Astronomical Observing Techniques

Lecture 6: Optics

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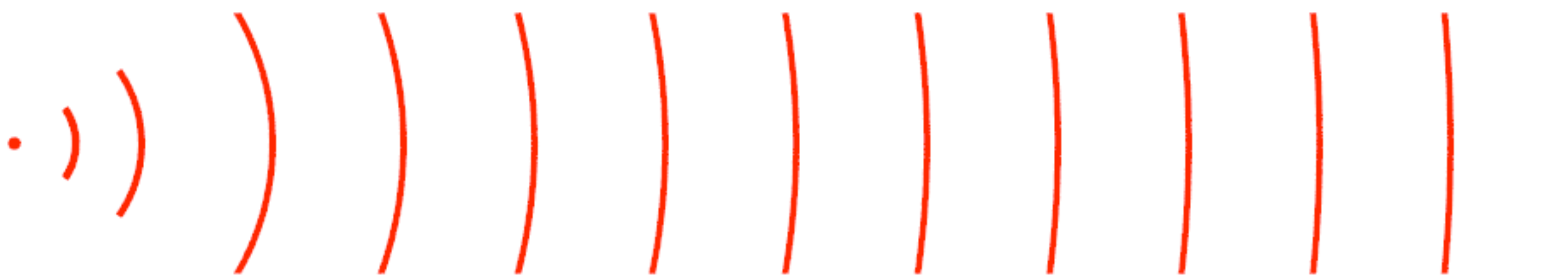
Outline

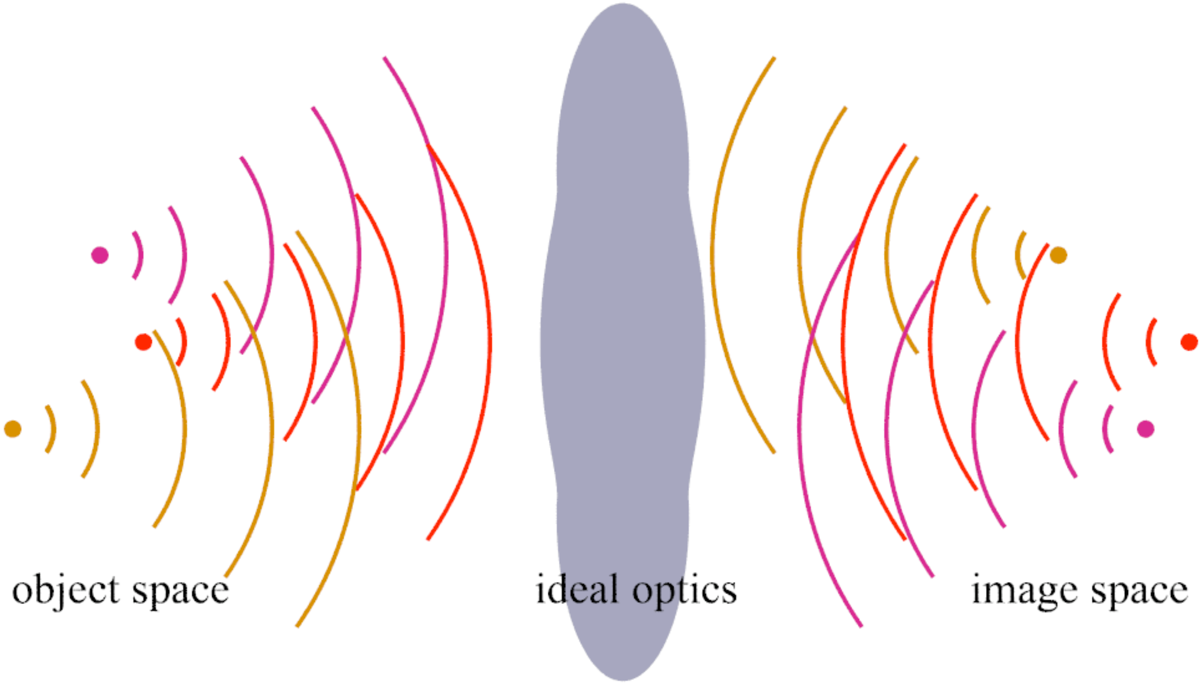
1. Geometrical Optics
    1. Definitions
    2. Aberrations
2. Diffraction Optics
    1. Fraunhofer Diffraction
    2. PSF, MTF
    3. SR & EE
    4. high contrast imaging

Spherical and Plane Waves

- light source: collection of sources of spherical waves
- astronomical sources: almost exclusively incoherent
- lasers, masers: coherent sources
- spherical wave originating at very large distance can be approximated by plane wave

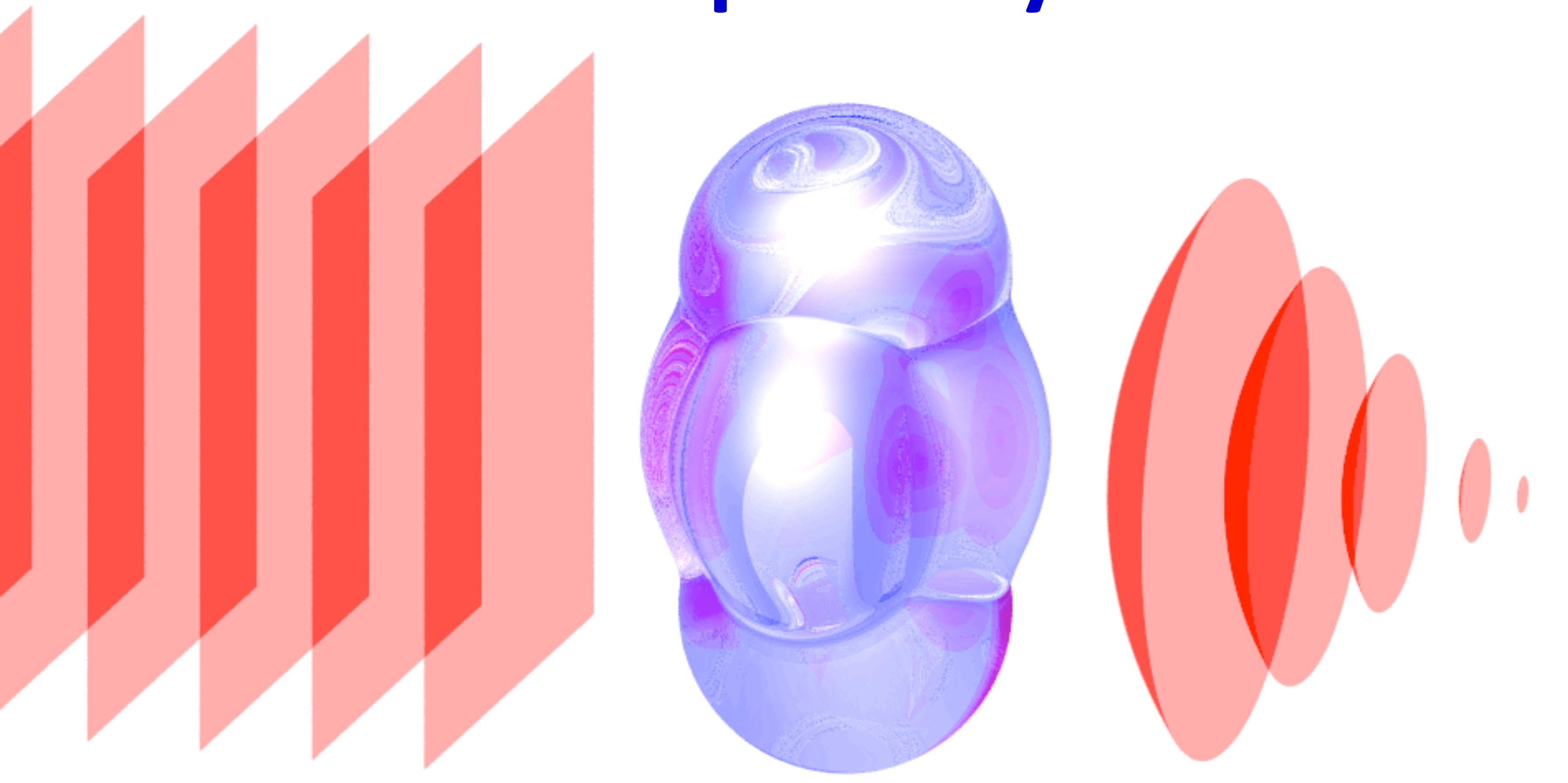


Ideal Optics



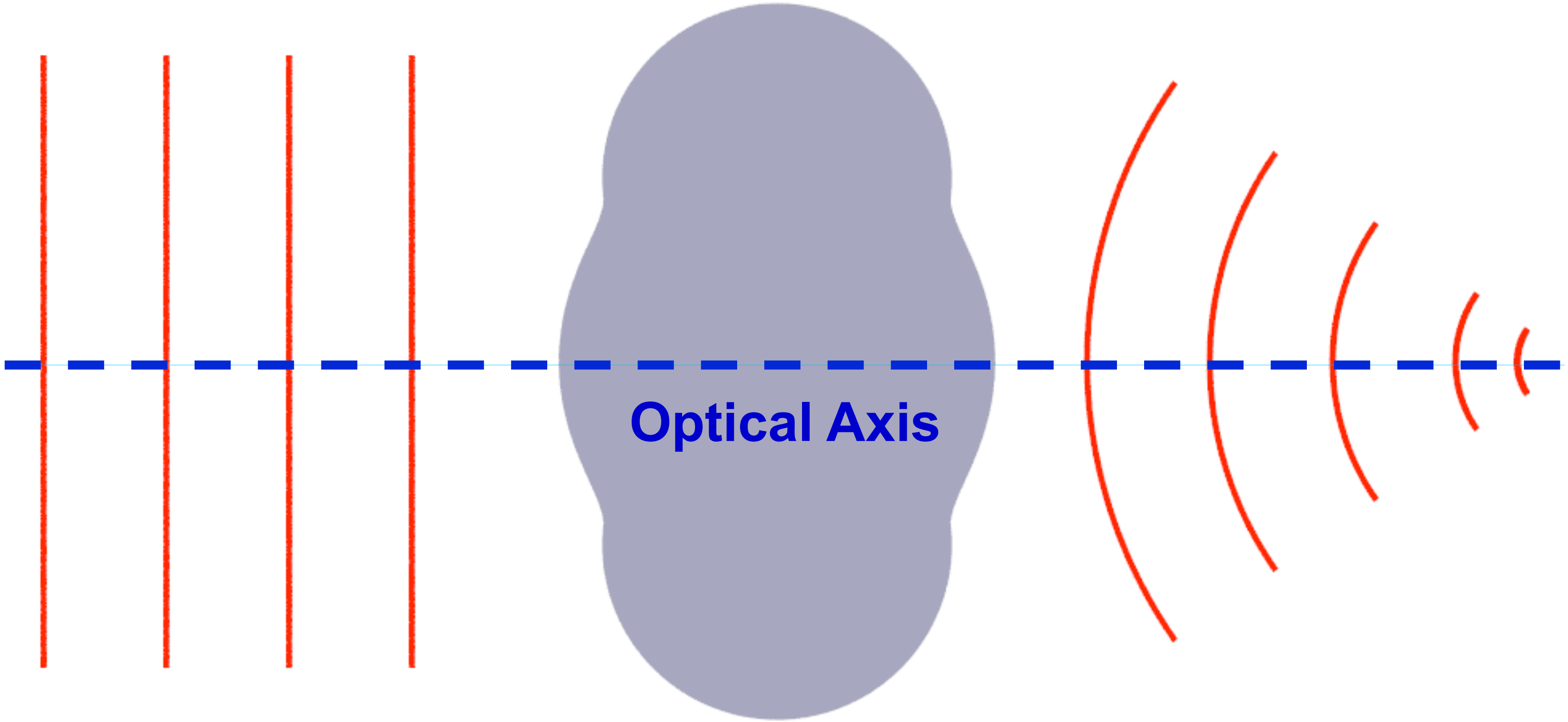
- ideal optics: spherical waves from any point in object space are
- imaged into points in image space
- corresponding points are called conjugate points
- focal point: center of converging or diverging spherical wavefront
- object space and image space are reversible

