# Lecture 8: Interferometry

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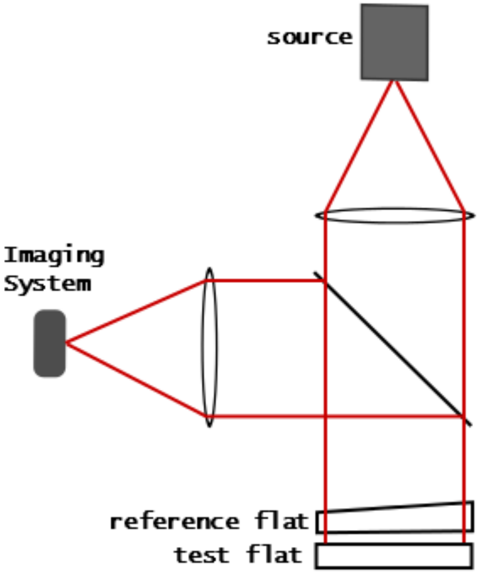
# Overview

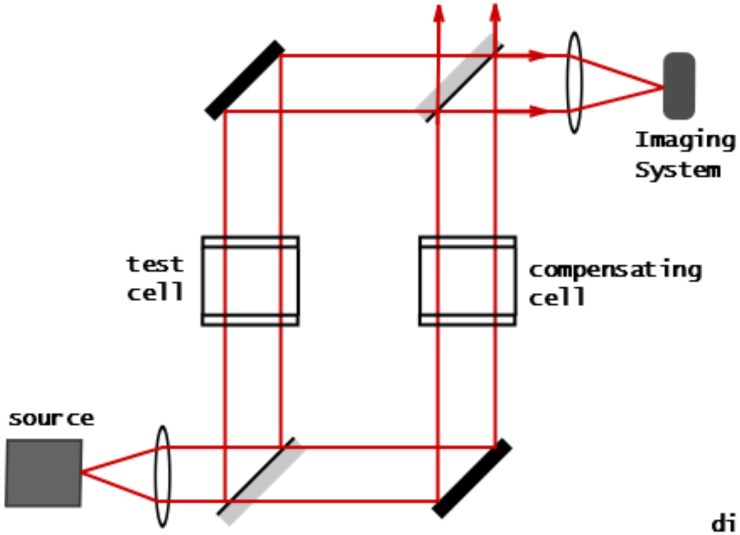
1. Basic Principle
2. Main Components
3. 1D Imaging and Fringes
4. Fringe Tracking
5. 2D Imaging
6. Fundamental Considerations
7. Radio Interferometers
8. Sub-mm Interferometers

# Interferometers

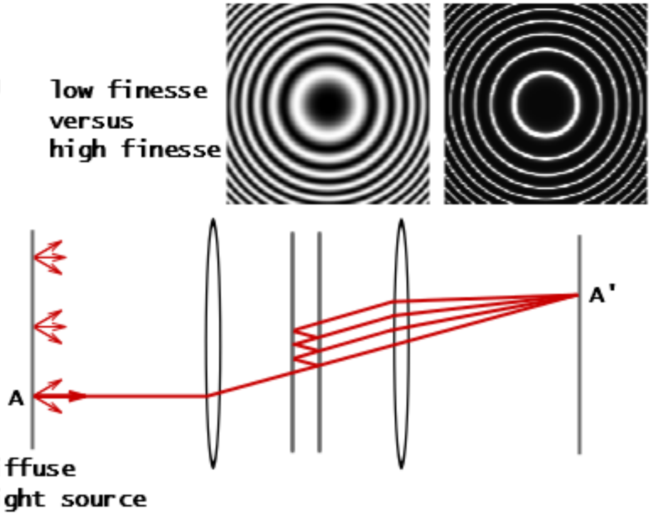
- General principle: Coherently combine two (or more) beams.
- Angular resolution determined by interference; interference does not require continuous aperture (e.g. Young's double slit experiment)!
- Hippolyte Fizeau (1868): basic concept of stellar interferometry
- Different types of interferometric concepts:



**Fizeau**



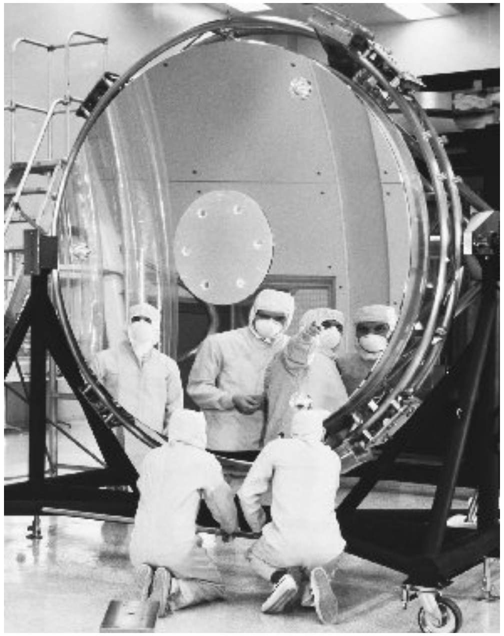
**Mach Zehnder**



**Fabry Perot**

# Goal: Increase Angular Resolution

$D = D_{tel}$

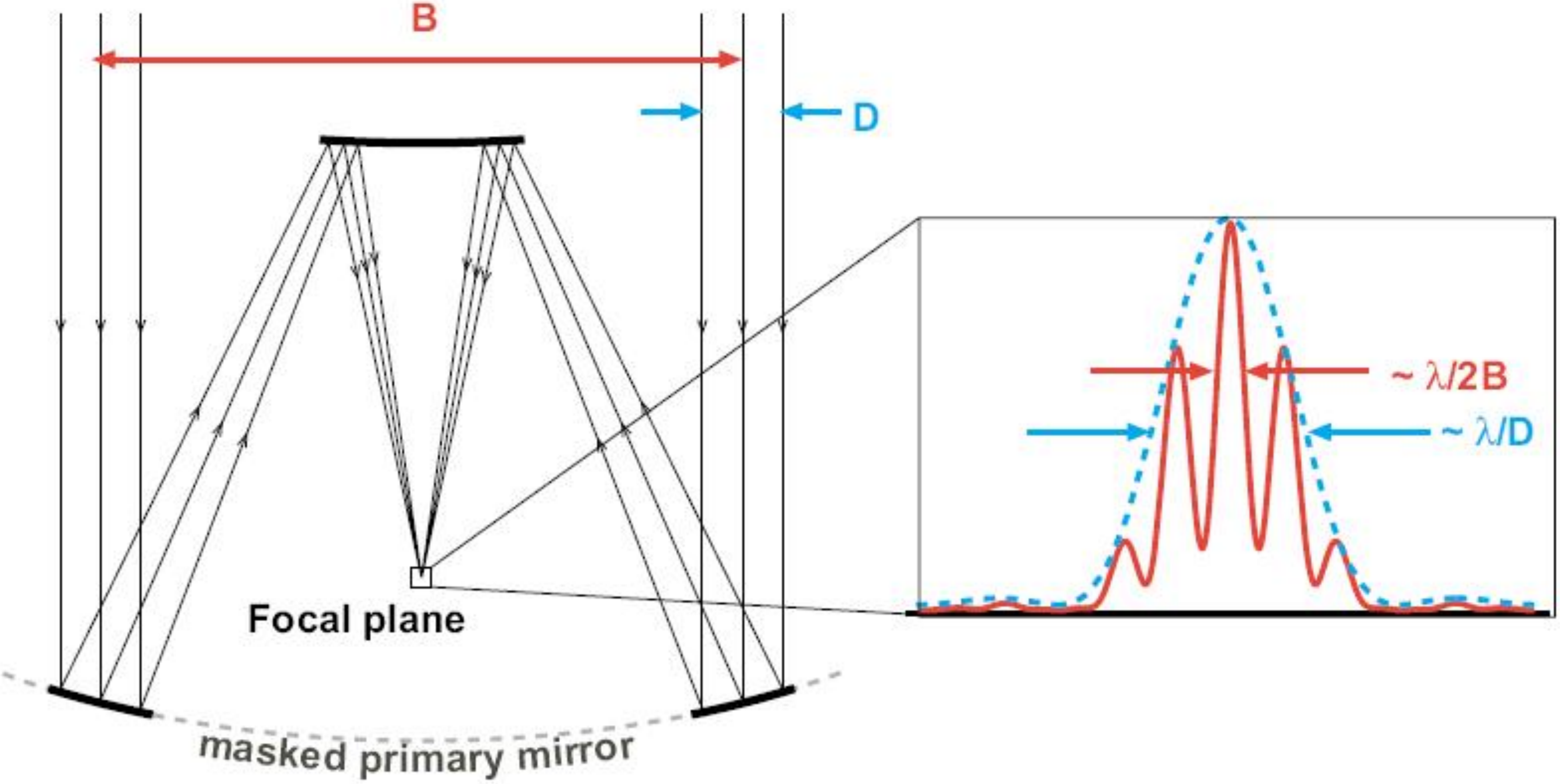


$D = d_{baseline} + D_{tel}$

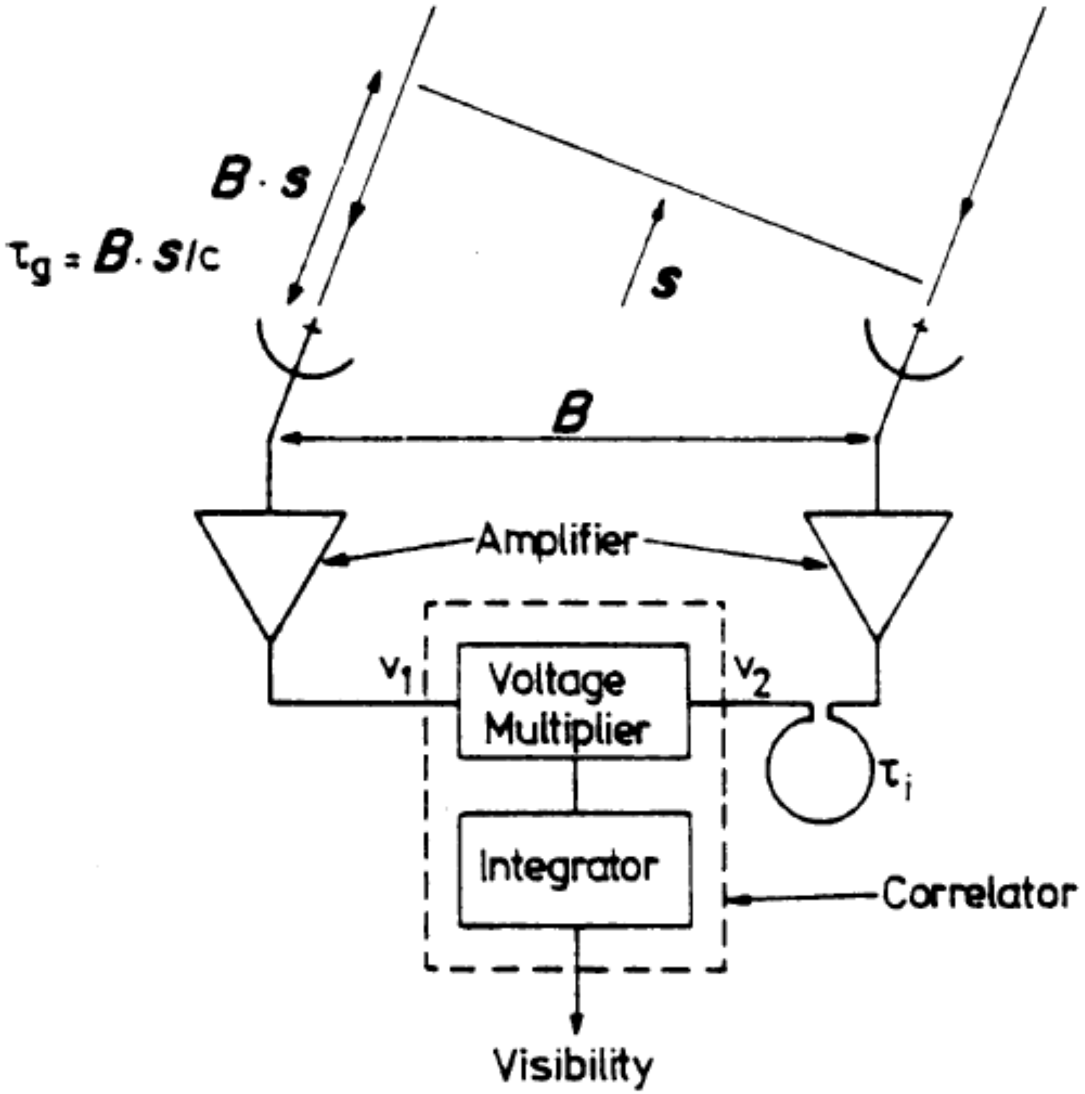
- angular resolution of 1.22λ/D only applies to filled aperture
- if only outer regions contribute, resolution is actually higher

# PSF of Masked Aperture

Interferometry is like masking a giant telescope:

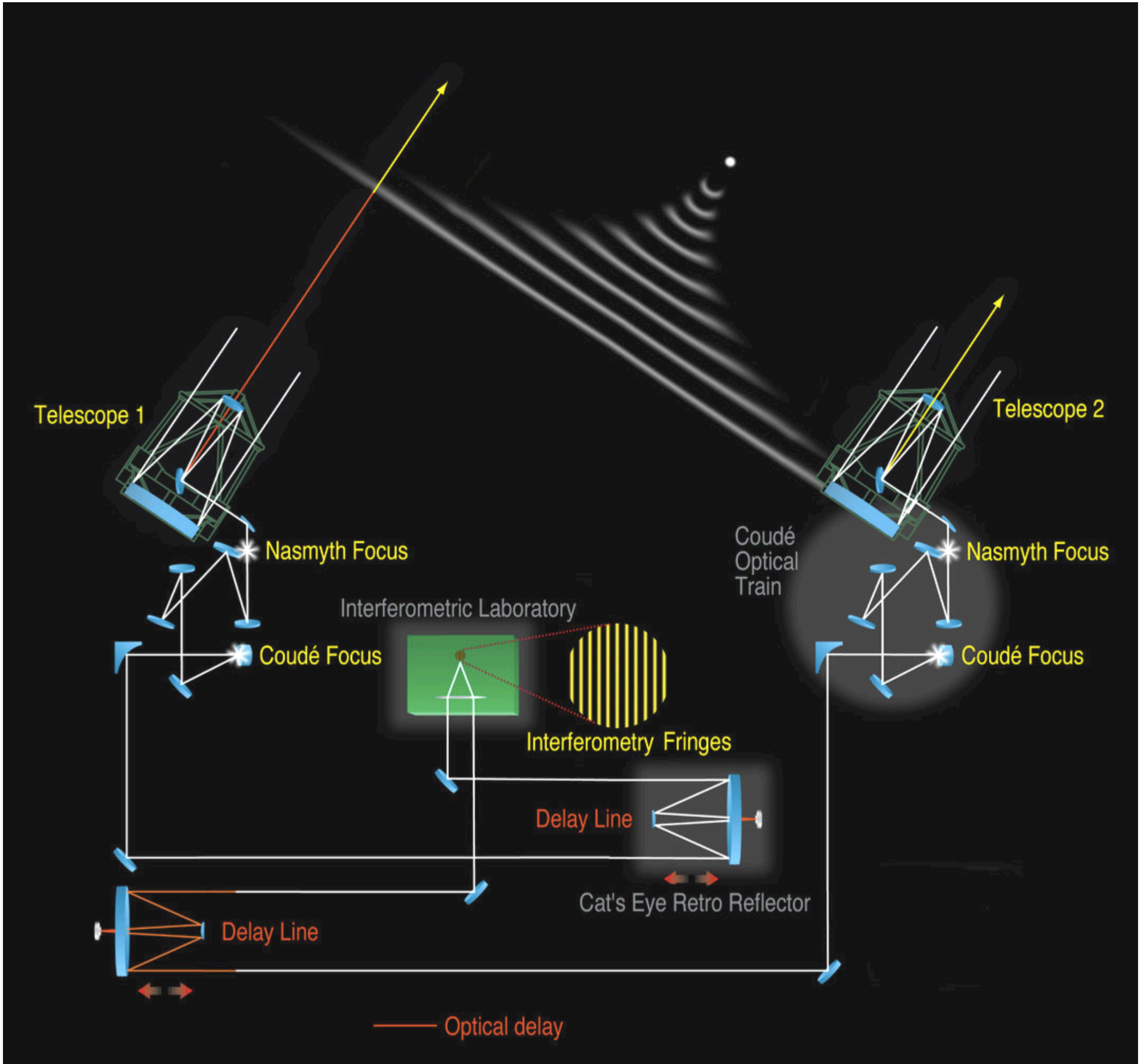


# Basic Principle - Radio



Radio heterodyne receivers provide great flexibility for interferometry because their outputs retain phase information of incoming signal

