Astronomische Waarneemtechnieken
(Astronomical Observing Techniques)

based on lectures by Bernhard Brandl

$$S_{cont} = \frac{\sigma h \lambda \sqrt{n_{pix}} 10^{30}}{SR\Delta\lambda A_{tel}\eta_D G \eta_{atm}\eta_{tot} t_{\text{int}}} \sqrt{\frac{2hc^2}{\lambda^5}\left(\frac{\varepsilon_T}{\exp\left[\frac{hc}{kT_T\lambda}\right]-1} + \frac{\varepsilon_A}{\exp\left[\frac{hc}{kT_A\lambda}\right]-1}\right)\eta_{tot}\cdot}$$

$$\cdot\sqrt{2\pi\left(1-\cos\left(\arctan\left(\frac{1}{2F\#}\right)\right)\right)D^2{}_{pix}\cdot\frac{\eta_D G\lambda}{hc}\cdot\Delta\lambda\cdot t_{\text{int}} + I_d t_{\text{int}} + N_{read}^2 n}$$

Lecture 4: Signal to Noise

1. Noise: Introduction
2. Noise: Distributions
3. Signal-to-noise (= f{t_{int}, D_{tel}})
4. Instrument sensitivities

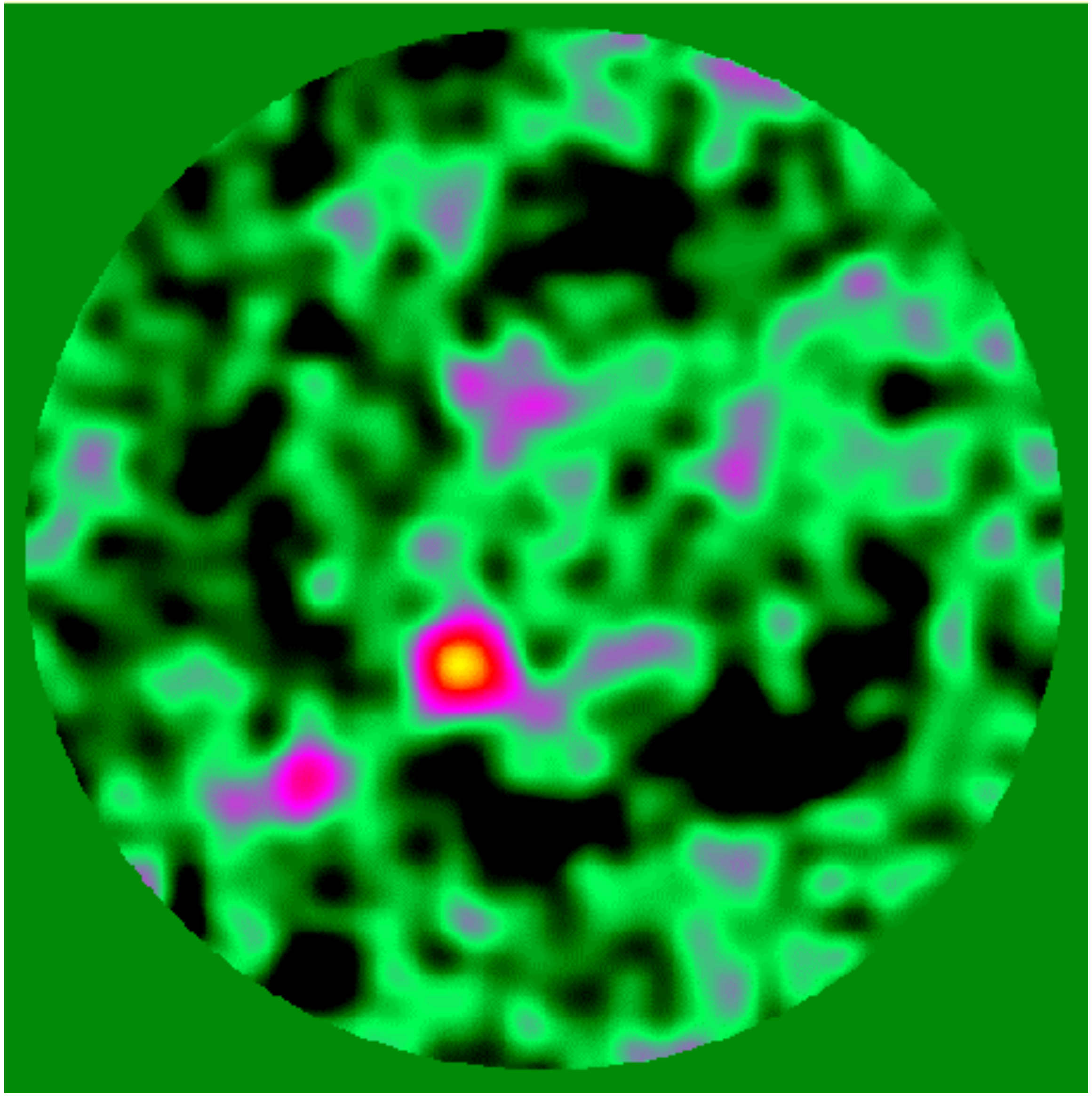
What is noise?

Wikipedia:

- Common use: unwanted sound
- Signal processing: random unwanted data without meaning
- "Signal-to-noise ratio" is sometimes used to refer to the ratio of useful to irrelevant information

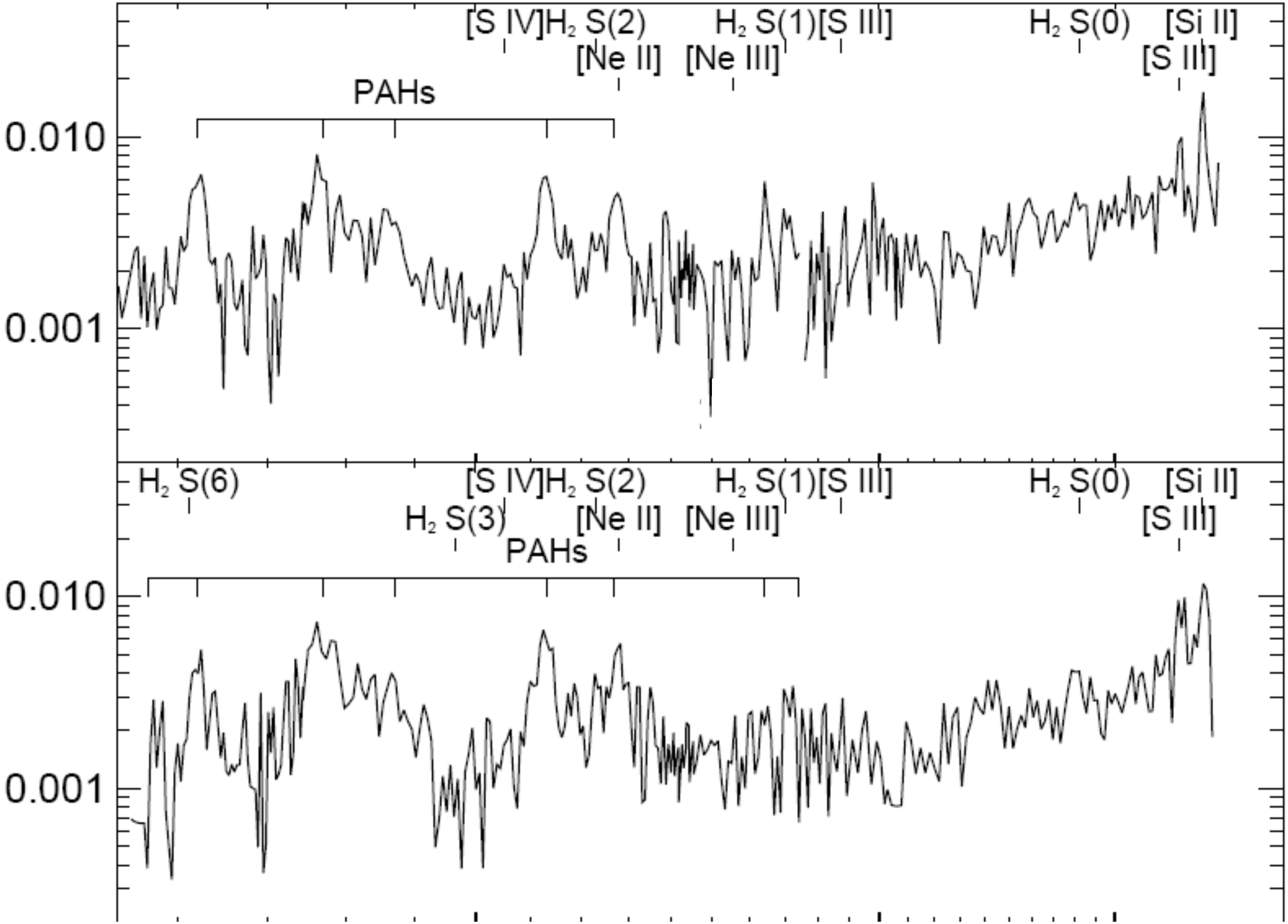
NASA researchers at Glenn Research Center conducting tests on aircraft engine noise in 1967