Ideal Optical System



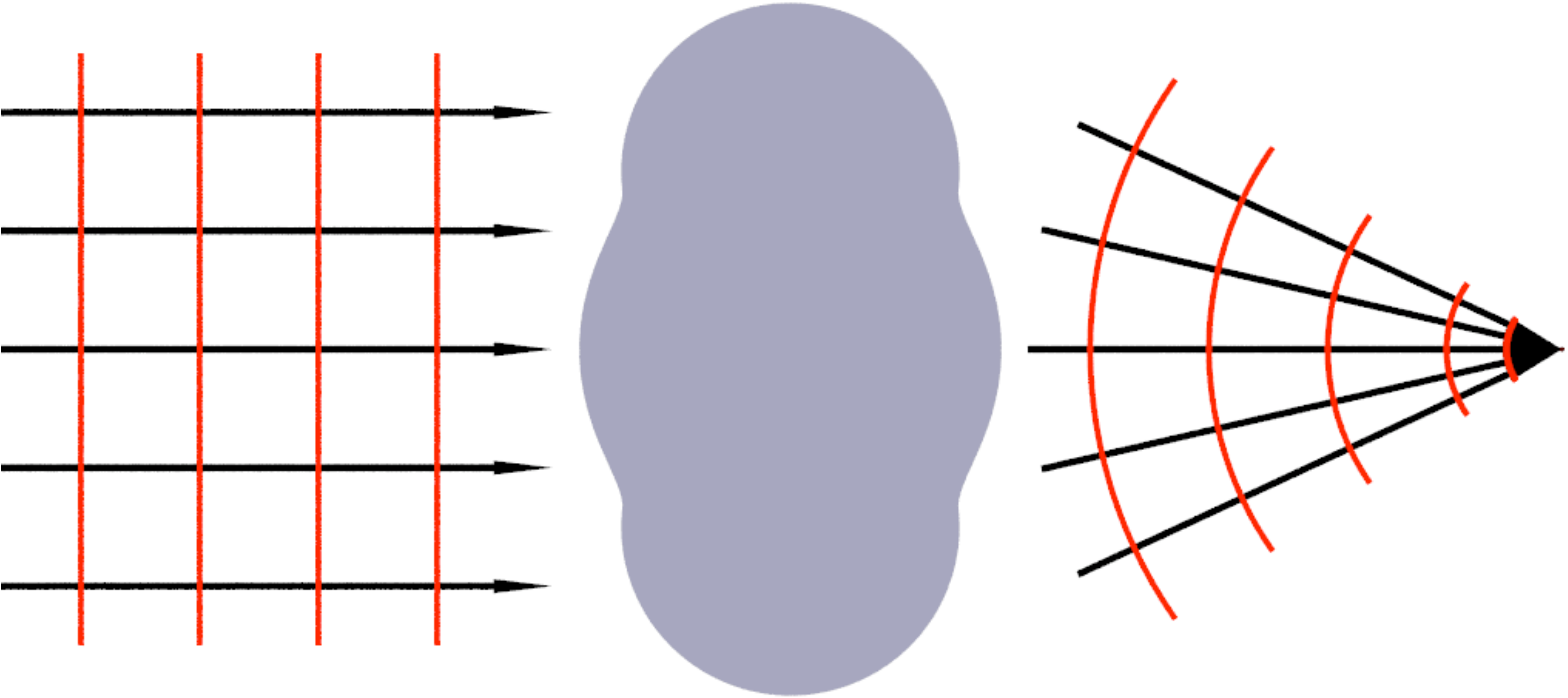
ideal optical system transforms plane wavefront into spherical, converging wavefront

Azimuthal Symmetry



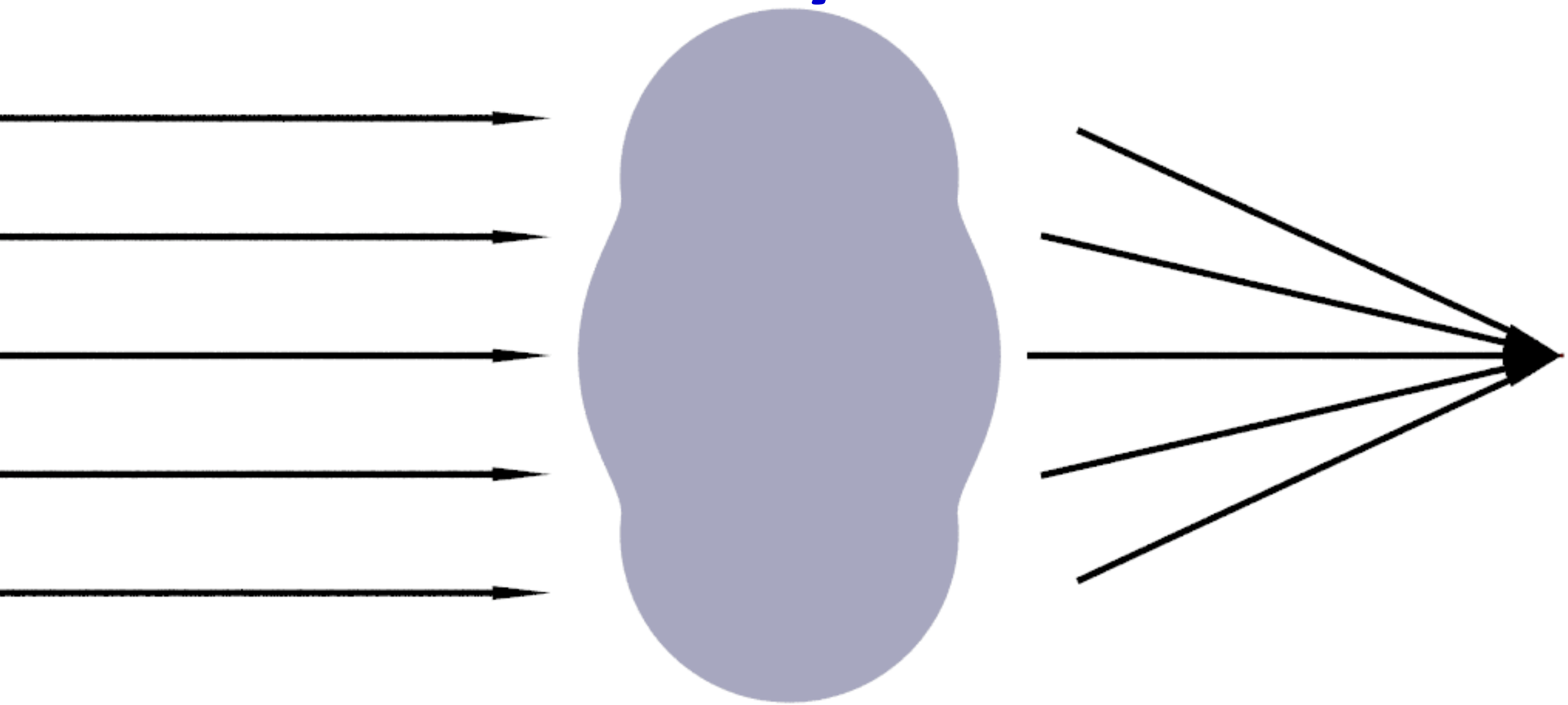
- most optical systems are azimuthally symmetric
- axis of symmetry is optical axis

Locally Flat Wavefronts



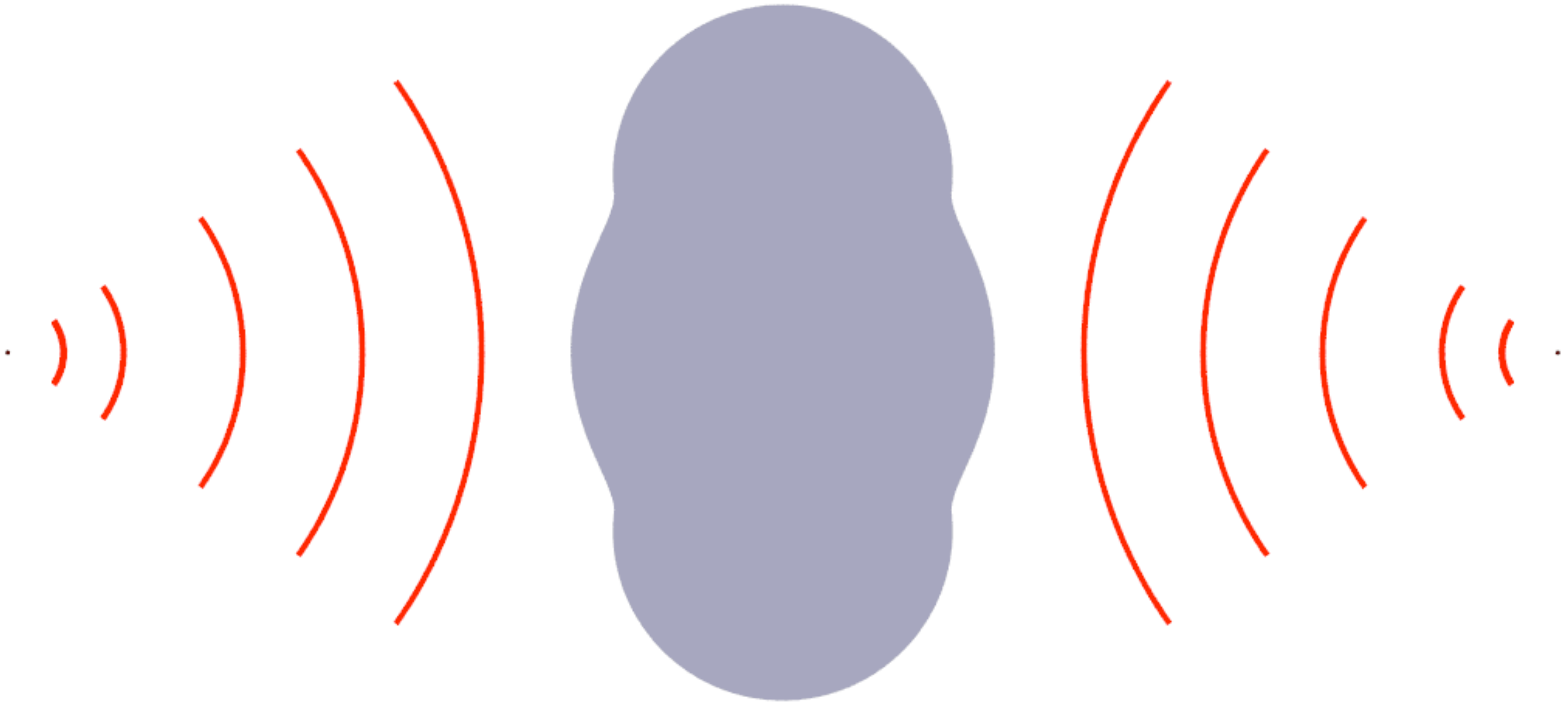
- rays normal to local wave (locations of constant phase)
- local wave around rays is assumed to be plane wave

Rays



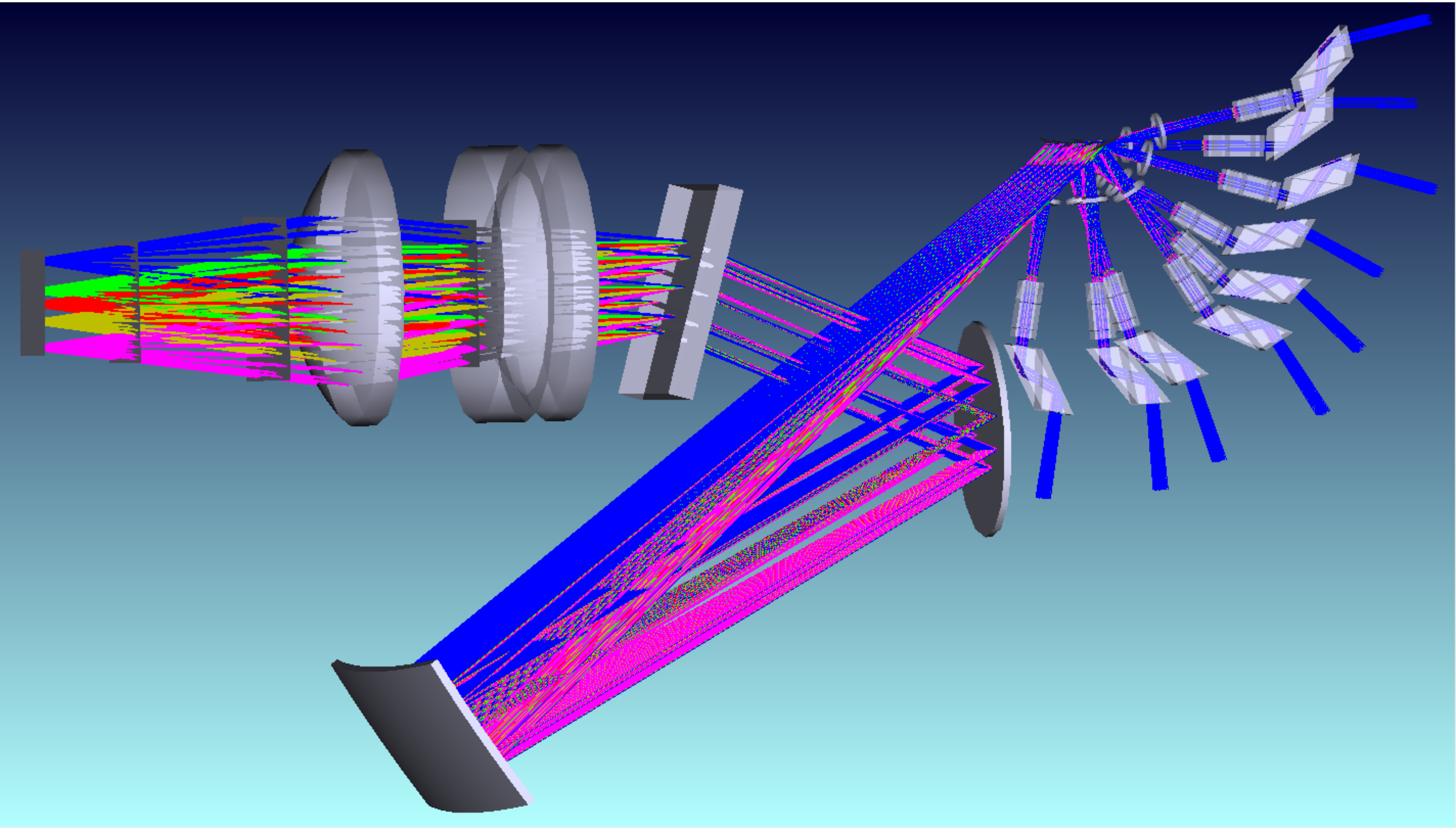
- geomtrical optics works with rays only
- rays reflected and refracted according to Fresnel equ.
- phase is neglected (incoherent sum)

