Astronomical Observing Techniques 2014:
Exercises on Detectors 1
(Due on 4 November 2014 at 09:00)

October 28, 2014

1 Bandgap

The bandgap of an intrinsic silicon photo-conductor is 1.11 eV.

1. Calculate the cut-off wavelength in μm.
2. How can you limit the bandpass of a detector system on the high energy (blue) side?

2 Backside Illuminated CCDs

There are two different ways to operate a CCD: in front and in back illumination mode (see figure below). In front illumination, the metal electrode is replaced by heavily doped silicon (since pure metal would block the incoming photons), but it is still not transparent in blue and UV wavelengths. Therefore, for detectors used at short wavelengths, back illumination is used.

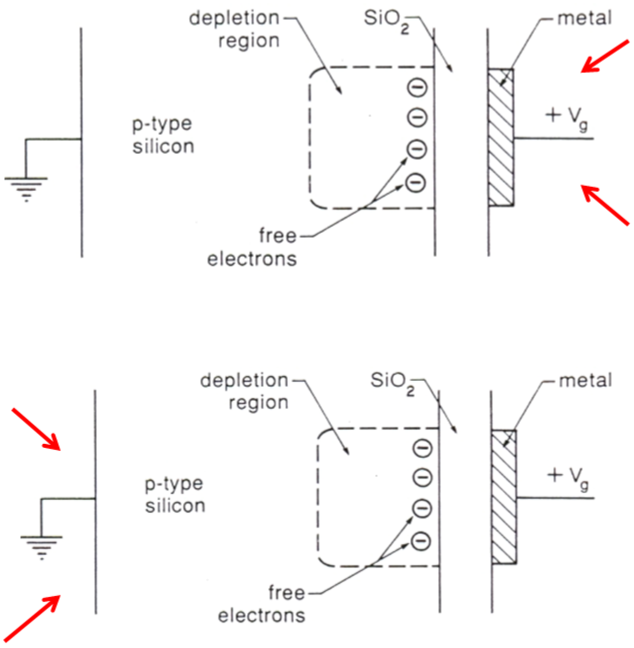

Figure 1: Front (top) and back illuminated (bottom) CCDs, arrows indicate the incoming photons.

1. Although the absorption of UV/blue photons in the p-type silicon is efficient, the quantum efficiency of a back illuminated CCD, as illustrated in the above figure, is generally low. Explain why this is the case.
2. Most backside-illuminated CCD are “thinned” (reduced thickness of the p-type silicon). Explain the objective of the “thinning” and list considerations on minimum/maximum thickness.

3 Fermi Distribution

The Fermi distribution for the probability of finding an electron with an energy E in a given physical system is given by:

$$f(E) = \frac{1}{1 + e^{(E-E_F)/kT}} \tag{1}$$

1. What is the probability of finding an electron in the conduction band at zero temperature? Explain this.
2. What is the probability of finding an electron with the Fermi energy at zero temperature? And at 100 K? What does this imply about the Fermi energy? Make a drawing of $f(E)$ for T=0 and T>0.

   The Fermi distribution gives a probability, but in a physical system, each energy has several states associated with it. To account for the actual number of electrons within an energy range dE, we need to know the density of states $N(E)$. The concentration of electrons n_0 in the energy range $[E_1, E_2]$ is

   $$n_0 = \int_{E_1}^{E_2} f(E)N(E)\mathrm{d}E \tag{2}$$

   For the conduction band, $N(E)$ can be derived as

   $$N(E) = \frac{1}{2\pi^2}(\frac{2m_{eff}}{\hbar^2})^{3/2}(E - E_c)^{1/2} \tag{3}$$

   with $\hbar = h/2\pi$.
3. Which famous equation do you use to derive a density of states?
4. What are (E_1, E_2) for the electron concentration in the conduction band?
5. Show that in the limit where $E - E_F >> kT$, the concentration of electrons is given by:

   $$n_0 = 2(\frac{2\pi m_{eff}kT}{h^2})^{3/2}e^{(E_F-E_c)/kT} \tag{4}$$

   Tip:

   $$\int_0^\infty x^{1/2}e^{-ax}\mathrm{d}x = \frac{\sqrt{\pi}}{2a\sqrt{a}} \tag{5}$$
6. The bandgap energy for pure silicon is 1.11 eV. What is $E_c - E_F$, using the definition of the Fermi energy?
7. Calculate the electron concentration in the conduction band at T = 300K and T = 30K for a “pixel” of volume 1 mm^3. Note that m$_{eff}$ for silicon is ∼1.1 m$_e$.
8. Explain why both the $f(E)$ and the $N(E)$ are essential for building a photo-conductor.