Astronomical Observing Techniques

Lecture 6: Everything You Always Wanted to Know About Optics

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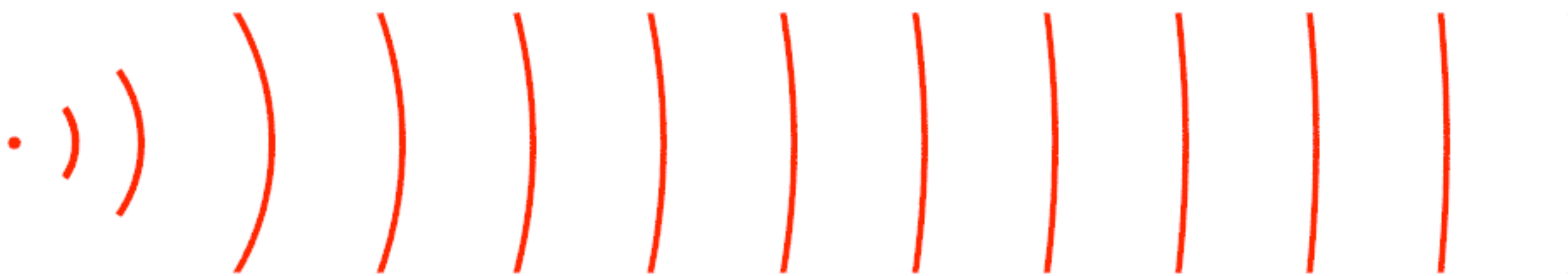
Outline

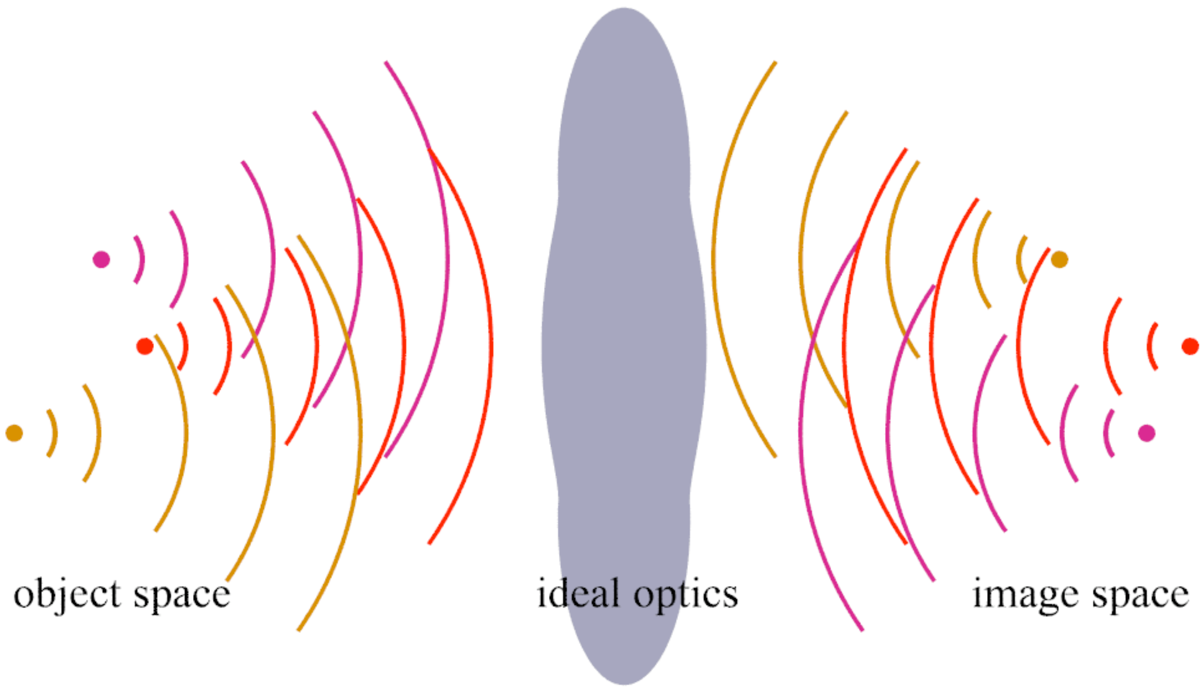
1. Geometrical Optics
   – Waves and Rays
   – Images and Pupils
   – Aberrations
2. Physical Optics
   – Diffraction
   – Transfer Functions
   – Image Metrics
3. High Contrast Imaging

Spherical and Plane Waves

- light source: collection of sources of spherical waves
- astronomical sources: almost exclusively incoherent
- lasers, masers: coherent sources
- spherical wave originating at very large distance can be approximated by plane wave

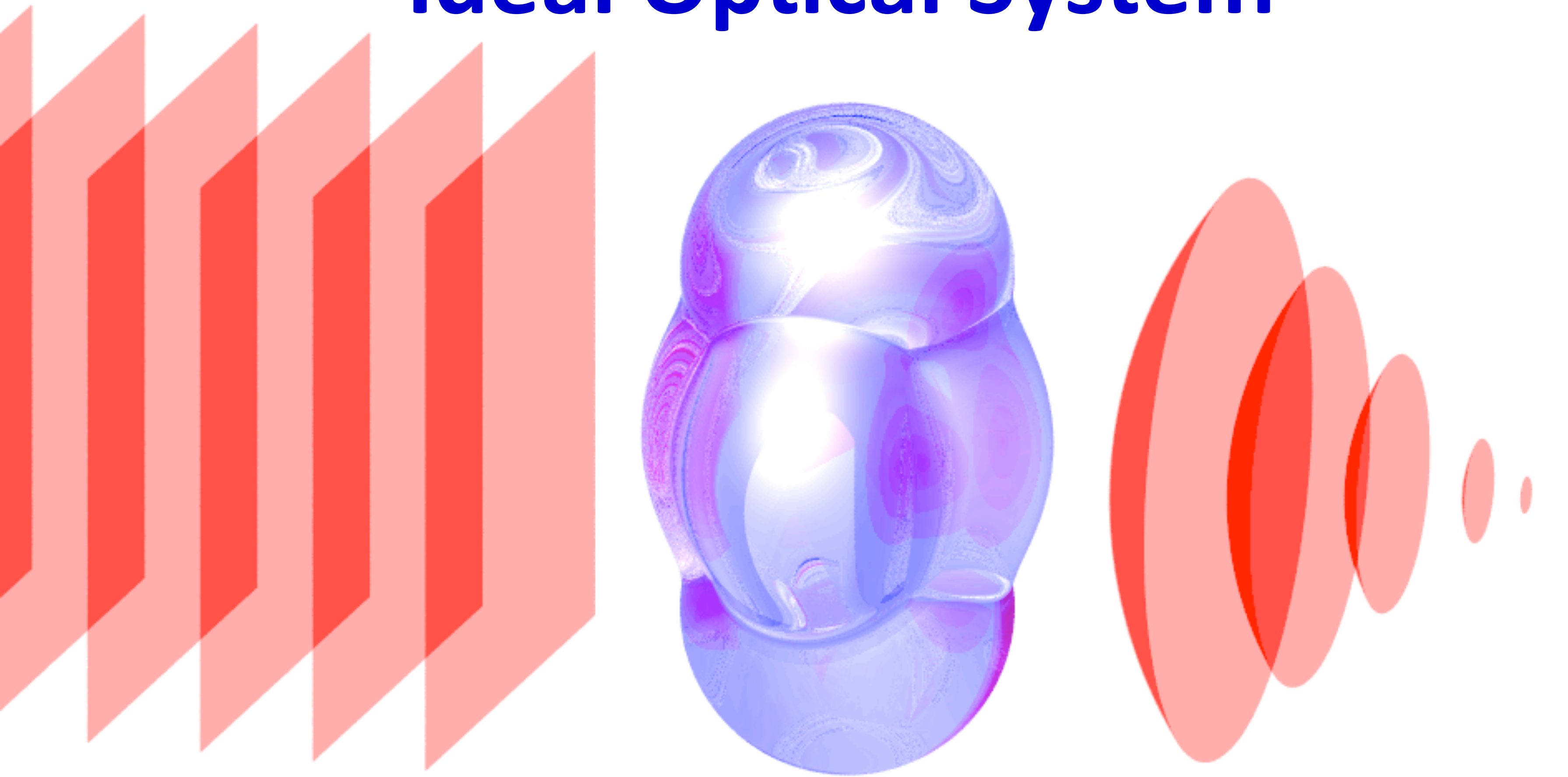


Ideal Optics



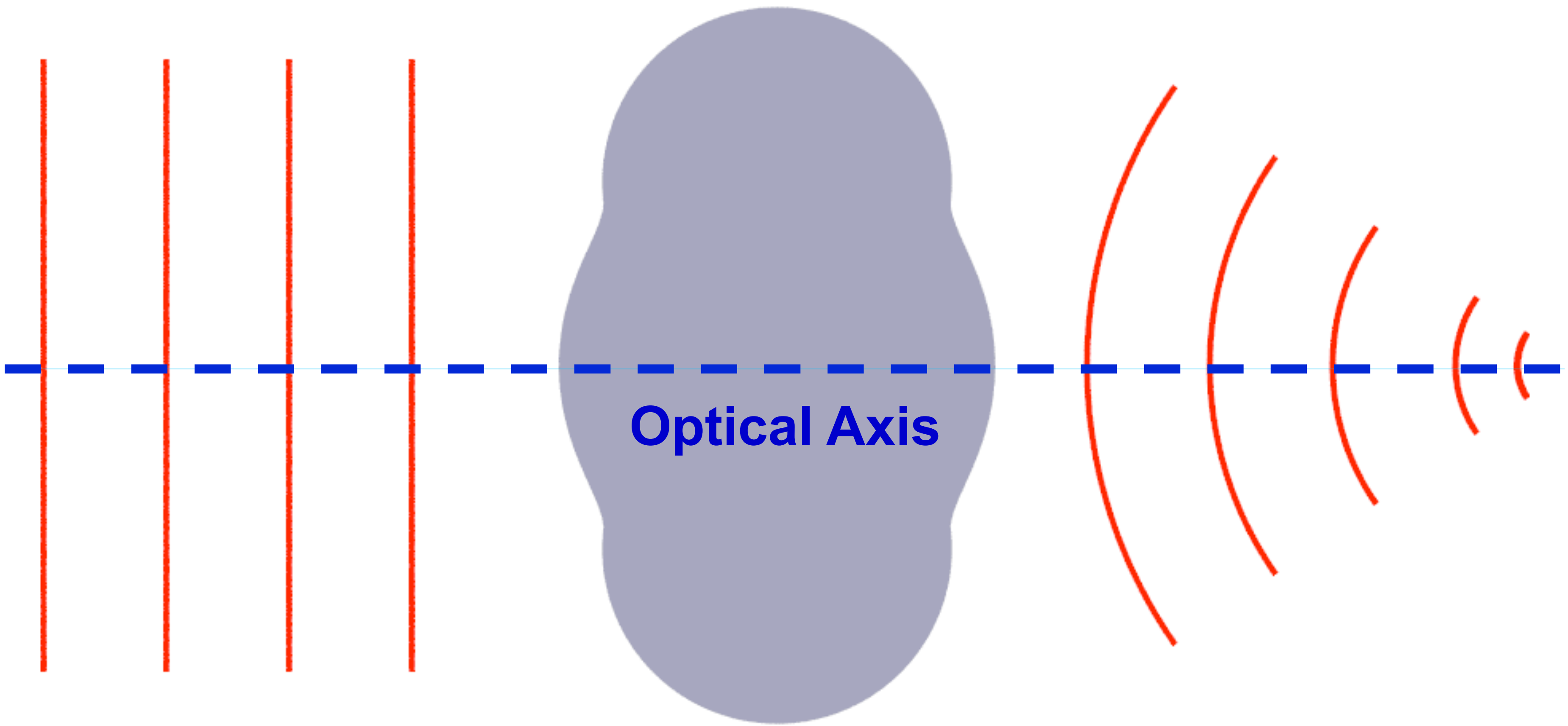
- ideal optics: spherical waves from any point in object space are
- imaged into points in image space
- corresponding points are called conjugate points
- focal point: center of converging or diverging spherical wavefront
- object space and image space are reversible

Ideal Optical System



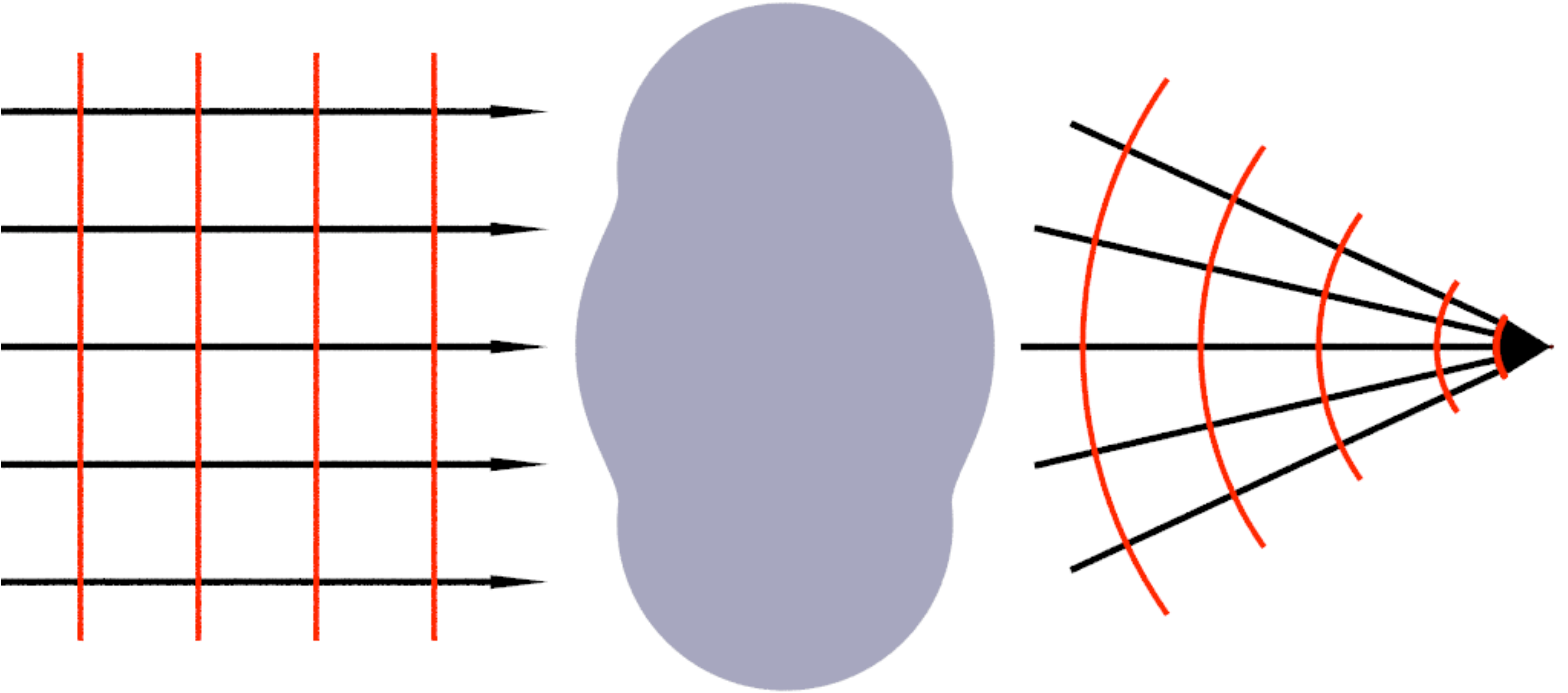
ideal optical system transforms plane wavefront into spherical, converging wavefront