Finite Object Distance

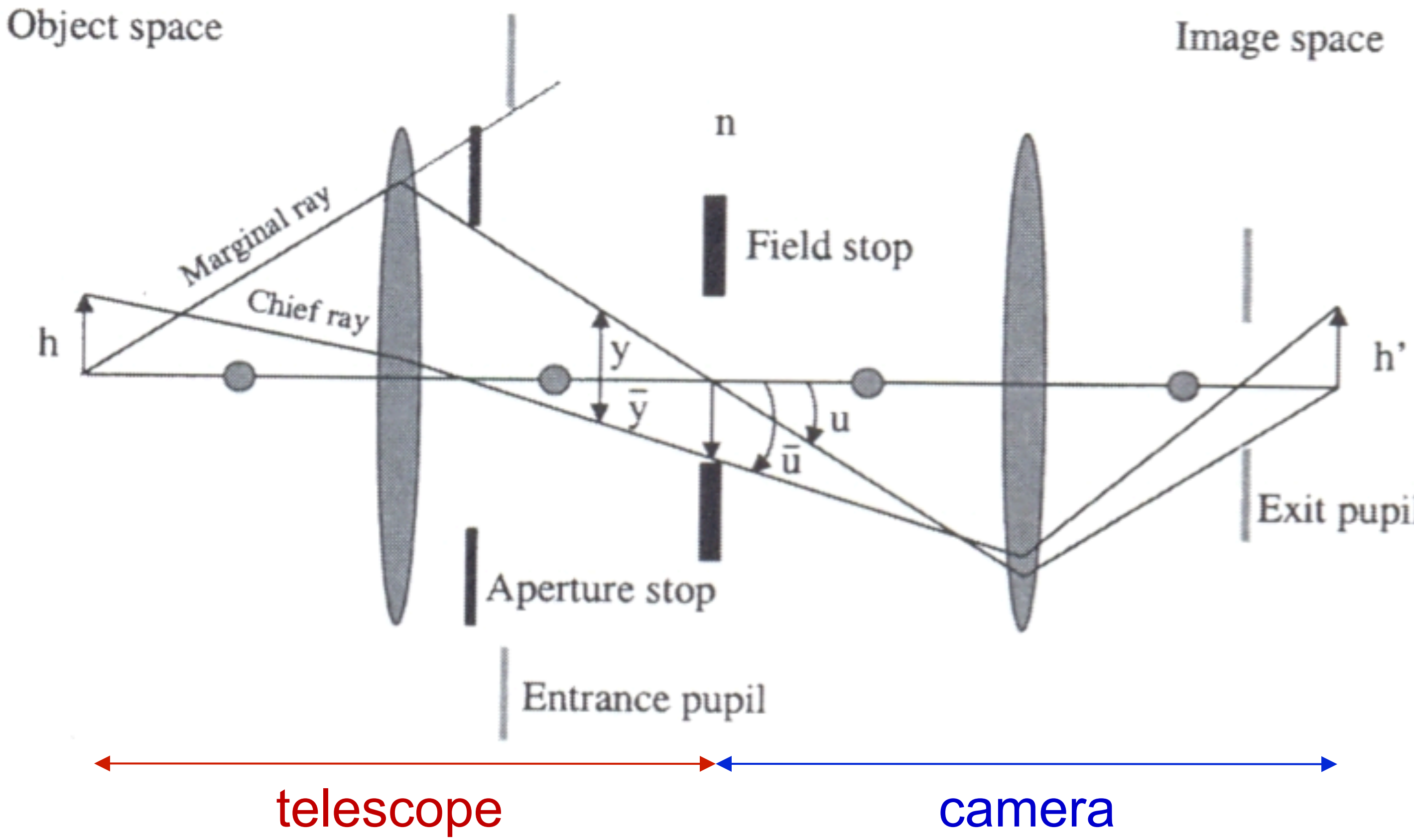


- object may also be at finite distance
- also in astronomy: reimaging within instruments and telescopes

Geometrical Optics Example: SPEX

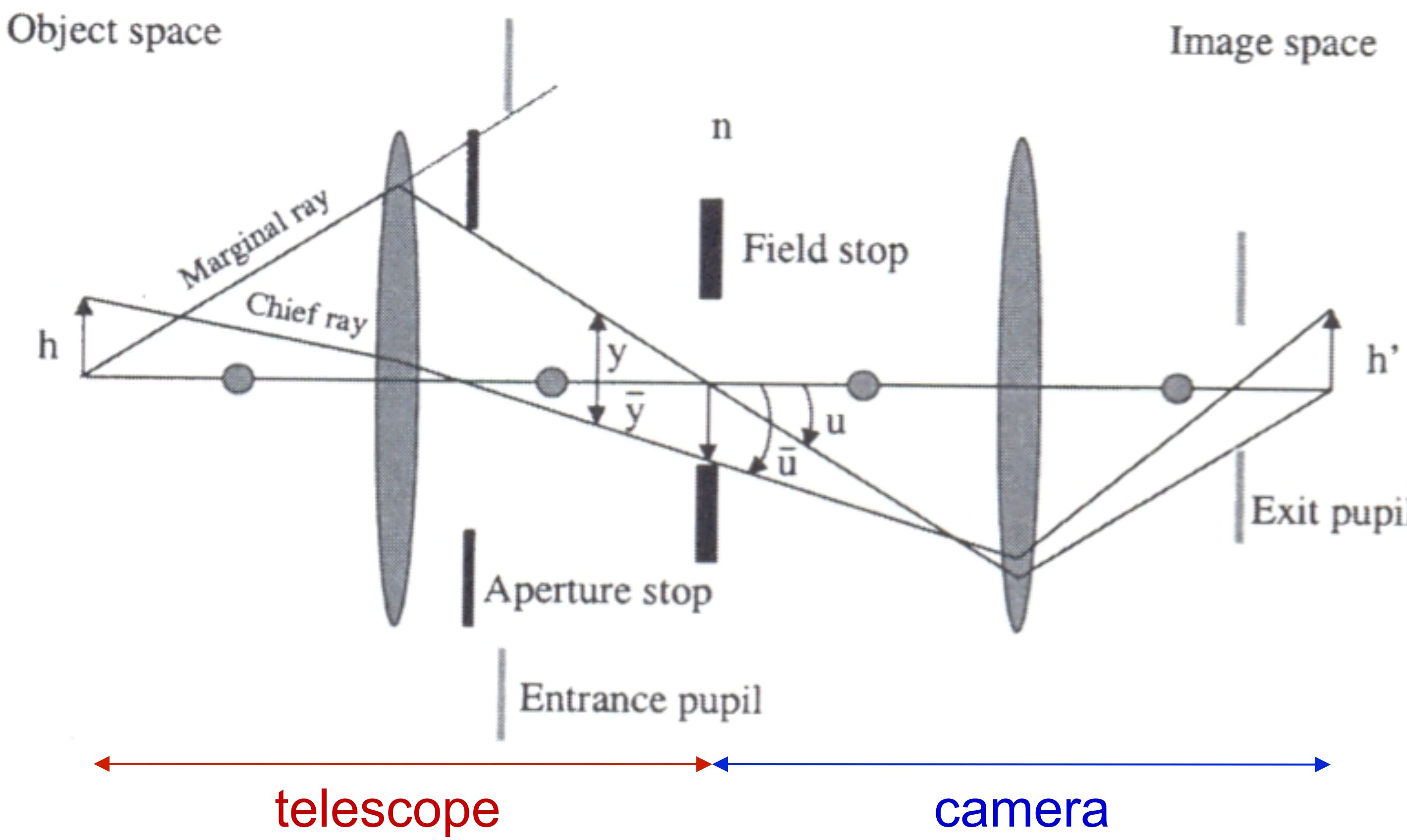


Aperture and Field Stops



- Aperture stop: determines diameter of light cone from axial point on object.
- Field stop: determines the field of view of the system.

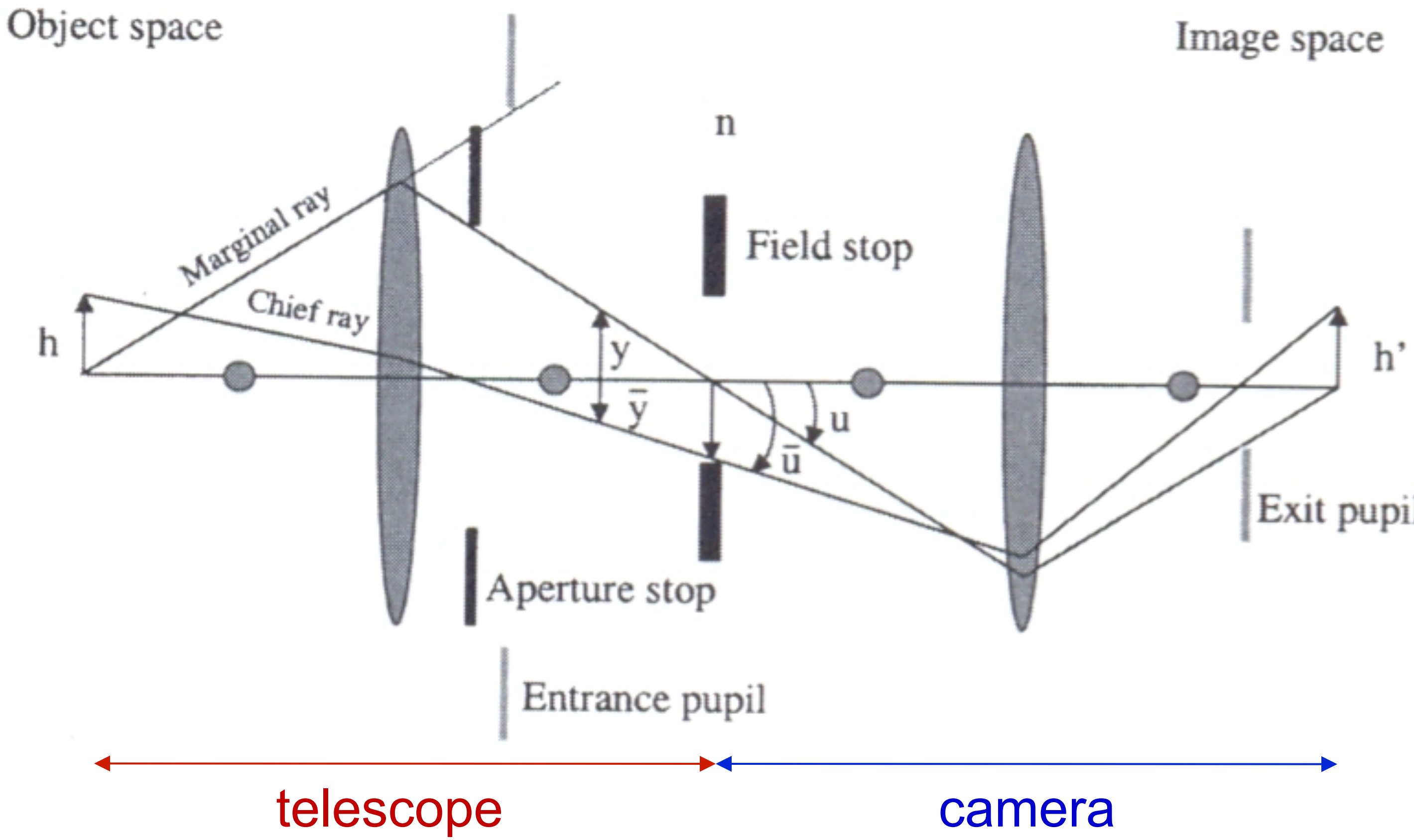
Entrance and Exit Pupils



Entrance pupil: image of aperture stop in object space

Exit pupil: image of aperture stop in image space

Marginal and Chief Rays



- Marginal ray: ray from object point on optical axis that passes at edge of entrance pupil
- Chief ray: ray from an object point at the edge of the field, passing through the center of the aperture stop.

