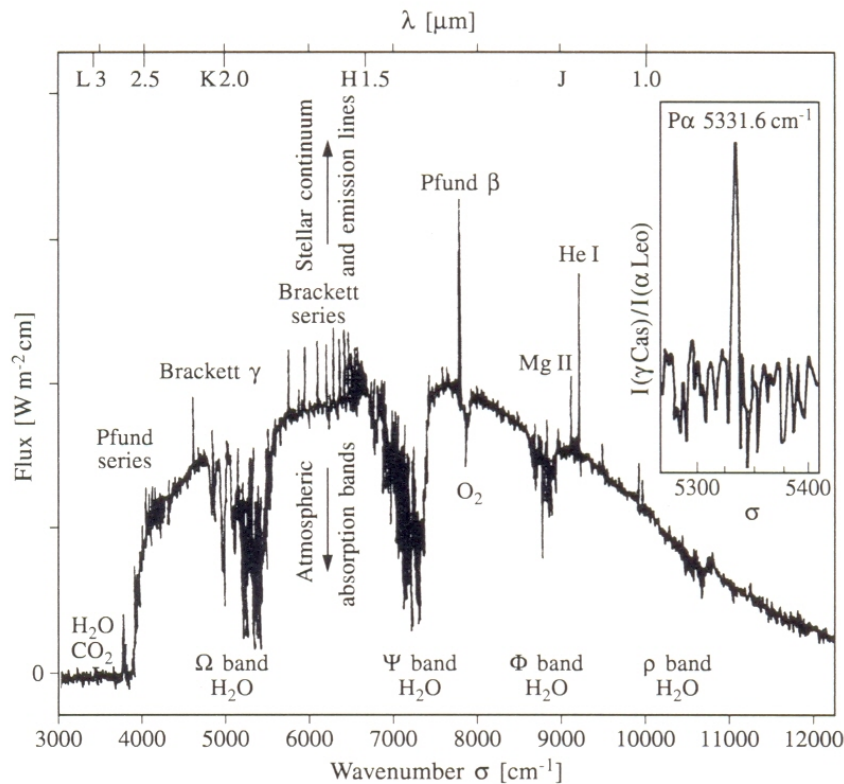


# Astronomische Waarneemtechnieken (Astronomical Observing Techniques)

based on lectures by Bernhard Brandl



## Lecture 2: Earth Atmosphere

1. Atmospheric Structure
2. Absorption
3. Emission
4. Scattering, Refraction & Dispersion
5. Turbulence & Seeing

# 1. Atmospheric Structure

Assumptions:

- atmosphere in local radiative equilibrium
- homogeneous composition

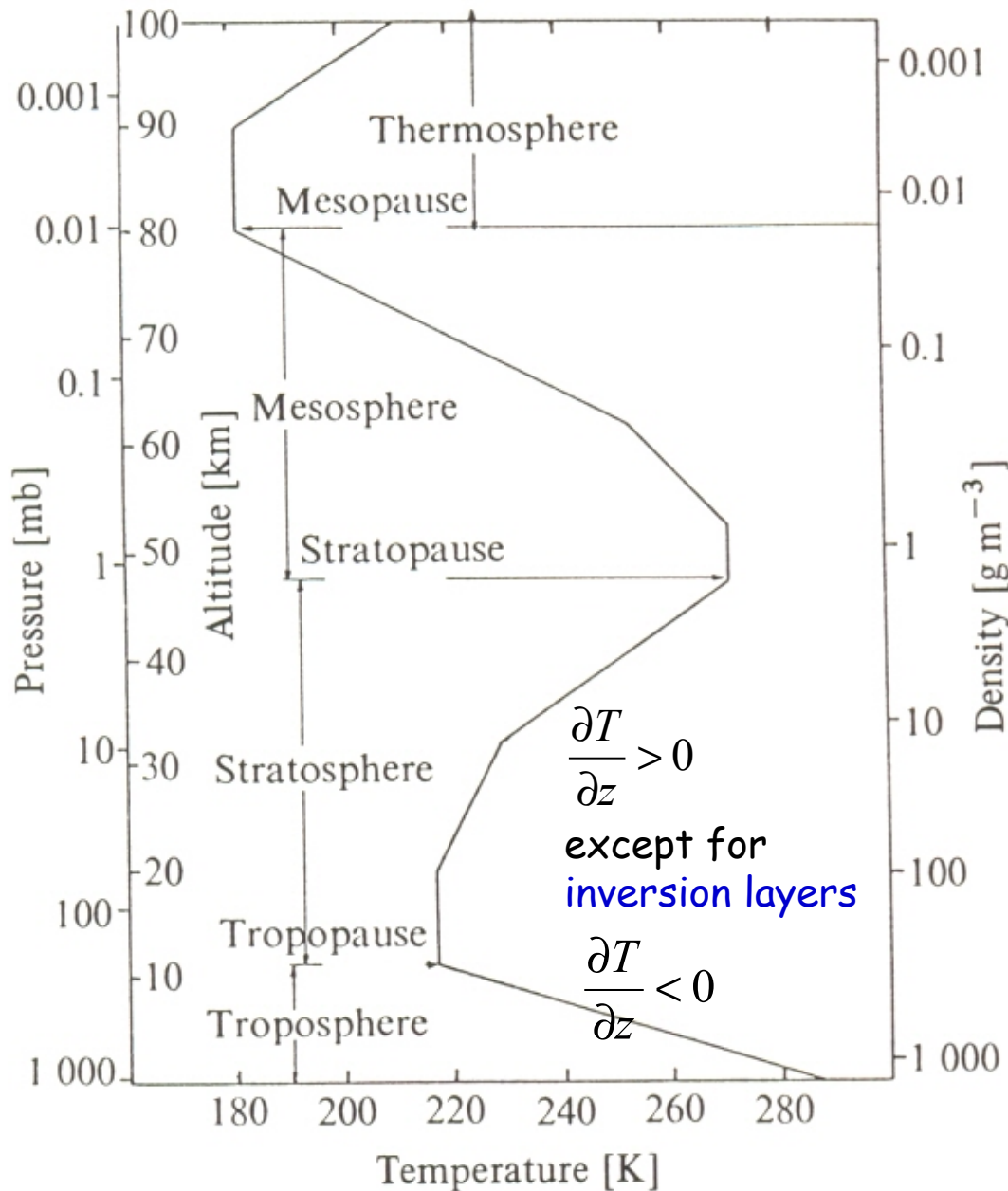
Hydrostatic equilibrium structure described by:

- altitude  $z$
- temperature  $T(z)$
- density  $\rho(z)$

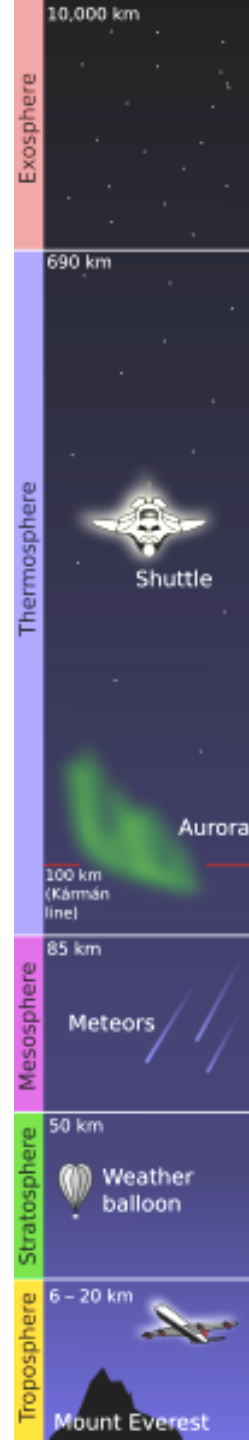
Pressure  $P(z)$  described by:  $P(z) = P_0 e^{-\frac{z}{H}}$ ,  $H = \frac{kT}{\mu g}$