# Basic Principle – Optical



# Main Components: 1) Telescopes

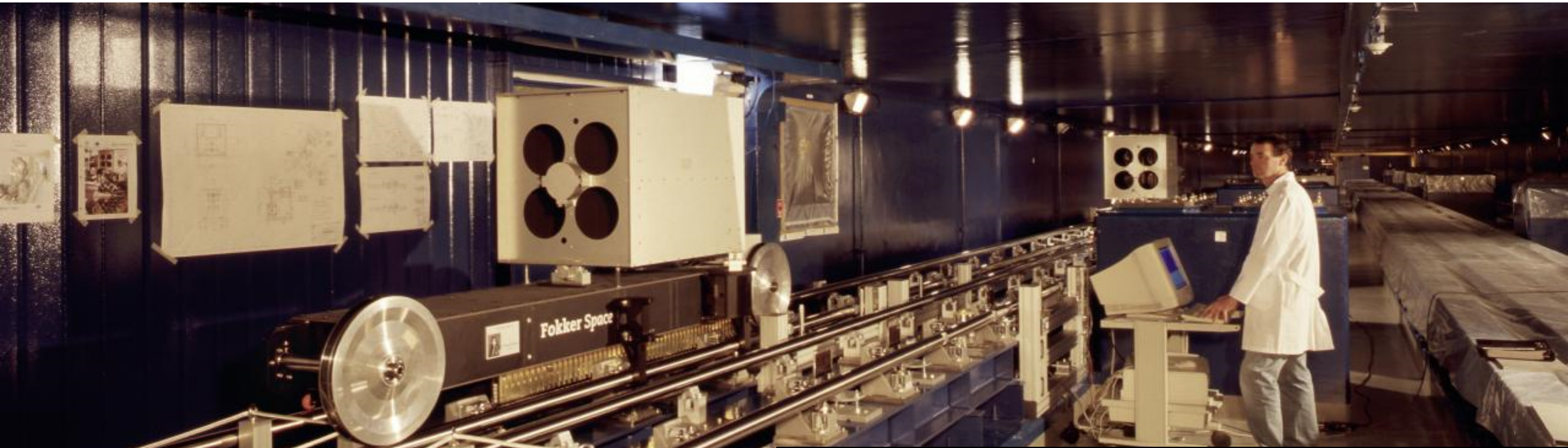
An optical interferometer typically consists of *n* telescopes of similar type and characteristics

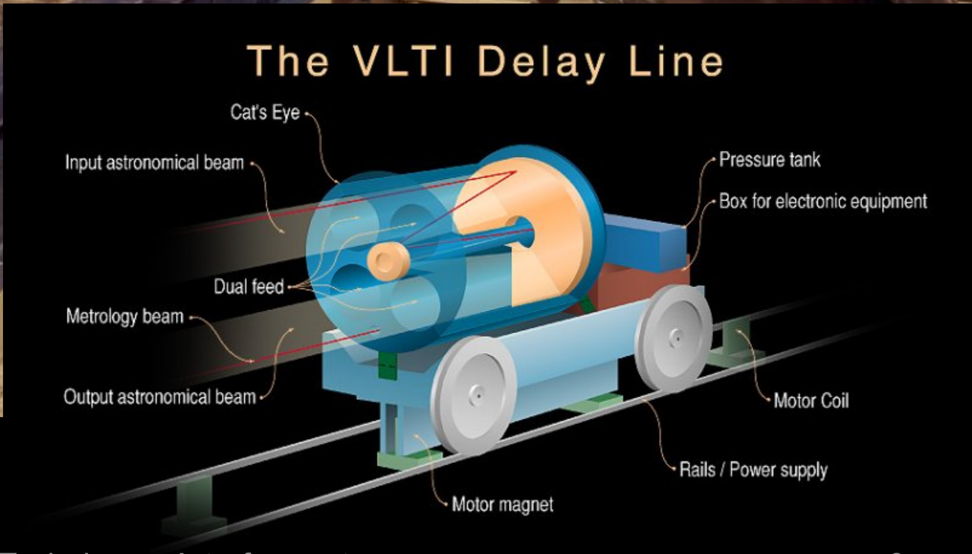
Keck interferometer (Hawaii) ↑

← VLTI (Paranal)

# Main Components: 2) Delay Lines

Compensate optical path difference between telescopes (depends on object location on sky)



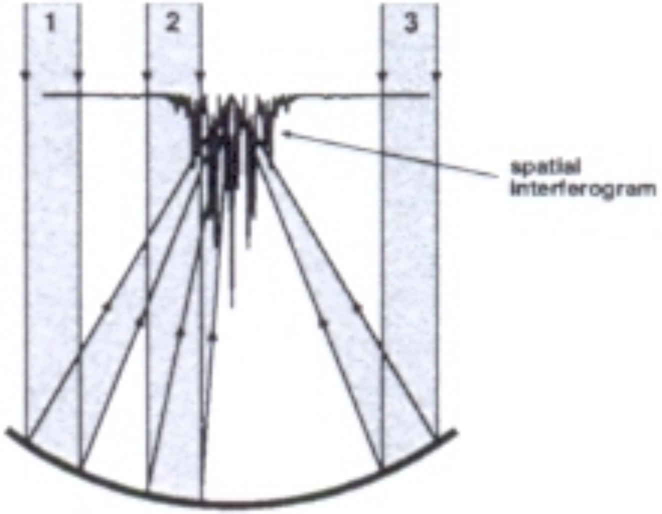


Challenge: travel over tens of meters, positioning to fractions of micrometers ➔ dynamic range of > $10^9$ !

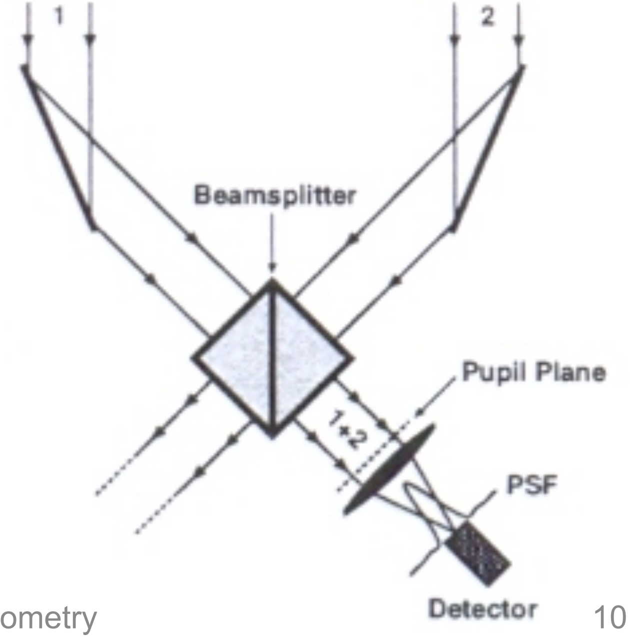
# Main Components: 3) Beam Combiners

Two main types:

- multi-axial (image plane): beams are placed adjacent to each other and form a fringe pattern in space.



- co-axial (pupil plane): beams are added on top of each other e.g. via a beam splitter.



- can also use single-mode fibers and integrated optics.

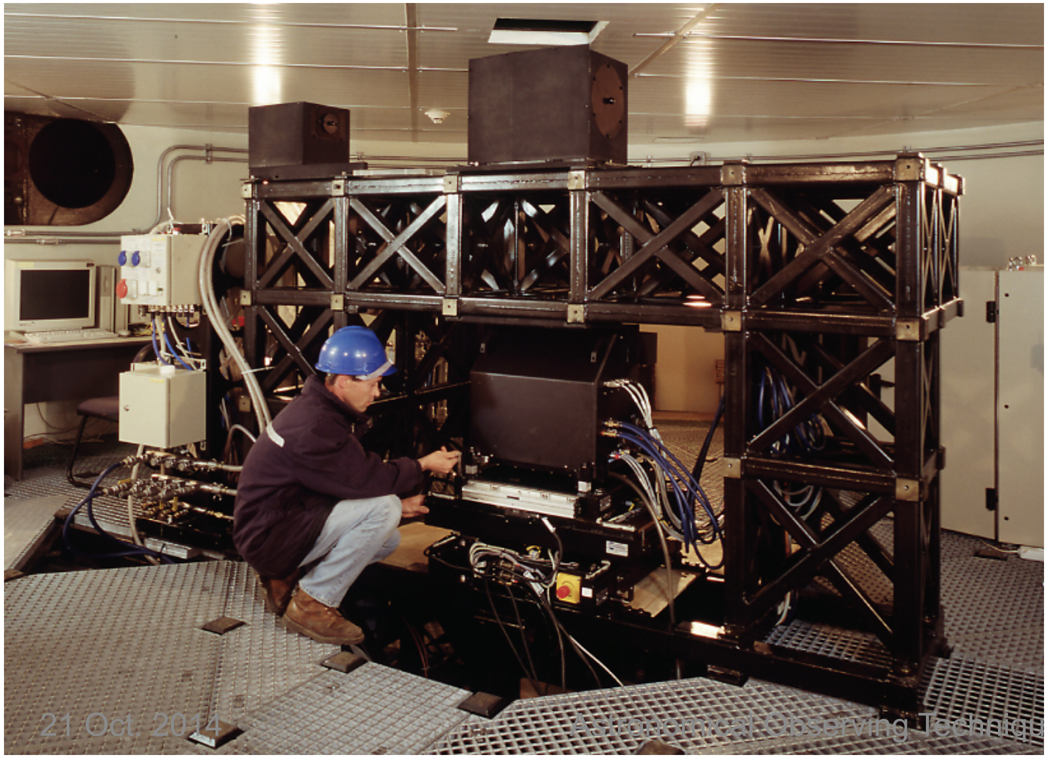
# Main Components: 4) Adaptive Optics

Adaptive optics (or for telescopes with $D < r_0$ tip-tilt correction) is essential to correct wavefront aberrations for good interference.

Path length fluctuations over distance B:

$$\sigma = \sqrt{6.88\left(\frac{B}{r_0}\right)^{5/6}} \quad \text{rads RMS}$$

Baseline B = 100m, seeing $r_0$=0.1 m at 0.5µm: 66µm !



*MACAO (Multi Application Curvature Adaptive Optics) system on a 8m VLT. Can be used with natural guide stars with $1 < V < 17$, seeing < 1.5", $\tau_0 > 1.5$ms and airmass < 2.*

# Fringe Visibility – Definition

Fringe visibility defined as

$$V = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

Van-Cittert Zernike theorem: Visibility is the absolute value of the Fourier transform of the object's brightness distribution

If dark regions in fringe pattern go to zero V = 1 → object is "unresolved"

If V = 0 then there are no fringes → object is completely "resolved"

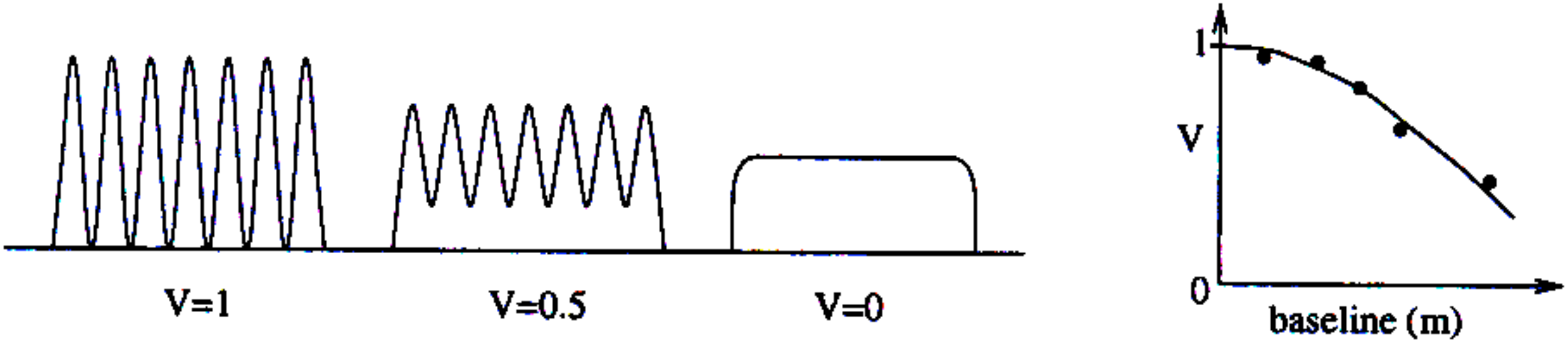
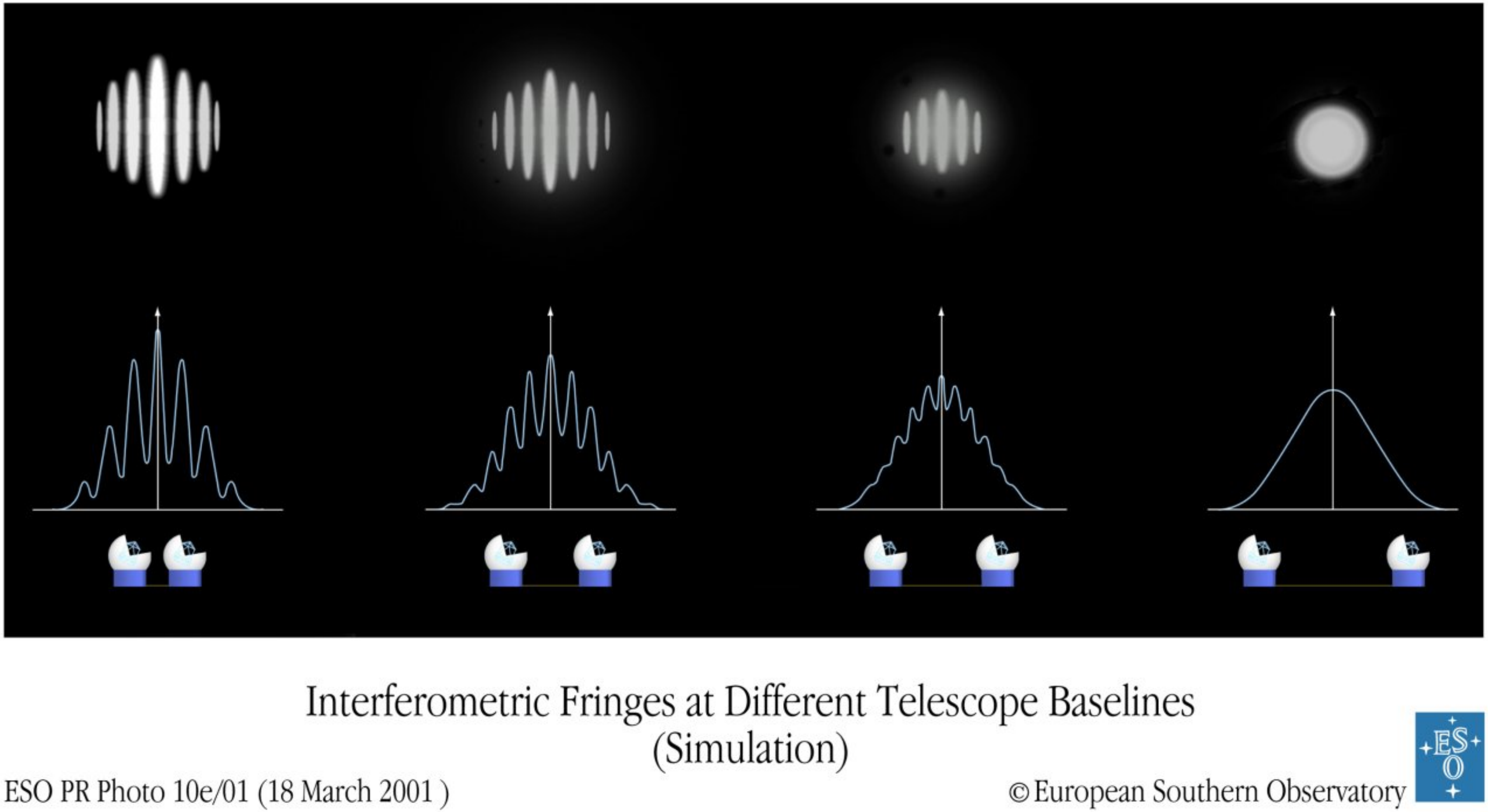