What is Noise? And what is real Signal?



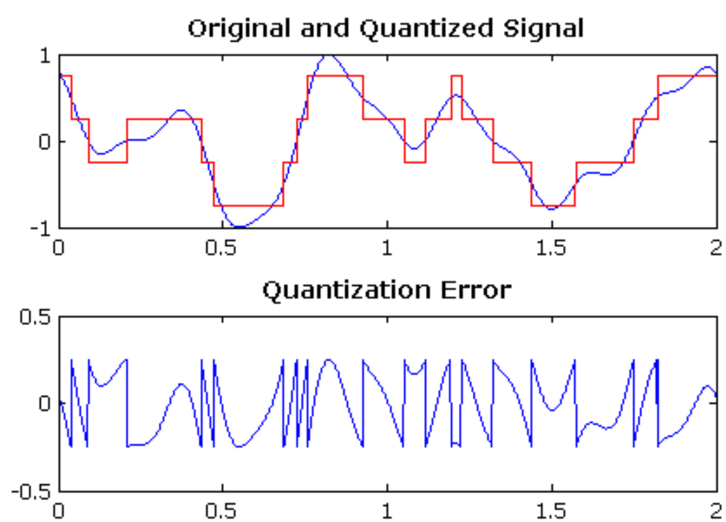
SCUBA 850μm map of the Hubble deep field

What is Noise? And what is real Signal?



Example: Digitization/Quantization Noise

- Quantization/Digitization = converting analog signal into digital signal with Analog-to-Digital Converter (ADC).
- Number of bits determines dynamic range of ADC
- Resolution is 2^n, n = number of bits
  Typical ADCs have 12 bit: 2^{12} = 4096 quantization levels
  16 bit: 2^{16} = 65636 quantization levels
- Too few bits → discrete, "artificial" steps in signal levels → noise



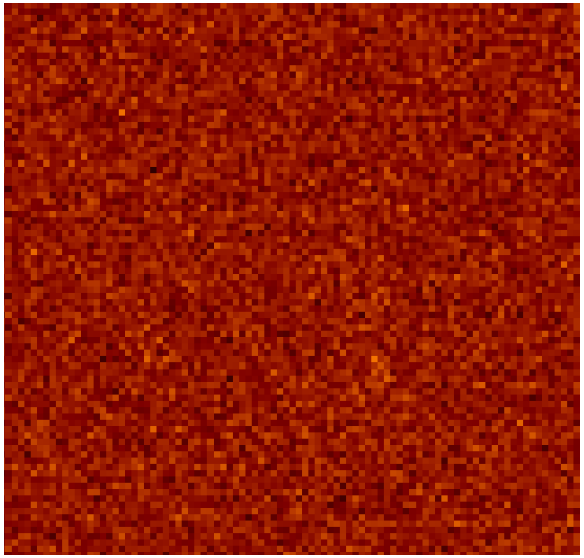
Some Sources of Noise in Astronomical Data

Noise type	Signal	Background
Photon shot noise	X	X
Scintillation	X	
Cosmic rays		X
Image stability	X	
Read noise	X	X
Dark current noise	X	X
CTE (CCDs)	X	X
Flat fielding (non-linearity)	X	X
Digitization noise	X	X
Other calibration errors	X	X
Image subtraction	X	X

Noise Distribution: 1.Gaussian Noise

Gaussian noise has Gaussian (normal) distribution

Somwtimes (incorrectly) called white noise (uncorrelated noise)

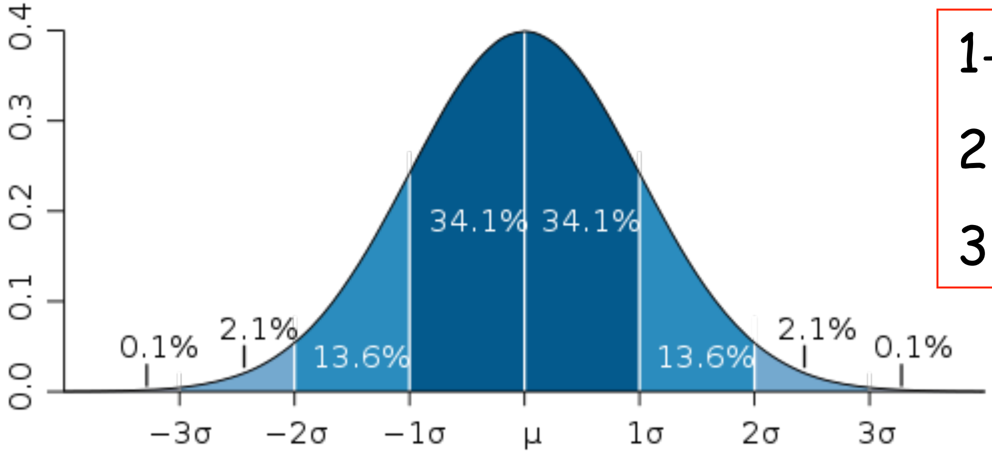


$$S = \frac{1}{\sqrt{2\pi\sigma}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]$$