Azimuthal Symmetry



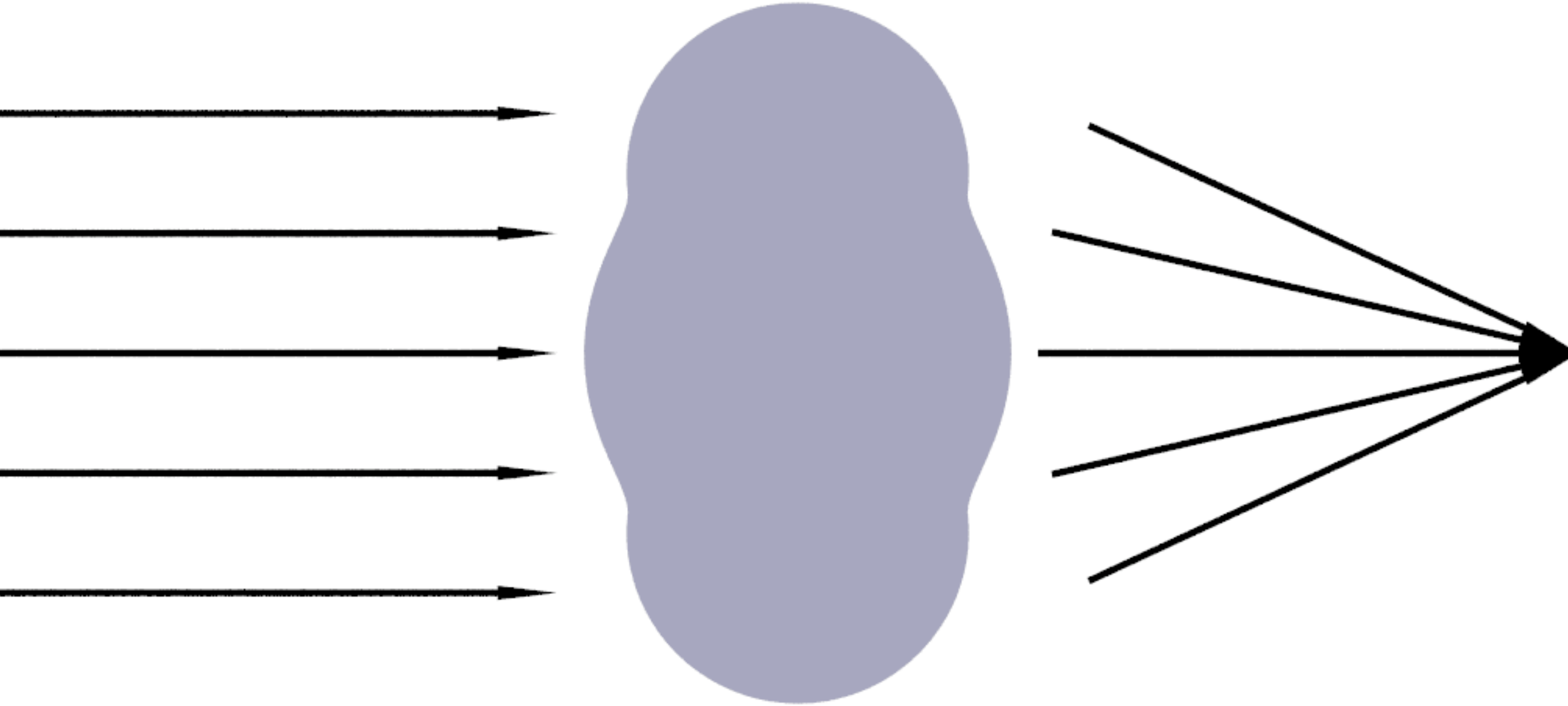
- most optical systems are azimuthally symmetric
- axis of symmetry is optical axis

Locally Flat Wavefronts



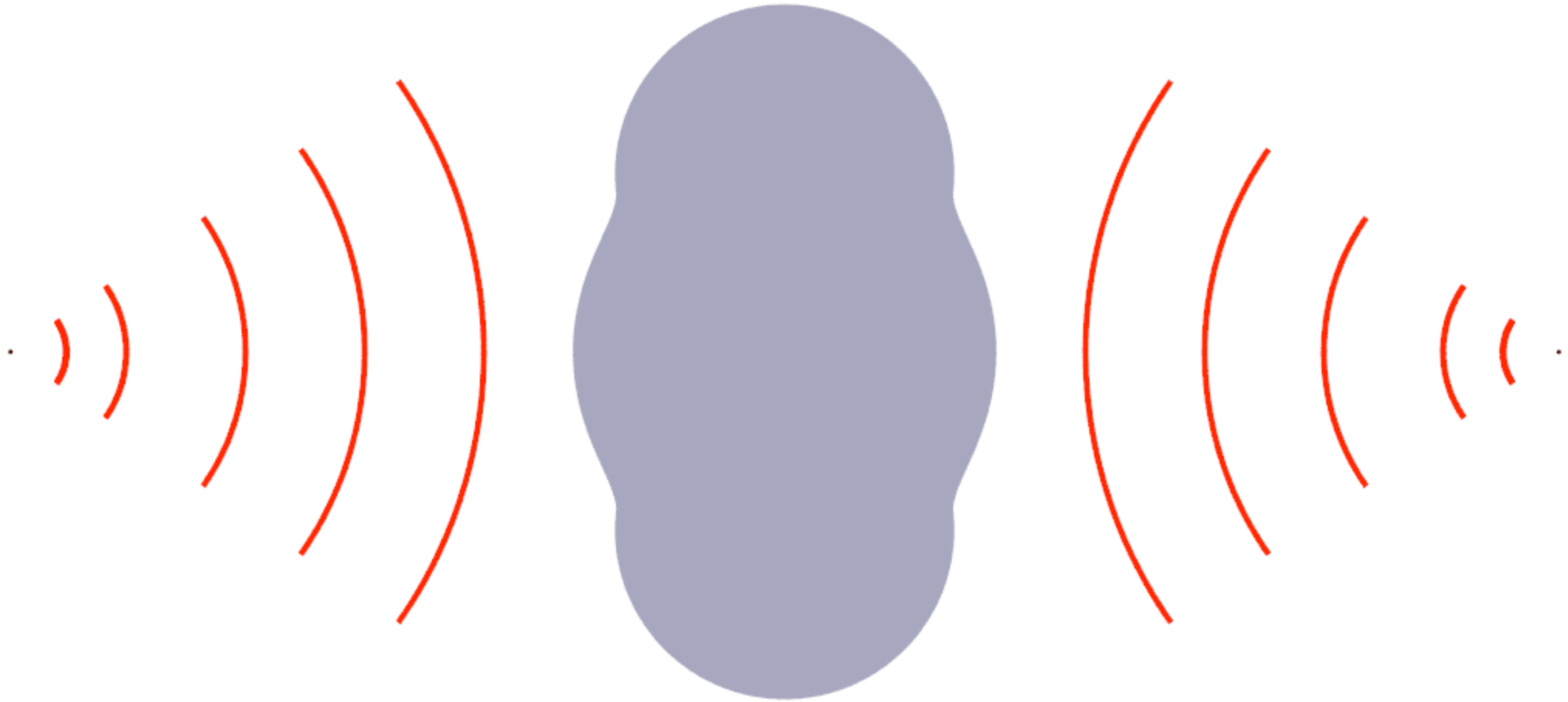
- rays normal to local wave (locations of constant phase)
- local wave around rays is assumed to be plane wave

Rays



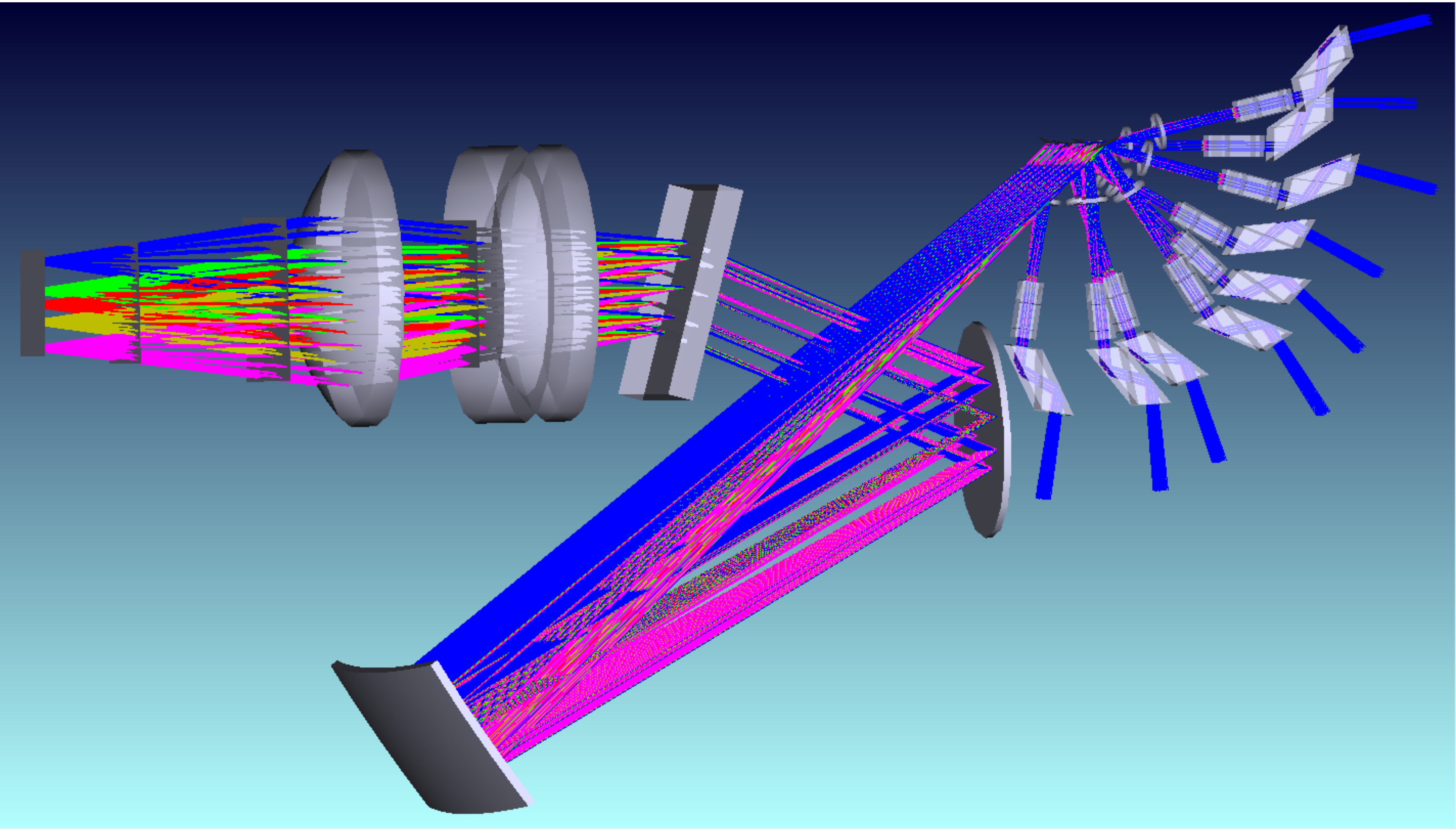
- geomtrical optics works with rays only
- rays reflected and refracted according to Fresnel equ.
- phase is neglected (incoherent sum)

Finite Object Distance



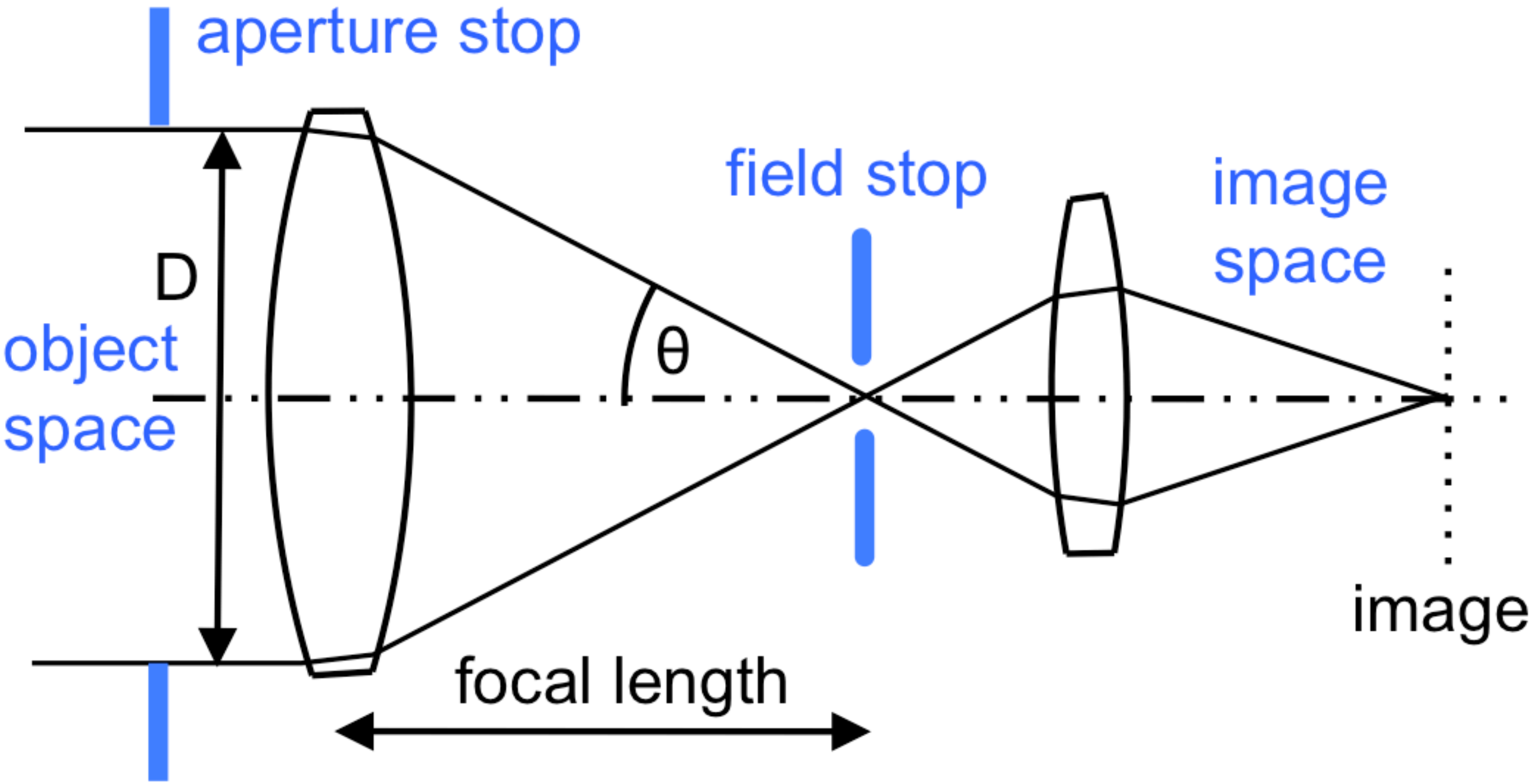
- object may also be at finite distance
- also in astronomy: reimaging within instruments and telescopes

