength interval has zero color. STMAGs are used by the HST photometry packages.

Color Indices

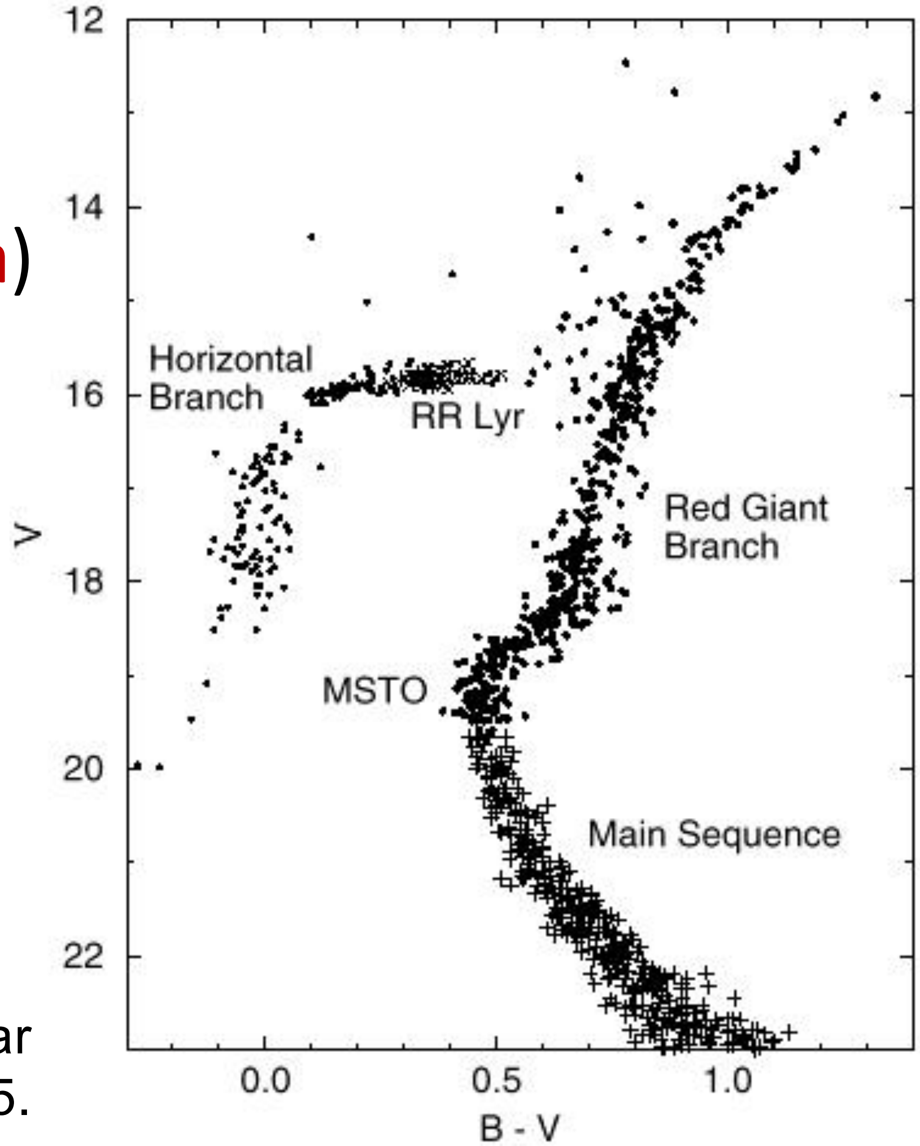
Color index = difference of magnitudes at different wavebands = ratio of fluxes at different wavelengths

- Color indices of A0V star (Vega) about zero longward of V
- Color indices of blackbody in Rayleigh-Jeans tail are:

  B-V = -0.46

  U-B = -1.33

  V-R = V-I = ... = V-N = 0.0



Color-magnitude diagram for a typical globular cluster, M15.

Color Index: Interstellar Extinction

- Absolute magnitude definition:

$$M = m + 5 - 5 \log D$$

- Interstellar extinction *E* or absorption *A* affects the apparent magnitudes

$$E(B-V) = A(B) - A(V) = (B-V)_{\text{observed}} - (B-V)_{\text{intrinsic}}$$

- Need to include absorption to obtain correct absolute magnitude:

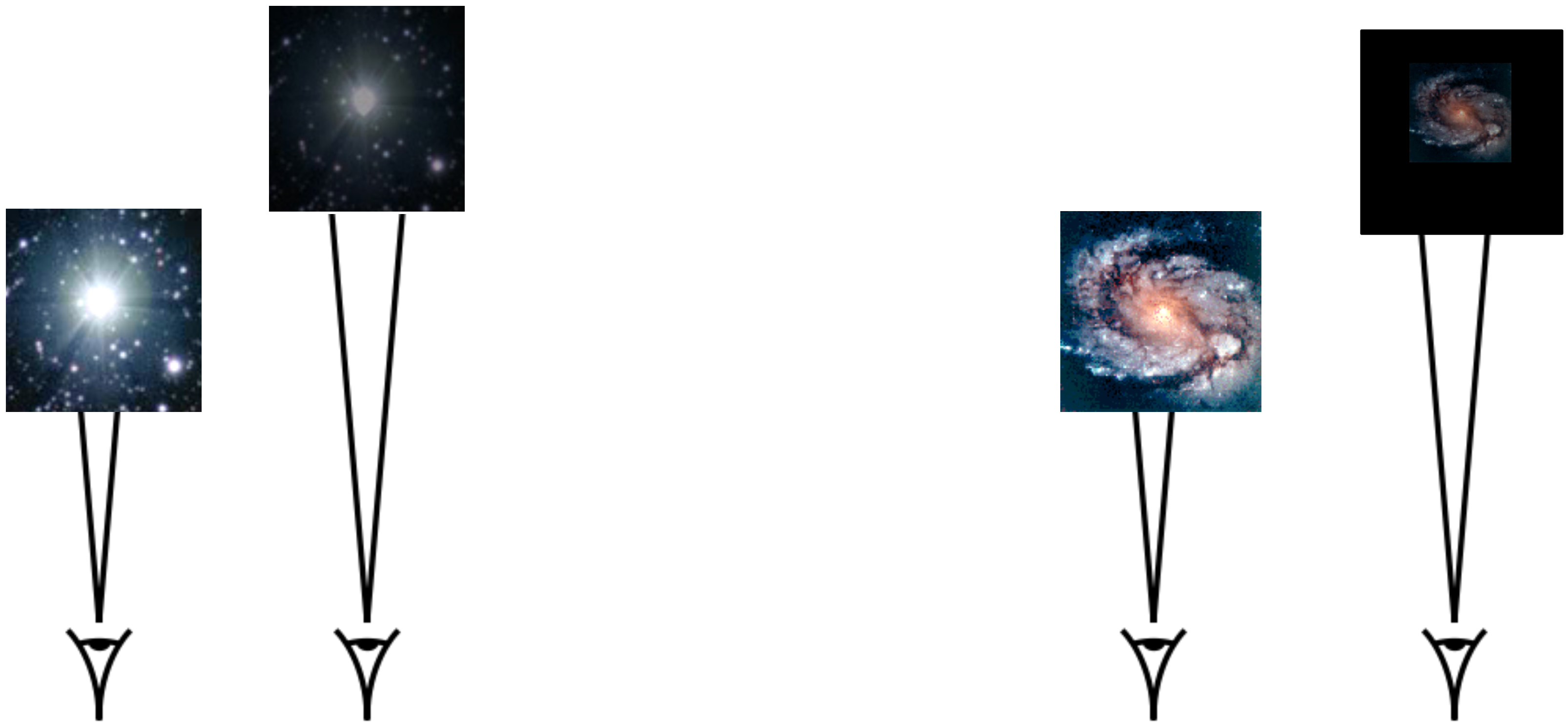
$$\boxed{M = m + 5 - 5 \log D - A}$$

Photometric Systems: Conversions

Name	λ_0 [μm]	$\Delta\lambda_0$ [μm]	F_λ [W m^{-2} μm^{-1}]	F_ν [Jy]	
U	0.36	0.068	4.35×10^{-8}	1 880	Ultraviolet
B	0.44	0.098	7.20×10^{-8}	4 650	Blue
V	0.55	0.089	3.92×10^{-8}	3 950	Visible
R	0.70	0.22	1.76×10^{-8}	2 870	Red
I	0.90	0.24	8.3×10^{-9}	2 240	Infrared
J	1.25	0.30	3.4×10^{-9}	1 770	Infrared
H	1.65	0.35	7×10^{-10}	636	Infrared
K	2.20	0.40	3.9×10^{-10}	629	Infrared
L	3.40	0.55	8.1×10^{-11}	312	Infrared
M	5.0	0.3	2.2×10^{-11}	183	Infrared
N	10.2	5	1.23×10^{-12}	43	Infrared
Q	21.0	8	6.8×10^{-14}	10	Infrared

1 Jy = 10^{-26} W m^{-2} Hz^{-1}.

Surface Brightness: Point & Extended Sources



Point sources = spatially unresolved

Brightness ~ 1 / distance^2

Size given by observing conditions

Extended sources = well resolved

Surface brightness ~ const(distance)

Brightness ~ $1/d^2$ and area size ~ $1/d^2$

Surface brightness [mag/arcsec^2] is constant with distance!

Surface Brightness: Calculating

Surface brightness of extended objects in units of mag/sr or mag/arcsec^2

Surface brightness *S* of area *A* in magnitudes:

$$S = m + 2.5 \cdot \log_{10} A$$

Observed surface brightness [mag/arcsec^2] converted into physical surface brightness units:

$$S\left[\mathrm{mag/arcsec^2}\right] = M_\Theta + 21.572 - 2.5 \cdot \log_{10} S\left[L_\Theta/\mathrm{pc}^2\right]$$

with $L_\Theta = 3.839 \times 10^{26}\ \mathrm{W} = 3.839 \times 10^{33}\ \mathrm{erg\ s^{-1}}$