x is the actual value

μ is the mean of the distribution

s is the standard deviation of the distribution



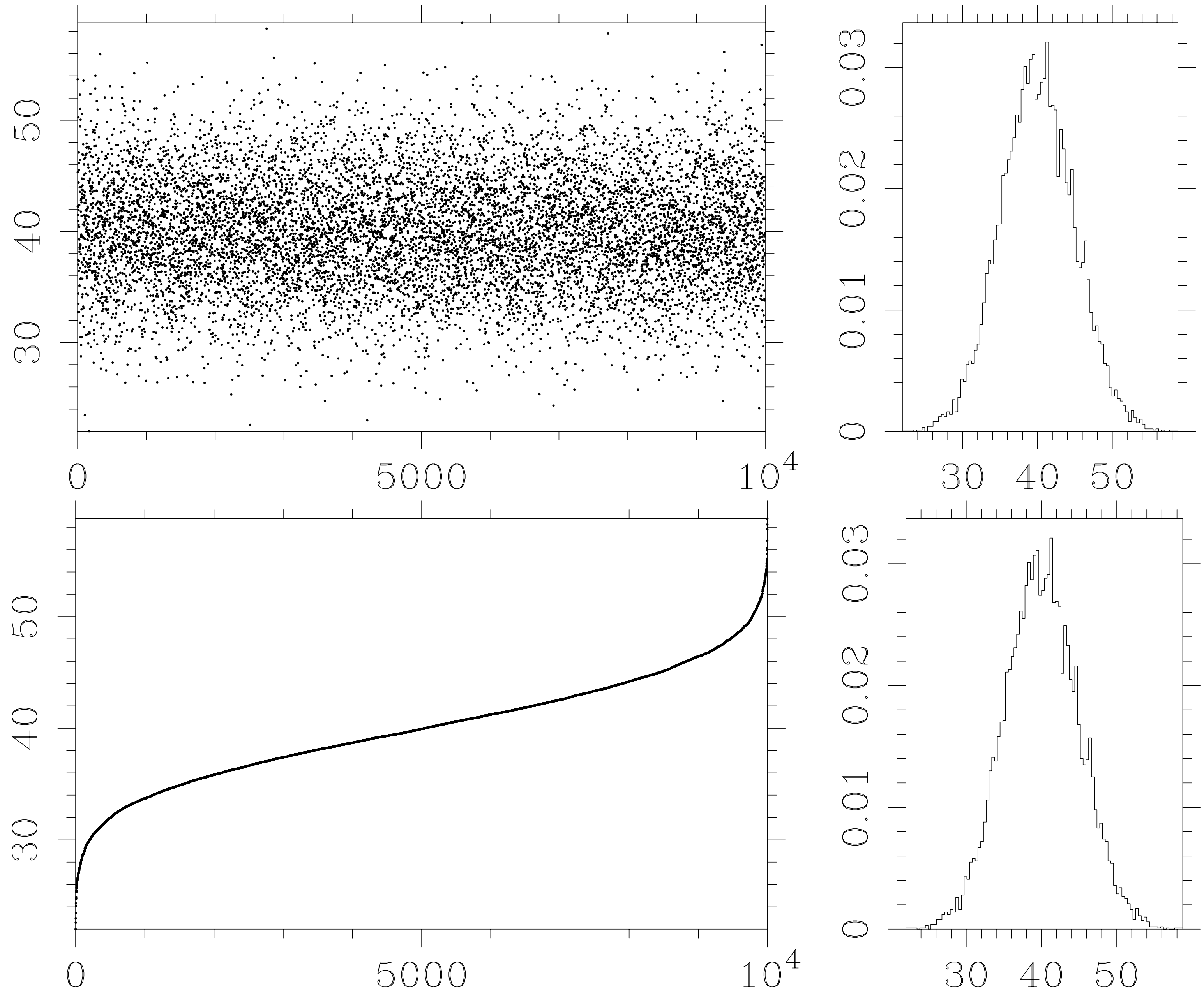
1-σ ~ 68%

2-σ ~ 95%

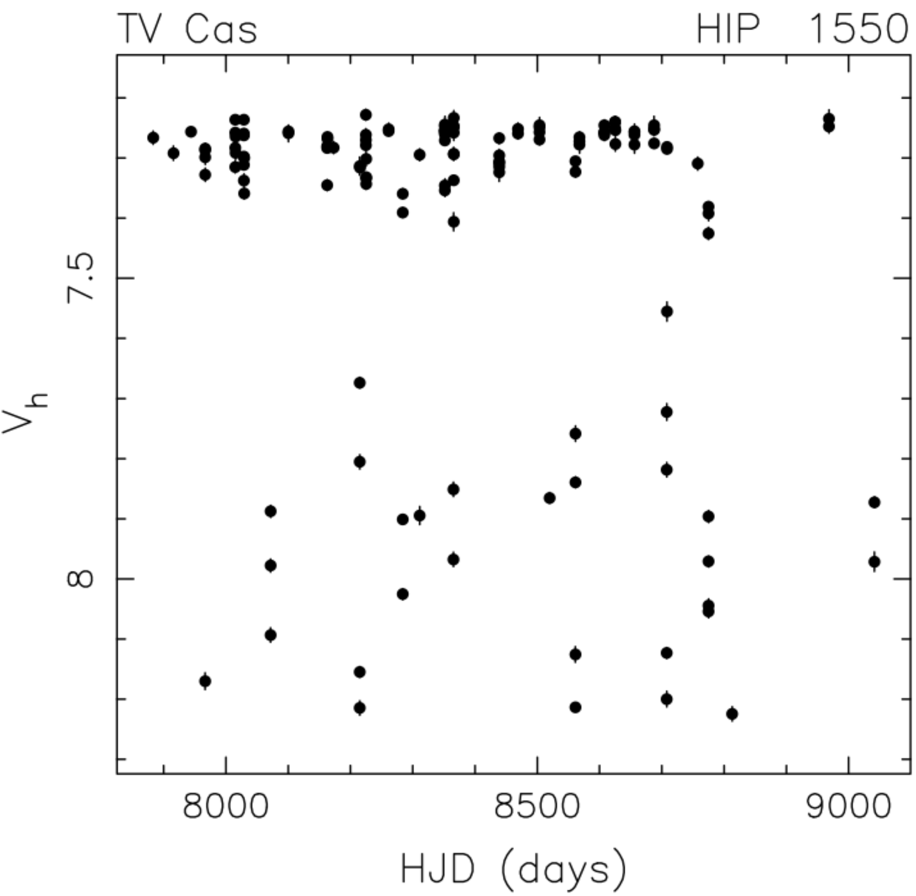
3-σ ~ 99.7%

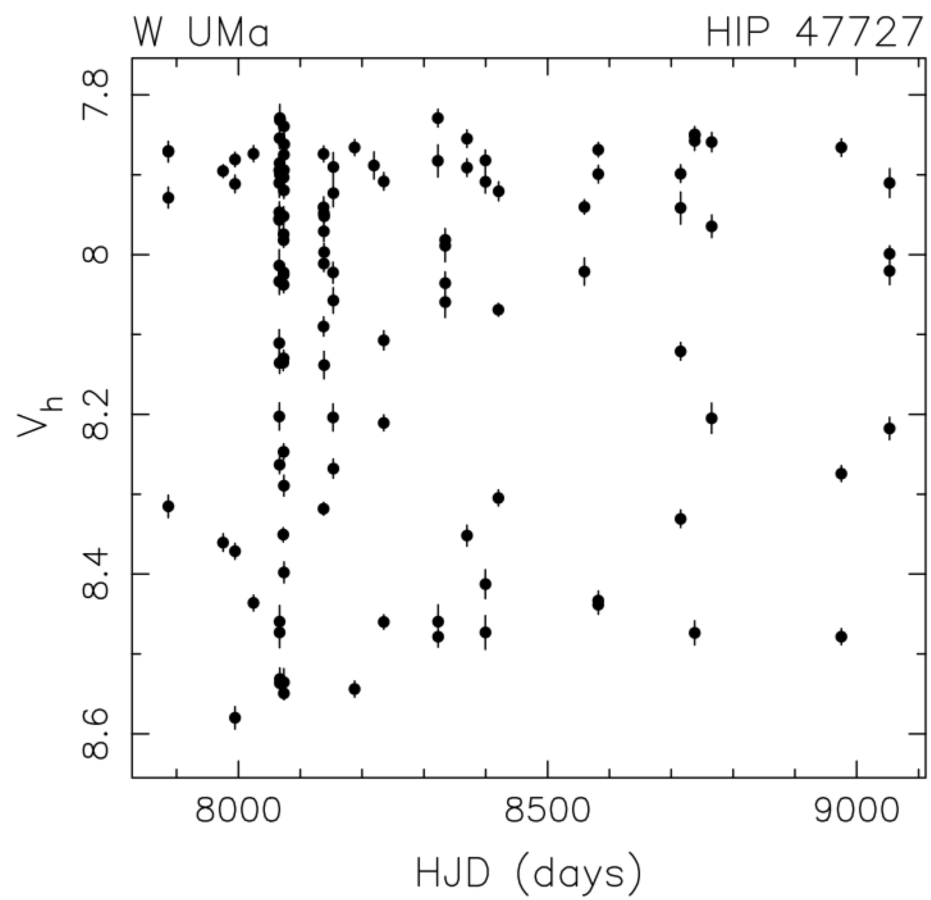