Geometrical Optics Example: SPEX



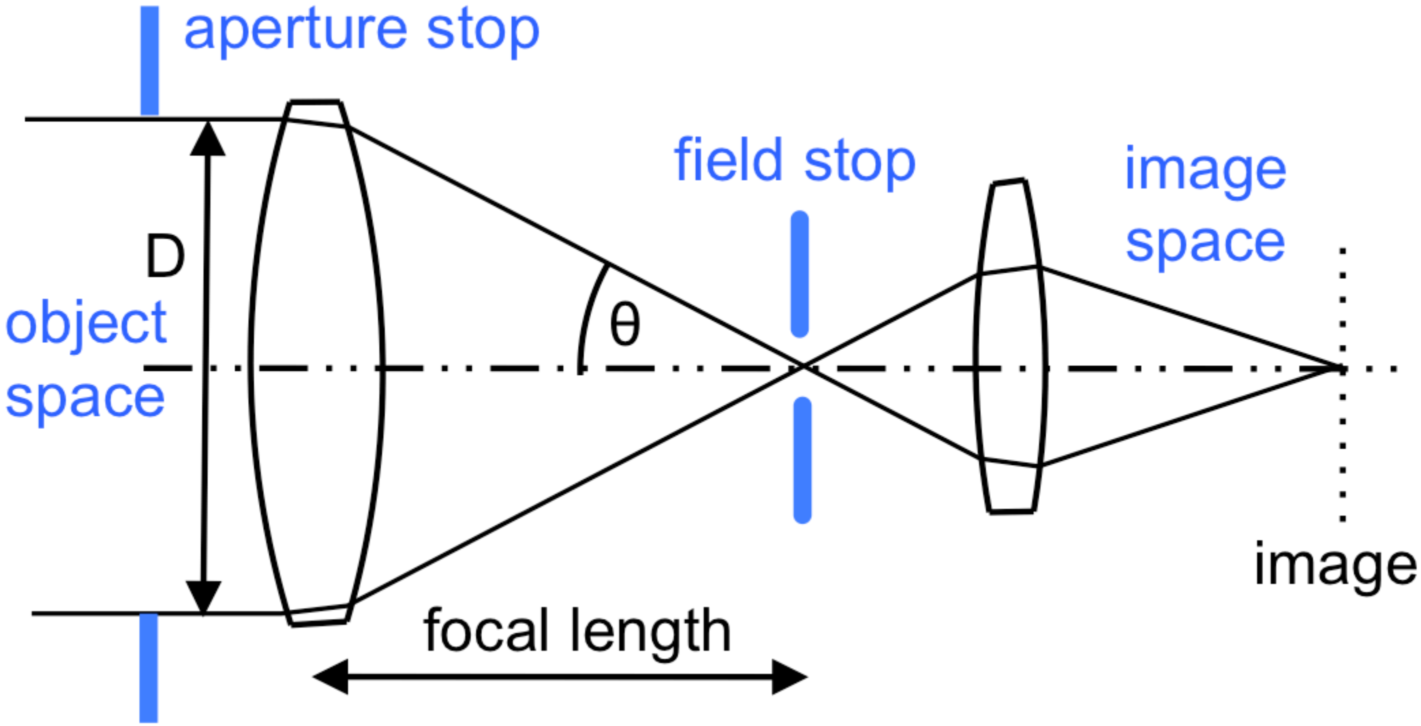
SPEX on NASA ER-2 plane

Aperture and Field Stops



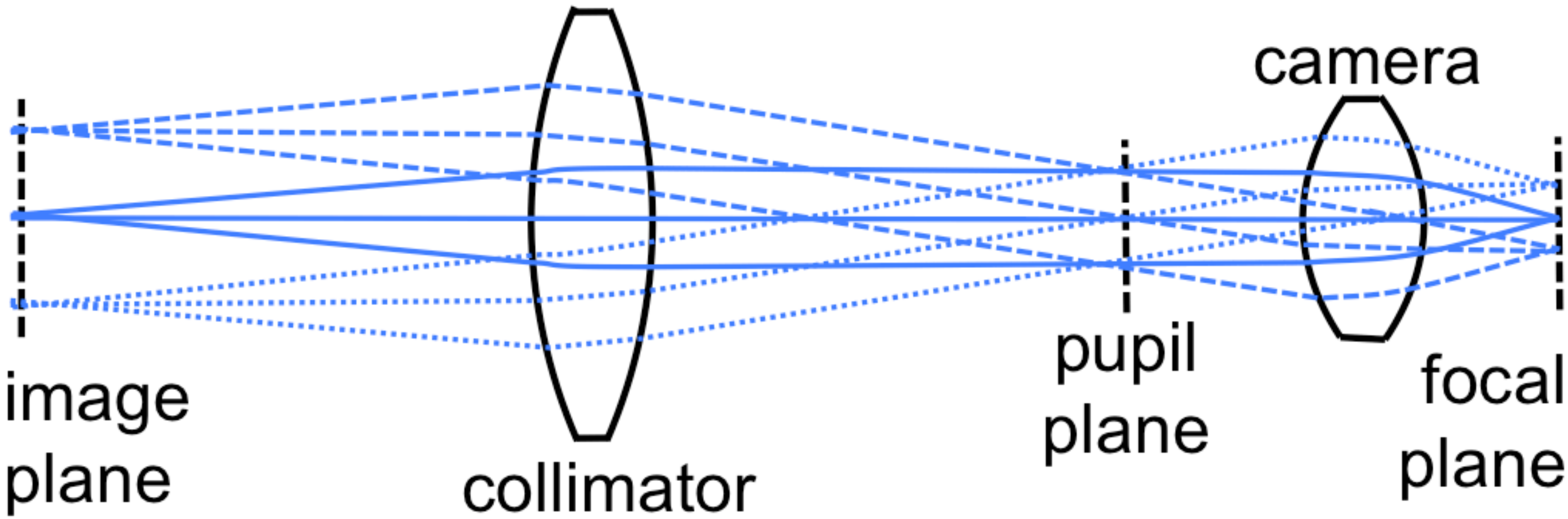
- Aperture stop: determines diameter of light cone from axial point on object.
- Field stop: determines the field of view of the system.

Images



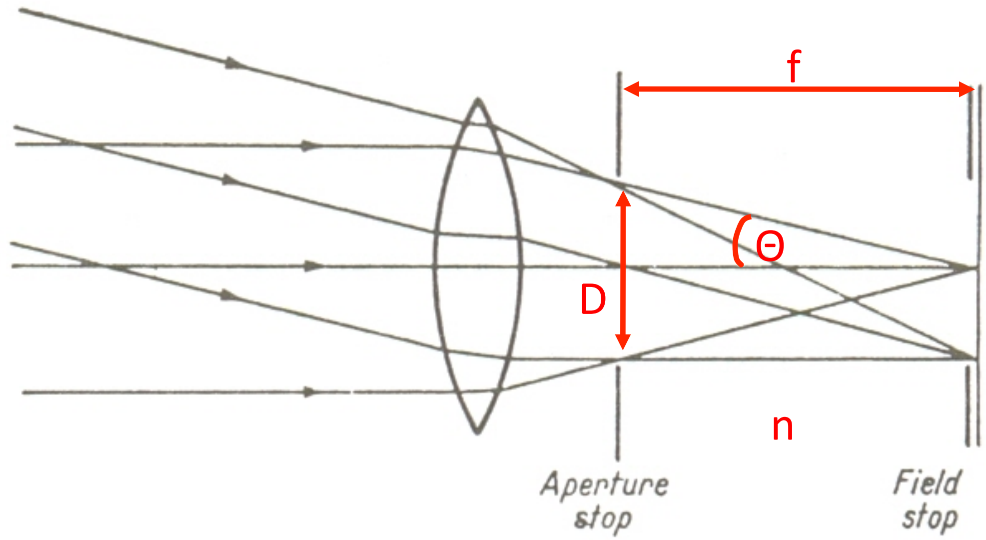
- every object point comes to focus in image plane
- light in image point comes from all pupil positions
- object information encoded in position, not angle

Pupils



- all object rays are smeared out over complete aperture
- light in one pupil point comes from different object positions
- object information is encoded in angle, not in position

Speed/F-Number/Numerical Aperture



Speed of optical system described by numerical aperture (NA) or *F* number:

$$\mathrm{NA} = n \cdot \sin\theta \qquad \text{and} \qquad F \equiv \frac{f}{D} = \frac{1}{2(\mathrm{NA})}$$

- fast optics (large NA)
- slow optics (small NA)

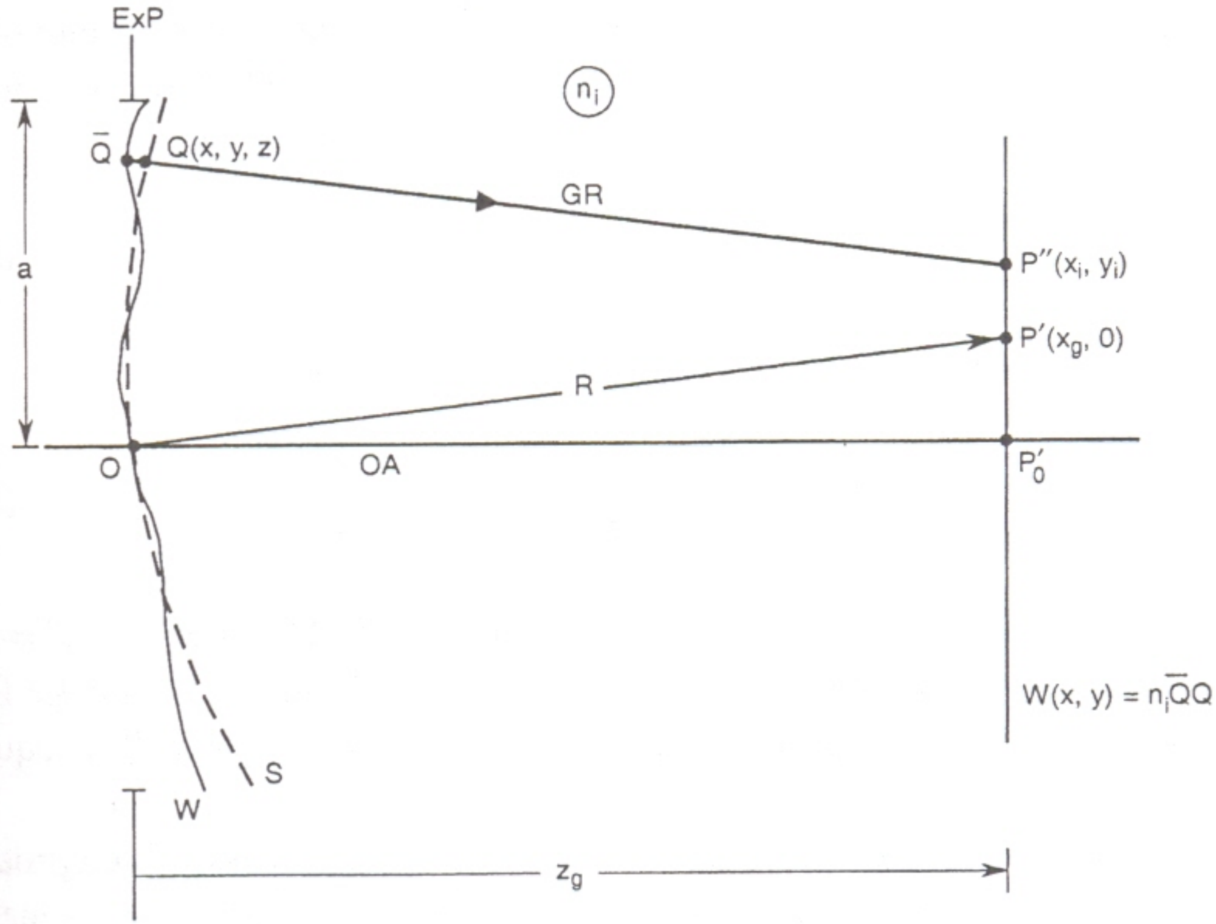
Aberrations

Aberrations are departures of the performance of an optical system from the ideal optical system.