**Fig. 2.** Left: examples of fringes with visibilities of 1, 0.5 and 0. Right: visibility as a function of baseline for a resolved star.

# Fringe Visibility – Baseline



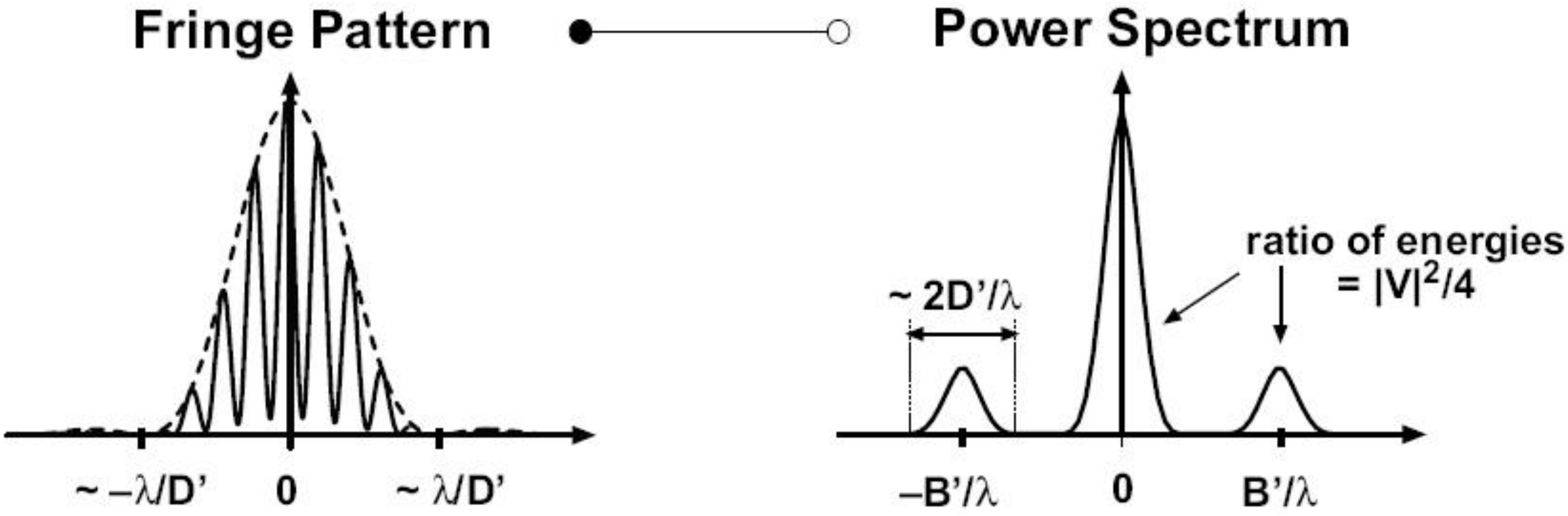
Interferometric Fringes at Different Telescope Baselines
(Simulation)

ESO PR Photo 10e/01 (18 March 2001 )

© European Southern Observatory

Pattern from single, unresolved star at focal plane changes as distance between 2 telescopes is increased. “Fringes" disappear completely when star is “resolved”.

# Fringe Visibility and Power Spectrum

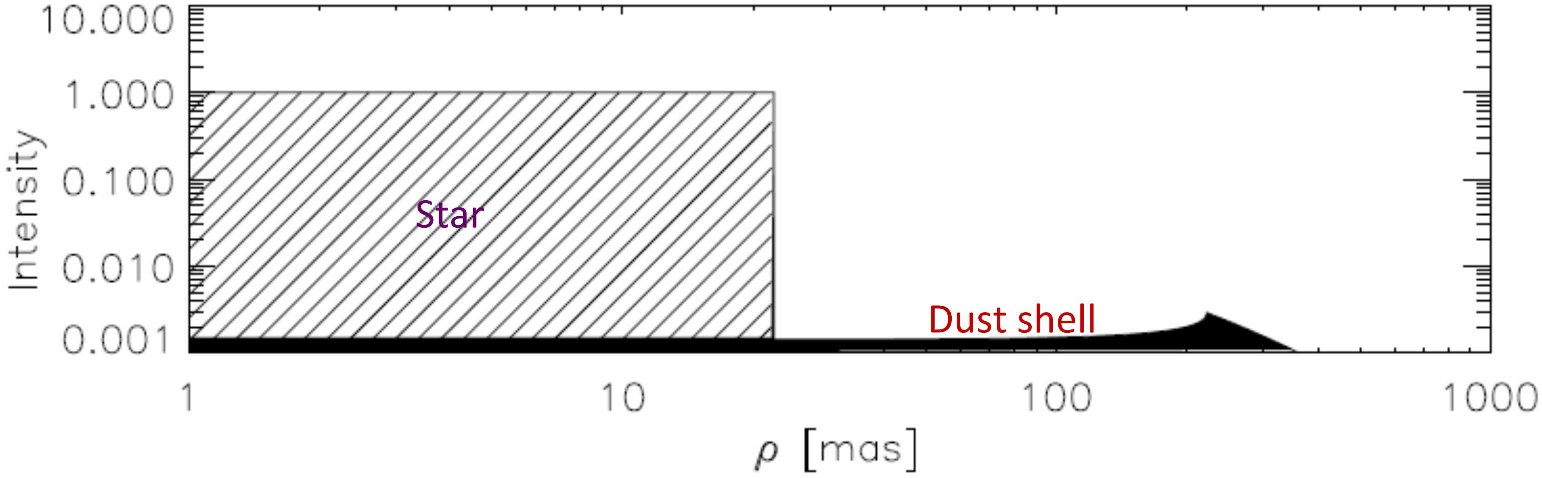


# Fringe Visibility – another Example

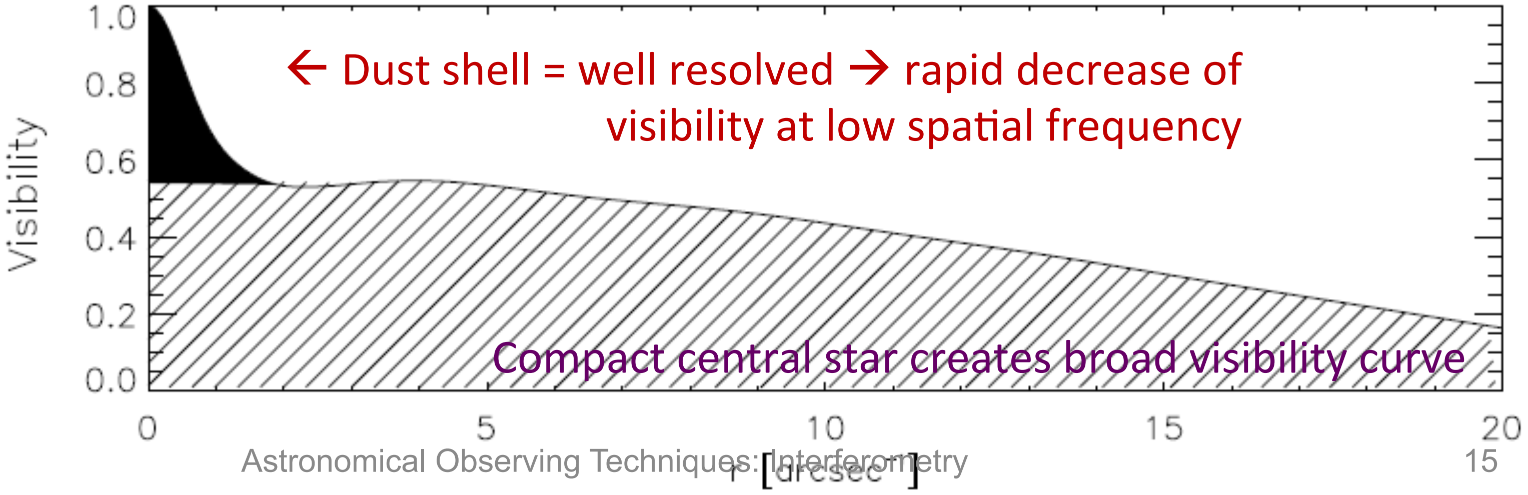
Intensity profile and corresponding visibility for a 2-component model of

(i) An (almost) unresolved central star (hatched), and

(ii) a well resolved dust shell (solid black).



Visibility = sum of visibilities of individual components



← Dust shell = well resolved → rapid decrease of visibility at low spatial frequency

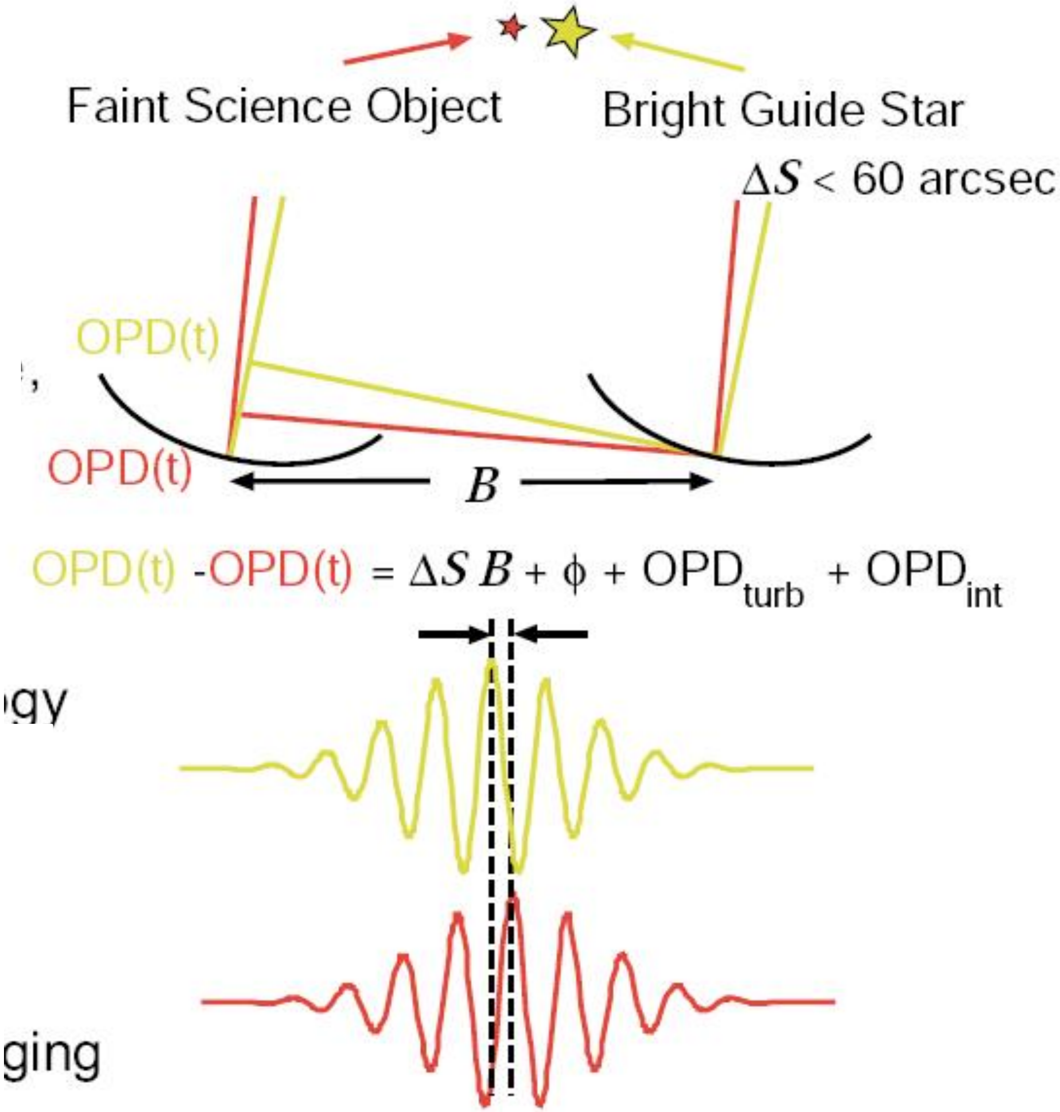
Compact central star creates broad visibility curve

*Example taken from PhD thesis Tijl Verhoelst, KU Leuven, 2005*

# Fringe Tracking (Co-Phasing)

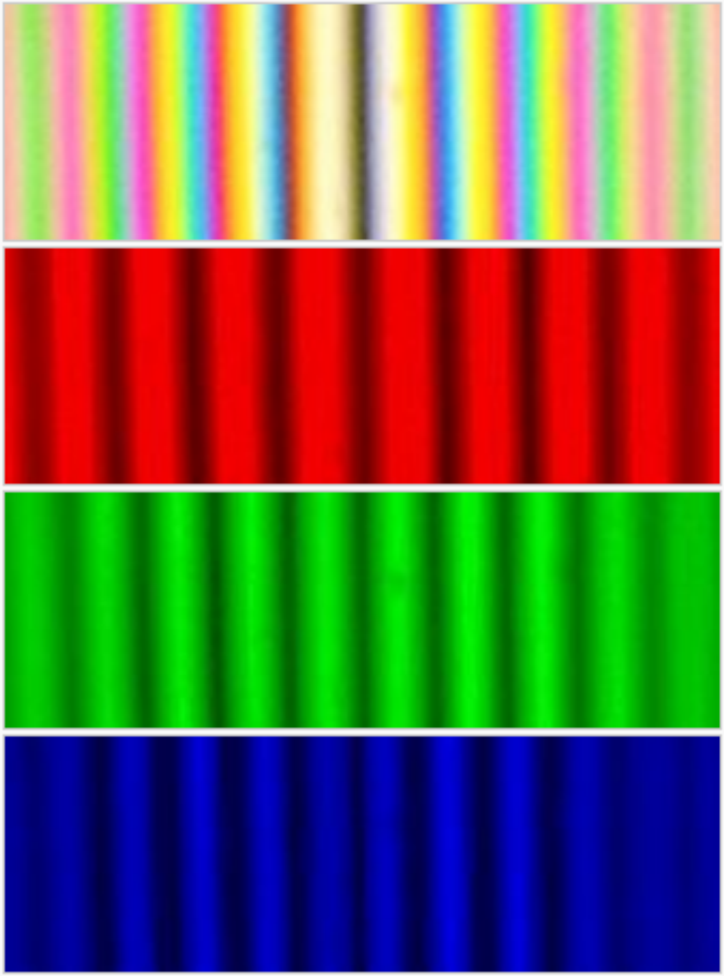
Fringes have to be actively tracked; requires tracking fluctuations within small fraction of wavelength in real-time

Example: ESO's FINITO scans center of fringe packet in H band with high speed and sends a co-phasing signal to the VLTI delay lines; FINITO operates on two channels, i.e. tracks three baselines.