Astronomers usually consider S/N > 3σ as significant.

Same Distribution, Different Signals

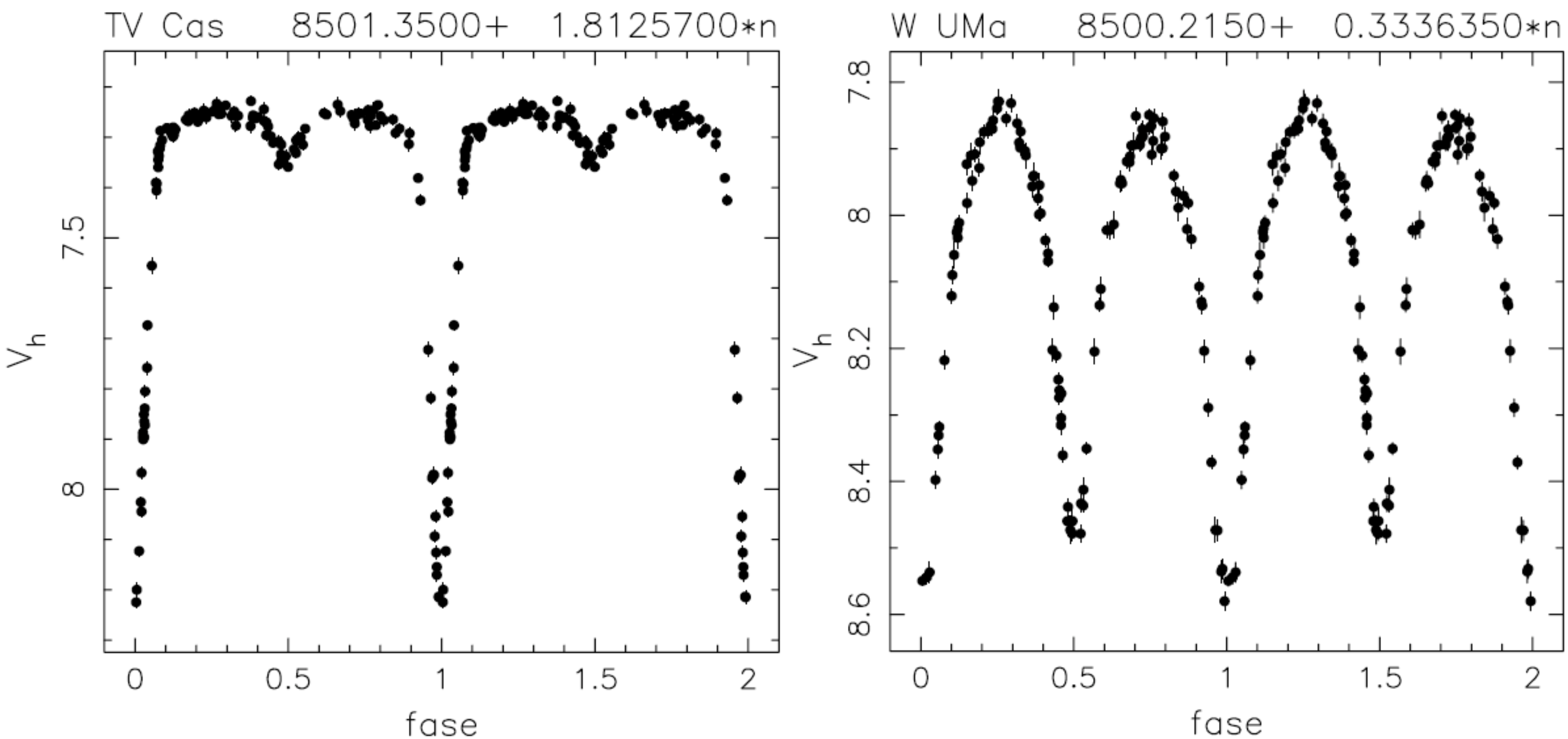


Time Series of Two Stars





Two Stars: Differently Sorted

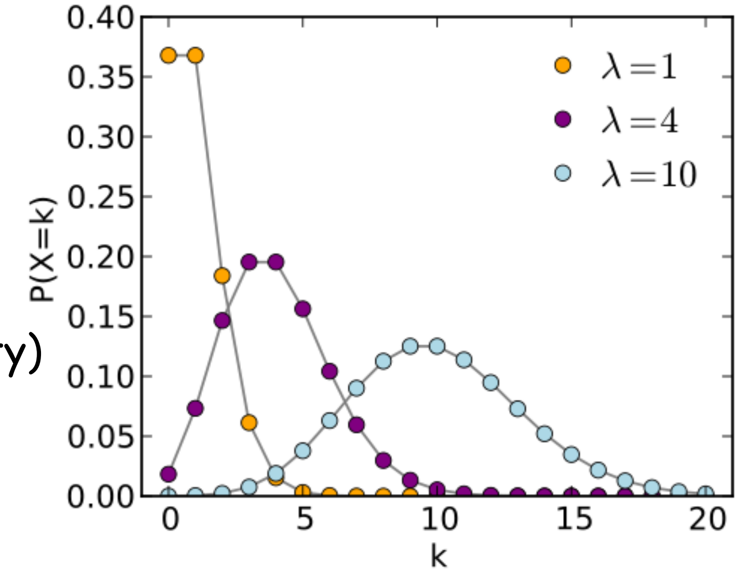


Noise Distribution: 2. Poisson Noise

Poisson noise has Poissonian distribution.