$H =$  scale height ( $\sim 8\text{km}$ ),  $\mu =$  mean molecular weight

# Vertical Profile



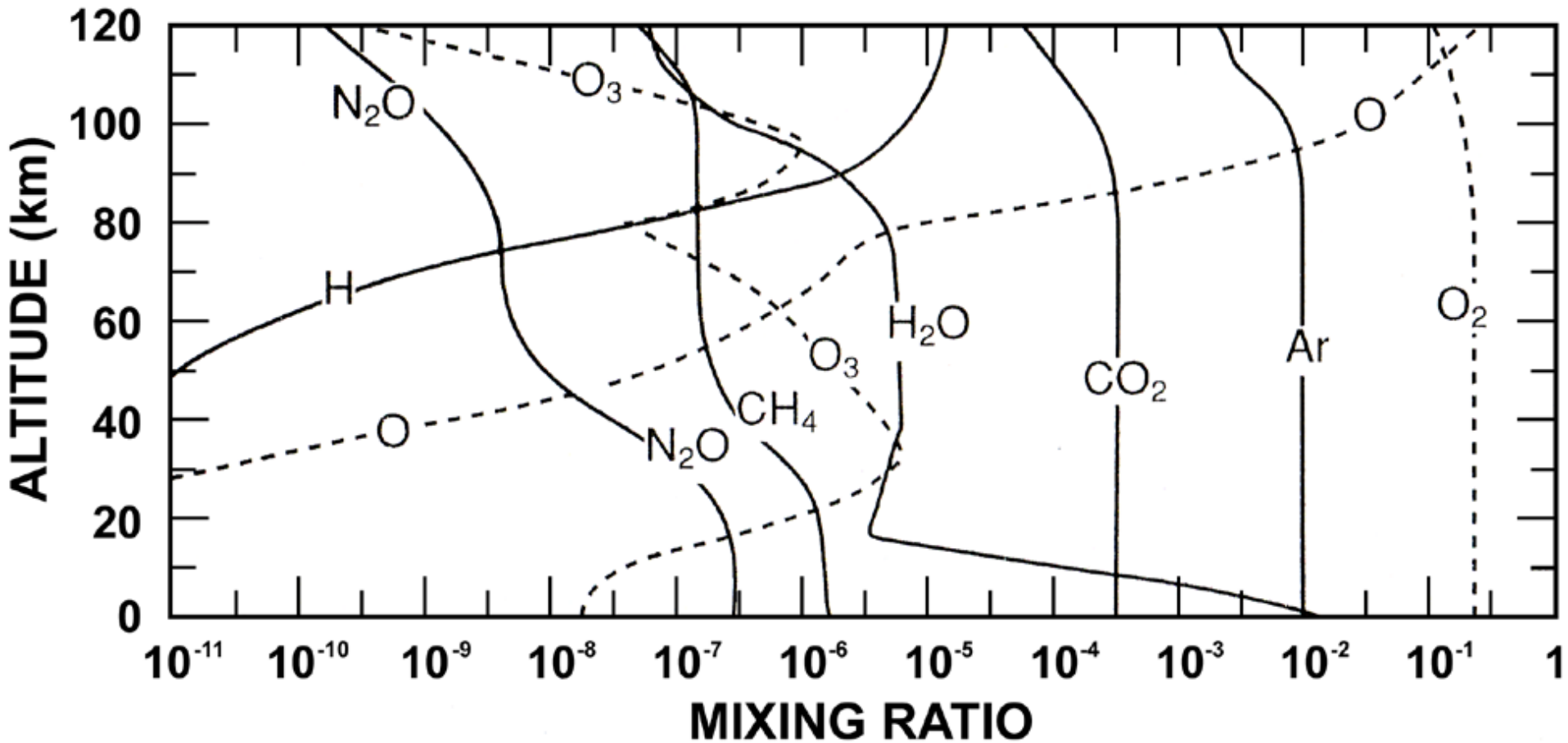
## Ionosphere



# Atmospheric Composition

- Main constituents:  $O_2$  and  $N_2$   
relative constant proportions (78.1%  $N_2$ , 20.9%  $O_2$ ) up to 100 km
- **Ozone** - mainly absorbs in UV
  - distribution depends on latitude and season
  - maximum concentration around 16 km height
- **CO<sub>2</sub>** - important component for (mid)IR absorption
  - mixing independent of altitude (similar to  $N_2$ ,  $O_2$ )
- **Ions** - varies strongly with altitude and solar activity
  - relevant above 60km where reactions with UV photons occur:  
$$O_2 + h\nu \rightarrow O_2^{+*} + e^- \quad \text{and} \quad O_2 + h\nu \rightarrow O^+ + O + e^-$$
  - electron showers along magnetic fields cause Aurora
  - at 100 - 300 km height:  $n_e \sim 10^5 - 10^6 \text{ cm}^{-3}$
- **Water vapour** - highly variable, causes very strong absorption bands