1. On-axis aberrations: aberrations that can be seen everywhere in the image, also on the optical axis (center of the image)
2. Off-axis aberrations: aberrations that are absent on the optical axis (center of the image)
   a) Aberrations that degrade the image
   b) Aberrations that alter the image position

Wave and Ray Aberrations

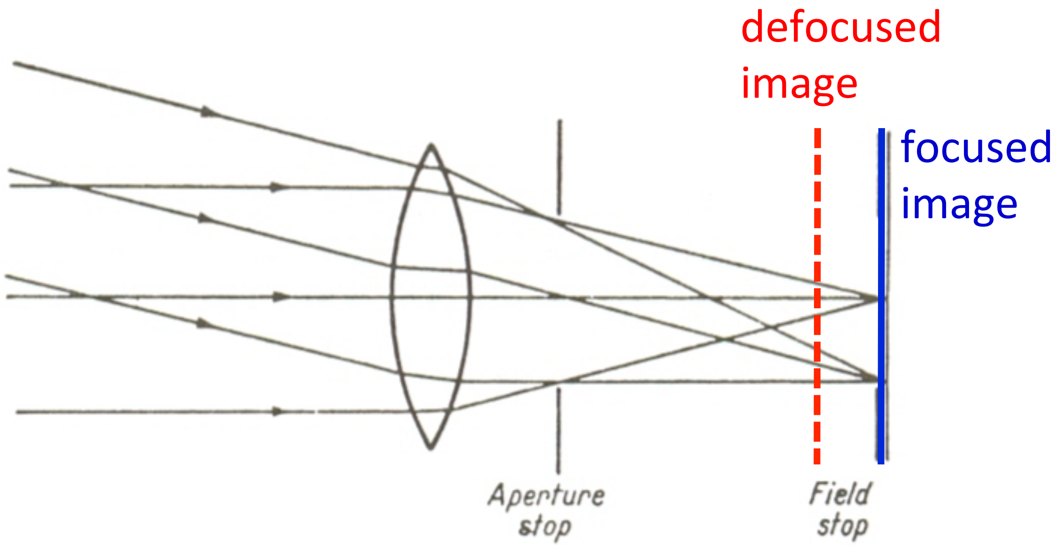
- Reference sphere S with radius R for off-axis point P’ and aberrated wavefront W
- “Aberrated” ray from object intersects image plane at P”
- Ray aberration is P’P”
- Wave aberration is n·QQ



Small FOV, radially symmetric wavefront W(r)

$$r_i = \frac{R}{n_i}\frac{\partial W(r)}{\partial r}$$

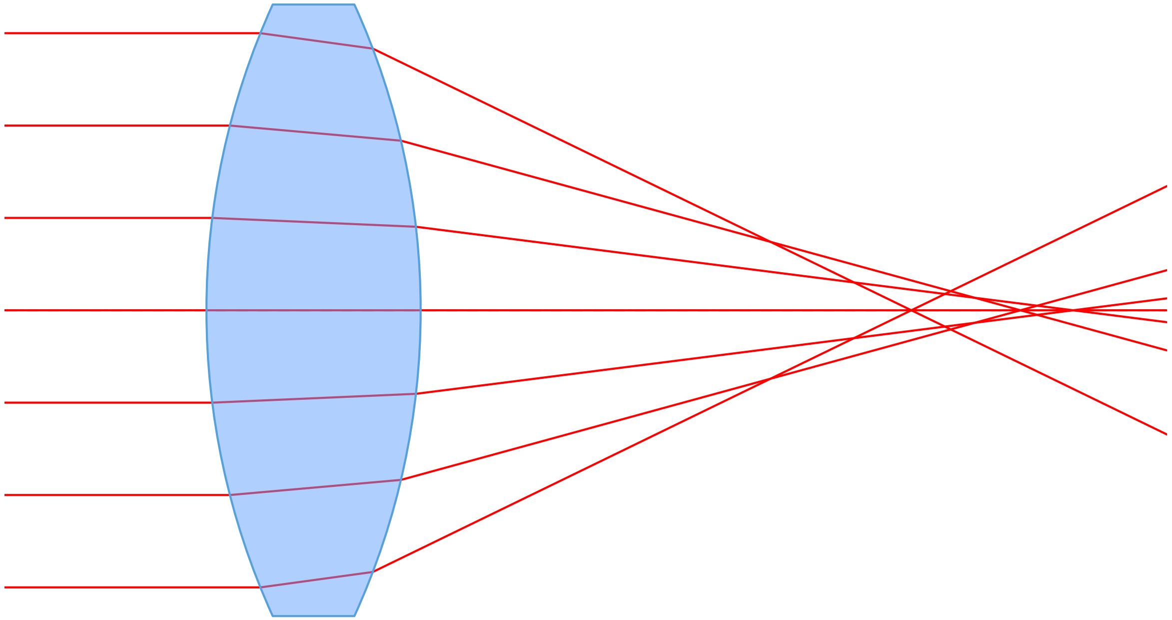
Defocus (Out of Focus)

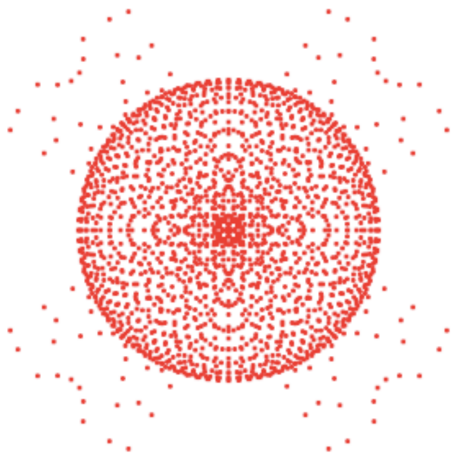


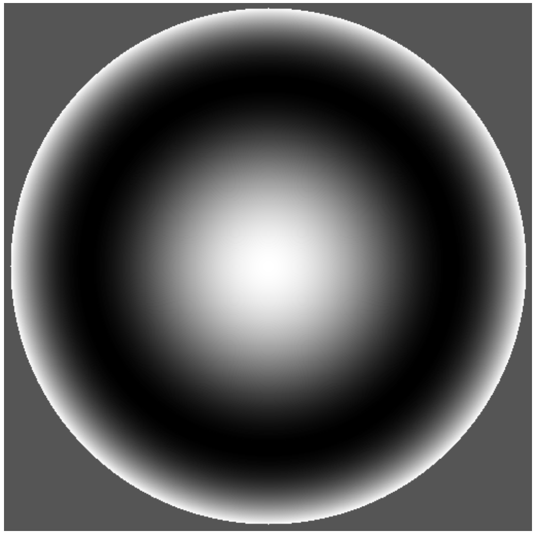
Depth of focus: $\delta = 2\lambda F^2 = \frac{\lambda}{2}\left(\frac{1}{\mathrm{NA}}\right)^2$

Usually refers to optical path difference of $\lambda/4$.

Spherical Aberration

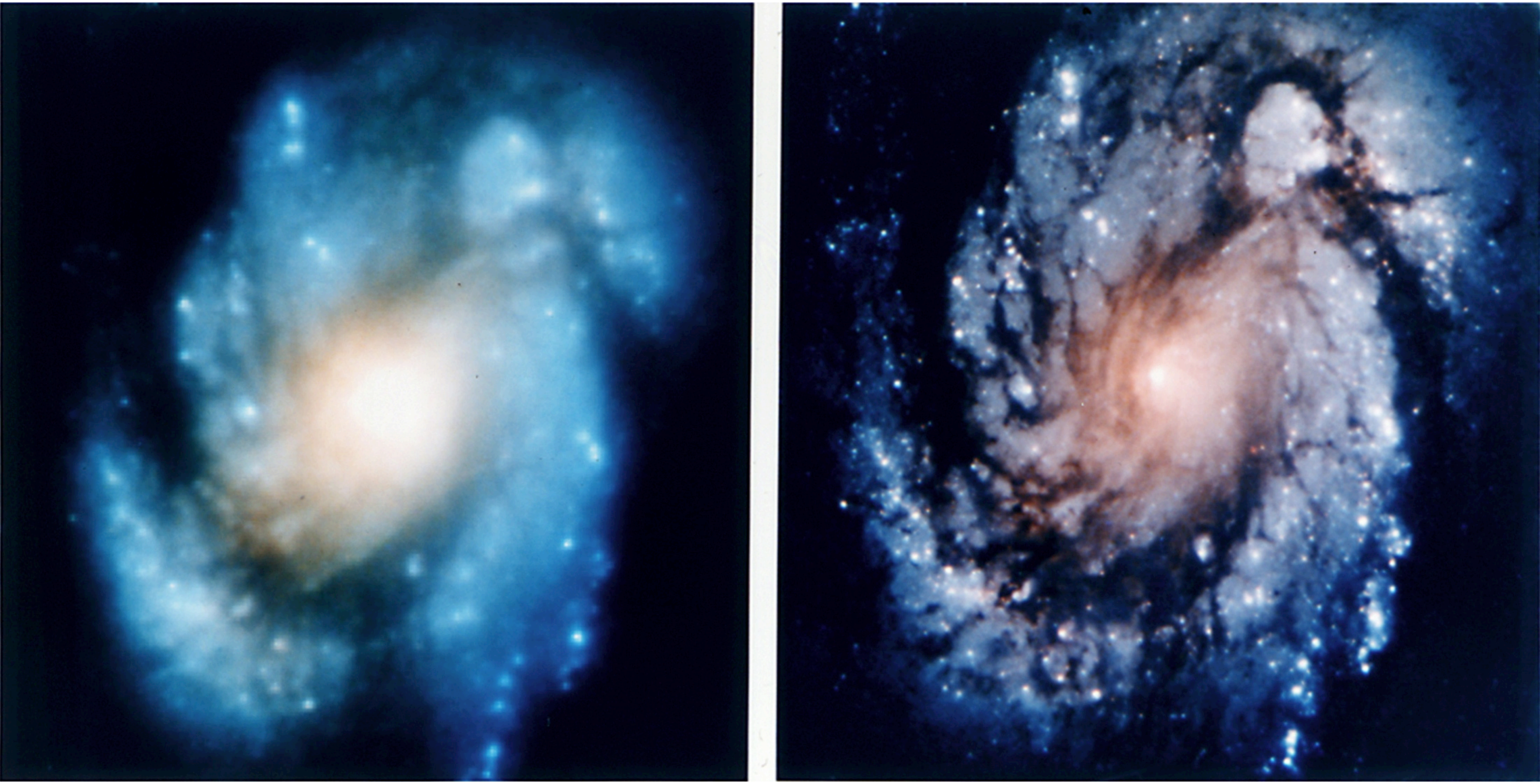






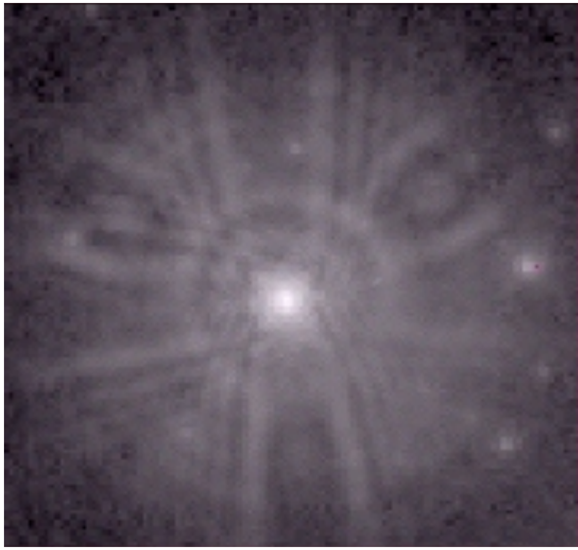
Rays further from the optical axis have a different focal point than rays closer to the optical axis.

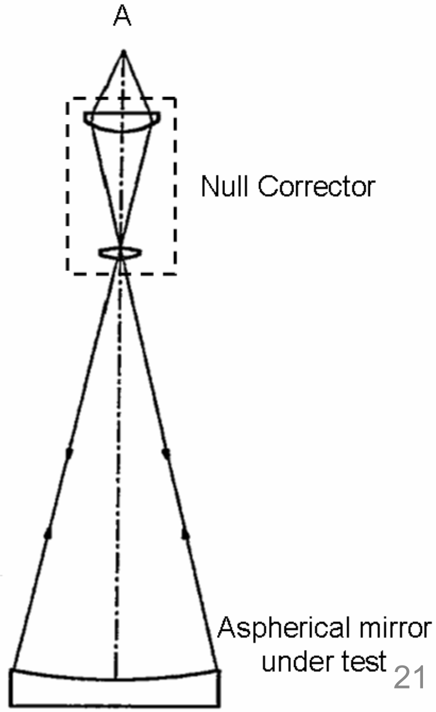
Hubble Trouble



HST Primary Mirror Spherical Aberration

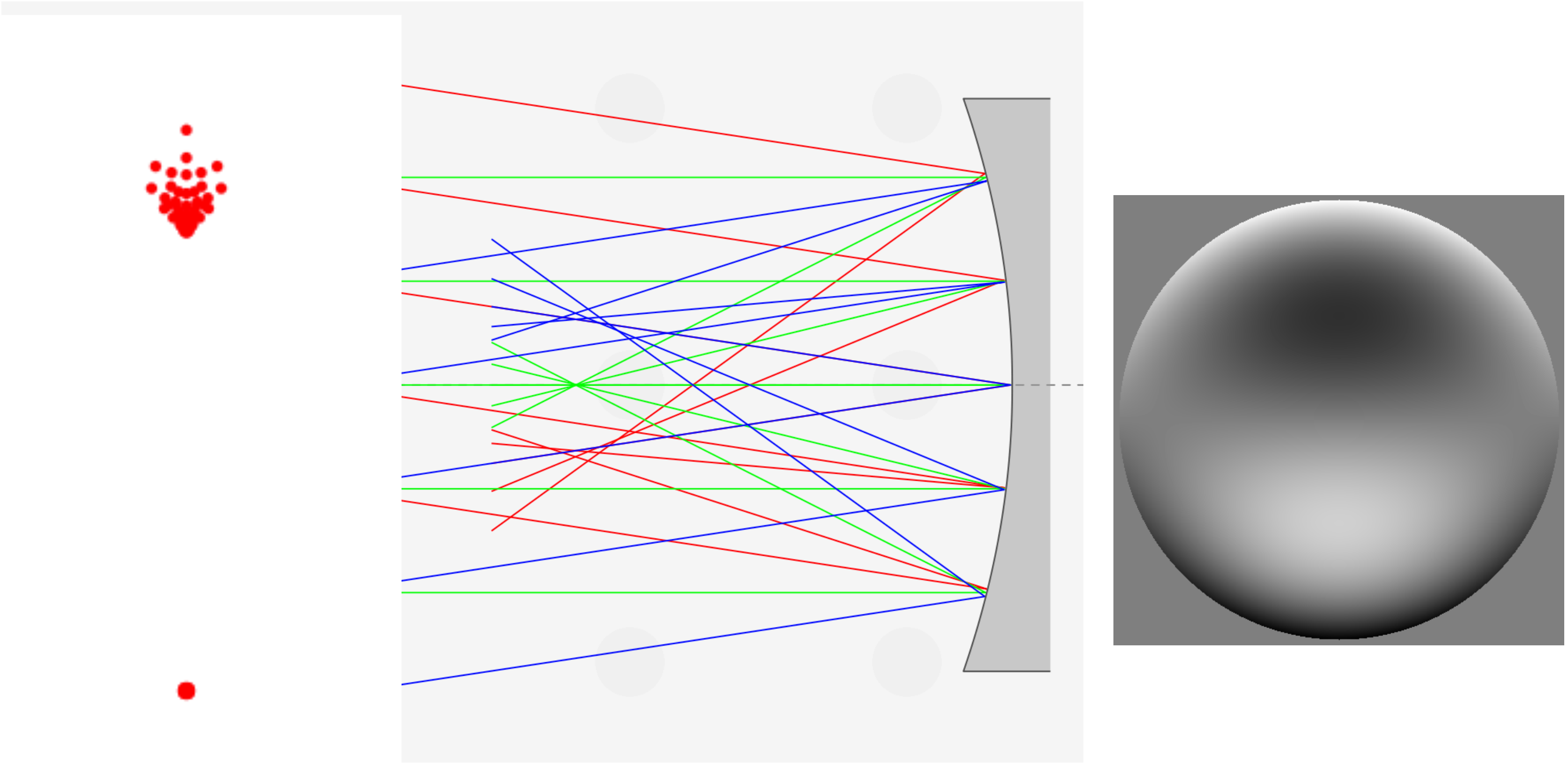
- *Null corrector* for measuring mirror shape was incorrectly assembled (one lens misplaced by 1.3 mm).
- A management problem: Mirror manufacturer had analyzed surface with other null correctors, which indicated the problem, but test results were ignored because they were believed to be less accurate.
- *Null corrector* cancels non-spherical portion of aspheric mirror figure. When correct mirror is viewed from point A, combination looks precisely spherical.