Probability of number of events occurring in constant interval of time/ space if events occur with known *average rate* and *independently* of each other.

$$P(k,\lambda)=\frac{\lambda^k e^{-\lambda}}{k!}$$



k is the number of occurrences of an event (probability)

λ is the *expected* number of occurrences

- the mean (average) of *P(k,λ)* is *λ*.
- the standard deviation of *P(k,λ)* is √*λ*.

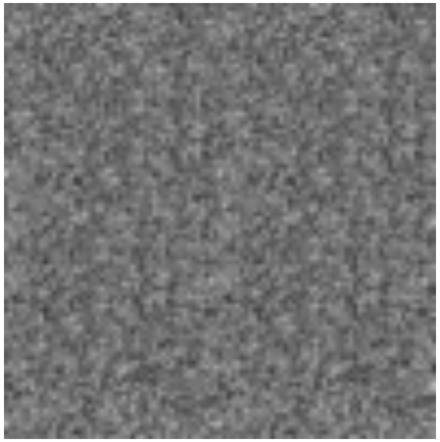
Example: fluctuations in the detected photon flux between finite time intervals Δt. Detected are k photons, while expected are on average λ photons.

Side note on Noise Measurement

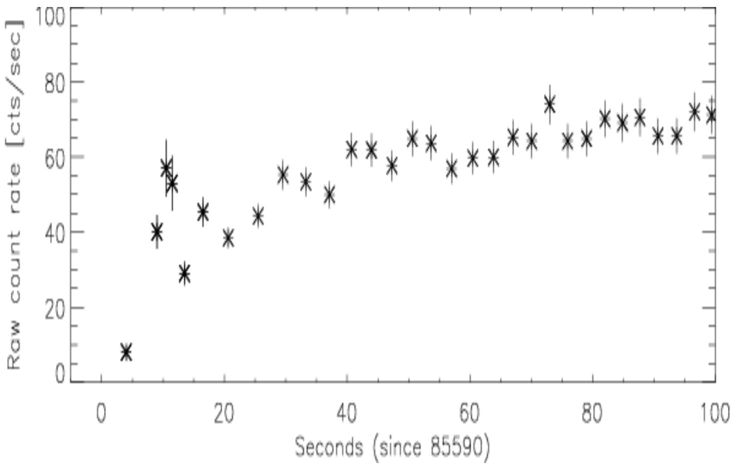
Let's assume the noise distribution is purely Gaussian or Poissonian, no other systematic noise, no correlations

Then the spatial distribution (neighbouring pixels) of the noise is equivalent to the temporal distribution (successive measurements with one pixel)

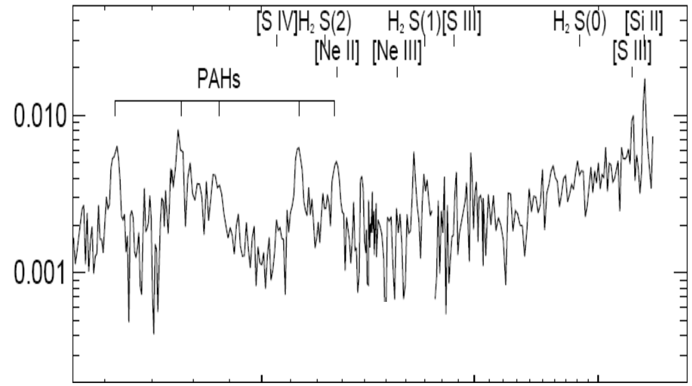
This is analogous to throwing 5 dices once versus throwing one dice 5 times.



Case 1: Spatial noise (detector pixels)



Case 2: Repeated measurements in time (time series)



Case 3: Spectrum (dispersed information)

Poisson Noise and Integration Time

- Integrate light from uniformly extended source on CCD
- In finite time interval Δt, expect average of λ photons.
- Due to statistical nature of photon arrival rate, some pixels will detect more, some less than λ photons.
- Noise of signal λ (i.e., between pixels) is $\sqrt{\lambda}$
- Integrate for $2\times\Delta t$ $\rightarrow$ expect average of $2\times\lambda$ photons
- Noise of that signal is now $\sqrt{(2\times\lambda)}$, i.e., increased by $\sqrt{2}$
- With respect to integration time t, noise will only increase $\sim\sqrt{t}$ while signal increases $\sim t$.

S/N Basics

Wikipedia:

Signal-to-noise ratio (often abbreviated SNR or S/N) is a measure used in science and engineering that compares the level of a desired signal to the level of background noise.

Signal = S; Background = B; Noise = N;

$$\sigma = \frac{\text{Signal}}{\text{Noise}}$$

← measured as (S+B) - mean{B}

← total noise = $\sqrt{\sum (N_i)^2}$ (if statistically independent)

Both S and N should be in units of events (photons, electrons, data numbers) per unit area (pixel, PSF size, arcsec^2).

S/N and Integration Time

Assuming the signal suffers from Poisson shot noise. Let's calculate the dependence on integration time t_{int}:

Integrating t_{int}: $\sigma = \frac{S}{N}$

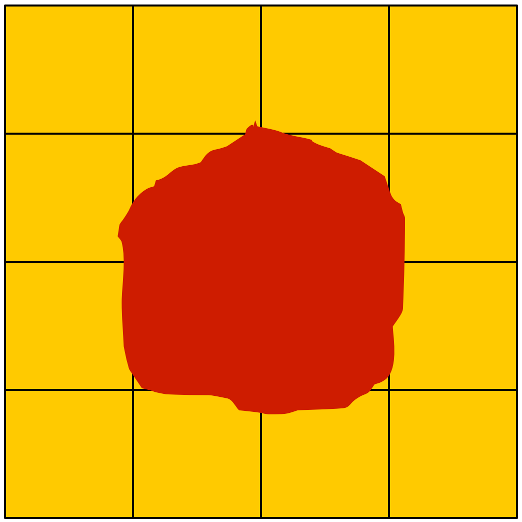
Integrating $n \times t_{int}$: $\sigma = \frac{n \cdot S}{\sqrt{n \cdot B}} \overset{N=\sqrt{B}}{=} \sqrt{n}\frac{S}{N} \quad \Rightarrow \quad \frac{S}{N} \propto \sqrt{t_{\text{int}}}$

Need to integrate four times as long to get twice the S/N.

Several Cases to Consider...