Images and Pupils

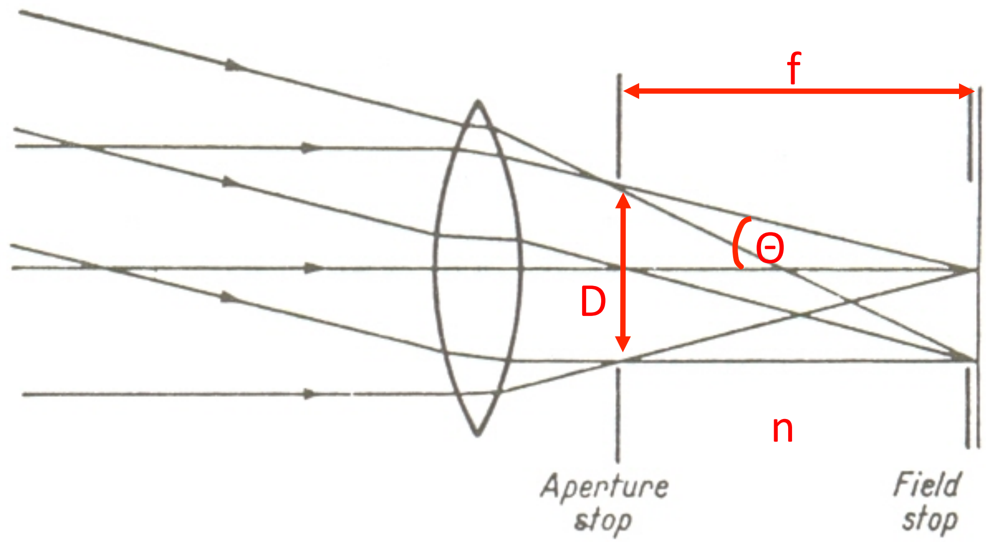
image

- every object point comes to a focus in an image plane
- light in one image point comes from pupil positions
- object information is encoded in position, not in angle

pupil

- all object rays are smeared out over complete aperture
- light in one pupil point comes from different object positions
- object information is encoded in angle, not in position

Speed/F-Number/Numerical Aperture



Speed of optical system described by numerical aperture (NA) or *F* number:

$$\mathrm{NA} = n \cdot \sin\theta \qquad \text{and} \qquad F \equiv \frac{f}{D} = \frac{1}{2(\mathrm{NA})}$$

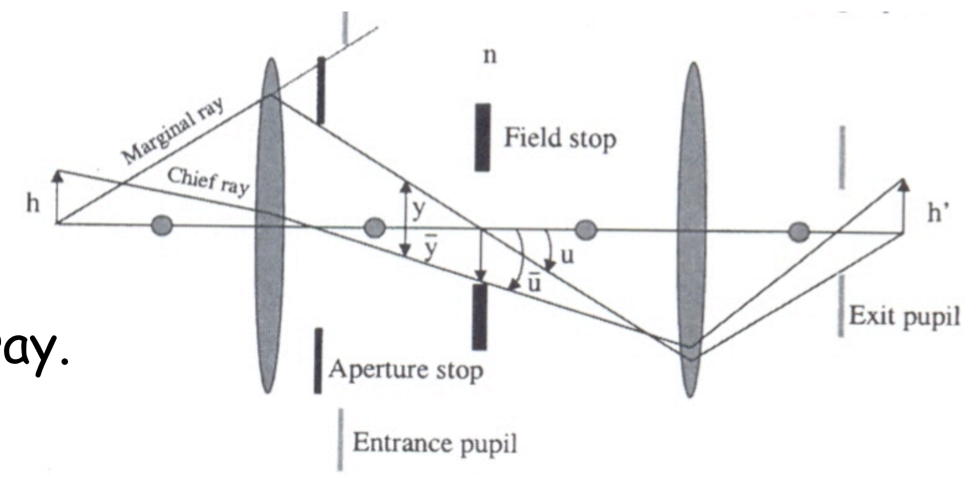
Generally, fast optics (large NA) has a high light power, is compact, but has tight tolerances and is difficult to manufacture.
Slow optics (small NA) is just the opposite.

Étendue and AxΩ

- Geometrical étendue (fr. `extent') is product of area A of source times solid angle Ω of optical system's entrance pupil as seen from source.
- Etendue is maximum beam size that instrument can accept.
- Hence, the étendue is also called acceptance, throughput, or A×Ω product.
- The étendue never increases in any optical system. A perfect optical system produces an image with the same étendue as the source.

Shrinking the field size A makes the beam faster (Ω bigger).

Area $A=h^2\pi$;
Solid angle Ω given by marginal ray.



Aberrations

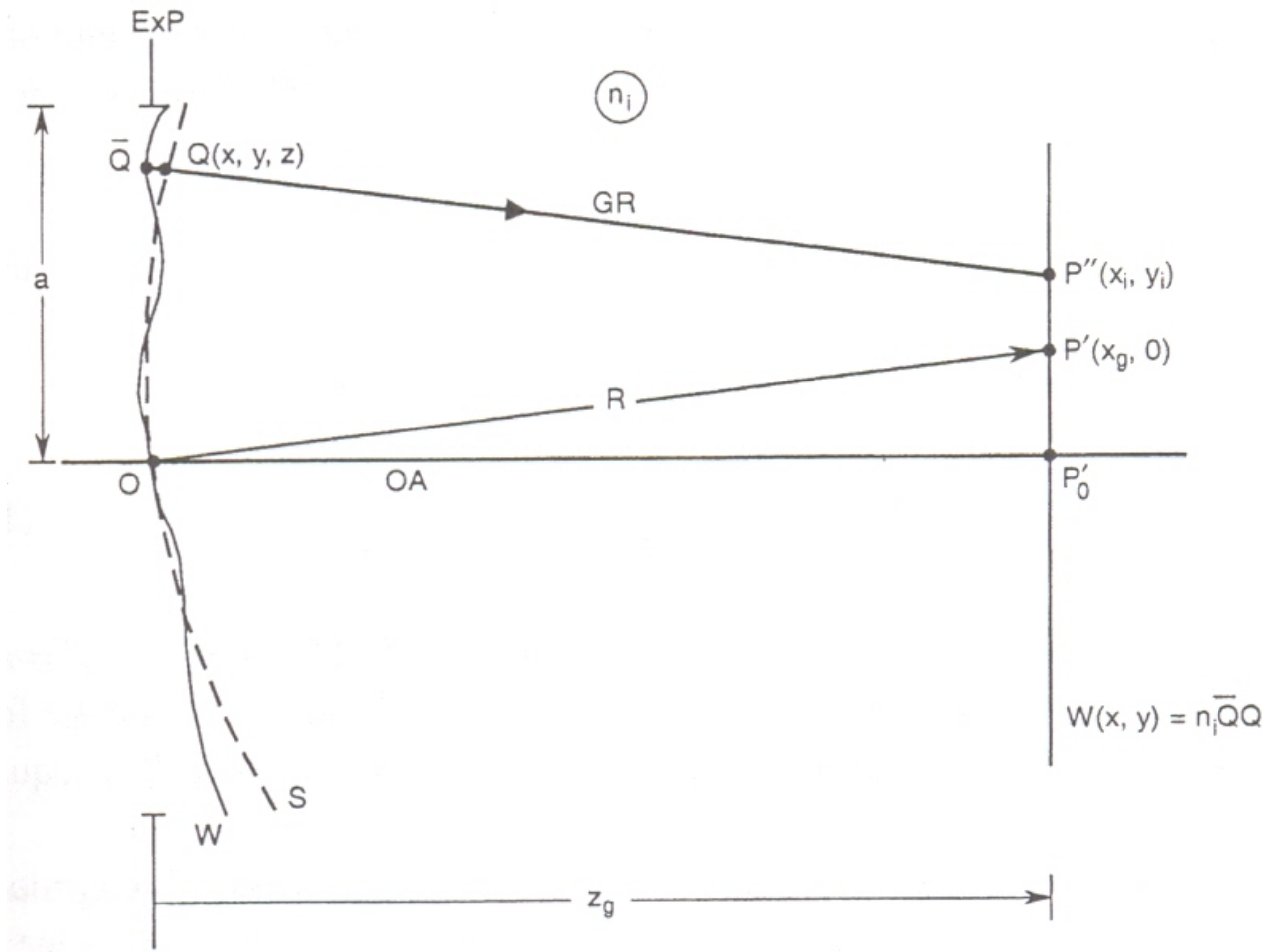
Aberrations are departures of the performance of an optical system from the predictions of paraxial optics.

Two categories of aberrations:

1. On-axis aberrations (defocus, spherical aberration)
2. Off-axis aberrations:
   a) Aberrations that degrade the image: coma, astigmatism
   b) Aberrations that alter the image position: distortion, field curvature

Wave and Ray Aberrations

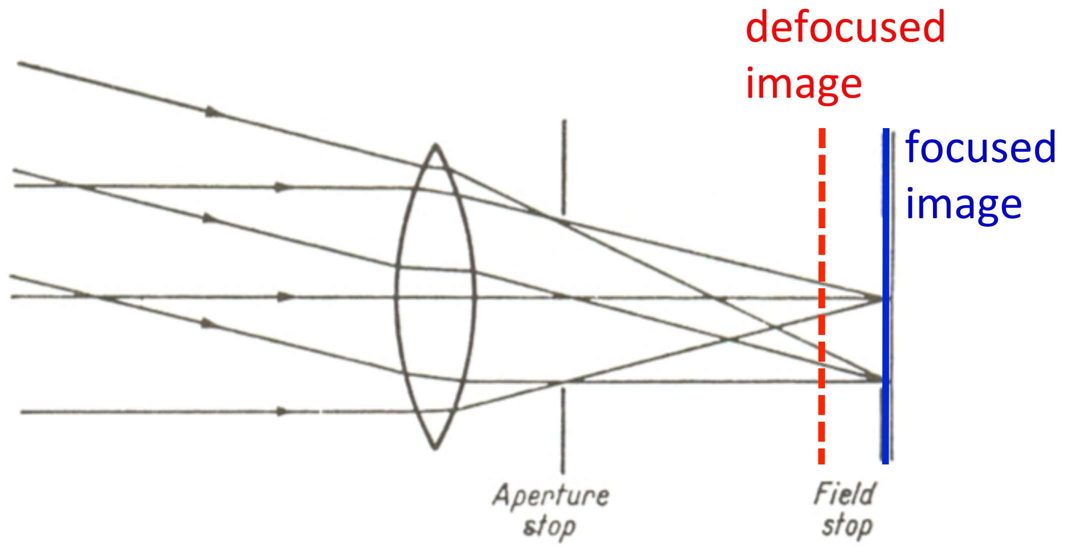
- Reference sphere S with radius R for off-axis point P’ and aberrated wavefront W.
- “Aberrated” ray from object intersects image plane at P”.
- Ray aberration is P’P”.
- Wave aberration is n·QQ



For small FOVs and radially symmetric aberrated wavefront W(r) we can approximate intersection with image plane:

$$r_i = \frac{R}{n_i}\frac{\partial W(r)}{\partial r}$$

Defocus (Out of Focus)



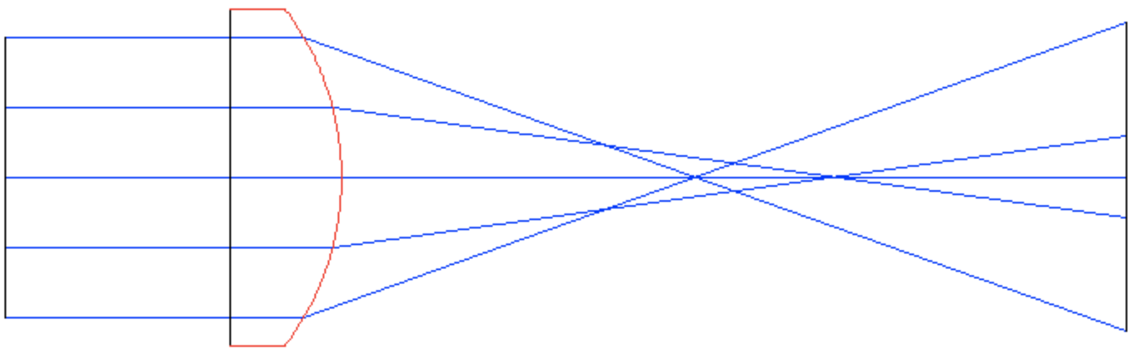
Depth of focus:

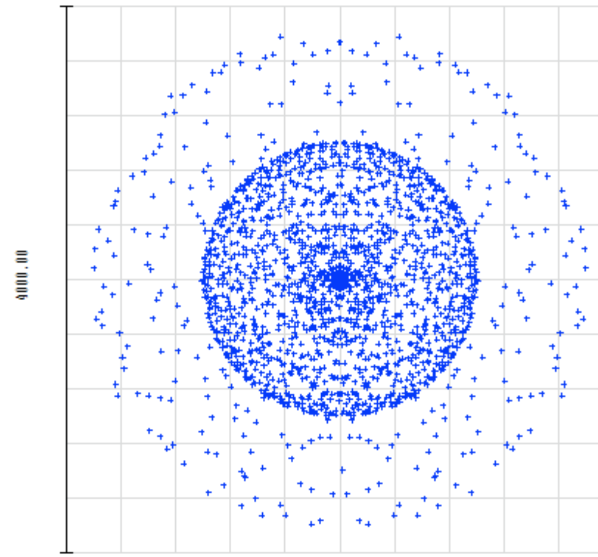
$$\delta = 2\lambda F^2 = \frac{\lambda}{2}\left(\frac{1}{\mathrm{NA}}\right)^2$$

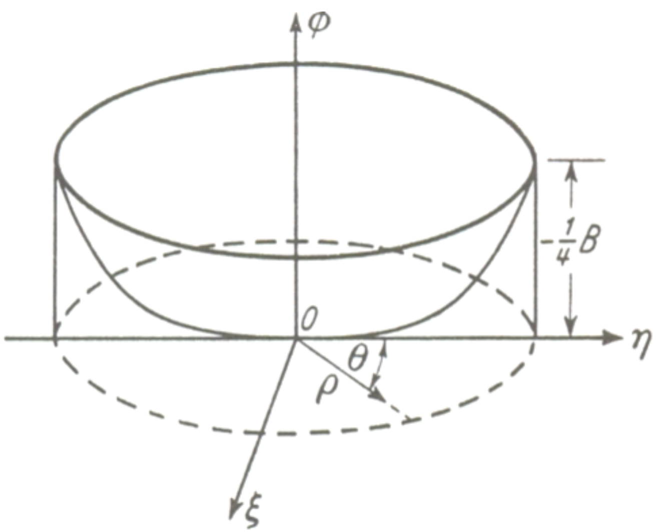
Usually refers to optical path difference of $\lambda/4$.