# White Light Fringes

- Use of white light (= typical astronomical signal) will result in a pattern of colored fringes
- *The term* ***white light fringe*** *refers to the central fringe*
- The central fringe representing equal path length may be light or dark depending on the number of phase inversions experienced by the two beams as they traverse the optical system



**Above**: White light Interferogram, **Below**: Red-, Green- and Blue channels of the White light interferogram shown above

# Closure Phase (1)

Fringe visibility tells one component of the objects Fourier transform = amplitude of the fringes

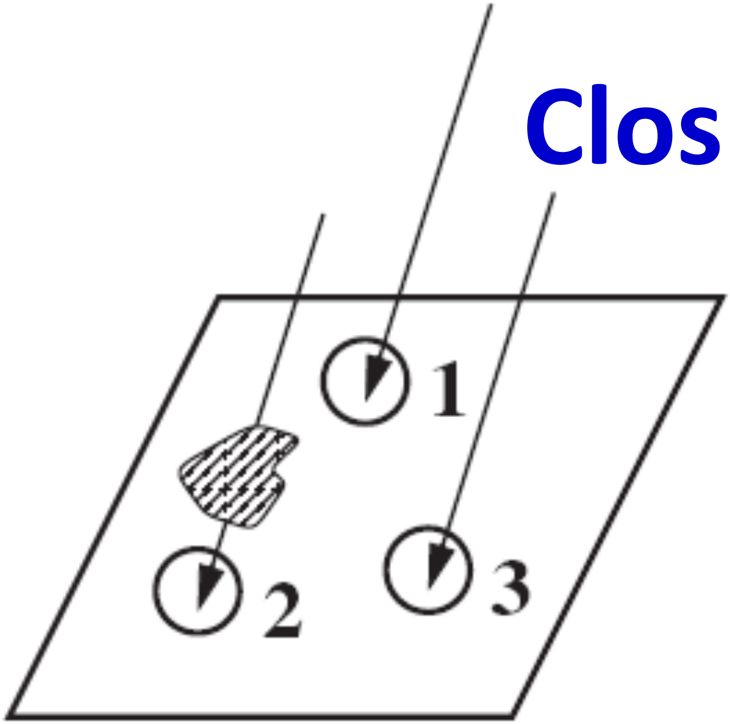
The phase is determined by the position of the fringes.

Problem: due to atmospheric turbulence (which changes the optical path length), the fringes move constantly forward and backward.

Idea: use three telescopes → three sets of fringes: (1-2), (2-3), (1-3)
In all three sets the fringes move, but not independently!

→ this information is called closure phase (or self-calibration in aperture synthesis imaging – the standard technique in radio interferometry) and can be used to cancel out phase error terms.

# Closure Phase (2)



Observed Intrinsic Atmosphere

$$\Phi(1\text{-}2) = \Phi_o(1\text{-}2) + [\phi(2)-\phi(1)]$$
$$\Phi(2\text{-}3) = \Phi_o(2\text{-}3) + [\phi(3)-\phi(2)]$$
$$\Phi(3\text{-}1) = \Phi_o(3\text{-}1) + [\phi(1)-\phi(3)]$$

$$\text{Closure Phase (1-2-3)} = \Phi_o(1\text{-}2)+\Phi_o(2\text{-}3)+\Phi_o(3\text{-}1)$$

Error terms cancel out!

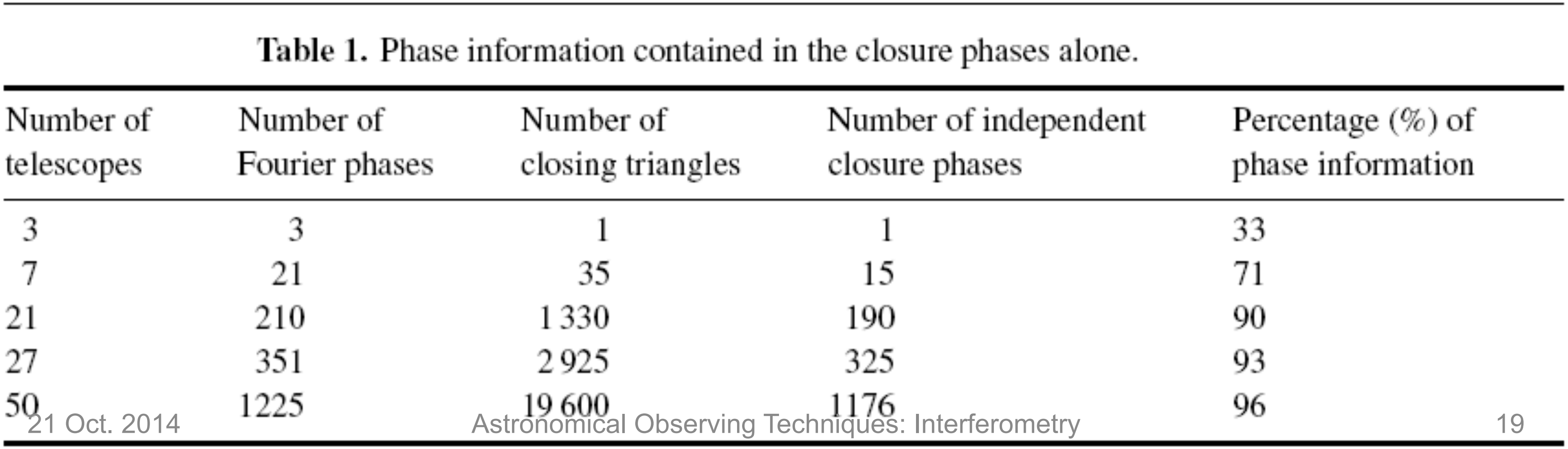


**Table 1.** Phase information contained in the closure phases alone.

| Number of telescopes | Number of Fourier phases | Number of closing triangles | Number of independent closure phases | Percentage (%) of phase information |
|---|---|---|---|---|
| 3 | 3 | 1 | 1 | 33 |
| 7 | 21 | 35 | 15 | 71 |
| 21 | 210 | 1 330 | 190 | 90 |
| 27 | 351 | 2 925 | 325 | 93 |
| 50 | 1225 | 19 600 | 1176 | 96 |

# (Radio) Aperture Synthesis

The limited information about the structure of a source provided by a two element interferometer can be expanded by moving the telescopes to change the baselines.

Even better: use N telescopes and combine their outputs: N telescopes provide N(N-1)/2 baselines. Each baseline adds a new Fourier component (or fringe spacing))

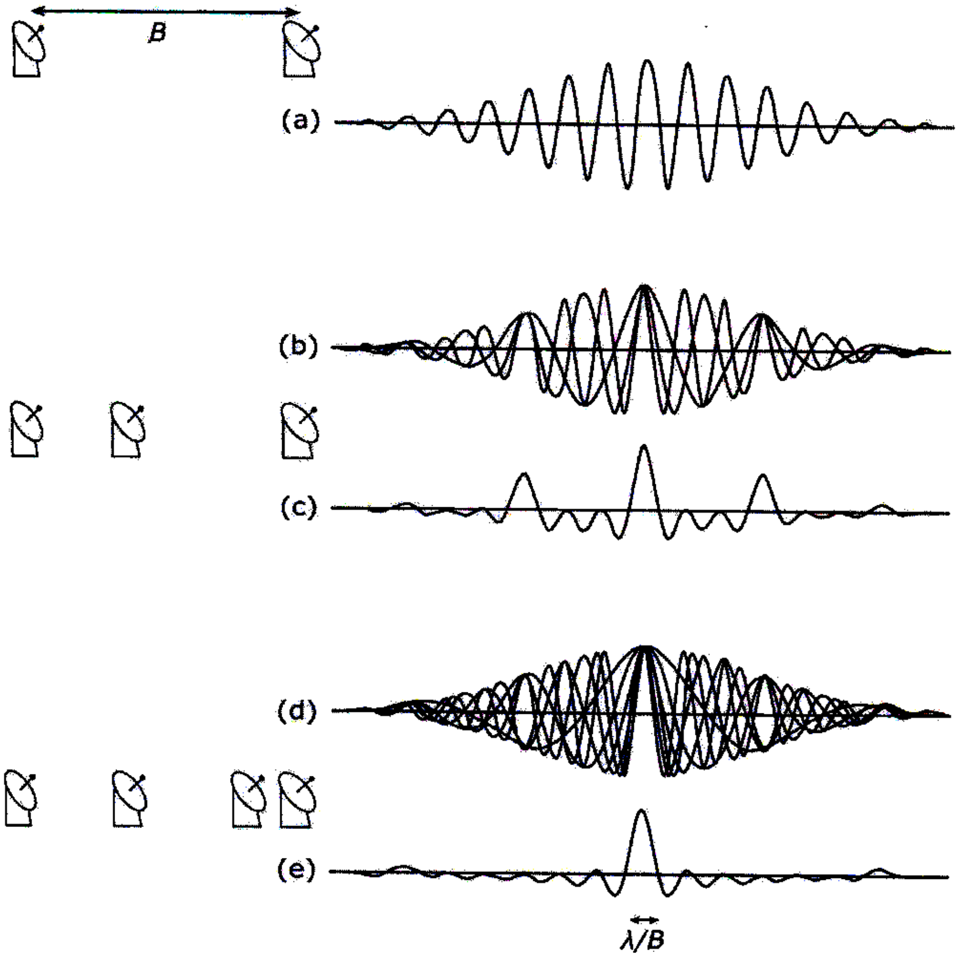
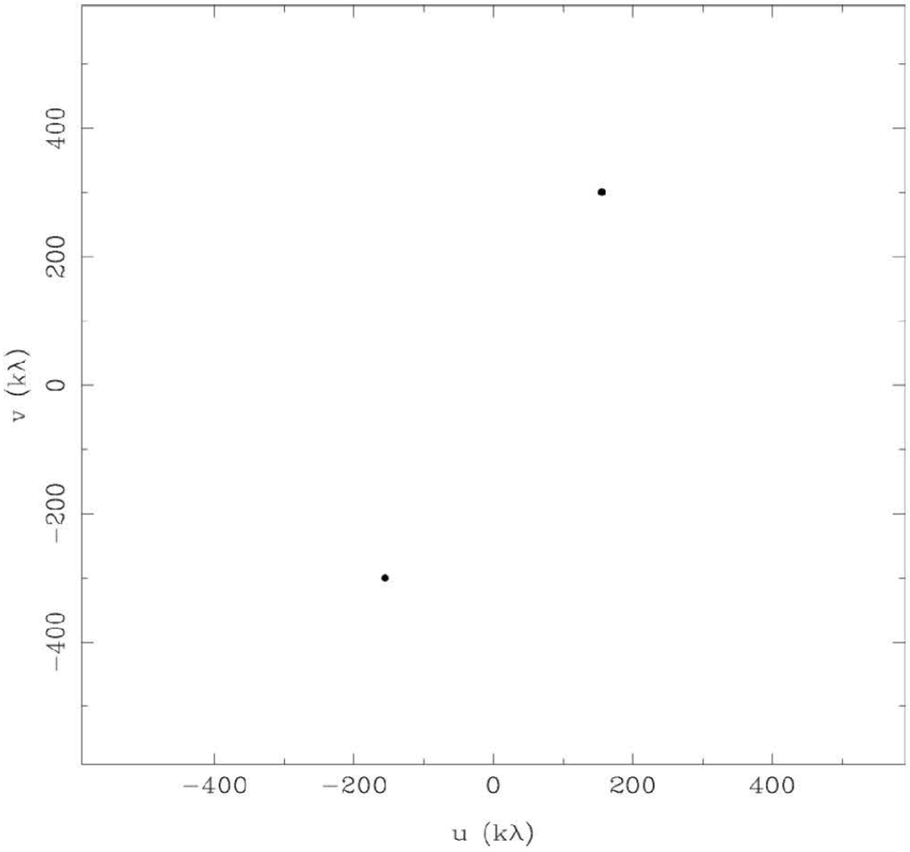
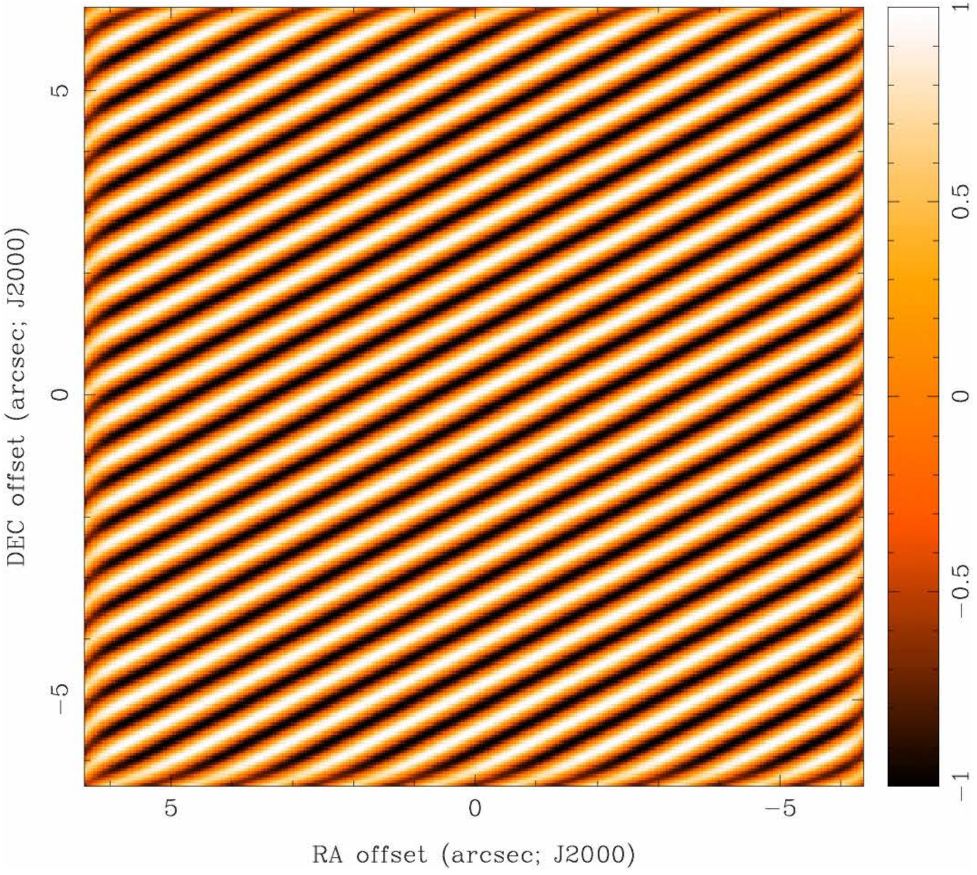


Figure 9.2. Improvement in field pattern quality (the images are the auto-correlation) with increasing number of interferometer baselines. Based on Condon and Ransom (2010).