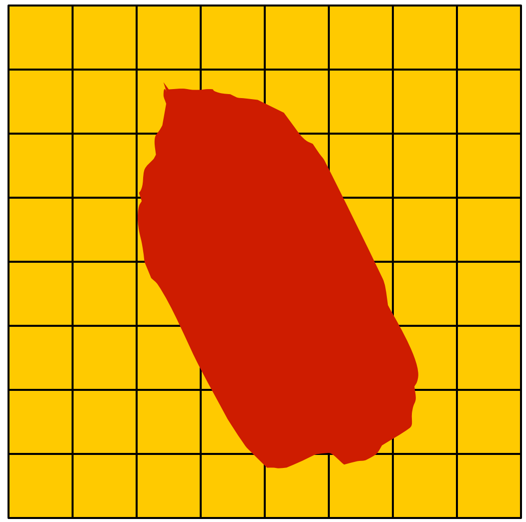
Background (=noise)

Target



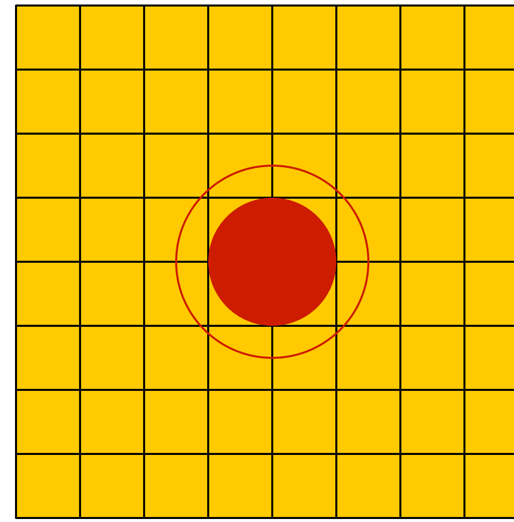
Seeing-limited
point source

- pixel size ~ seeing
- PSF ≠ f{D}



Diffraction-limited,
extended source

- pixel size ~ diff.lim
- PSF = f{D}
- target » PSF



Diffraction-limited,
point source

- pixel size ~ diff.lim
- PSF = f{D}
- target « PSF

Case 1: Seeing-limited "Point Source"

Signal = S; Background = B; Noise = N; Telescope diameter = D

θ_{seeing} ~ const

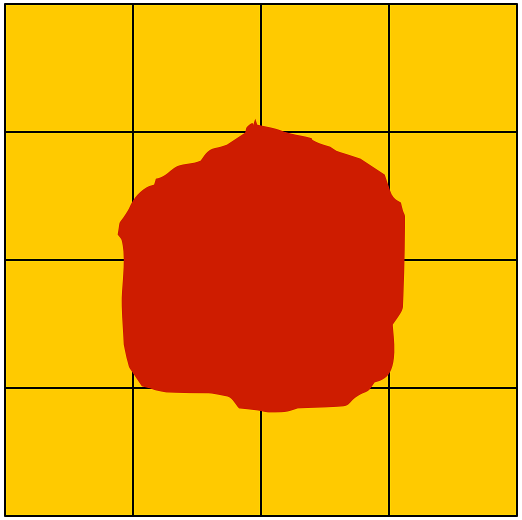
If detector is Nyquist-sampled to θ_{seeing} :

$S \sim D^2$ (area)

$B \sim D^2 \rightarrow N \sim D$ (Poisson std.dev)

➔ $S/N \sim D$

➔ $t_{int} \sim D^{-2}$



Case 2: Diffraction-limited extended Source

Signal = S; Background = B; Noise = N; Telescope diameter = D

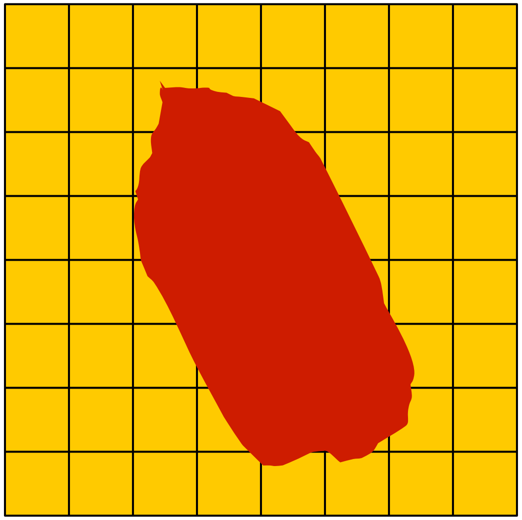
"Diameter" of PSF ~ const

If detector Nyquist sampled to θ_{diff}: pixel ~ D^{-2} but S ~ D^2

D^2 (telescope size) and D^{-2} (pixel FOV) cancel each other
→ no change in signal

same for the background flux

➔ S/N ~ const → t_{int} ~ const
→ *no gain for larger telescopes!*

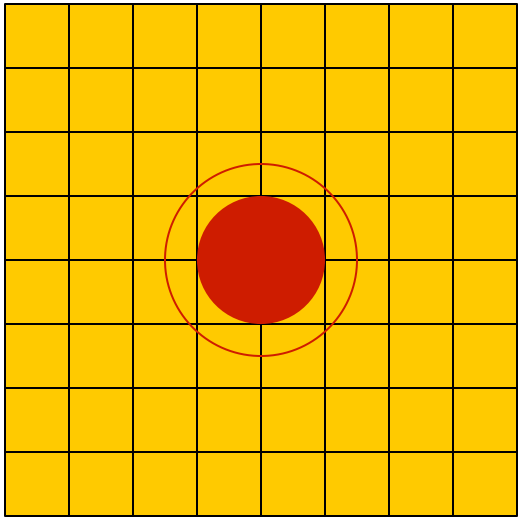


Case 2B: offline re-sampling by a factor x
(makes θ_{diff} x-times larger)

since $S/N \sim \sqrt{n_{pix}}$ → $S/N \sim \sqrt{x^2} = x$ → $t_{int} \sim x^{-2}$.

Case 3: Diffraction-limited "Point Source"

Signal = S; Background = B; Noise = N; Telescope diameter = D



"$S/N = (S/N)_{light\ bucket} \cdot (S/N)_{pixel\ scale}$"

(i) Effect of telescope aperture:

Signal $S \sim D^2$ → $S/N \sim D$

Background $B \sim D^2$ → $N \sim D$

(ii) Effect of pixel FOV (if Nyquist sampled to θ_{diff}):

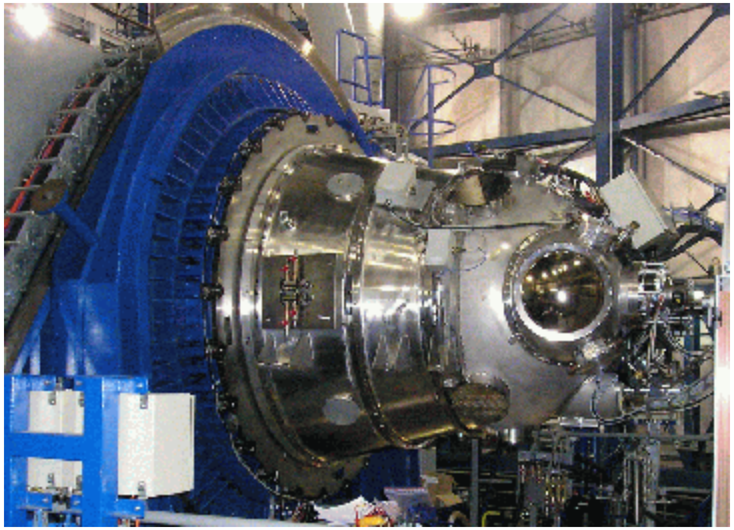
$S \sim$ const (pixel samples PSF = all source flux)