Spherical Aberration

Rays further from the optical axis have a different focal point than rays closer to the optical axis:





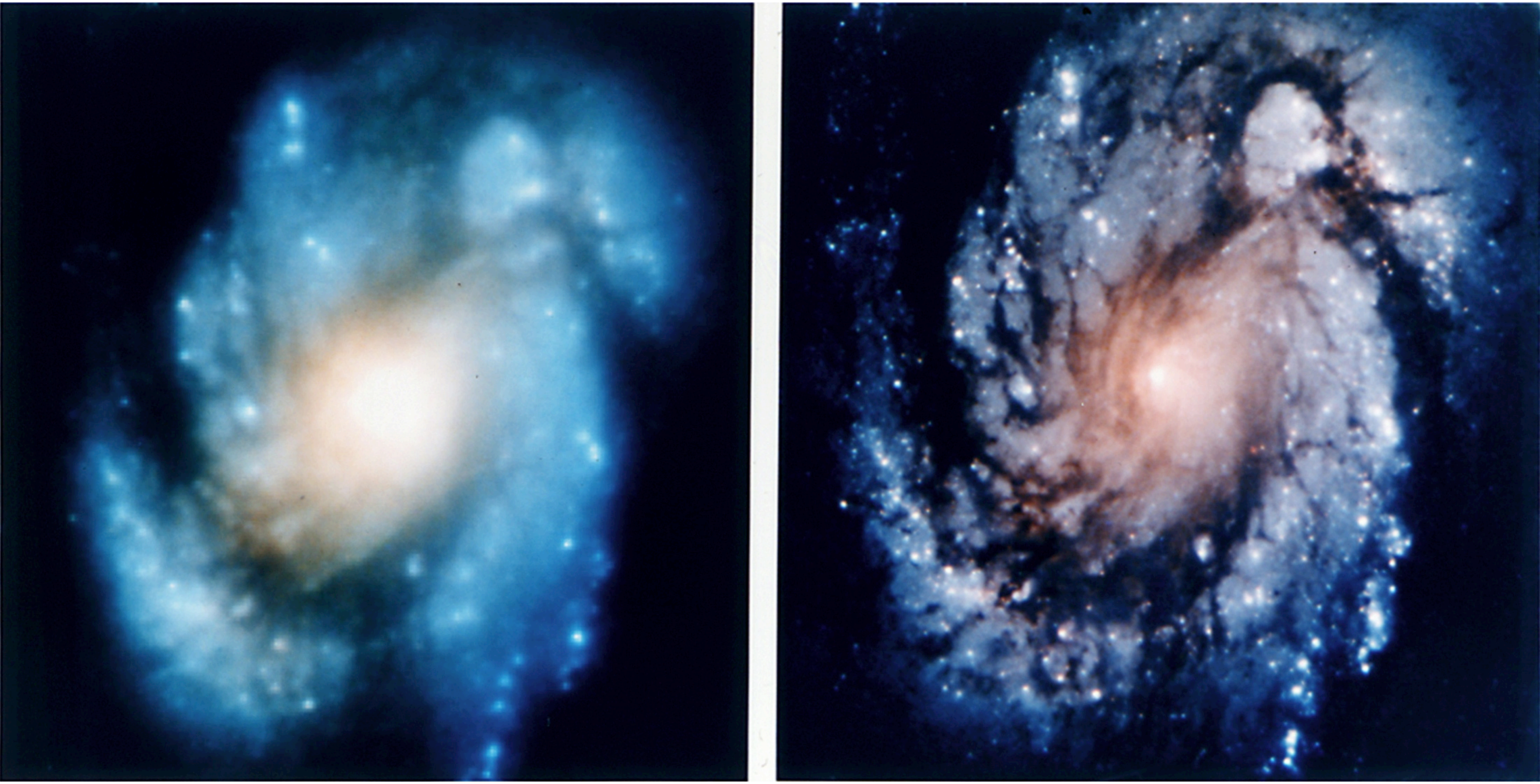


Spherical aberration

$\phi = -\frac{1}{4}B\rho^4$

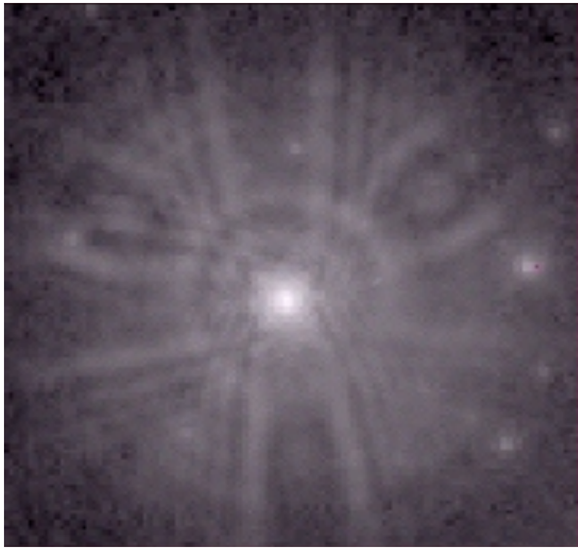
ϕ is the wave aberration
Θ is the angle in the pupil plane
ρ is the radius in the pupil plane
ξ = ρ sinΘ; η = ρ cosΘ

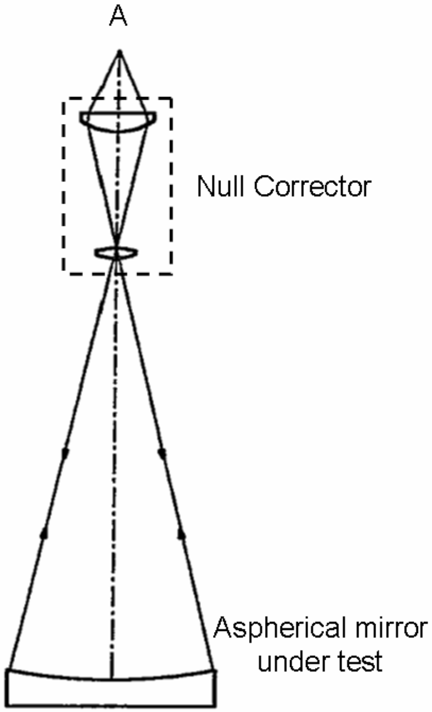
Hubble Trouble



HST Primary Mirror Spherical Aberration

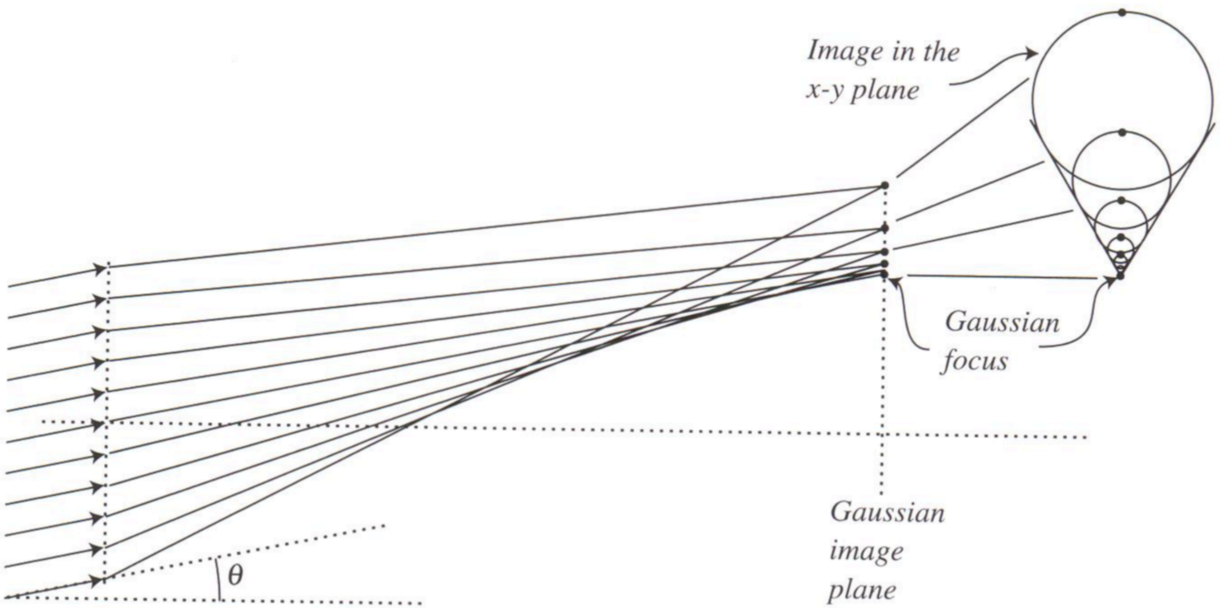
- *Null corrector* for measuring mirror shape was incorrectly assembled (one lens misplaced by 1.3 mm).
- A management problem: Mirror manufacturer had analyzed surface with other null correctors, which indicated the problem, but test results were ignored because they were believed to be less accurate.
- *Null corrector* cancels non-spherical portion of aspheric mirror figure. When correct mirror is viewed from point A, combination looks precisely spherical.

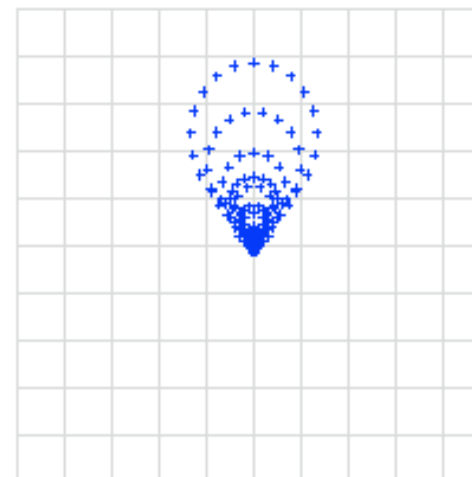


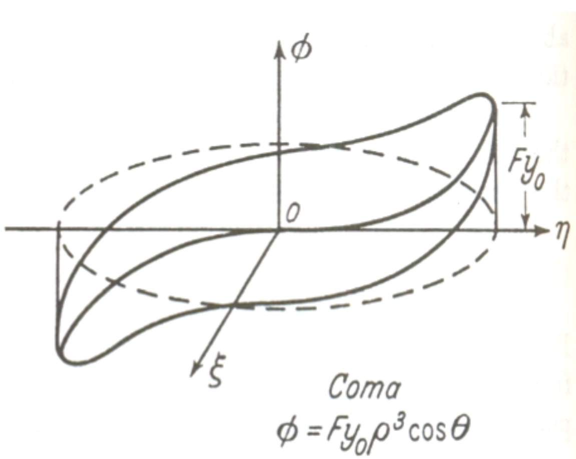


Coma

Variation of magnification across entrance pupil. Point sources will show a cometary tail. Coma is an inherent property of telescopes using parabolic mirrors.