# Dirty Beam Shape and N Antennas

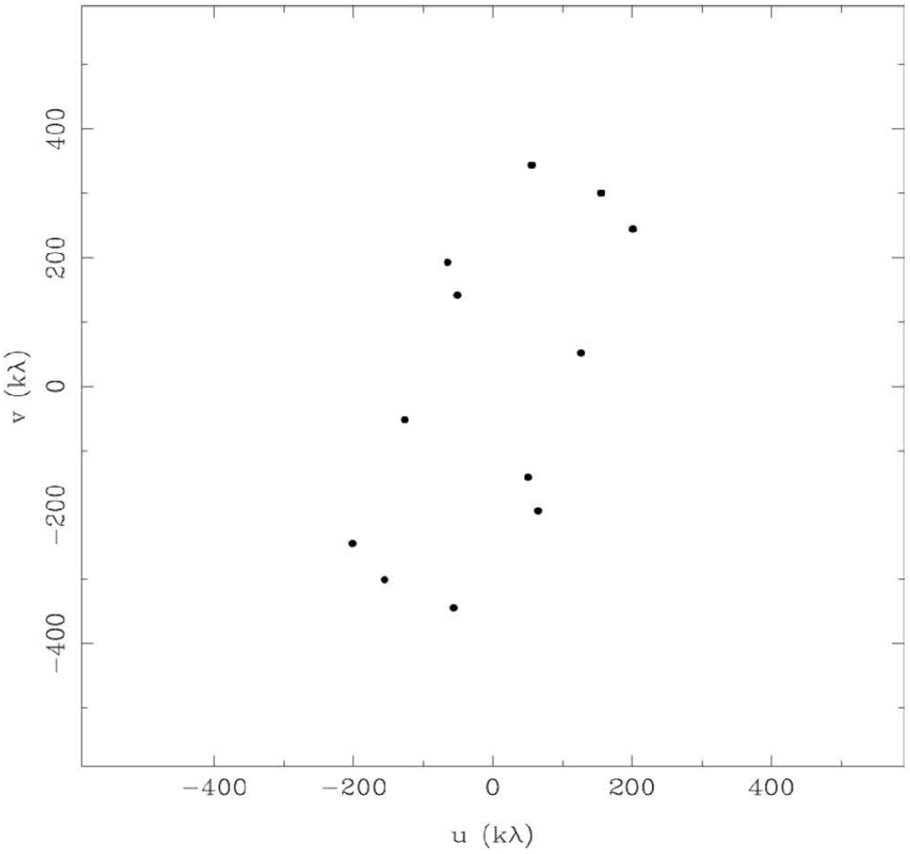
## 2 Antennas

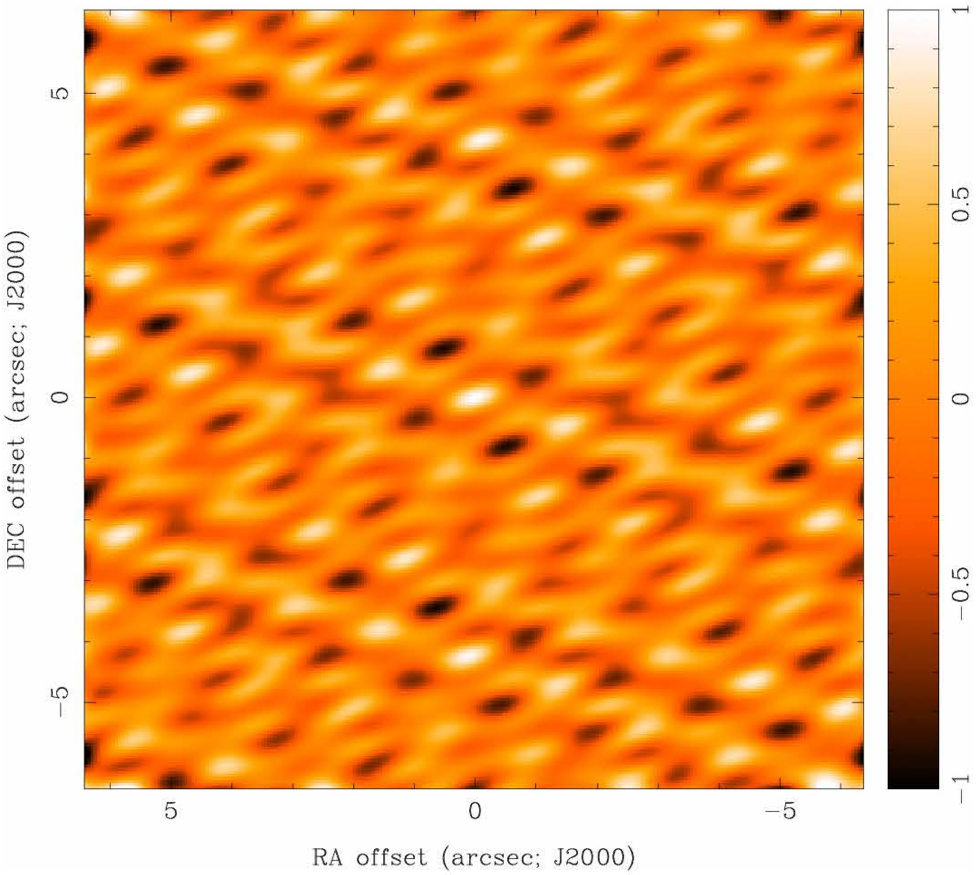




# Dirty Beam Shape and N Antennas

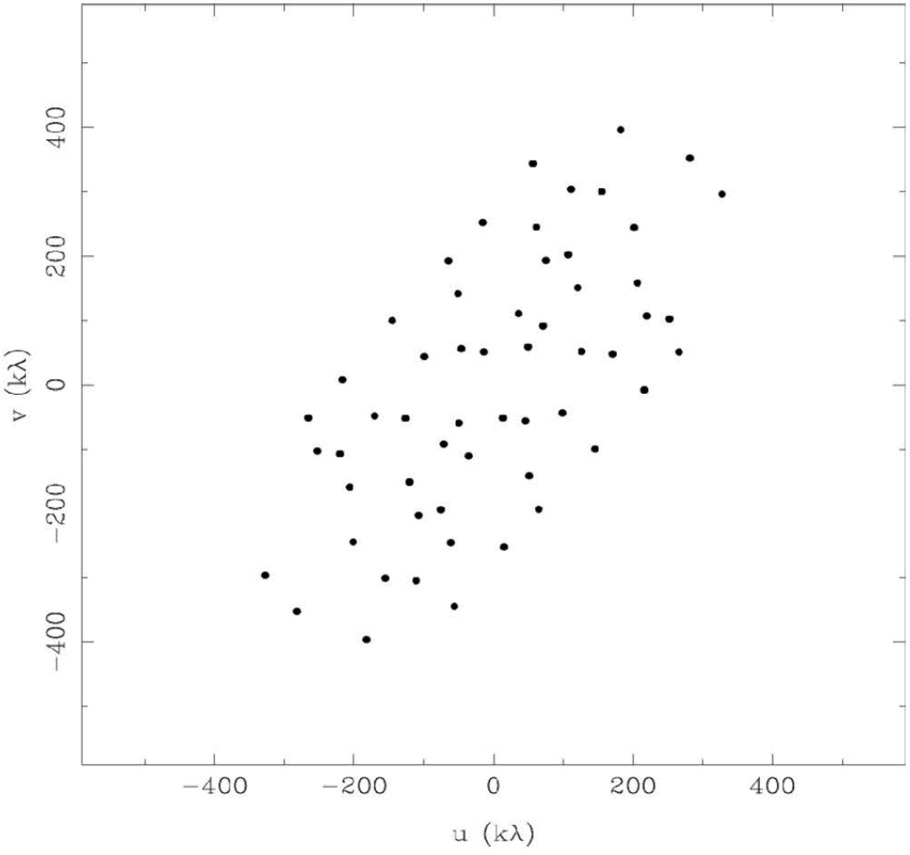
## 4 Antennas

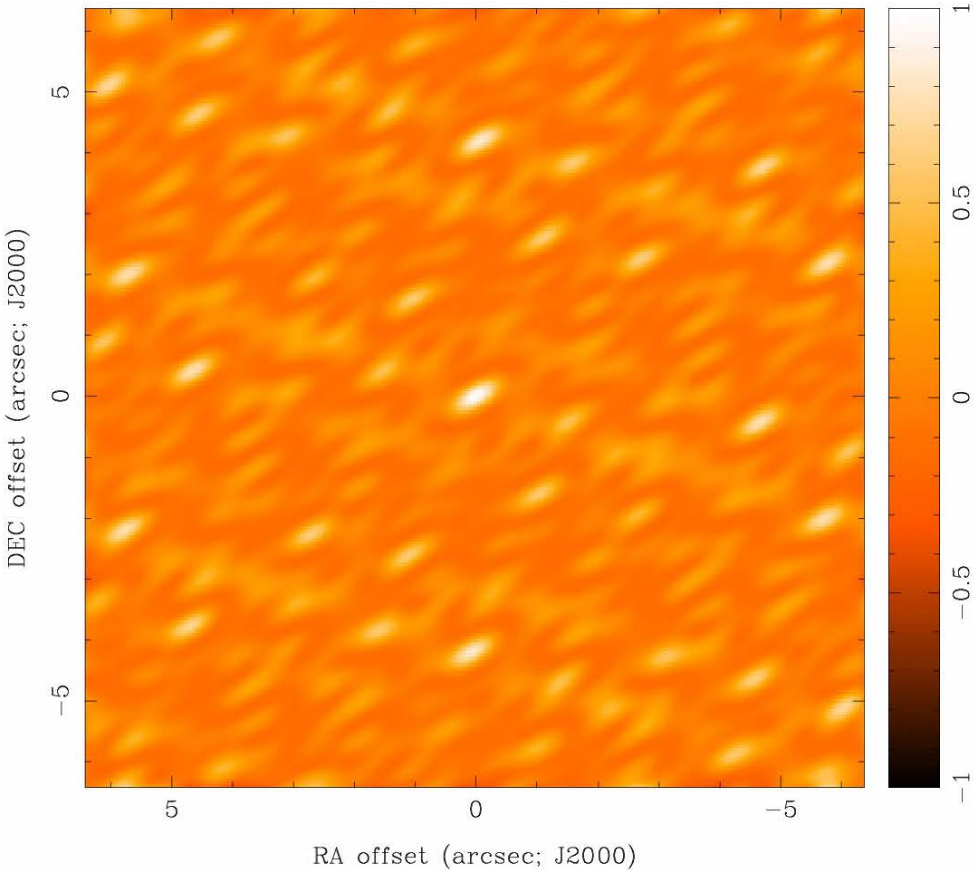




# Dirty Beam Shape and N Antennas

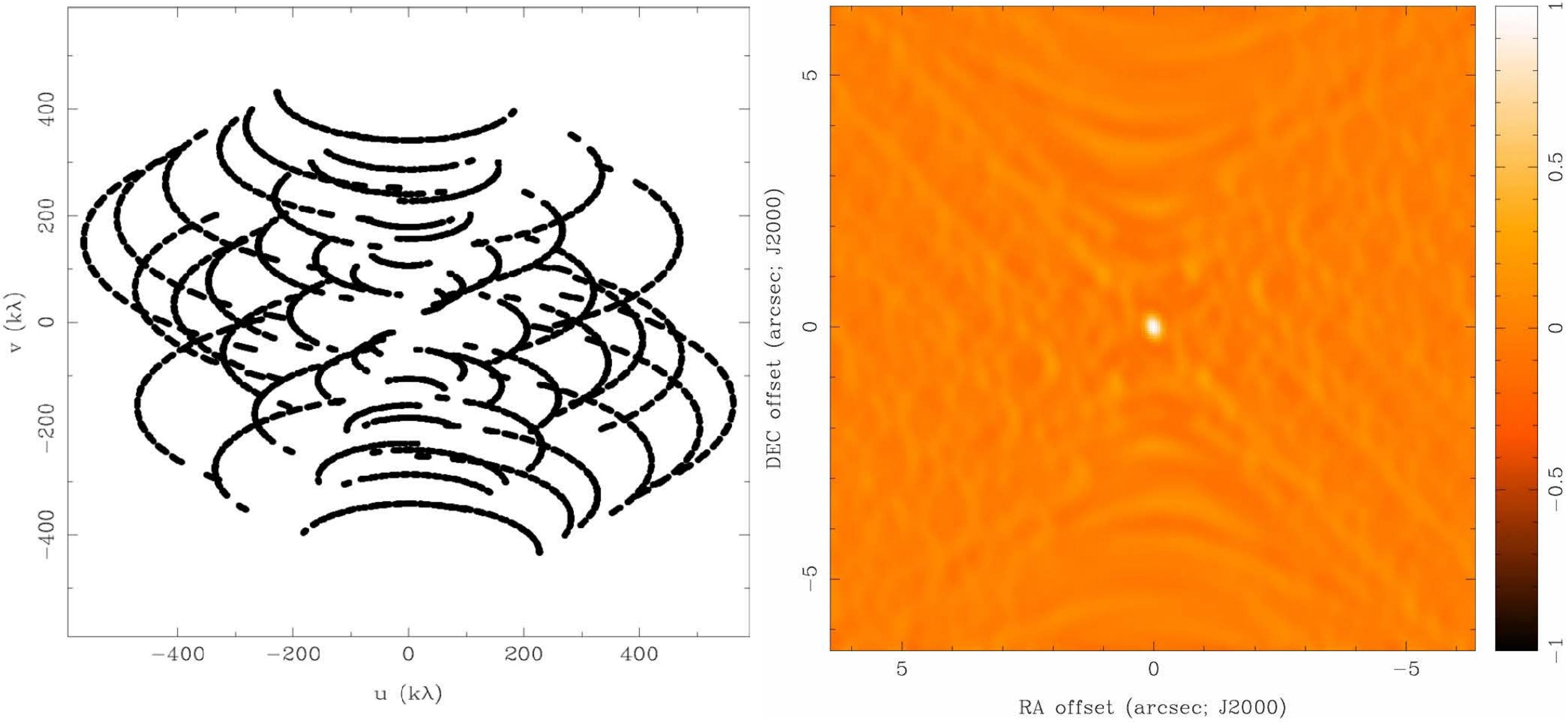
## 8 Antennas



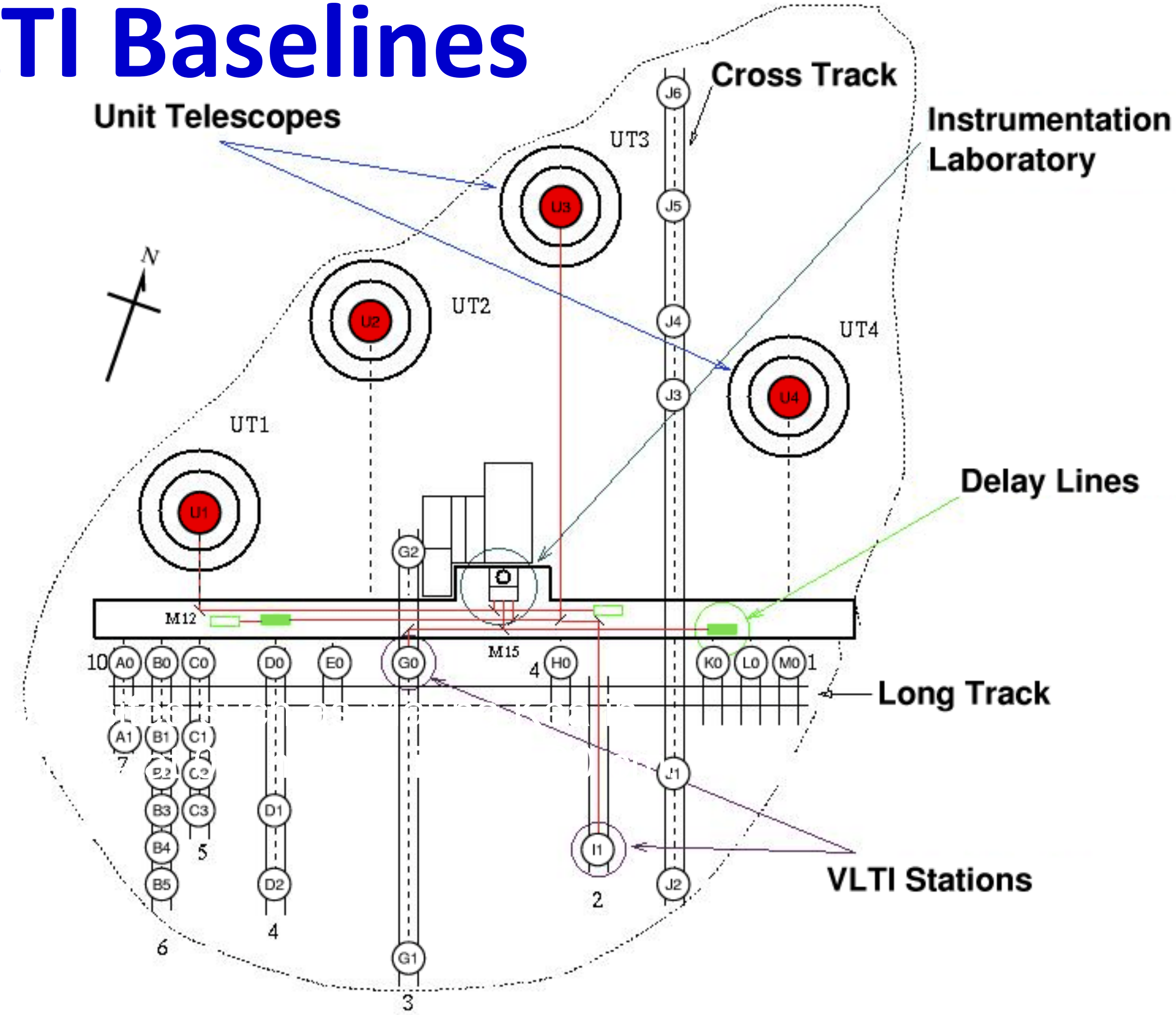


# Dirty Beam Shape and N Antennas

## 8 Antennas x 480 Samples



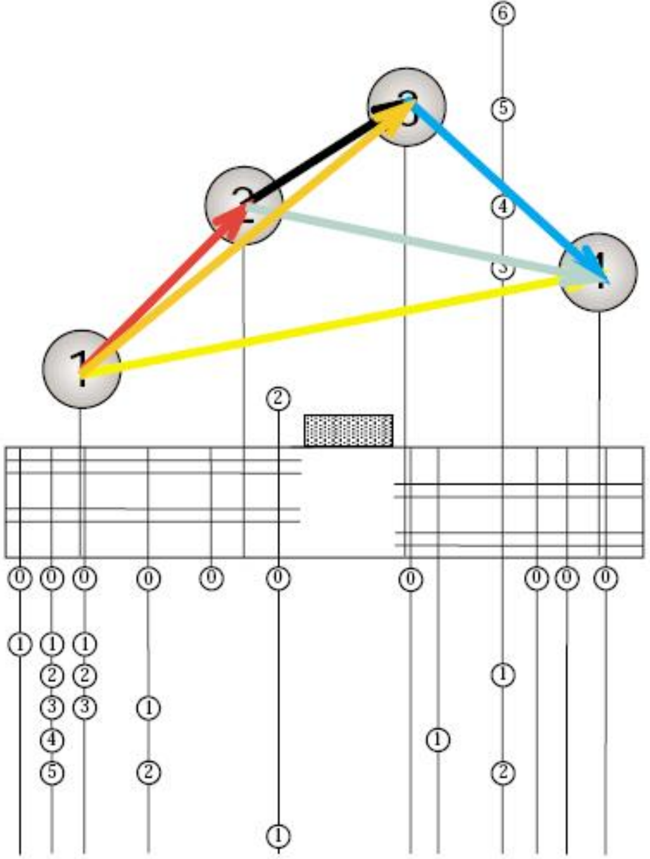
# Optical: VLTI Baselines

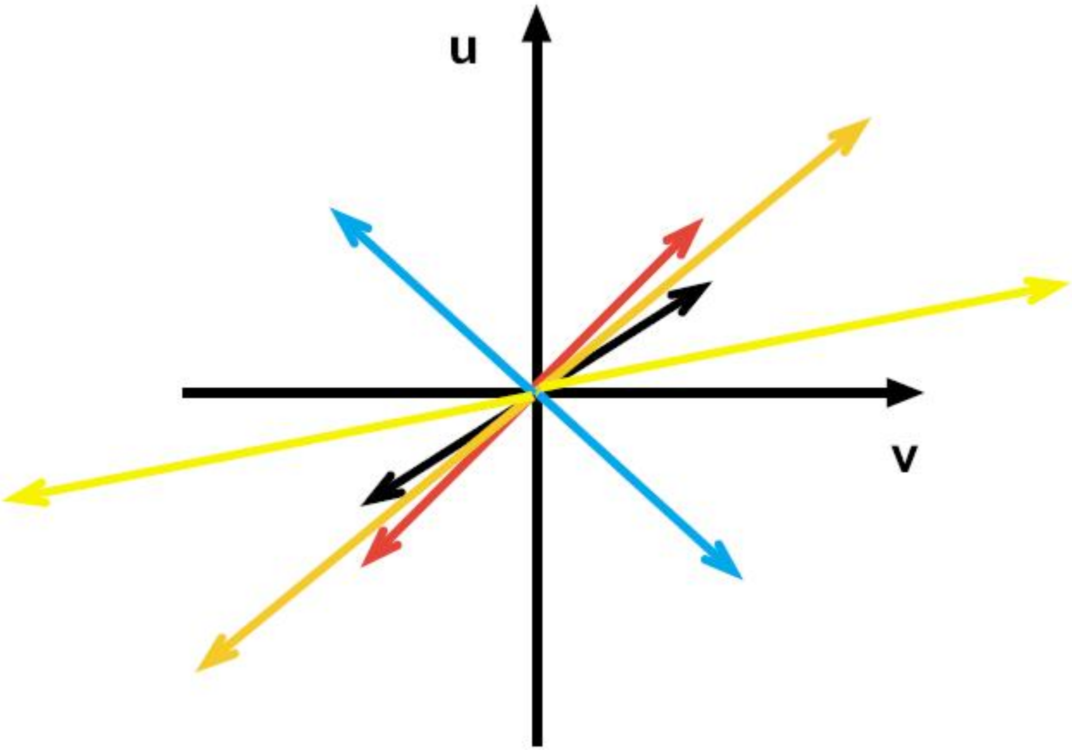


The three ATs move on rails between the thirty observing stations above the holes that provide access to the underlying tunnel system. The light beams from the individual telescopes are guided towards the centrally located, partly underground Interferometry Laboratory

# Baseline Coverage (1)

- Smooth reconstruction of object's intensity distribution requires good coverage of (u,v) plane
- *N telescopes provide N(N-1)/2 baselines*

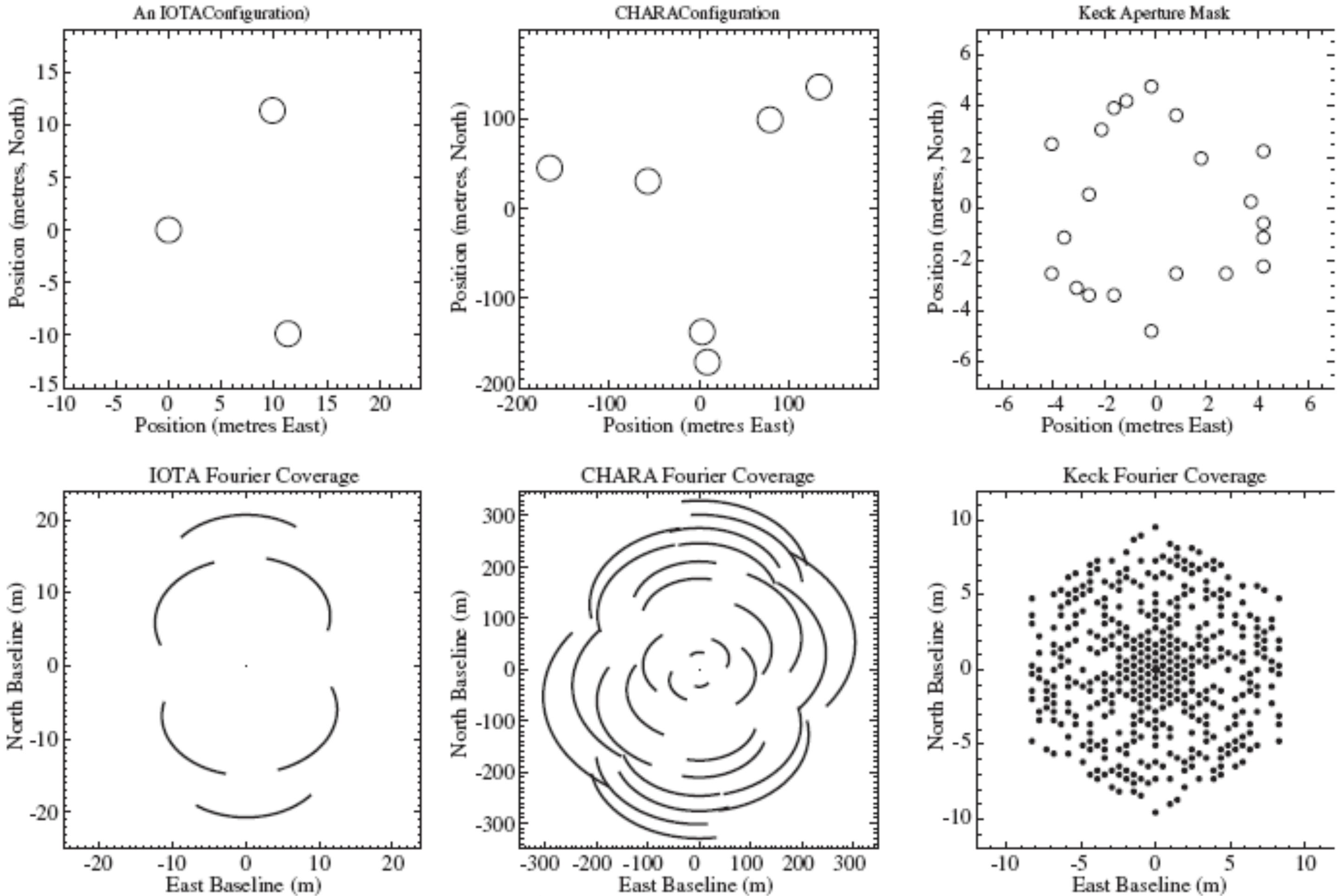