$B \sim D^{-2}$ → $N \sim D^{-1}$ → $S/N \sim D$

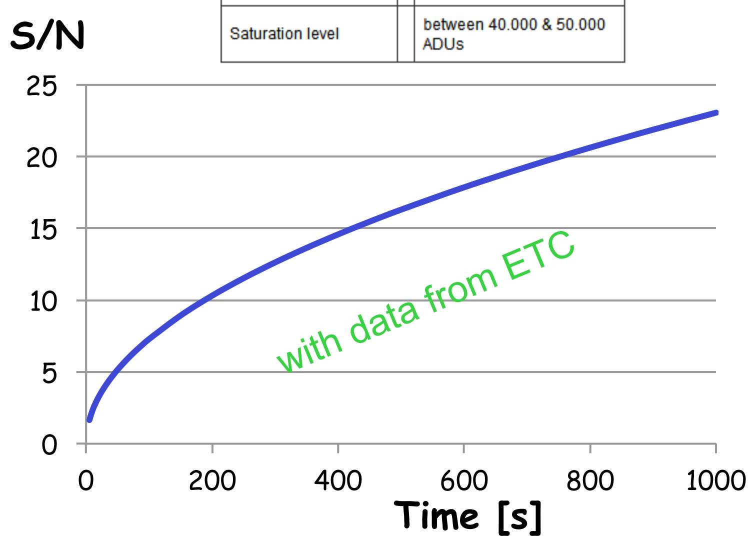
(i) and (ii) combined $S/N \sim D^2$ → $t_{int} \sim D^{-4}$

→ *huge gain: 1hr ELT = 3 months VLT*

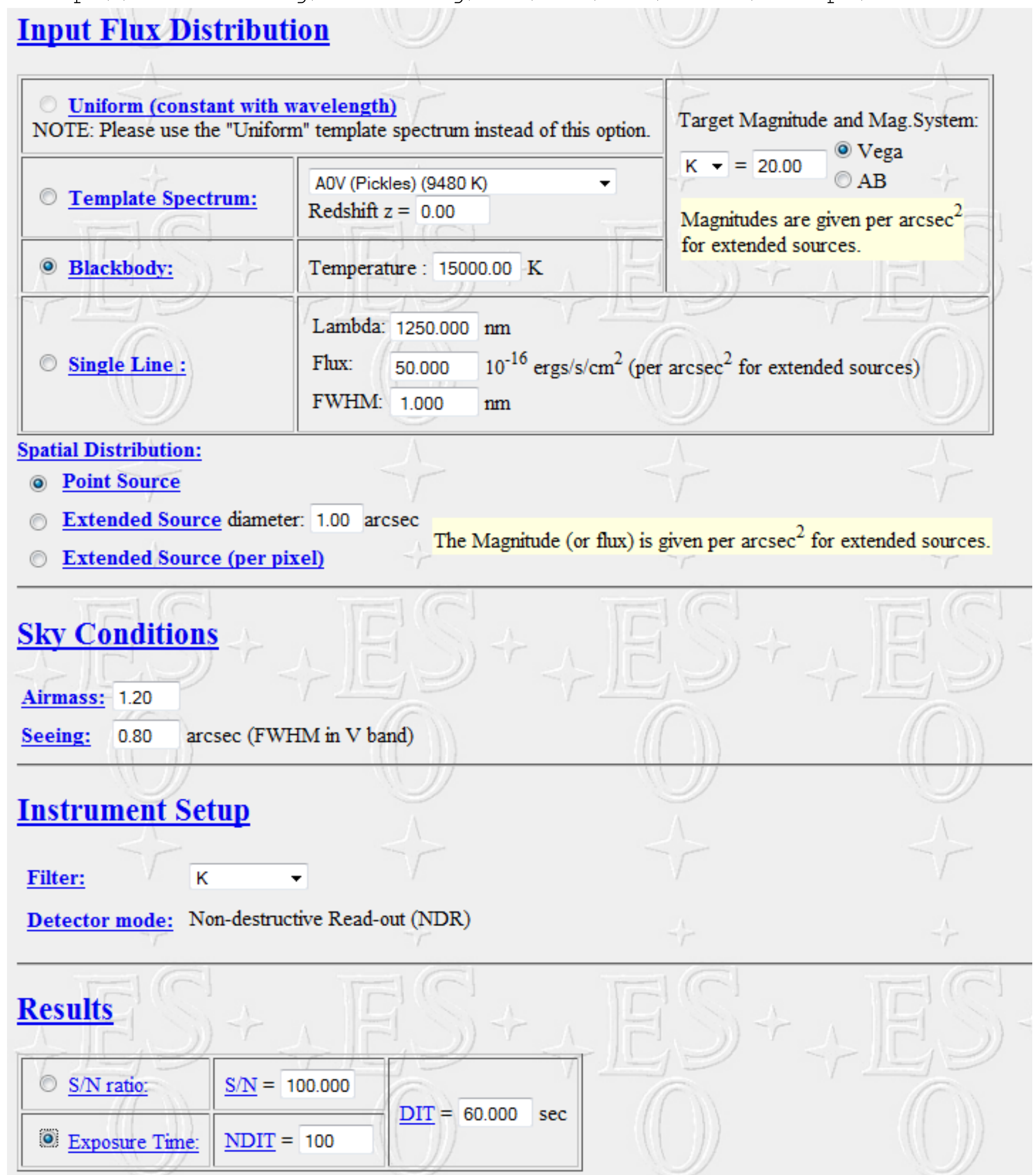
Instrument Sensitivity Example: HAWK-I



Operating temperature	75K, controlled to 1mK
Dark current [e-/s] (at 75K)	between 0.10 & 0.15
Read noise* (DCR)	~ 12 e-
Read noise* (NDR)	~ 5 e-
Linear range (1%)	60.000e- (~30.000 ADUs)
Saturation level	between 40.000 & 50.000 ADUs



http://www.eso.org/observing/etc/bin/ut4/hawki/script/hawkisimu



Input Flux Distribution

Uniform (constant with wavelength)
NOTE: Please use the "Uniform" template spectrum instead of this option.

Target Magnitude and Mag.System: K = 20.00 Vega / AB

Magnitudes are given per arcsec^2 for extended sources.

Template Spectrum: A0V (Pickles) (9480 K) Redshift z = 0.00

Blackbody: Temperature : 15000.00 K

Single Line : Lambda: 1250.000 nm Flux: 50.000 10^{-16} ergs/s/cm^2 (per arcsec^2 for extended sources) FWHM: 1.000 nm

Spatial Distribution:
- Point Source
- Extended Source diameter: 1.00 arcsec
- Extended Source (per pixel)

The Magnitude (or flux) is given per arcsec^2 for extended sources.