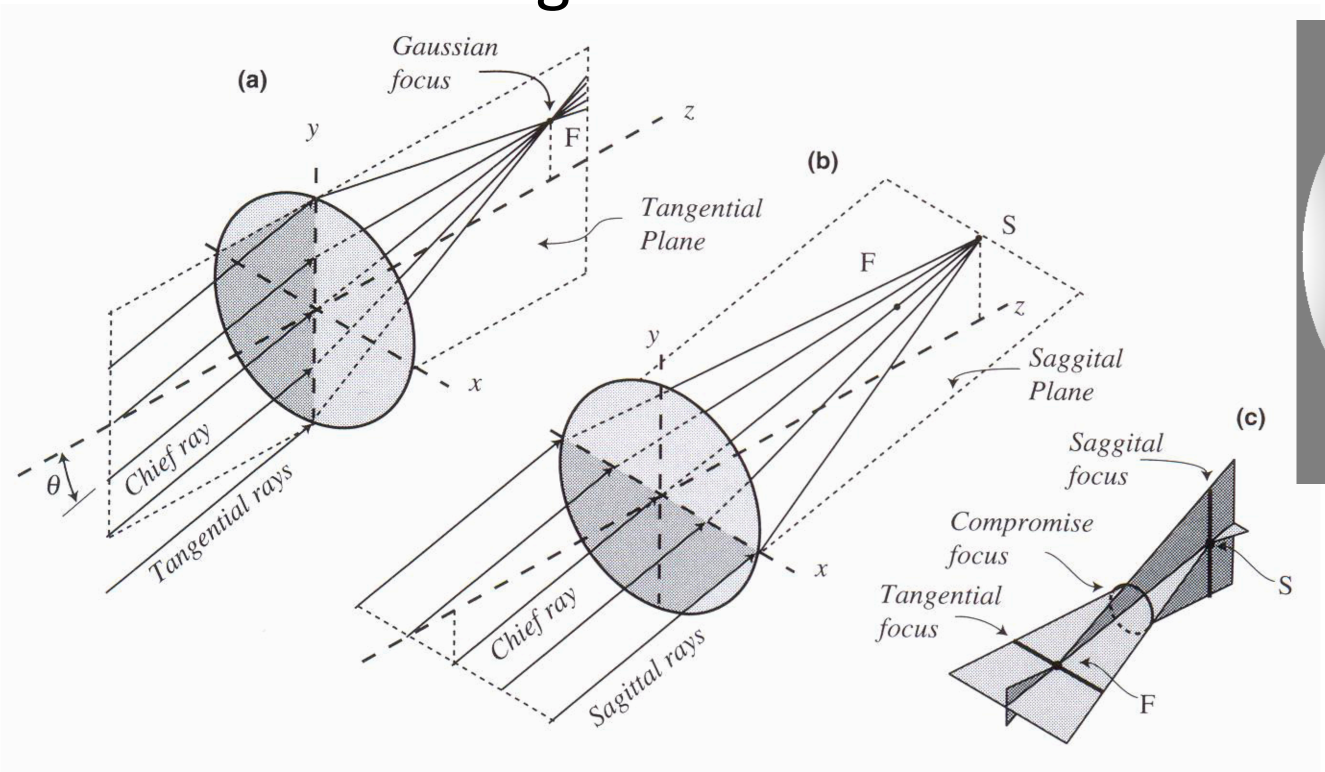
Coma

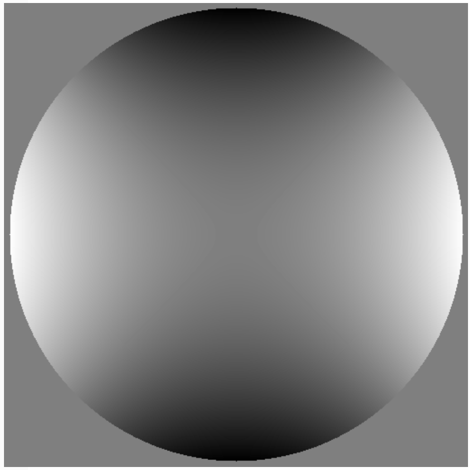
Variation of magnification across entrance pupil. Point sources will show a cometary tail. Coma is an inherent property of telescopes using parabolic mirrors.



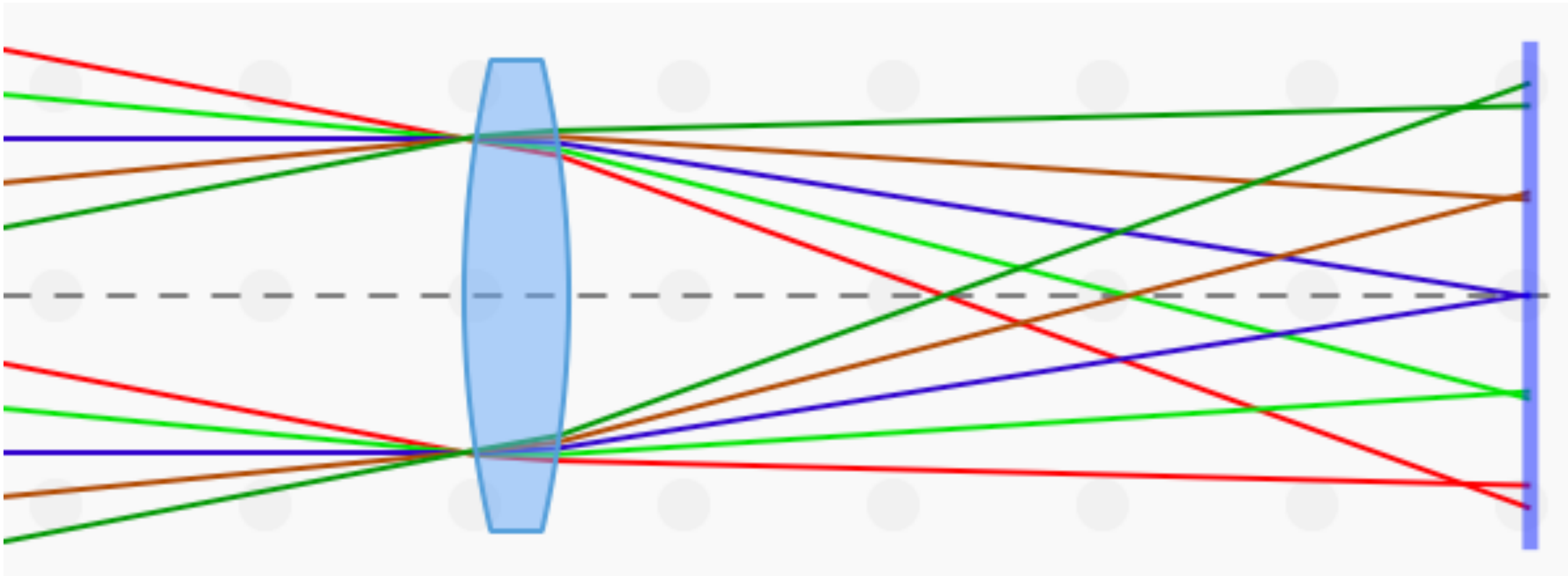
Astigmatism

From off-axis point A lens does not appear symmetrical but shortened in plane of incidence (tangential plane).
Emergent wave will have a smaller radius of curvature for tangential plane than for plane normal to it (sagittal plane) and form an image closer to the lens.





Field Curvature

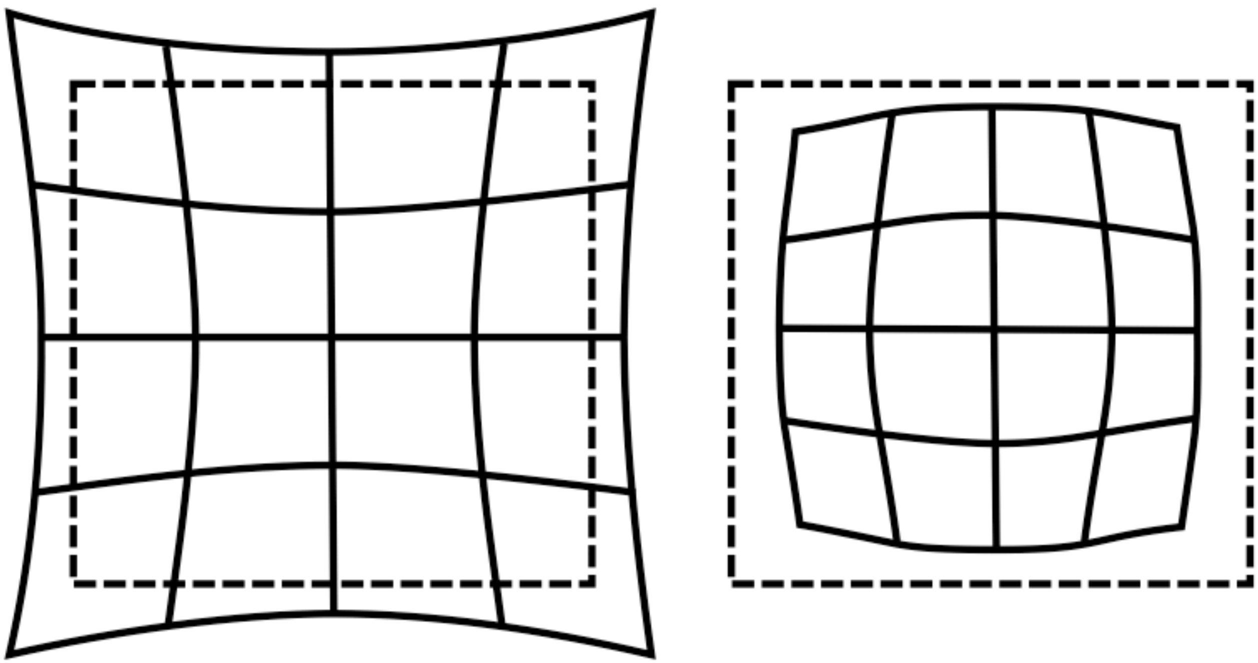


Only objects close to optical axis will be in focus on flat image plane. Off-axis objects will have different focal points.

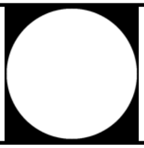
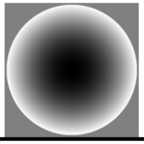
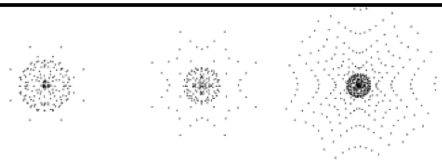
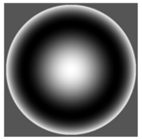
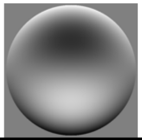
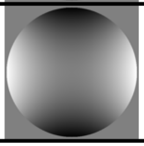
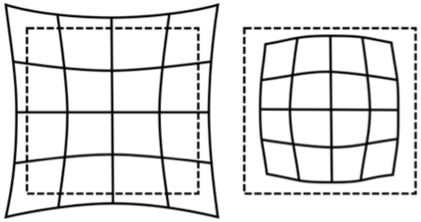
Distortion

Straight line on sky becomes curved line in focal plane because magnification depends on distance to optical axis.

1. Outer parts have larger magnification→ pincushion
2. Outer parts have smaller magnification → barrel

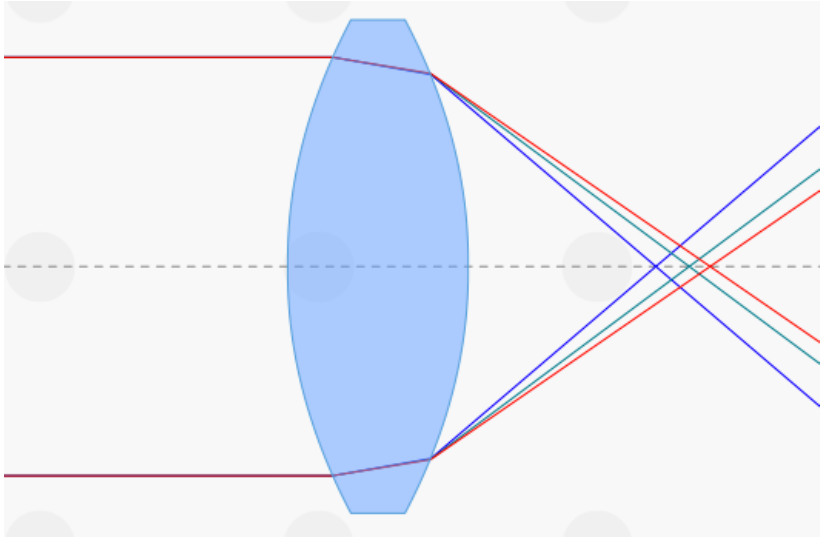


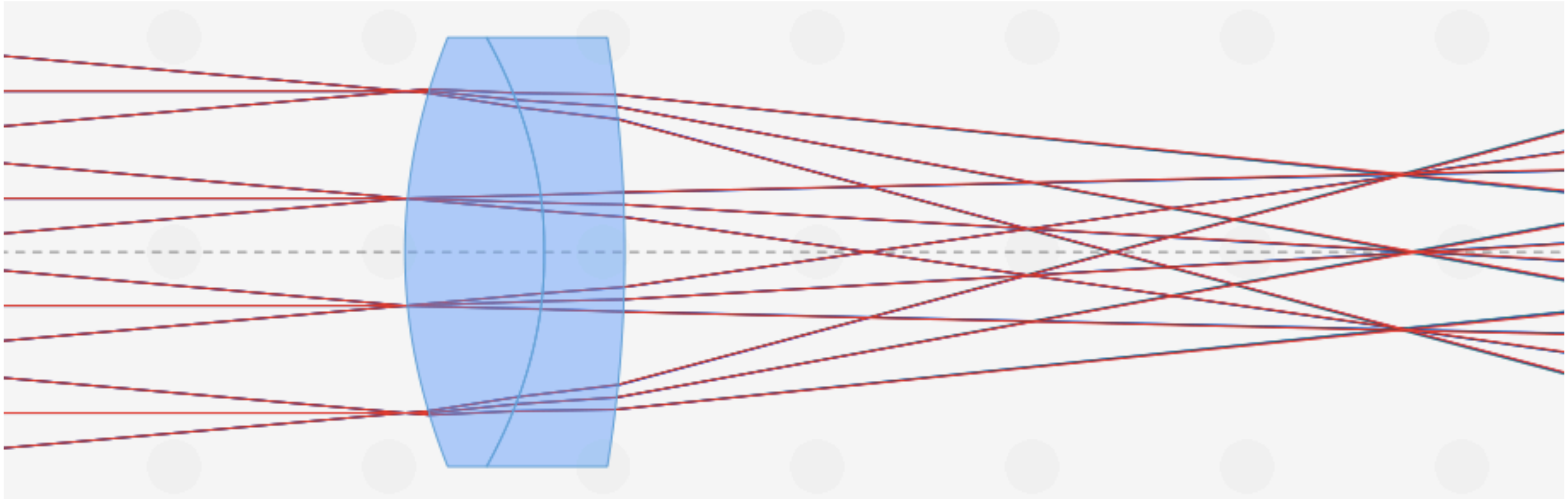
Aberrations Summary

aberration	spot diagram / image	wavefront	scaling	
perfect			-	-
focus			$1/F^2$	-
spherical			$1/F^3$	-
coma			$1/F^2$	y
astigmatism			$1/F^2$	y^2
field curvature			$1/F^2$	y^2
distortion			-	y^3