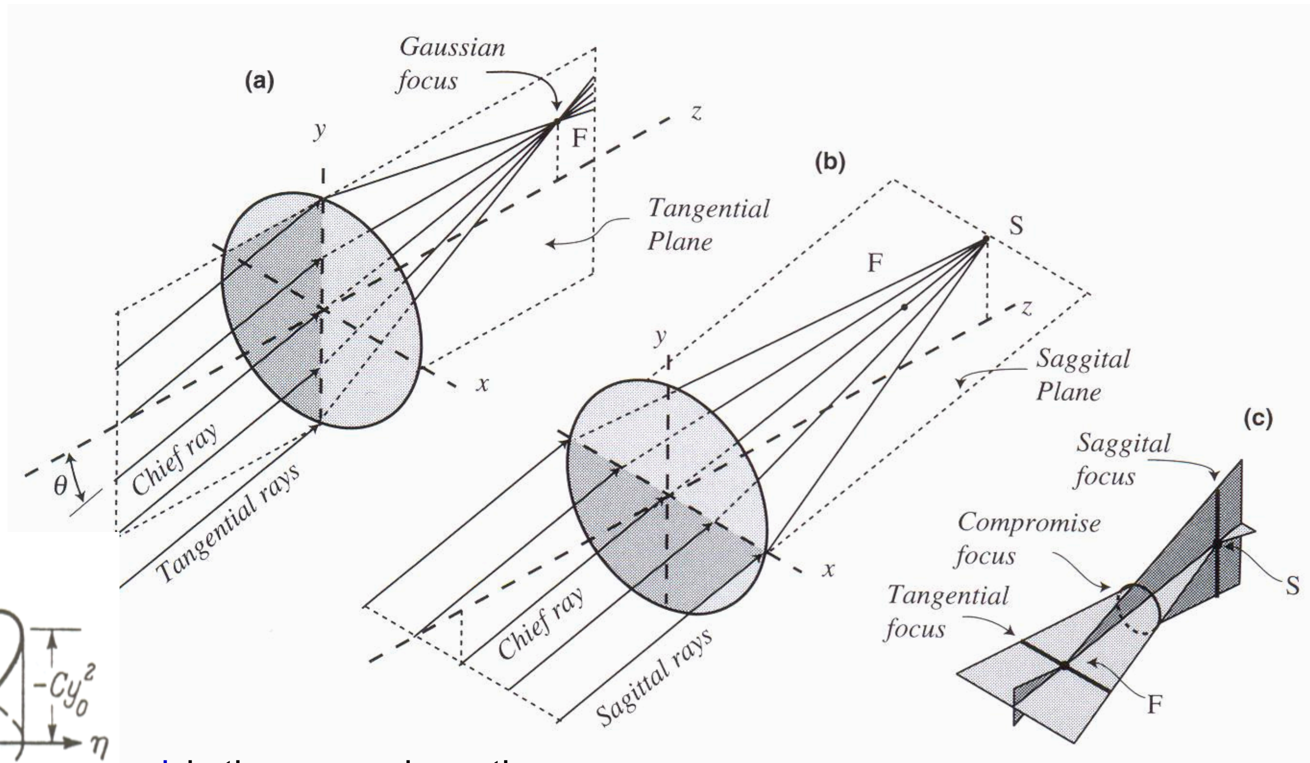


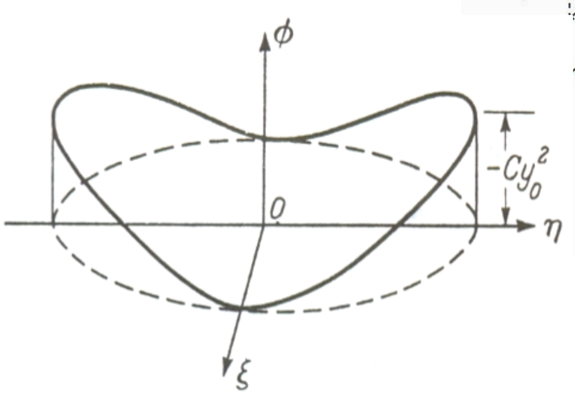


ϕ is the wave aberration
Θ is the angle in the pupil plane
ρ is the radius in the pupil plane
$\xi = \rho \sin\Theta$; $\eta = \rho \cos\Theta$
y_0 = position of the object in the field

Astigmatism

From off-axis point A lens does not appear symmetrical but shortened in plane of incidence (tangential plane). Emergent wave will have a smaller radius of curvature for tangential plane than for plane normal to it (sagittal plane) and form an image closer to the lens.



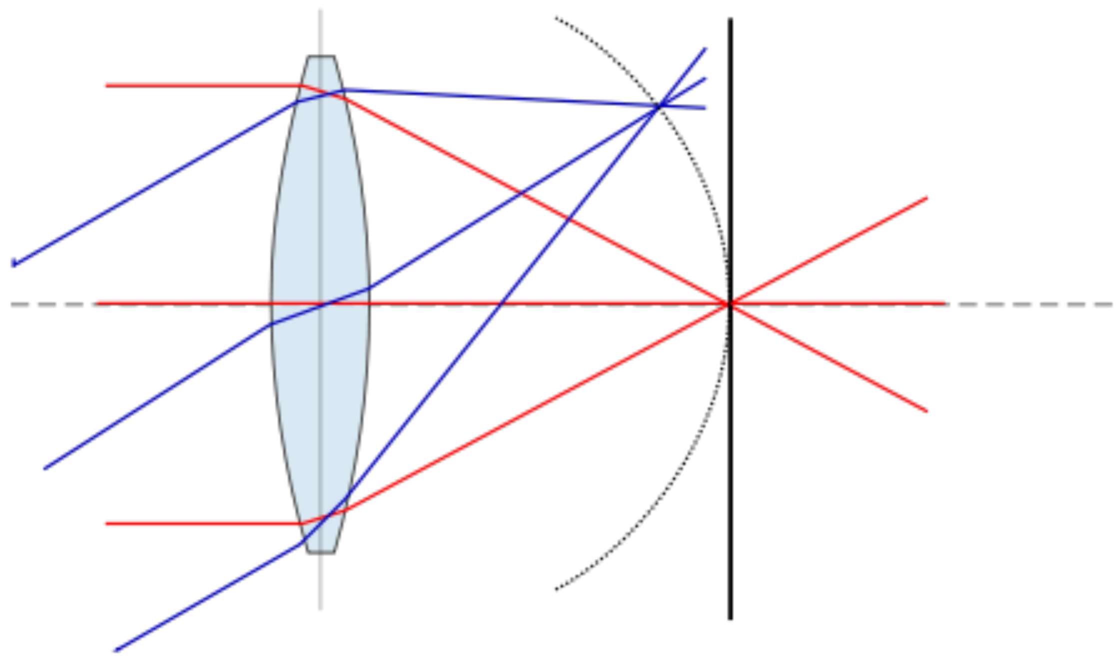


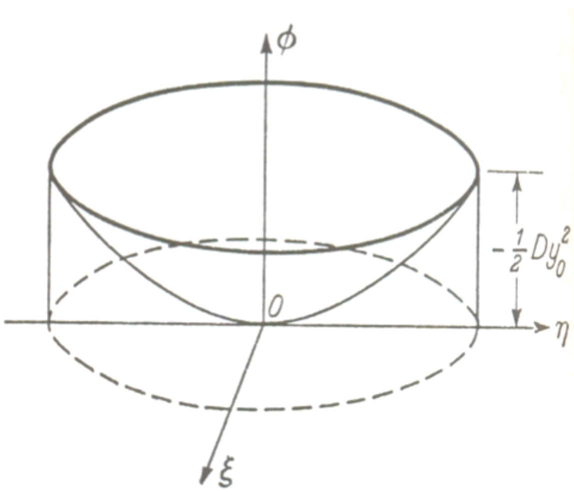
Astigmatism
$\phi = -Cy_0^2\rho^2\cos^2\theta$

ϕ is the wave aberration
Θ is the angle in the pupil plane
ρ is the radius in the pupil plane
$\xi = \rho \sin\Theta$; $\eta = \rho \cos\Theta$
y_0 = position of the object in the field

Field Curvature

Only objects close to optical axis will be in focus on flat image plane.   Close-to-axis and far off-axis objects will have different focal points due to OPL difference.



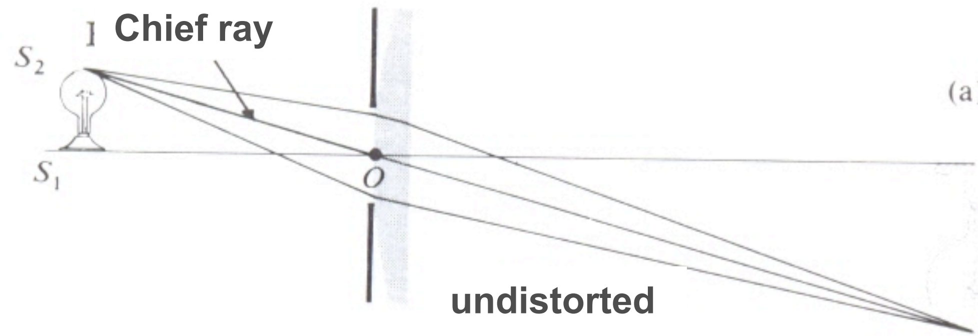


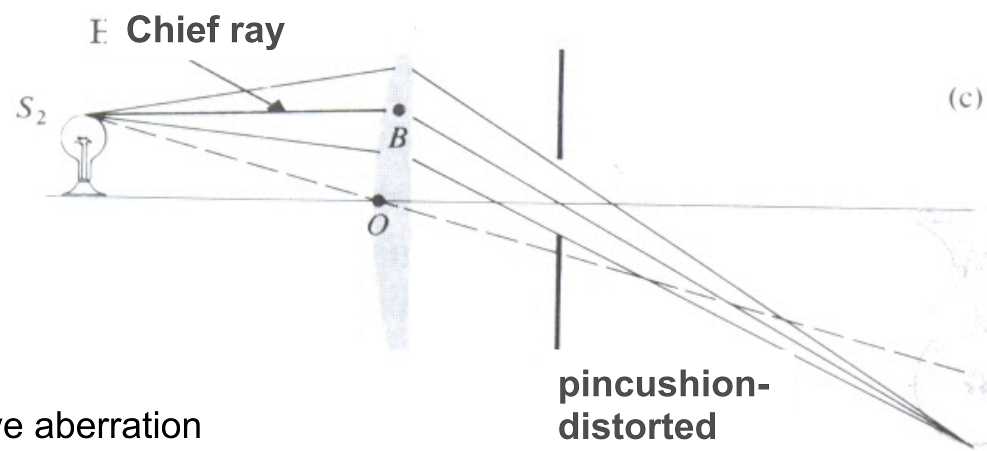
Curvature of field
$\phi = -\frac{1}{2}Dy_0^2\rho^2$

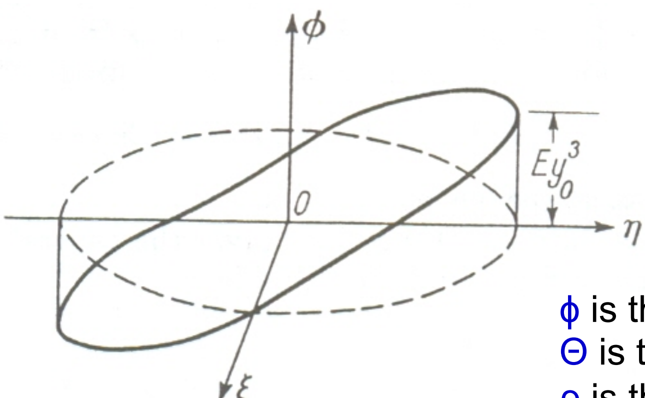
ϕ is the wave aberration
Θ is the angle in the pupil plane
ρ is the radius in the pupil plane
$\xi = \rho \sin\Theta$; $\eta = \rho \cos\Theta$
y_0 = position of the object in the field

Distortion (1)

Straight lines on sky become curved lines in focal plane. Transversal magnification depends on distance from optical axis.