Note: This is the uv-plane for an object at zenith.
In general, the projected baselines have to be used.

# Baseline Coverage (2)

Earth's rotation helps to fill (u,v) plane. *Example: source at 45˚ declination, observed for 3 hr both before and after transit.*

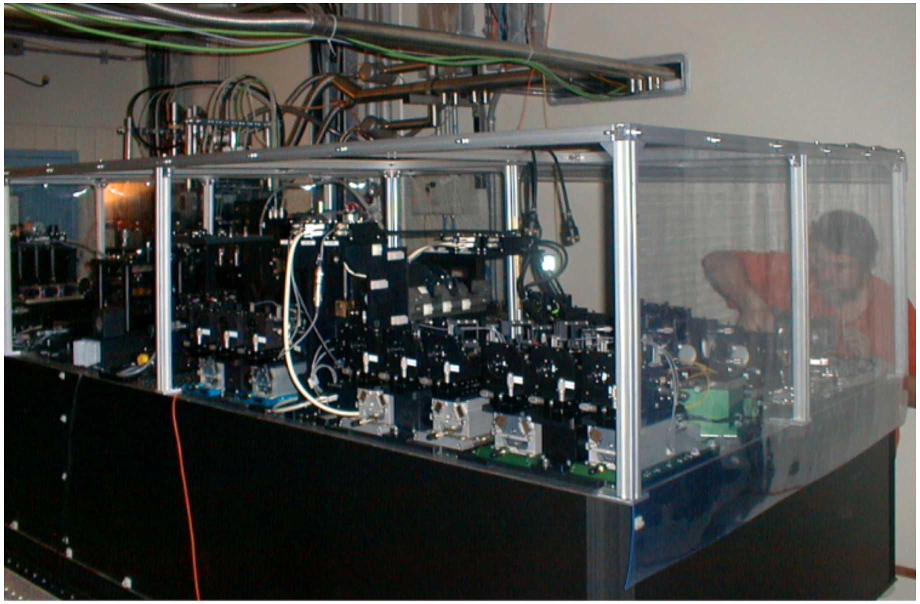


# Field of View

Typically, the field of view is limited to a few arcseconds only:

$$\theta_{\max} \leq \frac{\lambda}{D} \cdot \frac{\lambda}{\Delta\lambda}$$

- the size of the complex transfer optics.  Larger field = larger optical elements
- spatial filters, which limit the FOV



The AMBER Instrument at the VLTI interferometer

# Sensitivity

Signals $V_1$ and $V_2$ from telescopes 1 and 2 correspond to their respective areas $A_1$ and $A_2$ as:

$$\left\langle V_S^2 \right\rangle \approx \left\langle V_1 \cdot V_2 \right\rangle \propto \sqrt{A_1 A_2} = A_{\text{effective}}^{\text{interferometer}}$$

Thus, the effective area of an interferometer with two identical elements is that of one of the elements.