Chromatic Aberration

Refractive index variation with wavelength n(λ) results in focal length of lens f(λ) to depend on wavelength; different wavelengths have different foci

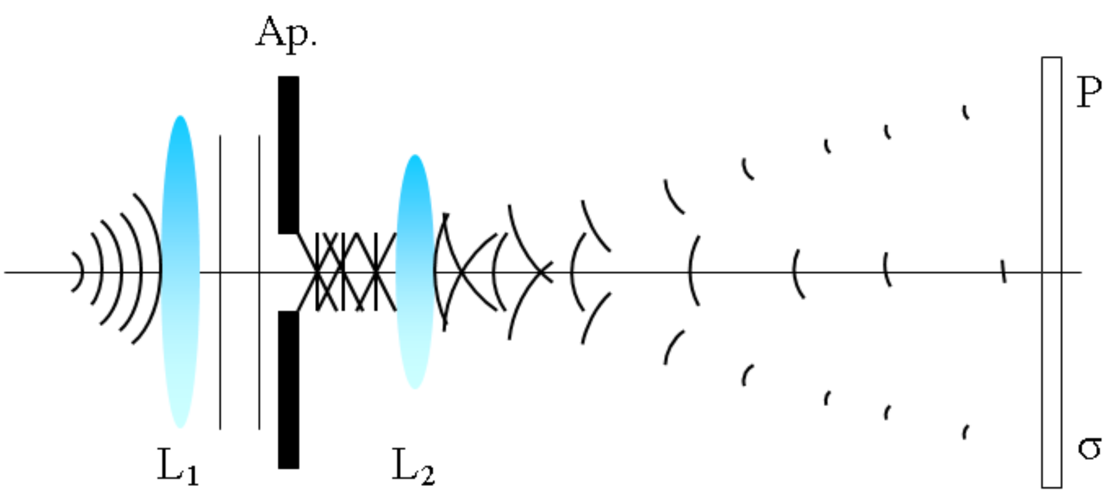




Fresnel and Fraunhofer Diffraction

Fresnel diffraction = near-field diffraction: When a wave passes through an aperture and diffracts in the near field it causes the observed diffraction pattern to differ in size and shape for different distances.

For Fraunhofer diffraction at infinity (far-field) the wave becomes planar.



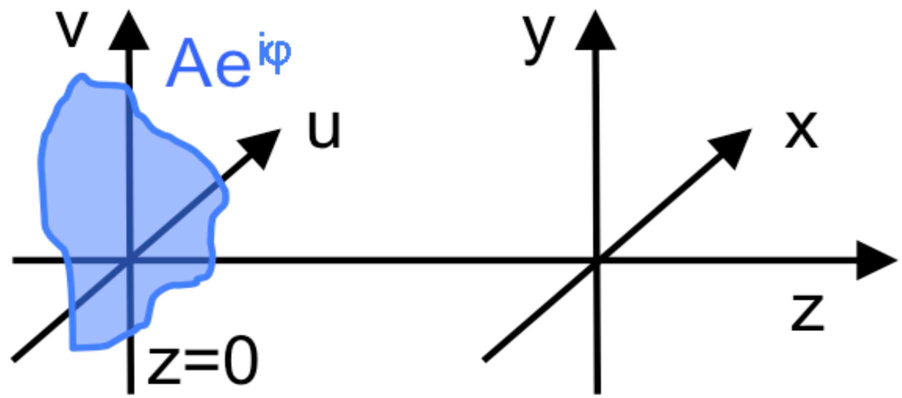
Fresnel: $F = \frac{r^2}{d \cdot \lambda} \geq 1$

Fraunhofer: $F = \frac{r^2}{d \cdot \lambda} << 1$

F = Fresnel number, r = aperture size and d = distance to screen

Fraunhofer Diffraction

- Electric field in image plane is Fourier transform
of electric field in aperture

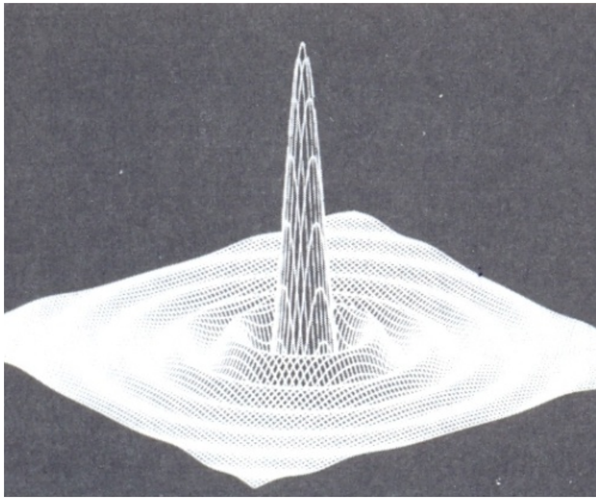


$$E(x,y,z) = \iint A(u,v)\, e^{i\varphi(u,v)}\, e^{-i\frac{2\pi}{\lambda z}(xu+yv)}\, du\, dv$$

Point Spread Function (1)

When the circular pupil is illuminated by a point source then the resulting PSF is described by a 1st order Bessel function:

$$I_1(\theta) = \left(\frac{2J_1(2\pi\, r_0\theta/\lambda)}{2\pi\, r_0\theta/\lambda} \right)^2$$



This is also called the Airy function.
The radius of the first dark ring (minimum) is at:

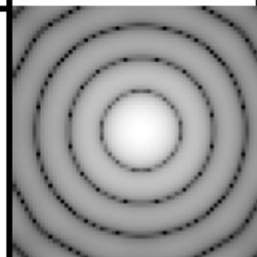
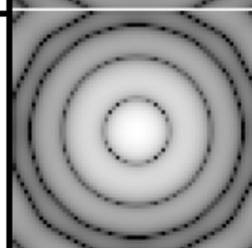
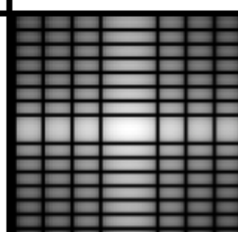
$$r_1 = 1.22\lambda F \qquad \text{or} \qquad \alpha_1 = \frac{r_1}{f} = 1.22\frac{\lambda}{D}$$

The PSF is often simply characterized by the half power beam width (HPBW) or full width half maximum (FWHM) in angular units.

According to the Nyquist-Shannon sampling theorem $I(\Theta)$ (or its FWHM) shall be sampled with a rate of at least:

$$\Delta\theta = 1/2\omega_c$$

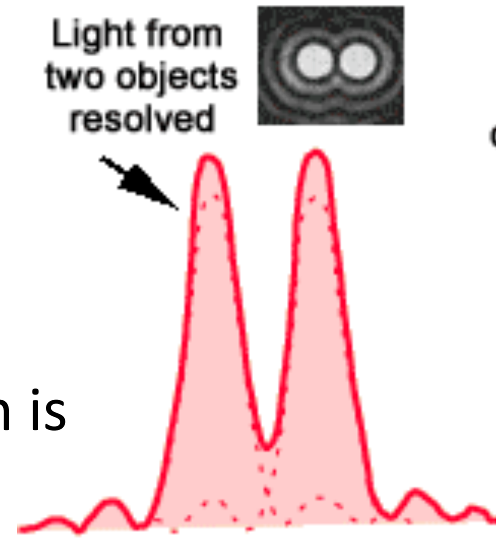
Point Spread Functions

Aperture	PSF	PSF equation
round, diameter d_x		$\left(\frac{2J_1(x)}{x}\right)^2$
obscured round, diameter d_x, obscuration ratio ε		$\frac{1}{(1-\varepsilon^2)^2}\left(\frac{2J_1(x)}{x}-\frac{2J_1(\varepsilon x)}{x}\right)^2$
rectangle, sides $d_{x,y}$		$\left(\frac{\sin x}{x}\right)^2\left(\frac{\sin y}{y}\right)^2$

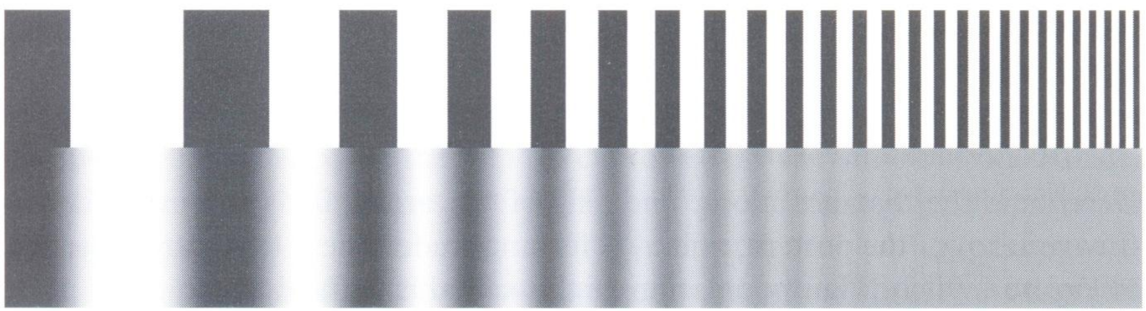
Optical/Modulation Transfer Function

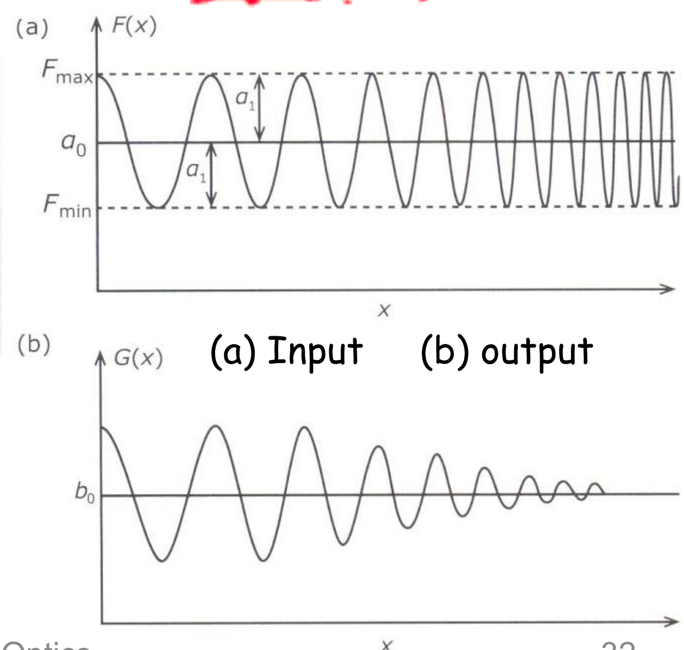
Rayleigh criterion: two sources can be resolved if the peak of the second source is no closer than the 1st dark Airy ring of the first source.

$$\sin\Theta = 1.22\frac{\lambda}{D}$$



A better measure of the resolution that the system is capable of is the optical transfer function (OTF):





$$MTF(f) = \frac{C(f)}{C_0}, \qquad \text{where } C = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

Optical/Modulation Transfer Function (2)

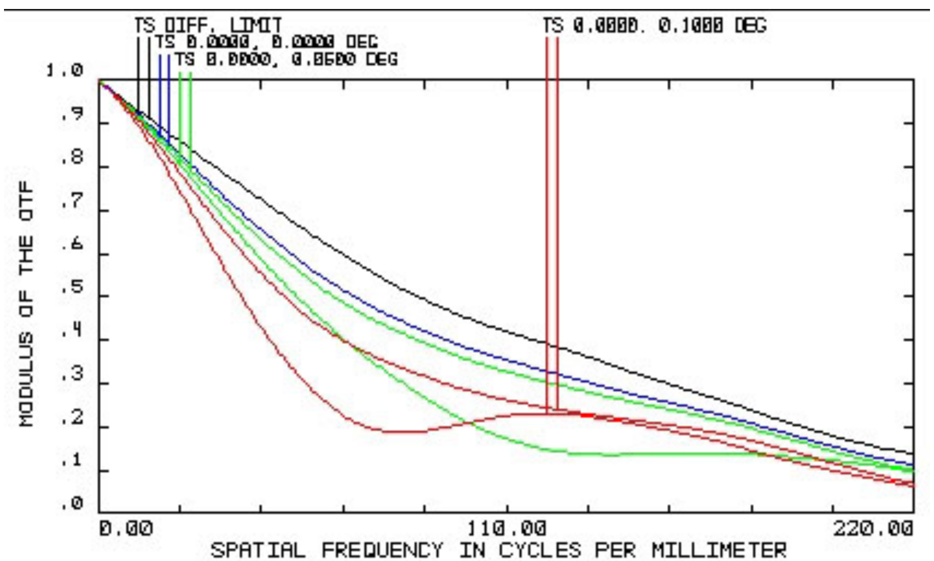
Optical Transfer Function (OTF) describes spatial signal variation as a function of spatial frequency. With spatial frequencies (ξ,η)

$$OTF(\xi,\eta)=MTF(\xi,\eta)\cdot PTF(\xi,\eta)$$

$$MTF(\xi,\eta)=\left|OTF(\xi,\eta)\right|$$

$$PTF(\xi,\eta)=e^{-i2\pi\lambda(\xi,\eta)}$$

Modulation Transfer Function (MTF) describes its magnitude, and the Phase Transfer Function (PTF) the phase.