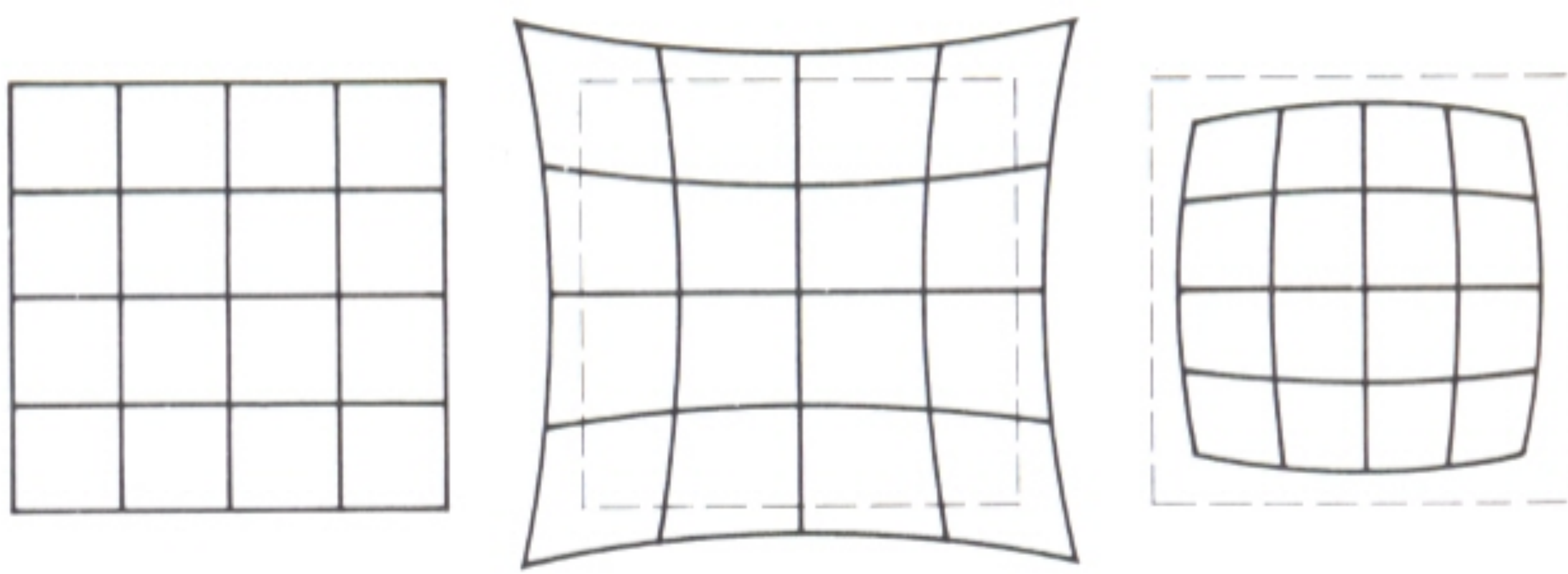
Distortion
$\phi = Ey_0^3 \rho \cos\theta$

ϕ is the wave aberration
Θ is the angle in the pupil plane
ρ is the radius in the pupil plane
$\xi = \rho \sin\Theta$; $\eta = \rho \cos\Theta$
y_0 = position of the object in the field

Distortion (2)

Two cases:

1. Outer parts have smaller magnification → barrel distortion
2. Outer parts have larger magnification → pincushion distortion

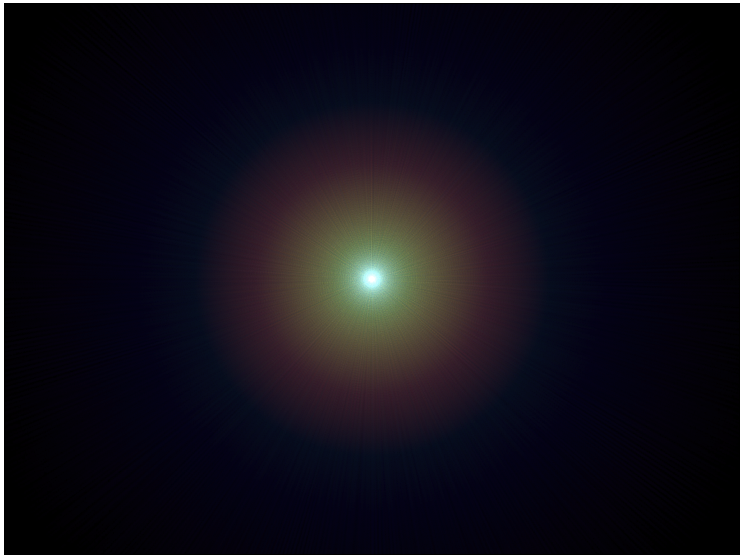


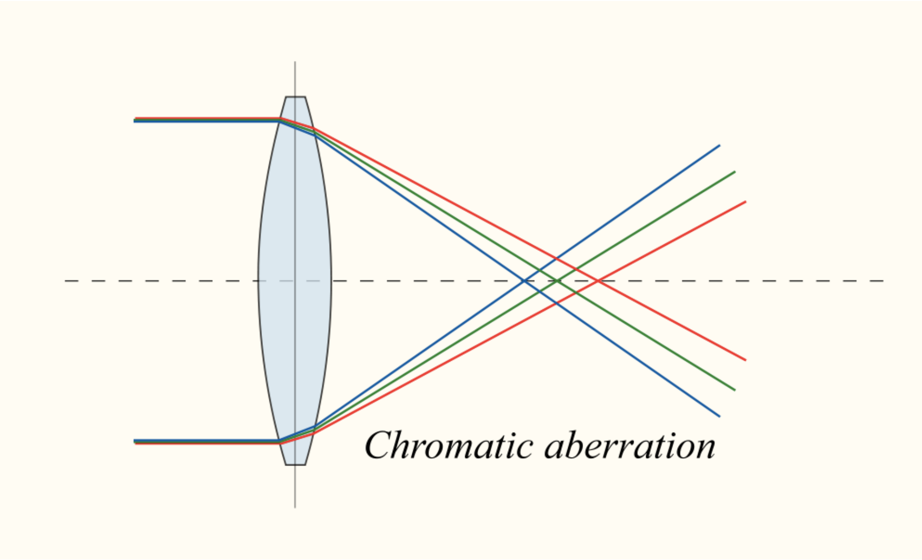
Summary: Primary Wave Aberrations

	On-axis focus	*On-axis defocus*	*Off-axis*	*Off-axis defocus*	*Dependence on pupil size*	*Dependence on image size*
Spherical aberration					$\sim\rho^4$	const.
Coma					$\sim\rho^3$	$\sim y$
Astigmatism					$\sim\rho^2$	$\sim y^2$
Field curvature					$\sim\rho^2$	$\sim y^2$
Distortion					$\sim\rho$	$\sim y^3$
Defocus					$\sim\rho^2$	const.

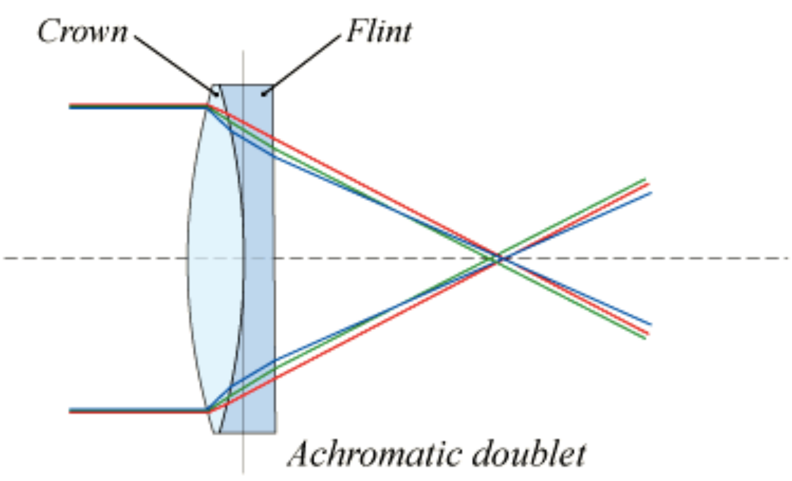
Chromatic Aberration

Refractive index n(λ), focal length of lens f(λ); different wavelengths have different foci. (Mirrors are usually achromatic).





Mitigation: use two lenses of different material with different dispersion → achromatic doublet

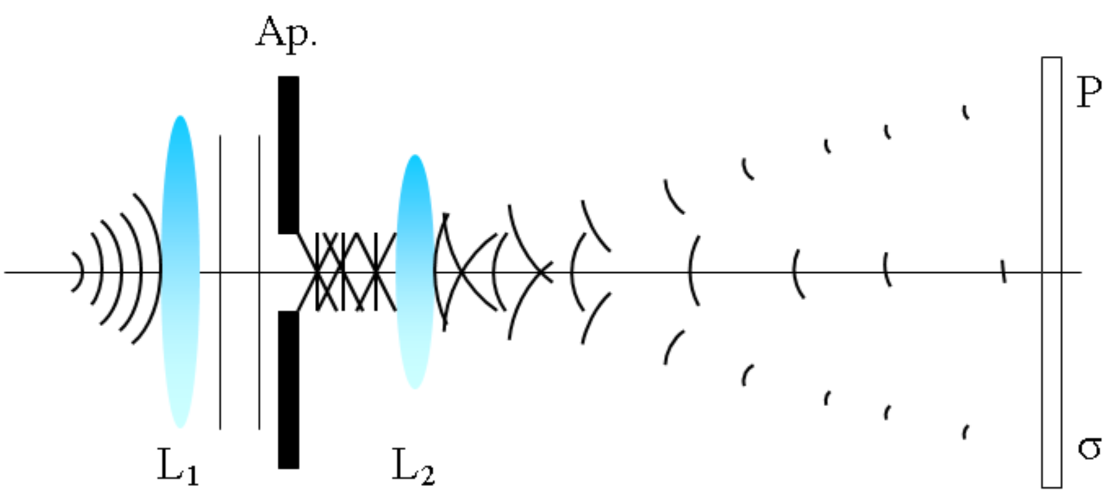


Fresnel and Fraunhofer Diffraction

Fresnel diffraction = near-field diffraction

When a wave passes through an aperture and diffracts in the near field it causes the observed diffraction pattern to differ in size and shape for different distances.

For Fraunhofer diffraction at infinity (far-field) the wave becomes planar.



*An example of an optical setup that displays Fresnel diffraction occurring in the **near-field**. On this diagram, a wave is diffracted and observed at point σ. As this point is moved further back, beyond the Fresnel threshold or in the **far-field**, Fraunhofer diffraction occurs.*

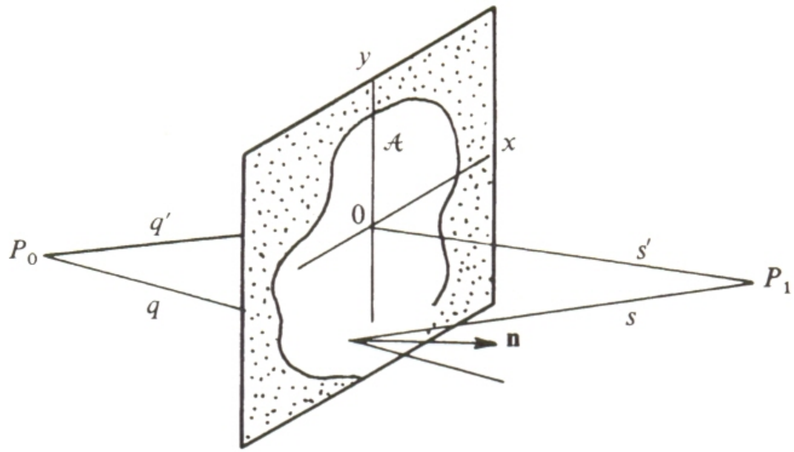
Fresnel: $$F = \frac{r^2}{d \cdot \lambda} \geq 1$$

Fraunhofer: $$F = \frac{r^2}{d \cdot \lambda} << 1$$

(where F = Fresnel number, r = aperture size and d = distance to screen).

Fraunhofer Diffraction at Pupil

Circular pupil function G(r) of unity within A and zero outside.



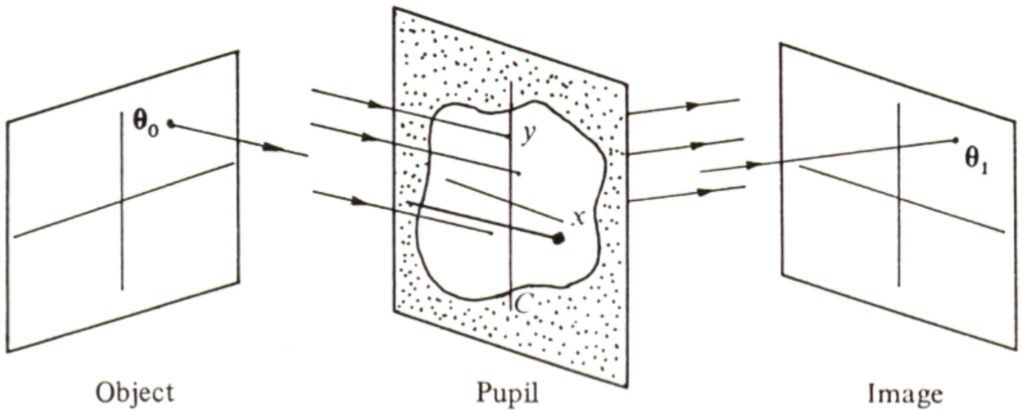
Theorem: *When a screen is illuminated by a source at infinity, the amplitude of the field diffracted in any direction is the Fourier transform of the pupil function characterizing the screen A.*

Mathematically, the amplitude of the diffracted field can be expressed as (see Lena book pp. 120ff for details):

$$V_1(\theta_1, t) = \lambda \sqrt{\frac{E}{A}} \iint_{screenA} G\left(\frac{r}{\lambda}\right) e^{-i2\pi(\theta_1 - \theta_0)\cdot\frac{r}{\lambda}} \frac{dr}{\lambda^2}$$

Imaging and Filtering

$V(\Theta_0)$, $V(\Theta_1)$: complex field amplitudes of points in object and image plane
$K(\Theta_0;\Theta_1)$: "transmission" of the system



Then the image of an *extended* object can be described by:

$$V(\theta_1) = \iint_{object} V_0(\theta_0) K(\theta_1 - \theta_0) d\theta_0 \quad \text{where} \quad K(\theta) = \iint G(r) e^{-i2\pi\theta\frac{r}{\lambda}} \frac{dr}{\lambda^2}$$

convolution

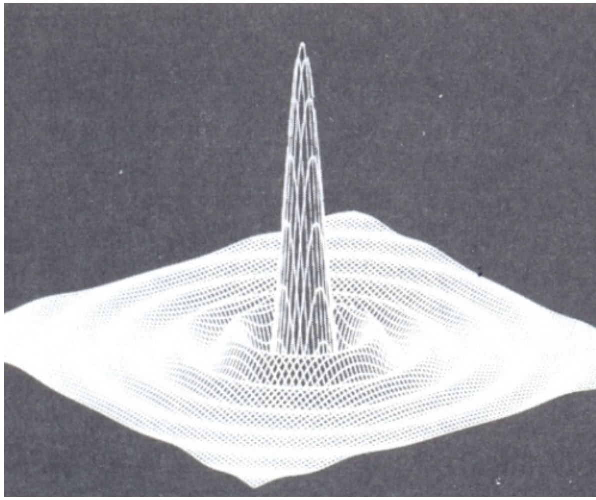
In Fourier space: $FT\{V(\theta_1)\} = FT\{V_0(\theta_0)\} \cdot FT\{K(\theta_0)\} = FT\{V_0(\theta_0)\} \cdot G(r)$

The Fourier transform of the image equals the product of Fourier transform of the object and the pupil function G, which acts as a linear spatial filter.