*However, the noise from the two elements is independent and hence the noise output from the correlator is reduced by √2*
*→ S/N of a two-element interferometer is √2 higher.*

Since an array of N identical telescopes can be seen as N(N-1)/2 two-element interferometers, their net sensitivity is given by:

$$\left(\frac{S}{N}\right)_c \approx \frac{T_S}{T_N^S}\left[N(N-1)(\Delta f_{IF}\Delta t)\right]^{1/2}$$

# Dirty Images & CLEAN

Ideally measure visibilities with densely covered (*u,v*) plane, just as for a filled aperture telescope, to get:

$$I(x,y)=\iint V(u,v)e^{2\pi j(ux+vy)}dudv$$

However, interferometer arrays leave gaps in the (u,v) plane, resulting in a "dirty" image:

$$I_D=\iint V(u,v)S(u,v)e^{2\pi j(ux+vy)}dudv$$

where S(*u,v*) describes the sampling of the (*u,v*) plane. S(*u,v*) = 1 where measurements exist, and is zero elsewhere.

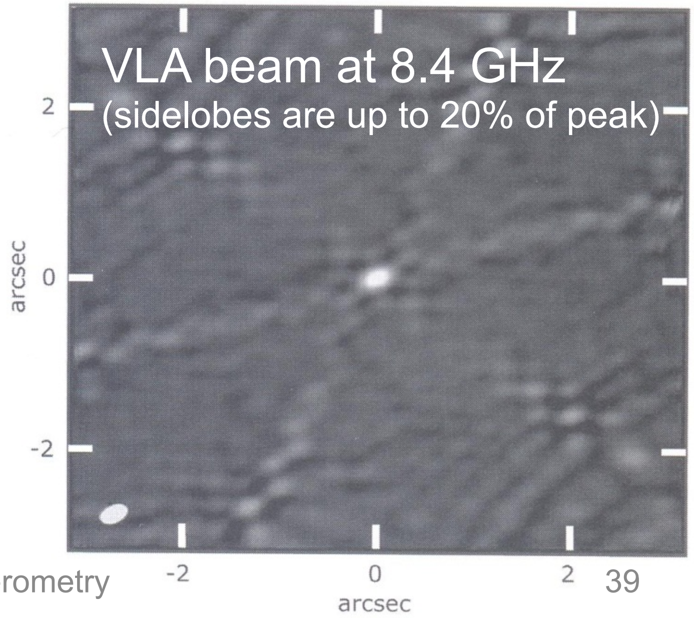
Hence, our image has artifacts and can be described by:

$$I_D(x,y)=I(x,y)\otimes B(x,y)$$

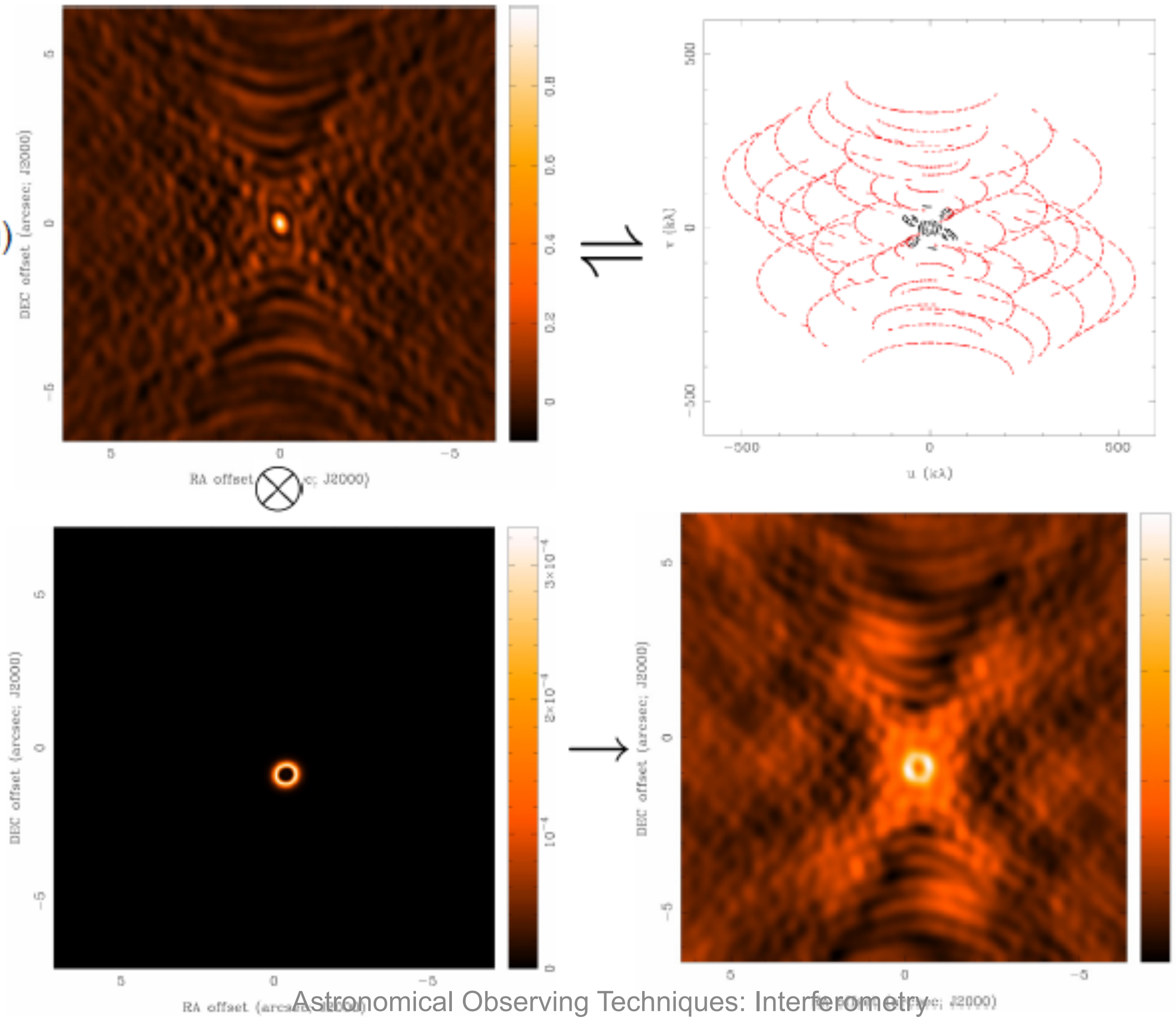
i.e., the true distribution is convolved with the PSF

$$B(x,y)=\iint S(u,v)e^{2\pi j(ux+vy)}dudv$$

Use the CLEAN (Högbom 1974) algorithm to iteratively remove the "dirty beam".


VLA beam at 8.4 GHz
(sidelobes are up to 20% of peak)

# Dirty Beam and Dirty Image



b(x,y)
(dirty beam)

B(u,v)

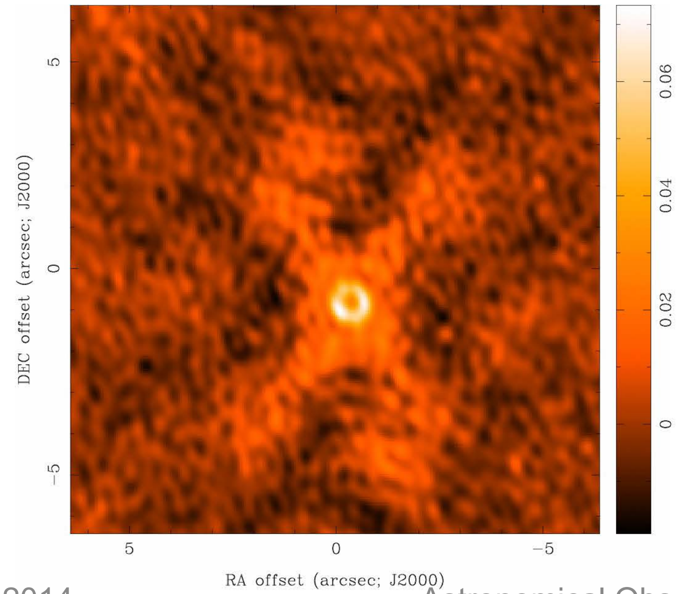
T(x,y)

$T^D(x,y)$
(dirty image)

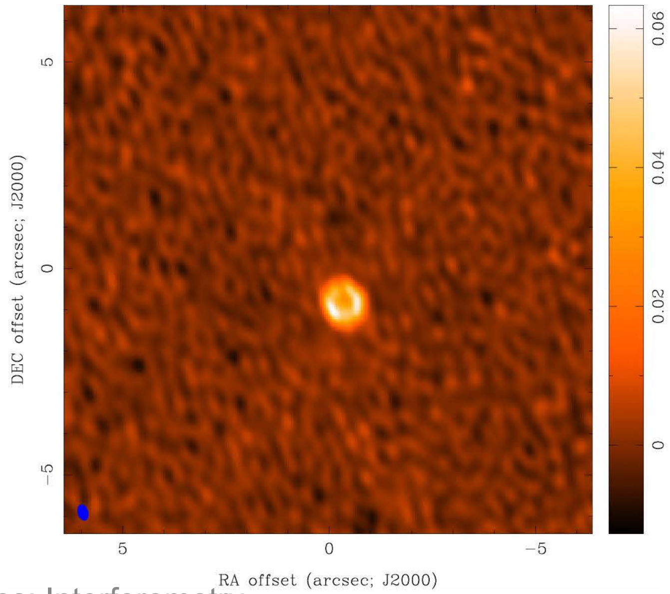
# Deconvolution

- difficult to do science with dirty image
- deconvolve b(x,y) from $Y^0$(x,y) to recover T(x,y)
- information is missing and noise is present -> difficult problem

dirty image



“CLEAN” image



# Very Large Array VLA

- *Y-shaped array of 27 telescopes moved on railroad tracks*
- *telescope diameter 25-m each*
- *located: high Plains of San Augustin in New Mexico*
- *“D", "C", "B", and "A" configurations, spanning 1.0, 3.4, 11, and 36 km, respectively*

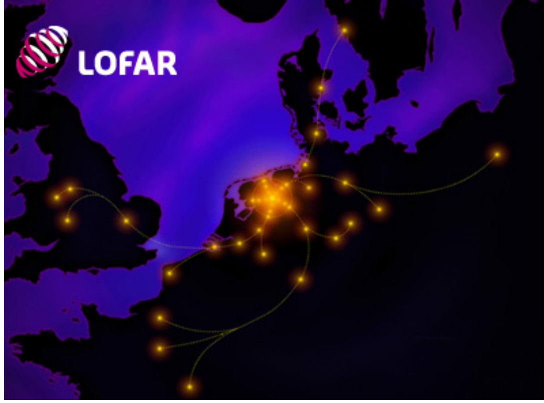
# Australia Telescope Compact Array ATCA

*Six 22 m telescopes on an east-west baseline*

# Westerbork

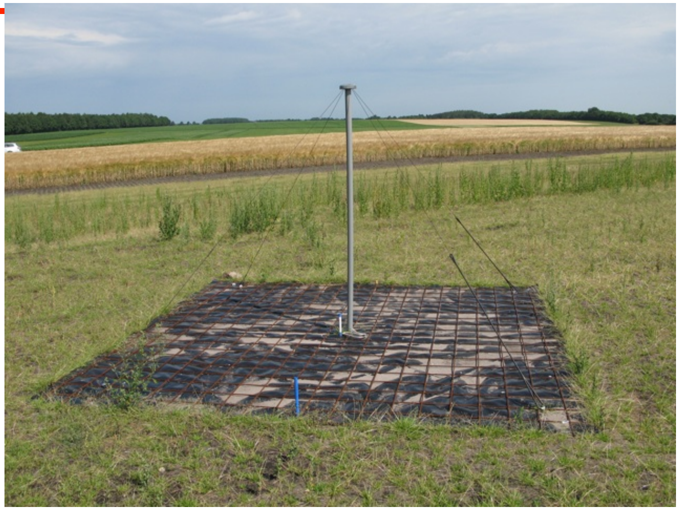
- *Westerbork Synthesis Radio Telescope (WSRT)*
- *14 telescopes*
- *25-meter each*
- *East-west baseline*
- *3 km in length*
- *effective collecting area of a 92 m dish*

# LOFAR in the Netherlands



LOw Frequency ARray uses two types of low-cost antennas:

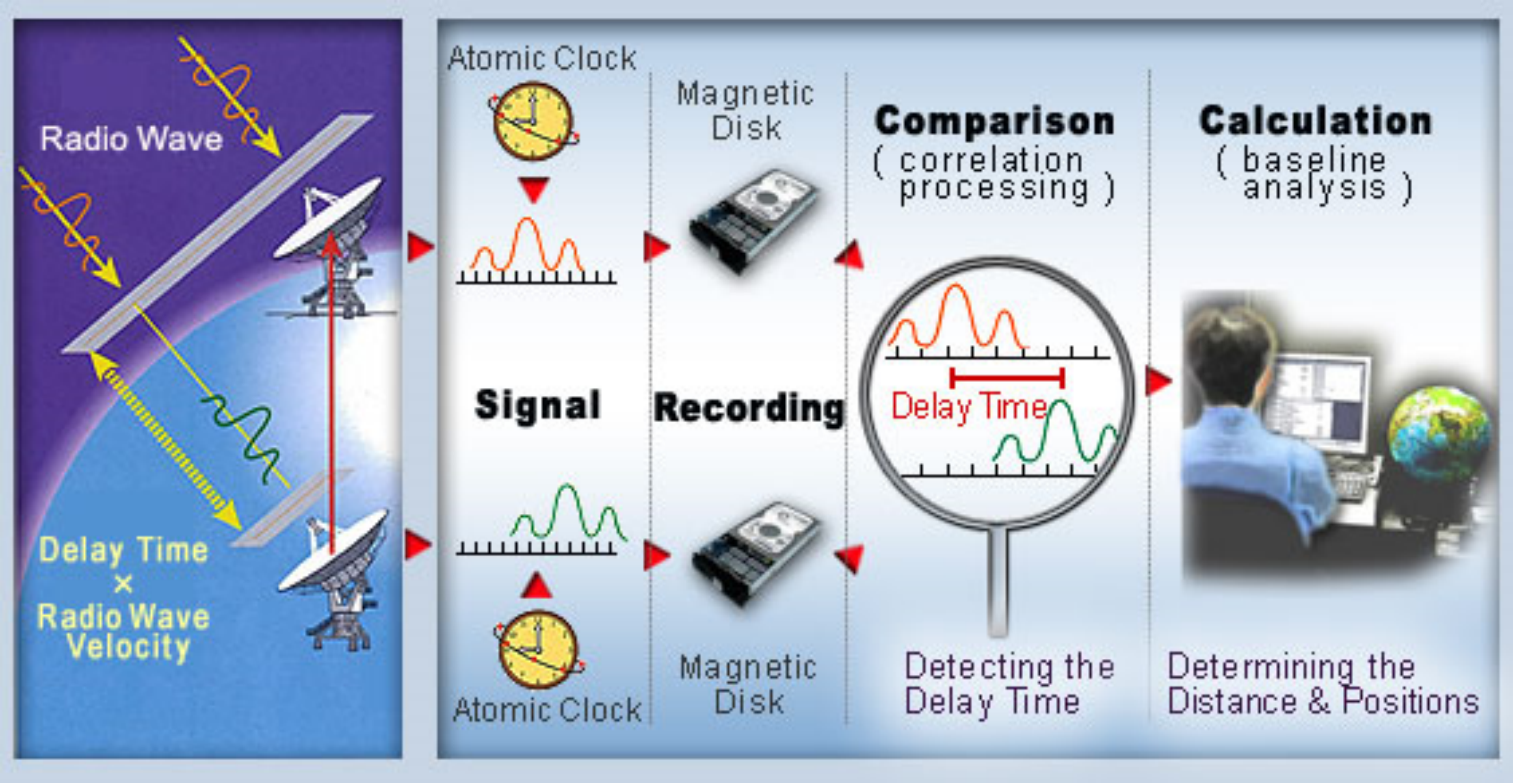
- Low Band Antenna (10-90 MHz)
- High Band Antenna (110-250 MHz)