Strehl Ratio

- Strehl ratio is convenient measure of optical quality.
- Strehl ratio (SR) is ratio of observed *peak intensity* of PSF compared to theoretical maximum peak intensity of point source seen with perfect imaging system
- With wave number k=2π/λ, RMS wavefront error ω

$$SR = e^{-k^2\omega^2} \approx 1 - k^2\omega^2$$

- <u>Examples:</u>
- SR > 80% considered diffraction-limited → WFE ~ λ/14
- typical adaptive optics system delivers SR ~ 10-80%
- seeing-limited PSF on 8m telescope has a SR~0.1-0.01%

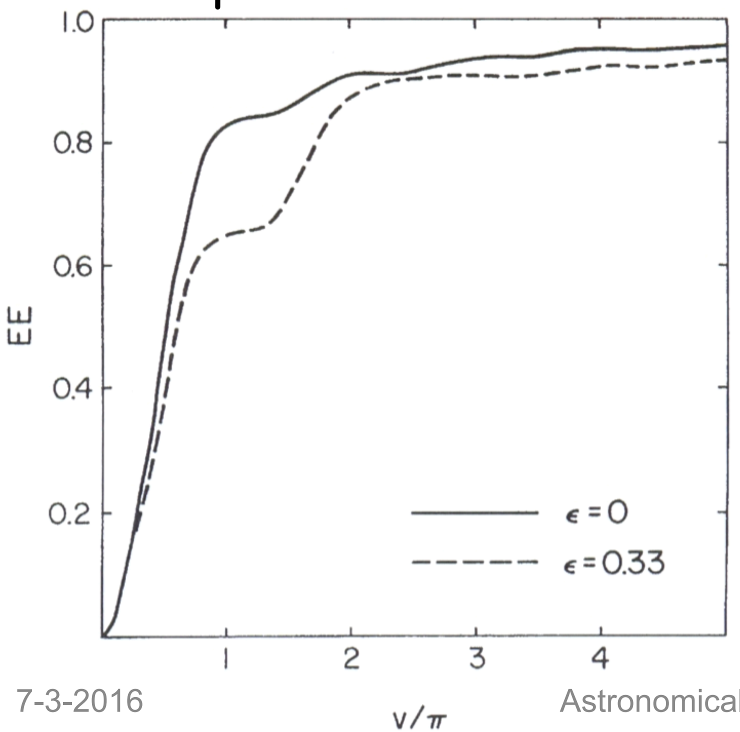
Encircled Energy

Q: What is the maximum concentration of light within a small area? The fraction of the total PSF intensity within a certain radius is given by the encircled energy (EE):

$$EE(r) = 1 - J_0^2\left(\frac{\pi r}{\lambda F}\right) - J_1^2\left(\frac{\pi r}{\lambda F}\right)$$

F is the f/# number

Note that the EE depends strongly on the central obscuration ε of the telescope:

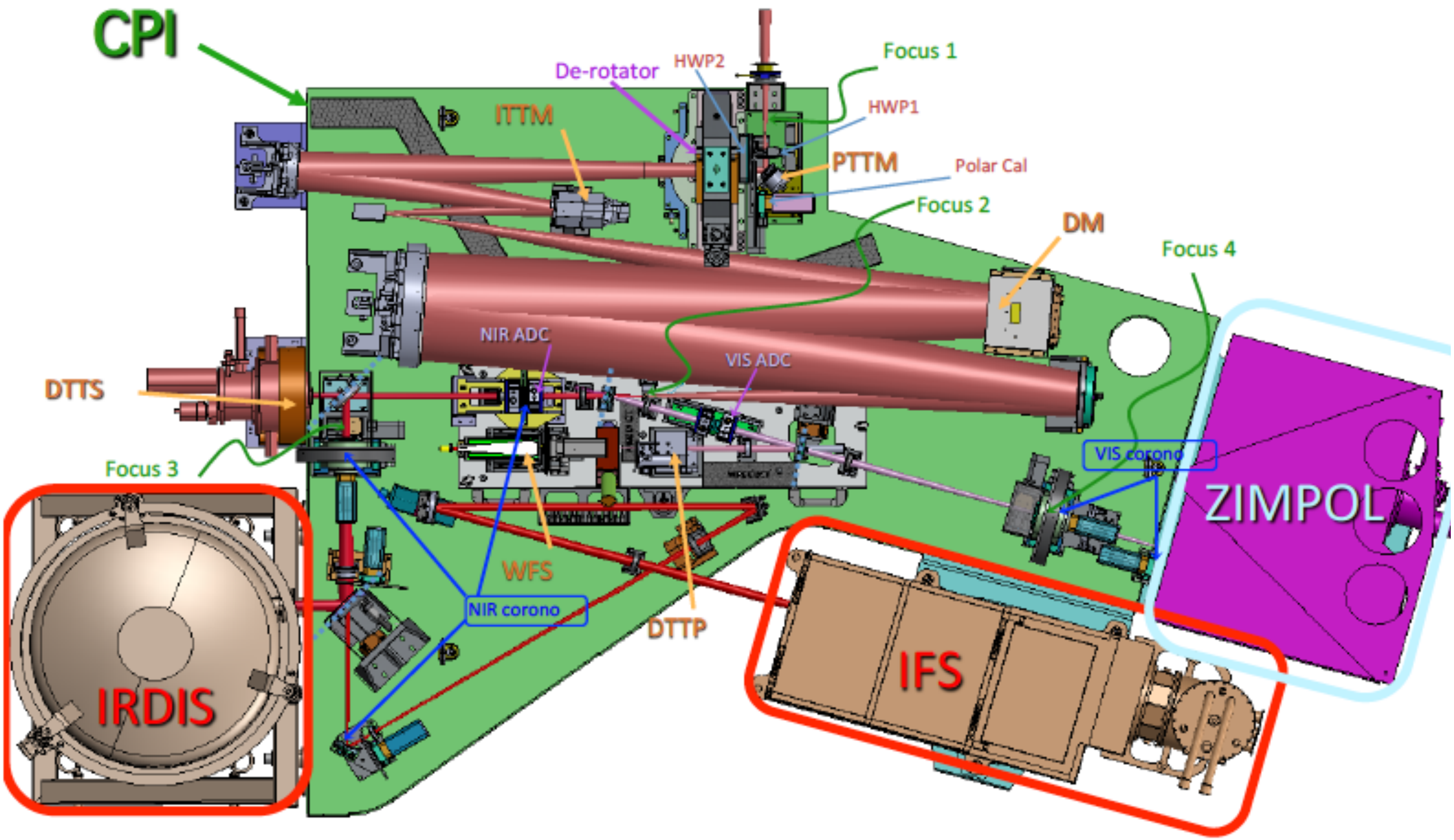


Encircled Energy Fraction within Airy Dark Rings[a]

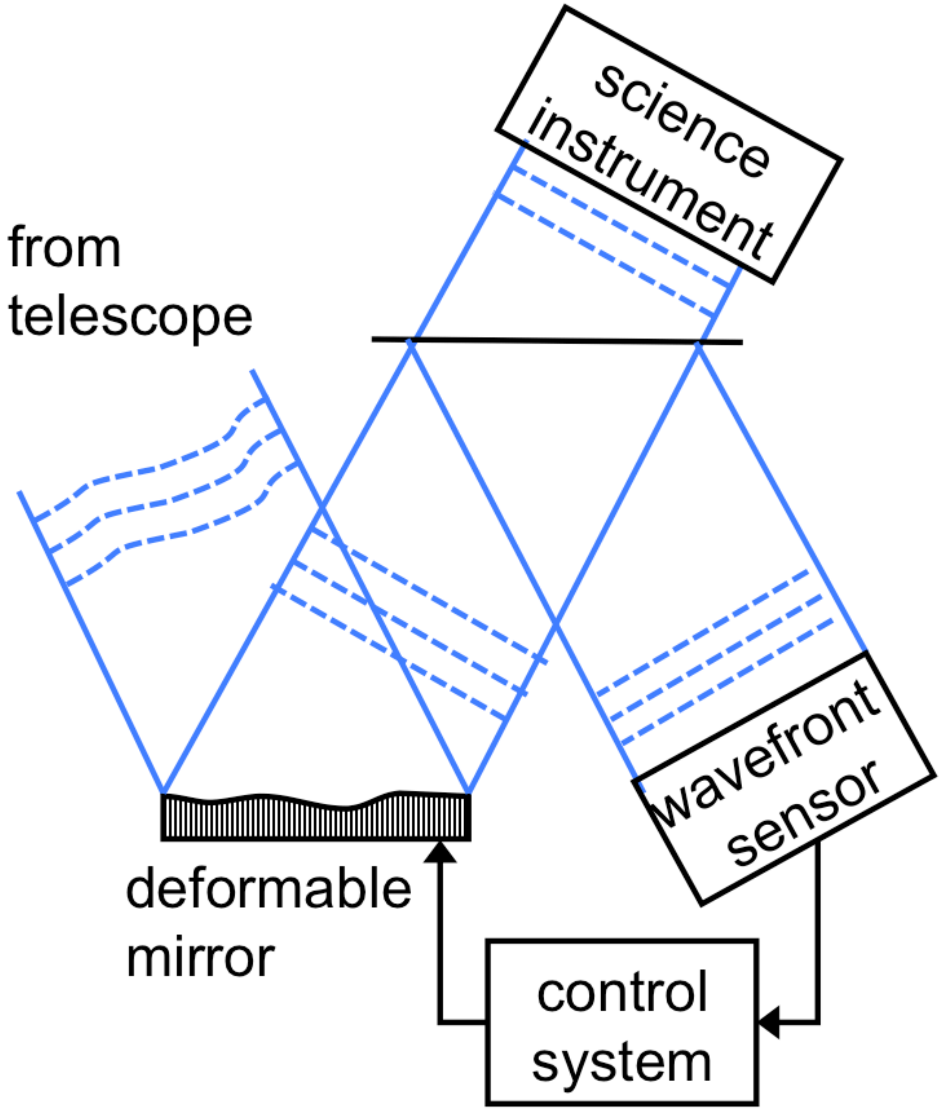
ε	EE_1	EE_2	EE_3
0.00	0.838	0.910	0.938
0.10	0.818	0.906	0.925
0.20	0.764	0.900	0.908
0.33	0.654	0.898	0.904
0.40	0.584	0.885	0.903
0.50	0.479	0.829	0.901
0.60	0.372	0.717	0.873

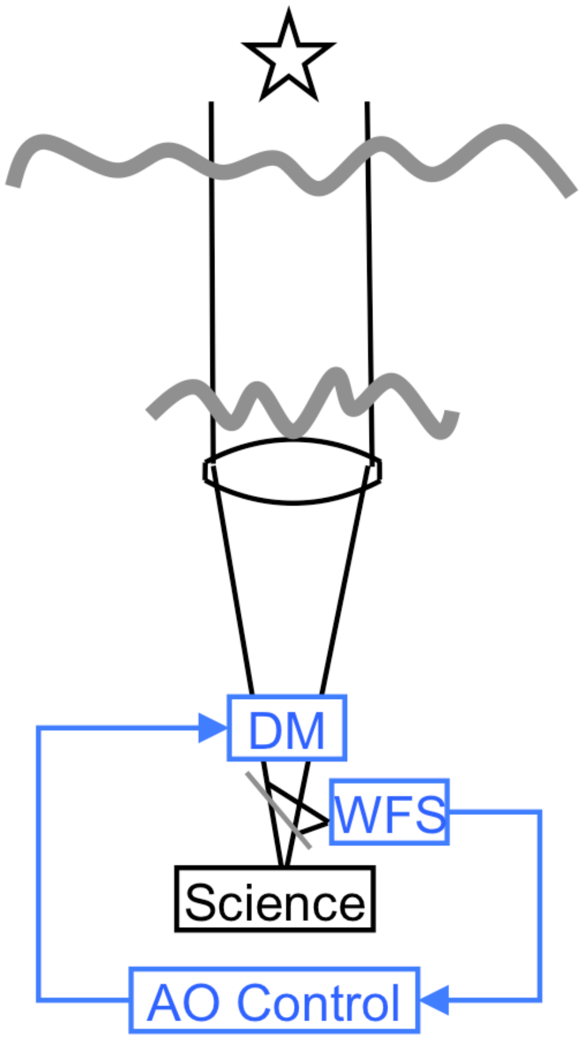
[a] Subscript on EE is number of dark ring starting at innermost ring.

SPHERE at VLT

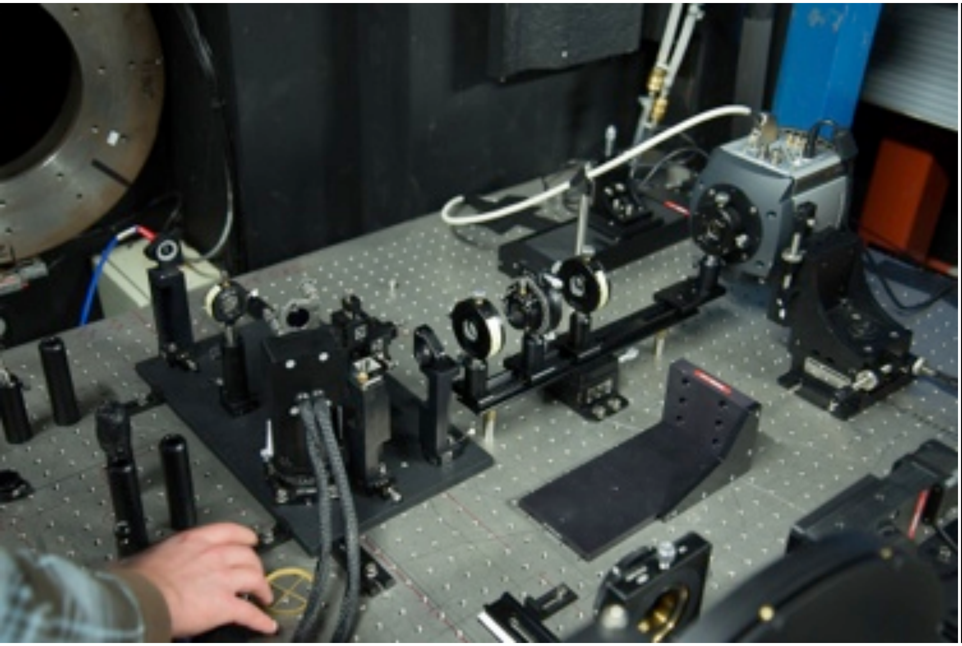


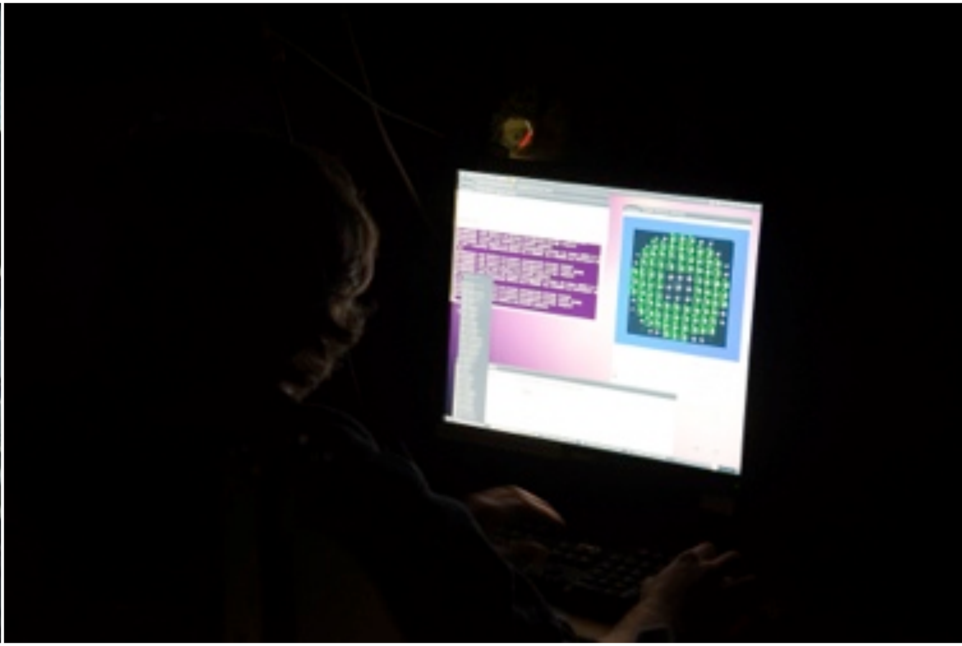
Adaptive Optics (lecture 13)