Point Spread Function (1)

When the circular pupil is illuminated by a point source $I_0(\theta) = \delta(\theta)$ then the resulting PSF is described by a 1st order Bessel function:

$$I_1(\theta) = \left(\frac{2J_1(2\pi\, r_0\theta/\lambda)}{2\pi\, r_0\theta/\lambda} \right)^2$$



This is also called the Airy function.
The radius of the first dark ring (minimum) is at:

$$r_1 = 1.22\lambda F \qquad \text{or} \qquad \alpha_1 = \frac{r_1}{f} = 1.22\frac{\lambda}{D}$$

The PSF is often simply characterized by the half power beam width (HPBW) or full width half maximum (FWHM) in angular units.

According to the Nyquist-Shannon sampling theorem I(Θ) (or its FWHM) shall be sampled with a rate of at least:

$$\Delta\theta = {}^{1}\!/_{2\omega_c}$$

Point Spread Function (2)

Most "real" telescopes have a central obscuration, which modifies our simplistic pupil function $G(r) = \Pi(r/2r_0)$

The resulting PSF can be described by a modified Airy function:

$$I_1(\theta) = \frac{1}{(1-\varepsilon^2)^2}\left(\frac{2J_1(2\pi\, r_0\theta/\lambda)}{2\pi\, r_0\theta/\lambda} - \varepsilon^2\,\frac{2J_1(2\pi\, r_0\varepsilon\theta/\lambda)}{2\pi\, r_0\varepsilon\theta/\lambda}\right)^2$$

where ε is the fraction of central obscuration to total pupil area.

Radii of Dark Rings in Airy Pattern[a,b]

ε	w_1	w_2	w_3
0.00	1.220	2.233	3.238
0.10	1.205	2.269	3.182
0.20	1.167	2.357	3.087
0.33	1.098	2.424	3.137
0.40	1.058	2.388	3.300
0.50	1.000	2.286	3.491
0.60	0.947	2.170	3.389

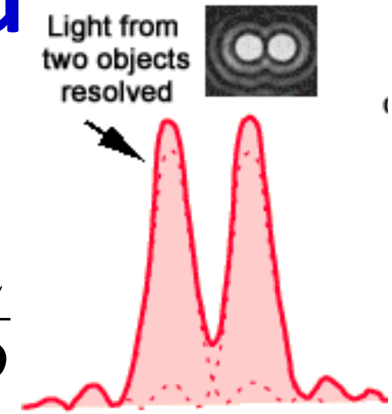
[a] Subscript on w is the number of the dark ring starting at the innermost ring.
[b] $w = v/\pi$.

Astronomical instruments sometimes use a phase mask to reduce the secondary lobes of the PSF (from diffraction at "hard edges"). Phase masks introduce a position dependent phase change. This is called apodisation.

Optical/Modulation Transfer Function

Remember: so far we have used the Rayleigh criterion to describe resolution: two sources can be resolved if the peak of the second source is no closer than the 1st dark Airy ring of the first source.

$$\sin\Theta = 1.22\frac{\lambda}{D}$$



A better measure of the resolution that the system is capable of is the optical transfer function (OTF):

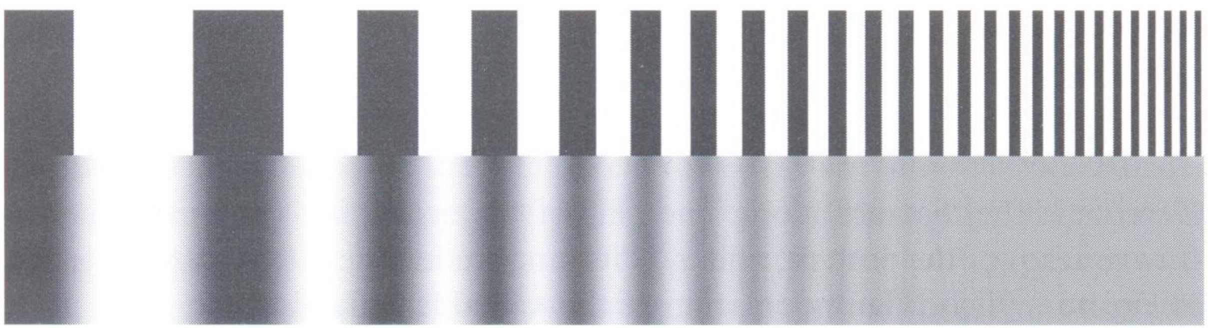
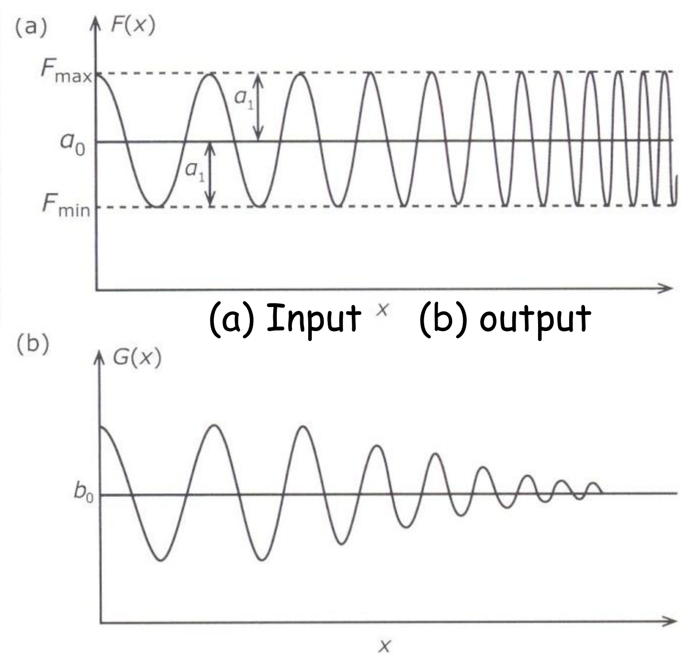


Figure 2.10. Bar chart test of resolution. The upper half shows the object imaged, while the lower half is the image and shows the blurring due to the optical system. Based on material from Norman Koren (n.d.), with permission.



$$MTF(f) = \frac{C(f)}{C_0}, \qquad \text{where } C = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

Optical/Modulation Transfer Function (2)

The Optical Transfer Function (OTF) describes the spatial signal variation as a function of spatial frequency. With the spatial frequencies (ξ,η), the OTF can be written as…

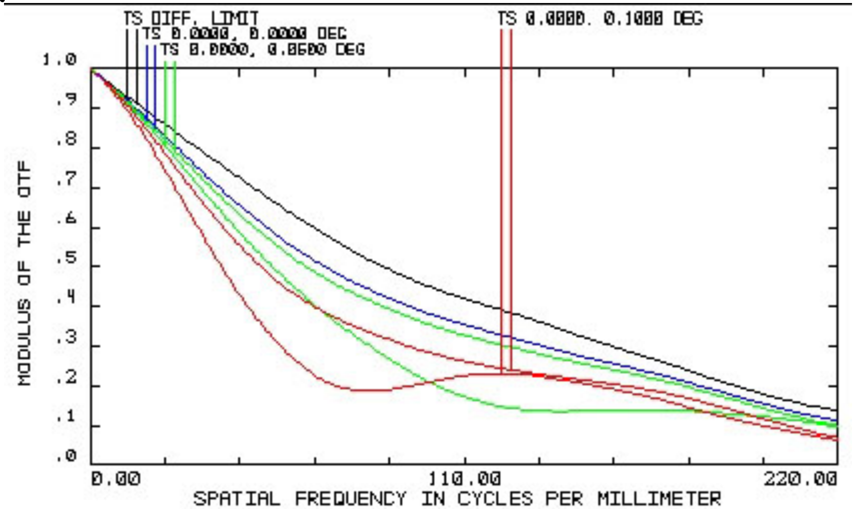
$$OTF(\xi,\eta) = MTF(\xi,\eta) \cdot PTF(\xi,\eta)$$

$$MTF(\xi,\eta) = \left| OTF(\xi,\eta) \right|$$

$$PTF(\xi,\eta) = e^{-i2\pi\lambda(\xi,\eta)}$$

…where the Modulation Transfer Function (MTF) describes its magnitude, and the Phase Transfer Function (PTF) the phase.

Example:



Strehl Ratio

Strehl ratio is convenient measure to assess quality of optical system.

Strehl ratio (SR) is the ratio of the observed *peak intensity* of the PSF compared to the theoretical maximum peak intensity of a point source seen with a perfect imaging system working at the diffraction limit.

Using the wave number k=2π/λ and the RMS wavefront error ω one can calculate that:

$$SR = e^{-k^2\omega^2} \approx 1 - k^2\omega^2$$

Examples:

- A SR > 80% is considered diffraction-limited → average WFE ~ λ/14
- A typical adaptive optics system delivers SR ~ 10-50% (depends on λ)
- A seeing-limited PSF on an 8m telescope has a SR ~ 0.1-0.01%.

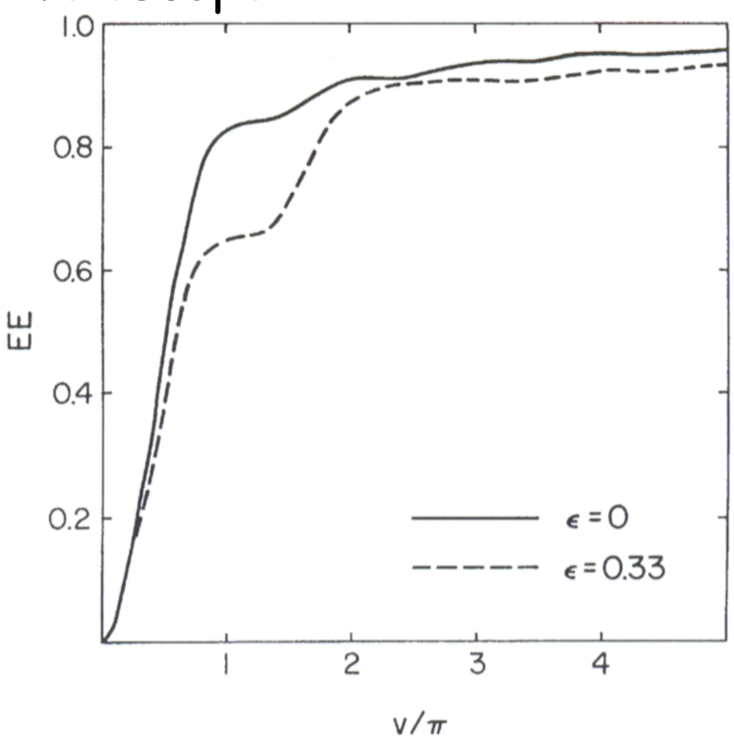
Encircled Energy

Q: What is the maximum concentration of light within a small area?
The fraction of the total PSF intensity within a certain radius is given by the encircled energy (EE):

$$EE(r) = 1 - J_0^2\left(\frac{\pi r}{\lambda F}\right) - J_1^2\left(\frac{\pi r}{\lambda F}\right)$$

F is the f/# number

Note that the EE depends strongly on the central obscuration ε of the telescope:



Encircled Energy Fraction within Airy Dark Rings[a]

ε	EE_1	EE_2	EE_3
0.00	0.838	0.910	0.938
0.10	0.818	0.906	0.925
0.20	0.764	0.900	0.908
0.33	0.654	0.898	0.904
0.40	0.584	0.885	0.903
0.50	0.479	0.829	0.901
0.60	0.372	0.717	0.873

[a] Subscript on EE is number of dark ring starting at innermost ring.