Antennae are organized in 36 stations over ~100 km. Each station contains 96 LBAs and 48 HBAs

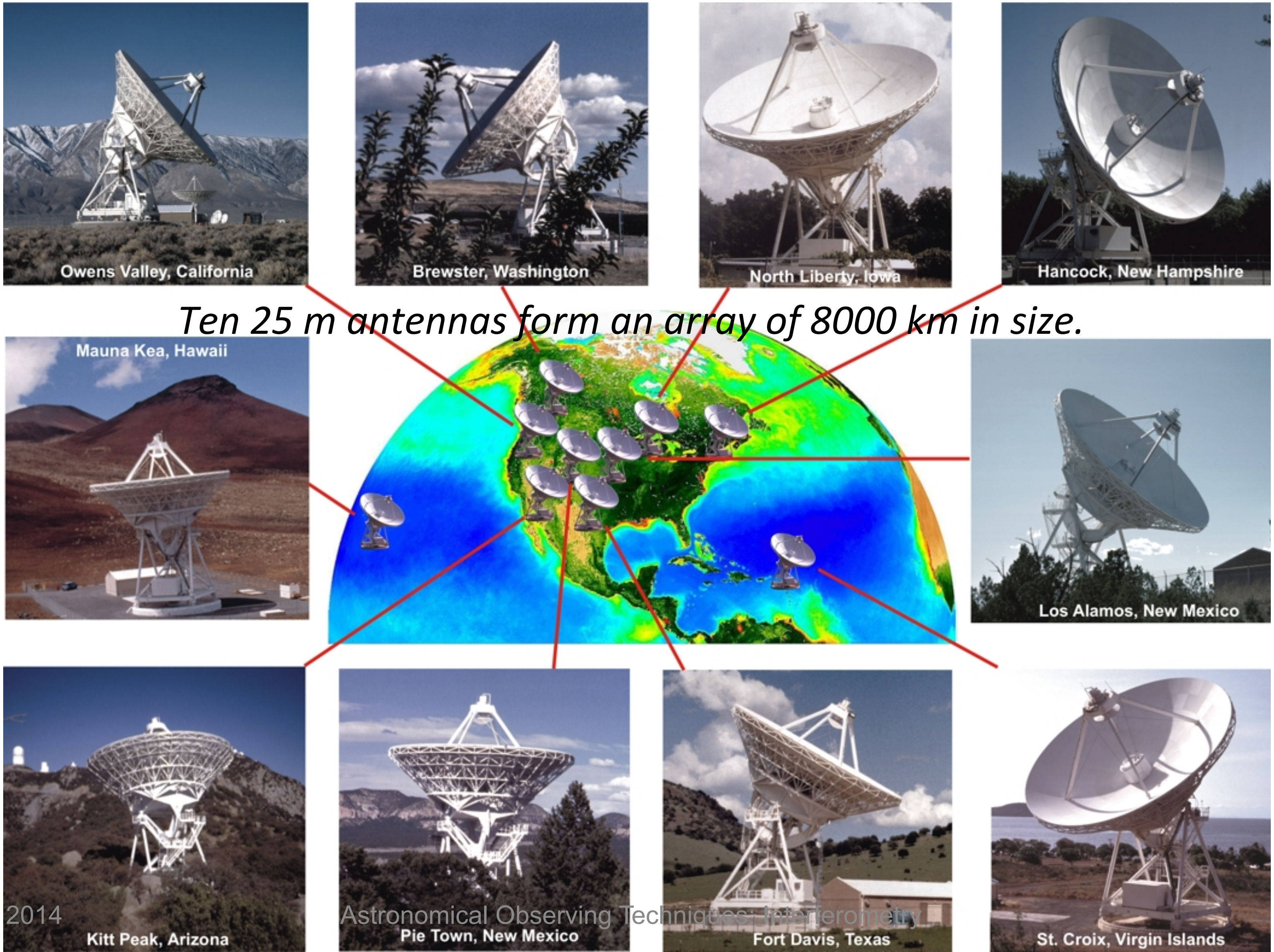
Baselines: 100m – 1500km

# The Very Long Baseline Interferometer (VLBI) Technique



vldb.gsi.go.jp/sokuchi/vlbi/en/whatisvlbi/principle.html

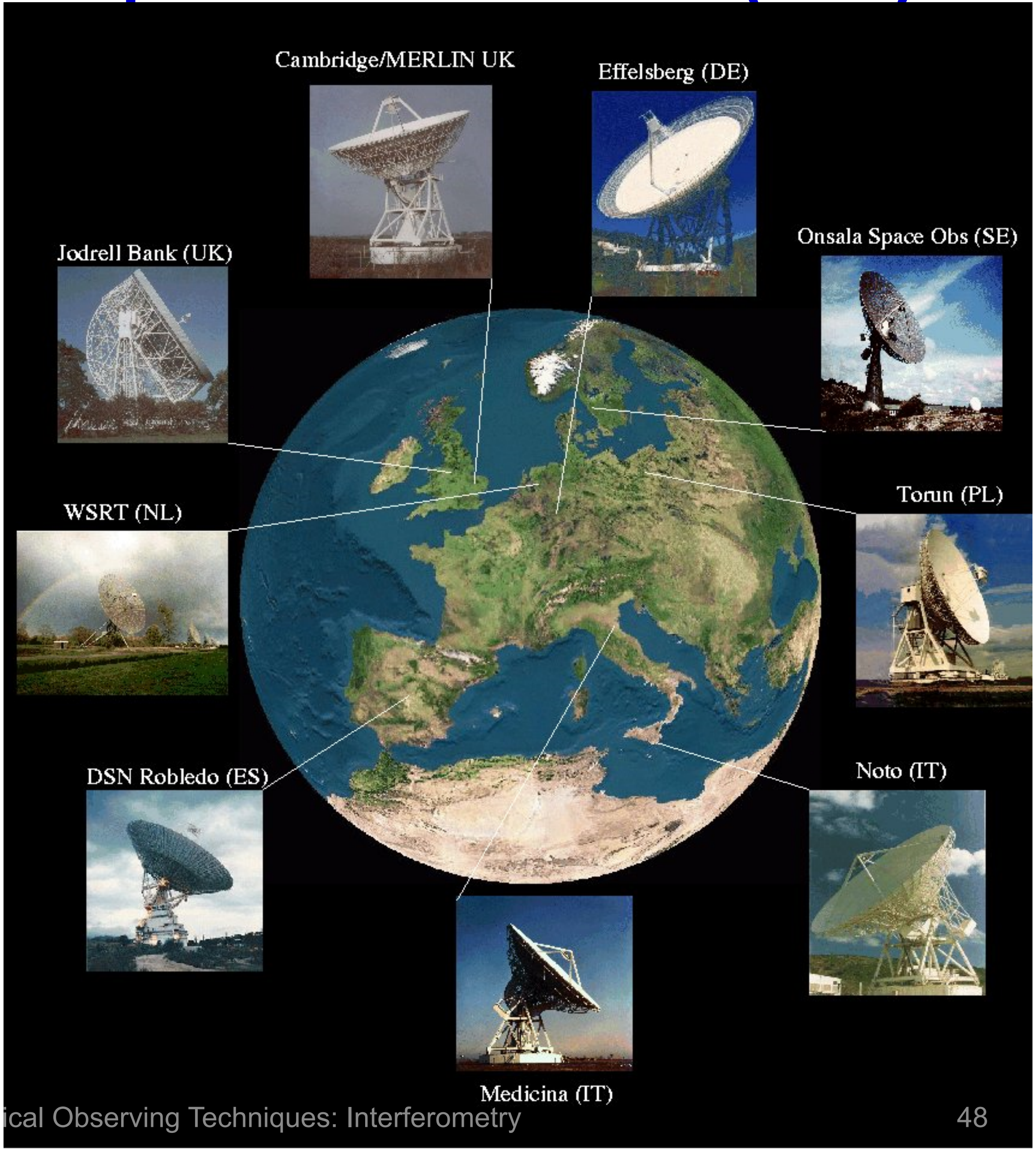
# Very Long Baseline Array VLBA (USA)



*Ten 25 m antennas form an array of 8000 km in size.*

# VLBI in Europe: European VLBI Network (EVN)

- now possible to connect VLBI radio telescopes in real-time → e-VLBI
- In Europe, six radio telescopes of the EVN are now connected with Gbit/s-links
- Data processing in real time at the European Data Processing centre at JIVE (Astron/ Dwingeloo)



# Plateau de Bure

*Interferometer with six 15 m antennas*

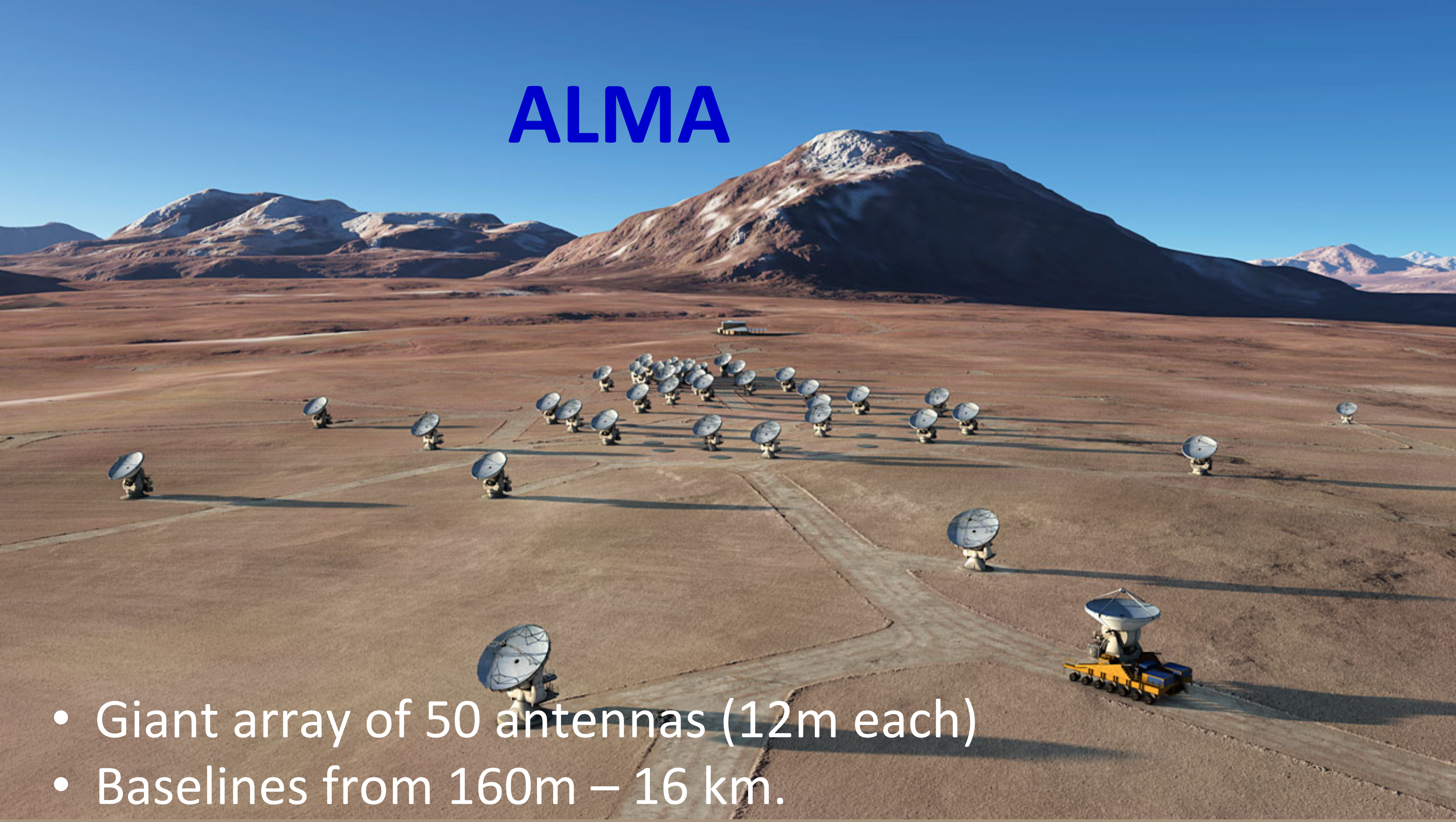
# Combined Array for Research in Millimeter-wave Astronomy (CARMA)

*CARMA = six 10-meter telescopes from Caltech's Owens Valley Radio Observatory + nine 6-meter telescopes from the Berkeley-Illinois-Maryland Association → Cedar (CA)*

# Sub-Millimeter Array (SMA)

*The SMA consists of eight 6 m antennas on Mauna Kea (HI)*

# ALMA



- Giant array of 50 antennas (12m each)
- Baselines from 160m – 16 km.
- Additional compact array of twelve 7m and four 12m antennas
- Located on the Chajnantor plain at 5000m altitude
- Wavelength range 3 mm – 400 μm (84 to 720 GHz)

# ALMA

- Frequencies: Band 3 (>84 GHz) to band 9 (<720 GHz).
- Field of view depends on antenna diameter and frequency
  - independent of array configuration!
  - FWHM of beam: 21" at 300 GHz
  - *uniform sensitivity over larger field requires mosaicking*
- Spatial resolution depends on frequency & maximum baseline
  - Most extended configuration (~16 km): 6 mas at 675 GHz
  - *Structures >* $0.6 » \lambda/b_{min}$ *(*$b_{min}$*=shortest baseline) are not well reproduced in reconstructed images → measure with the ALMA Compact Array (ACA) using the 7-m antennae (come closer)*
- Spectral resolution: up to 8192 frequency channels (spectral resolution elements). At 110 GHz, R=30,000,000 or 10m/s velocity resolution.
- Sensitivity: noise levels or required integration times (**almascience.eso.org/call-for-proposals/sensitivity-calculator**)