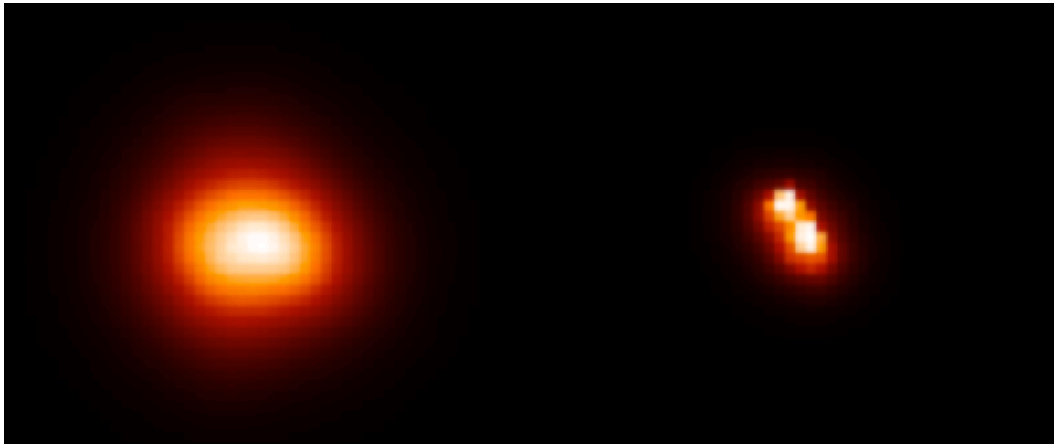




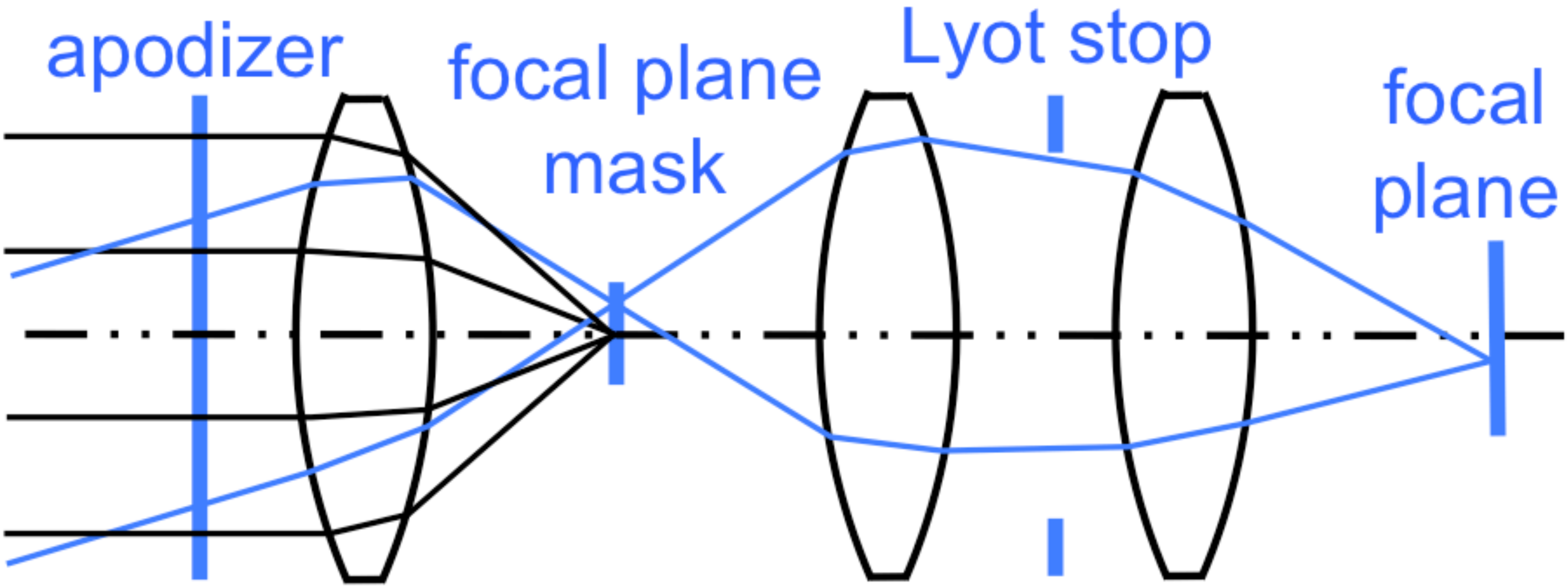
Student-Built Leiden AO





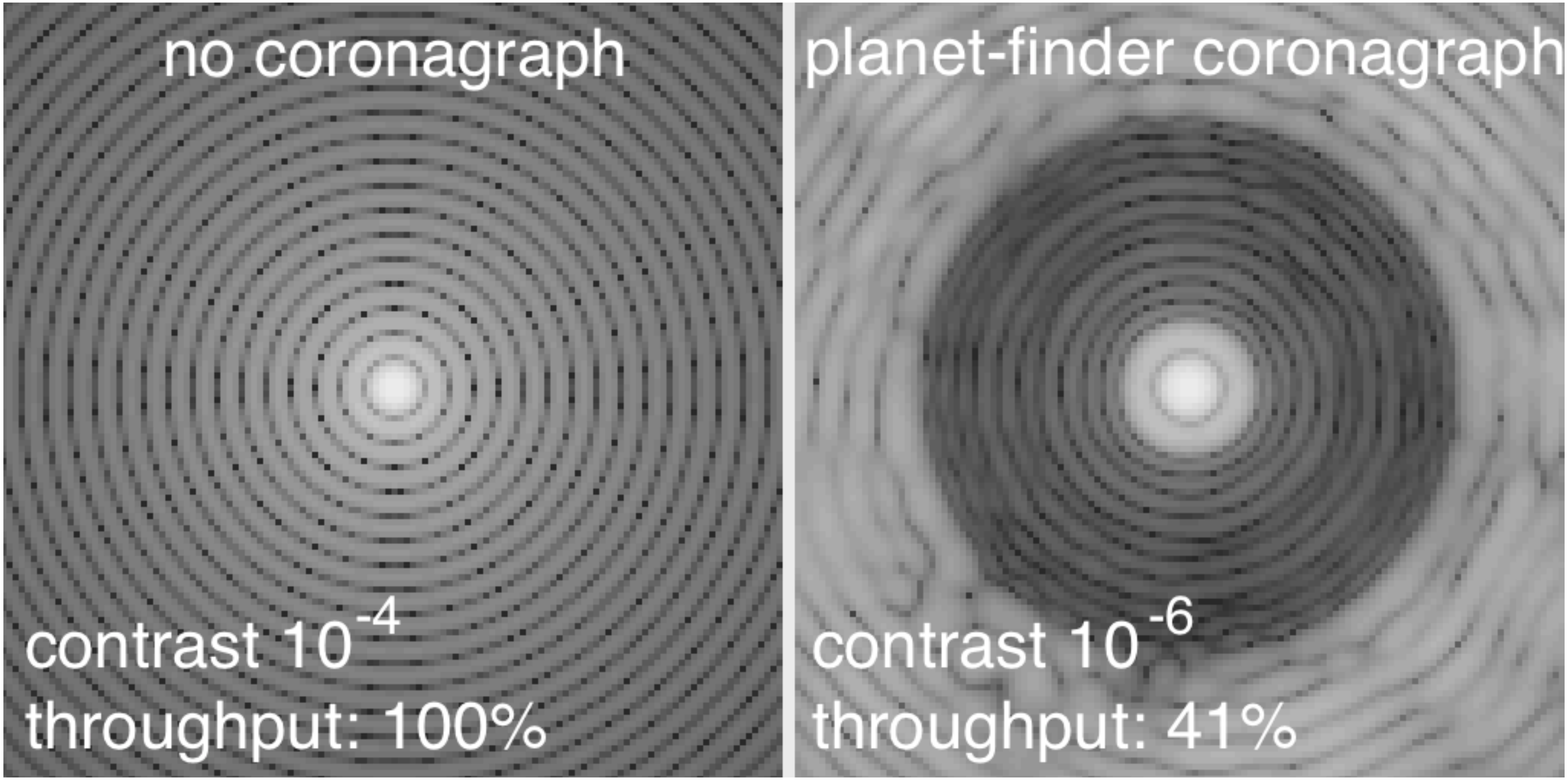


Coronagraph

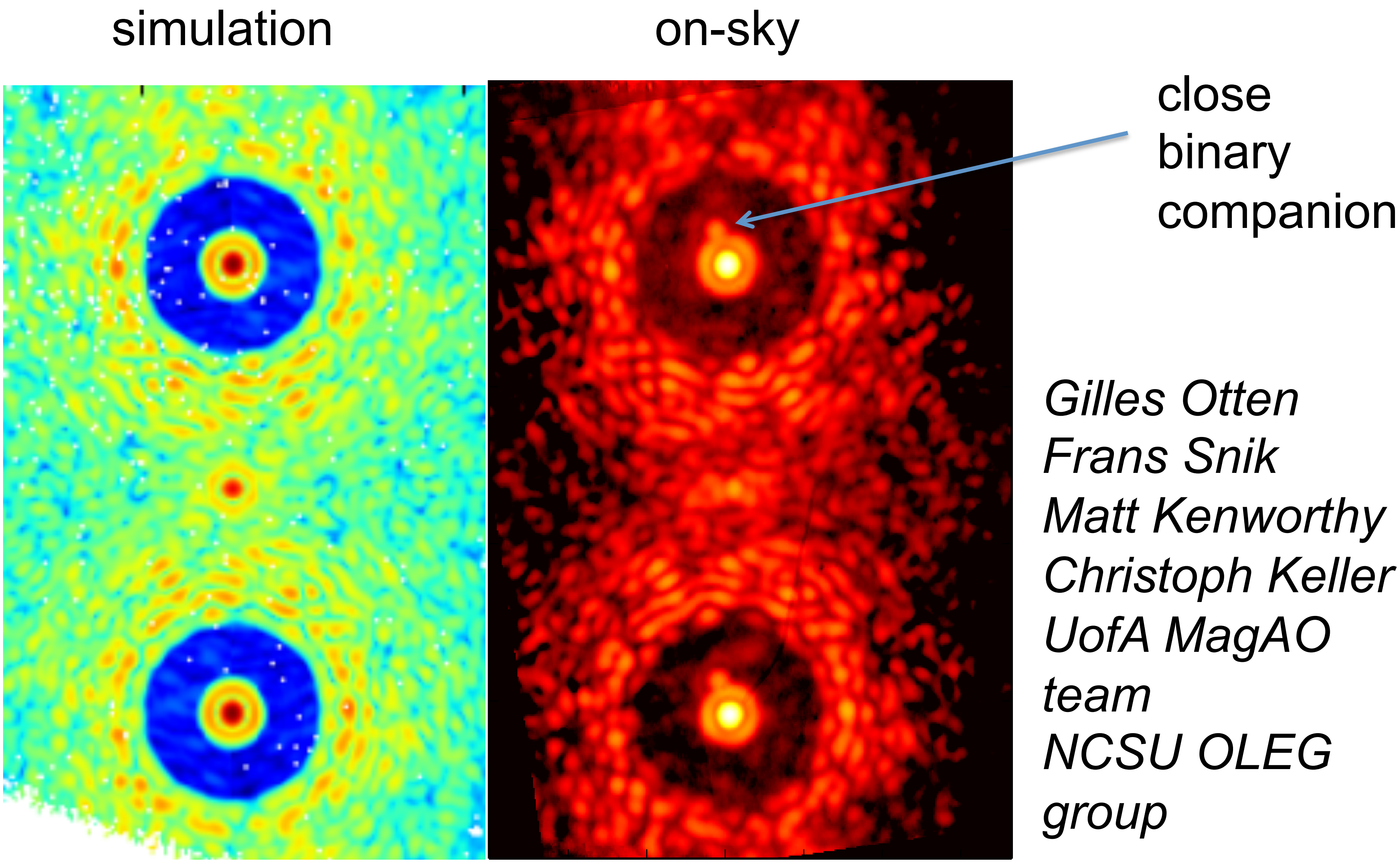


- goal: minimize diffracted light close to star
- can introduce optics in pupil and/or focal planes

Apodizing Pupil Phase Coronagraph



360° vAPP on-sky at MagAO



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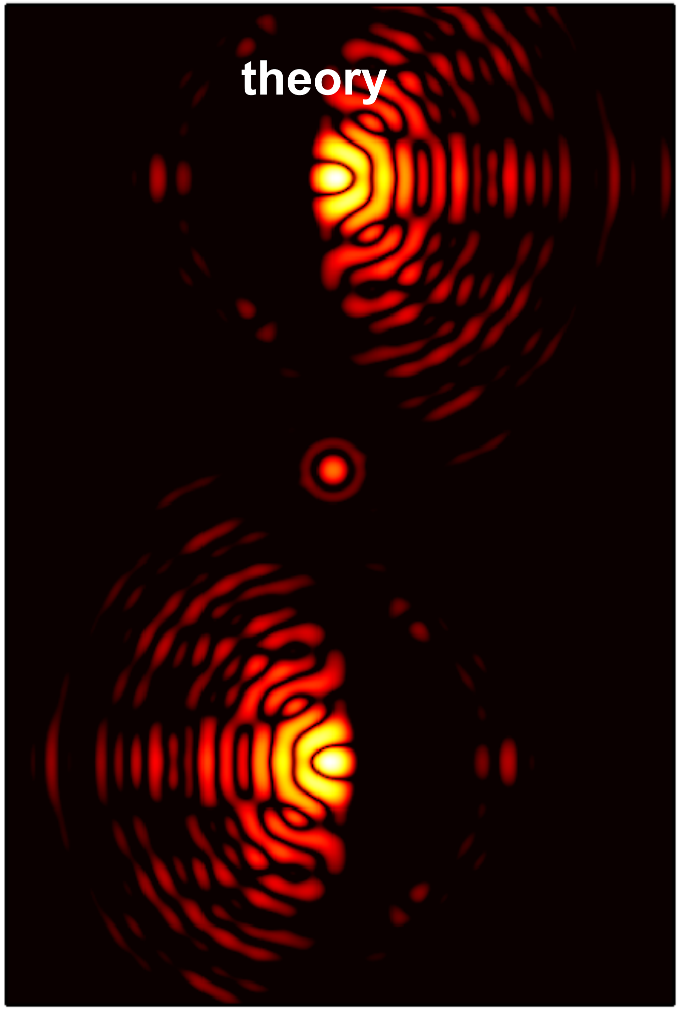
gvAPP on MagAO/Clio2



Otten et al. 2015