Astronomical Observing Techniques 2014:
Exercises on Radio
(Due on 21 October 2014 at 09:00)

October 13, 2014

1 Rayleigh-Jeans Approximation

Derive the Rayleigh-Jeans approximation for the brightness $B_{RJ}(\nu, T)$ from the Planck formula:

$$B_{\nu,T} = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1} \tag{1}$$

2 Solar Radio Observations

The quiet sun has a brightness temperature $T_b \sim 11000$ K at 10 GHz and an angular diameter of 32 arcminutes.
a) What is the flux density we receive from the sun?

b) We observe the sun with a radio telescope of 25 meter diameter. What is its beam size at 10 GHz?

c) What flux density will this telescope measure at the center of the sun?

3 Atmospheric Transmission

a) Which molecule in the atmosphere is responsible for the transmission curve shown below?

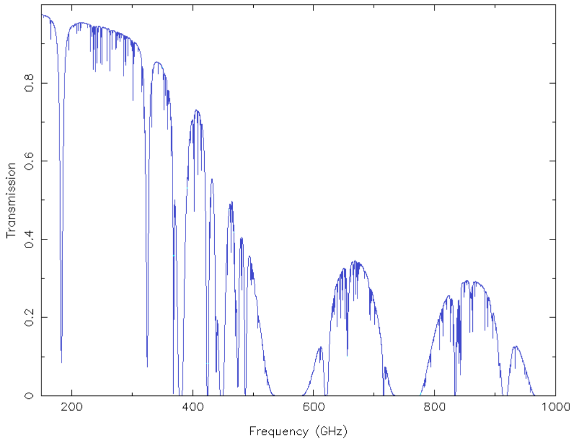

Figure 1: Atmospheric transparancy

b) How do astronomers minimize this issue of absorption bands when building telescopes?

c) For a certain type of objects in the sky, we are still able to observe this molecule by making use of the velocity of the object. What type of object is this?

4 Frequency Mixing

The figure below shows the principle of frequency mixing in a heterodyne receiver.
a) Why do we need to down-convert the frequency for (sub)millimeter radio observations?

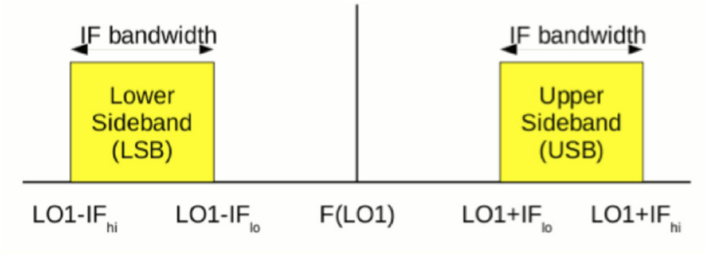

Figure 2: Principle of frequency mixing.

The resulting mixed signal from a heterodyne receiver contains the signal from the upper side band (USB) and lower side band (LSB), both separated by the intermediate frequency (IF) from the local oscillator (see Figure 1). The IF is usually fixed when you choose the settings for observations at a heterodyne instrument. For the FLASH instrument at the APEX telescope, the IF ranges from 4 to 8 GHz to cover a 4 GHz bandwidth per sideband. By choosing your local oscillator frequency wisely, you can get useful lines in both sidebands simultaneously. An example of a nice combination is the 12CO 3–2 (345.79599 GHz) and its isotope 13CO 3–2 (330.58796 GHz).
b) Calculate the local oscillator frequency to get these lines simultaneously. Make a drawing of the frequency ranges.

In the previous calculation you have derived the setting for the rest frequencies of CO. However, everything in the sky is moving at a certain speed, so we need to correct for that.
c) Show that the frequency shift $\Delta\nu = \nu_{obs} - \nu_{rest} = -\frac{\Delta v}{c}\nu_{rest}$. Use the general formula for Doppler shift

$$\nu_{obs} = \frac{c + v_{rec}}{c + v_{source}}\nu_{rest} \tag{2}$$

assuming $\Delta v = v_{source} - v_{rec}$ (you can assume the latter to be zero) and $v_{source} << c$.

d) Let's assume we want to observe a galaxy in this setting that is moving with 2000 km/s away from us. Calculate the new local oscillator frequency.

e) A galaxy CO spectrum is very broad (∼1000 km/s), due to the high velocity dispersion. Can you think of two reasons why this is inconvenient when we want to observe ^{12}CO and ^{13}CO simultaneously?

f) Go back to 3c and calculate the velocity required to observe the molecule from exercise 3a that is emitting at 325 GHz (rest frequency).

5 Heterodyne Receiver

a) Determine a suitable local oscillator frequency (LOF) for the WSRT radio telescope operating a 1.4 GHz, where we would like to down-convert the signal to a 50 MHz (carrier) signal.

b) An instrument uses a high frequency heterodyne receiver operating a 300 GHz with a system temperature (T_{sys}) of 30 K. We want to increase the sensitivity of the instrument. What properties of the receiver do we need to change to achieve this?

c) same as b) but now for a system temperature of 14.4 K.