Sky Conditions

Airmass: 1.20

Seeing: 0.80 arcsec (FWHM in V band)

Instrument Setup

Filter: K

Detector mode: Non-destructive Read-out (NDR)

Results

S/N ratio: S/N = 100.000

Exposure Time: NDIT = 100

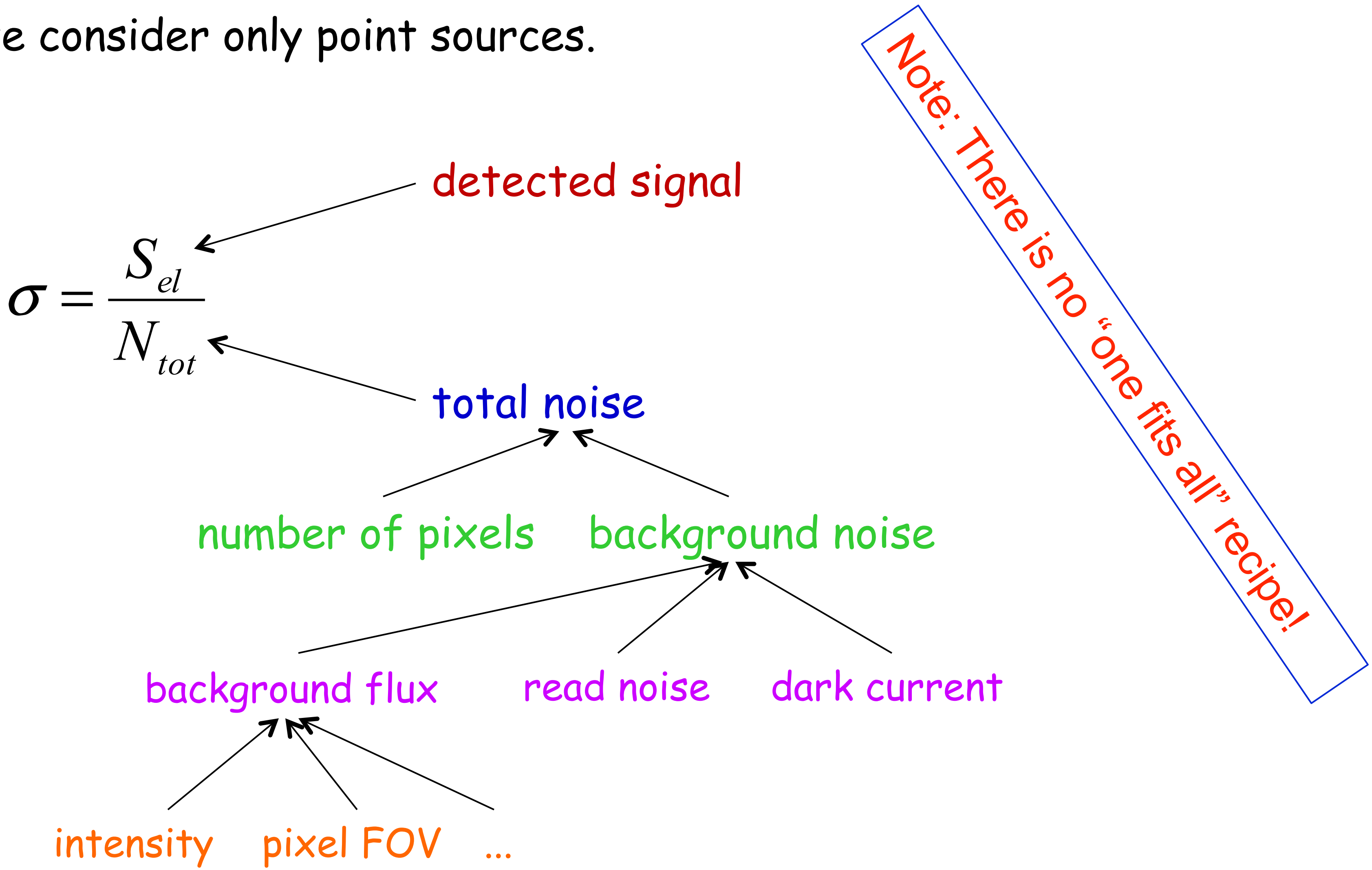
DIT = 60.000 sec

Instrument Sensitivity: Preface

(i) in this discussion we neglect quantum (shot) noise from source.

(ii) we consider only point sources.

$$\sigma = \frac{S_{el}}{N_{tot}}$$



Detected Signal

Detected signal S_{el} depends on:

- source flux density S_{src} [photons s^{-1} cm^{-2} μm^{-1}]
- integration time t_{int} [s]
- telescope aperture A_{tel} [m^2]
- transmission of the atmosphere η_{atm}
- total throughput of the system η_{tot}, which includes:
  - reflectivity of all telescope mirrors
  - reflectivity (or transmission) of all instrument components, such as mirrors, lenses, filters, beam splitters, grating efficiencies, slit losses, etc.
- Strehl ratio SR (ratio of actual to theoretical maximum intensity)
- detector responsivity $\eta_D G$
- spectral bandwidth $\Delta\lambda$ [µm]

$$S_{el} = S_{src} \cdot SR \cdot \Delta\lambda \cdot A_{tel} \cdot \eta_D G \cdot \eta_{atm} \cdot \eta_{tot} \cdot t_{\text{int}}$$

Total Noise (1)

Total noise N_{tot} depends on:

- number of pixels n_{pix} of one resolution element
- background noise per pixel N_{back}

$$N_{tot} = N_{back}\sqrt{n_{pix}}$$

total background noise N_{back} depends on:

- background flux density S_{back}
- integration time t_{int}
- detector dark current I_d
- pixel read noise (N) and detector frames (n)

$$N_{back} = \sqrt{S_{back} \cdot t_{\text{int}} + I_d \cdot t_{\text{int}} + N_{read}^2 \cdot n}$$

Total Noise (2)

Background flux density S_{back} depends on:

- the total background intensity $B_{tot} = \left(B_T + B_A\right)\cdot \eta_{tot}$

  where B_T and B_A are the thermal emissions from telescope and atmosphere, approximated by black body emission

$$B_{T,A} = \frac{2hc^2}{\lambda^5}\left[\frac{\varepsilon}{\exp\left[\frac{hc}{kT\lambda}\right]-1}\right]$$

- the spectral bandwidth $\Delta\lambda$
- the pixel field of view $A\times\Omega = 2\pi\left(1-\cos\left(\arctan\left(\frac{1}{2F\#}\right)\right)\right)D^2{}_{pix}$
- the detector responsivity $\eta_D G$, and
- the photon energy hc/λ

$$S_{back} = B_{tot}\cdot A\times\Omega\cdot\frac{\eta_D G\lambda}{hc}\cdot\Delta\lambda$$

Resulting Instrument Sensitivity

Putting it all together, the minimum detectable source signal is:

$$\sigma = \frac{S_{el}}{N_{tot}} = \frac{S_{src} \cdot SR \cdot \Delta\lambda \cdot A_{tel} \cdot \eta_D G \cdot \eta_{atm} \cdot \eta_{tot} \cdot t_{int}}{N_{back}\sqrt{n_{pix}}}$$

$$\Rightarrow S_{src} = \frac{\sigma \cdot \sqrt{S_{back} \cdot t_{int} + I_d \cdot t_{int} + N_{read}^2 \cdot n} \cdot \sqrt{n_{pix}}}{SR \cdot \Delta\lambda \cdot A_{tel} \cdot \eta_D G \cdot \eta_{atm} \cdot \eta_{tot} \cdot t_{int}}$$

Now we can calculate the unresolved line sensitivity S_{line} [W/m^2] from the source flux S_{src} [photons/s/cm^2/µm]:

$$S_{line} = \frac{hc}{\lambda} S_{src} \Delta\lambda \cdot 10^4$$

and with the relation $S_\lambda\left[\frac{W}{m^2 \mu m}\right] = S_\nu [Jy] \cdot 10^{-26} \frac{c}{\lambda^2}$

we can calculate the continuum sensitivity S_{cont}:

$$S_{cont} = \frac{hc}{\lambda} S_{src} \cdot 10^4 \cdot \frac{\lambda^2}{c} \cdot 10^{26} = 10^{30} h \lambda S_{src}$$