# Mixing Ratio of Atmospheric Gases



# 2. Absorption

Atomic and molecular transitions causing absorption features:

- **pure rotational** molecular transitions:  $\text{H}_2\text{O}$ ,  $\text{CO}_2$ ,  $\text{O}_3$ ,
- **rotation-vibrational** molecular transitions:  $\text{CO}_2$ ,  $\text{NO}$ ,  $\text{CO}$
- **electronic** molecular transitions:  $\text{CH}_4$ ,  $\text{CO}$ ,  $\text{H}_2\text{O}$ ,  $\text{O}_2$ ,  $\text{O}_3$ ,  $\text{OH}$
- **electronic** atomic transitions:  $\text{O}$ ,  $\text{N}$ , ...

Attenuation at altitude  $z_0$ : 
$$I(z_0) = I_0(\infty) \cdot \exp\left[-\frac{1}{\cos\theta} \sum_i \tau_i(\lambda, z_0)\right]$$

for  $i$  absorbing species with **optical depth** 
$$\tau_i(\lambda, z_0) = \int_{z_0}^{\infty} r_i(z) \rho_0(z) \kappa_i(\lambda) dz$$

( $\theta$  is the zenith distance;  $\kappa$  is the absorption coefficient;  $\rho_0$  is the mass density of air, and  $r_i(z)$  the mixing ratio).

# Atmospheric Bands

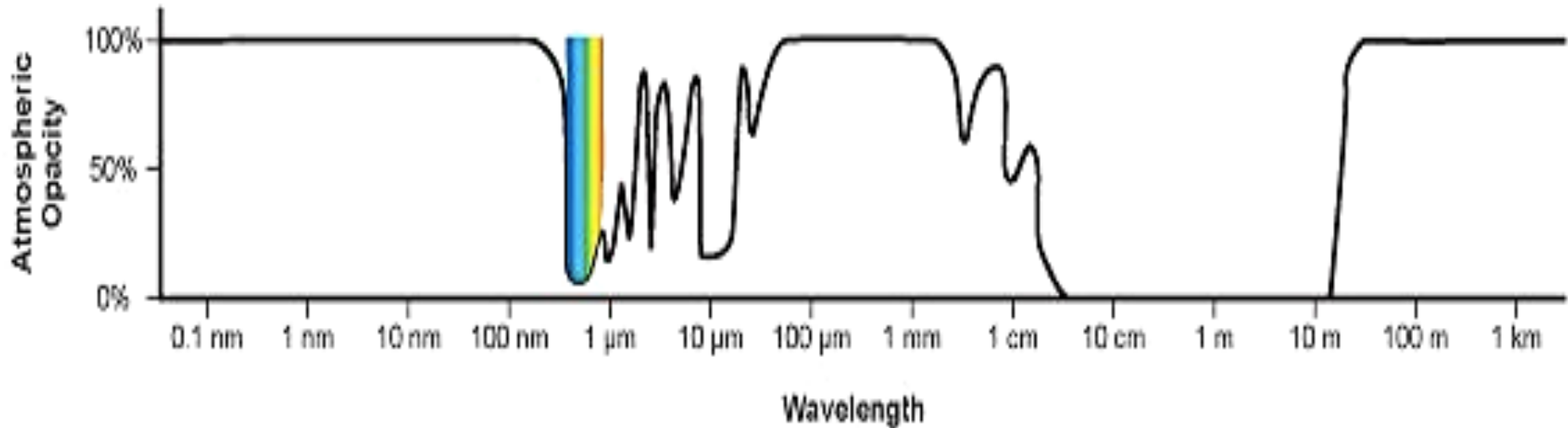
Two cases of absorption:

**total** absorption → atmospheric **transmission windows**

**partial** absorption → reduced transmission due to narrow **telluric\*** absorption features

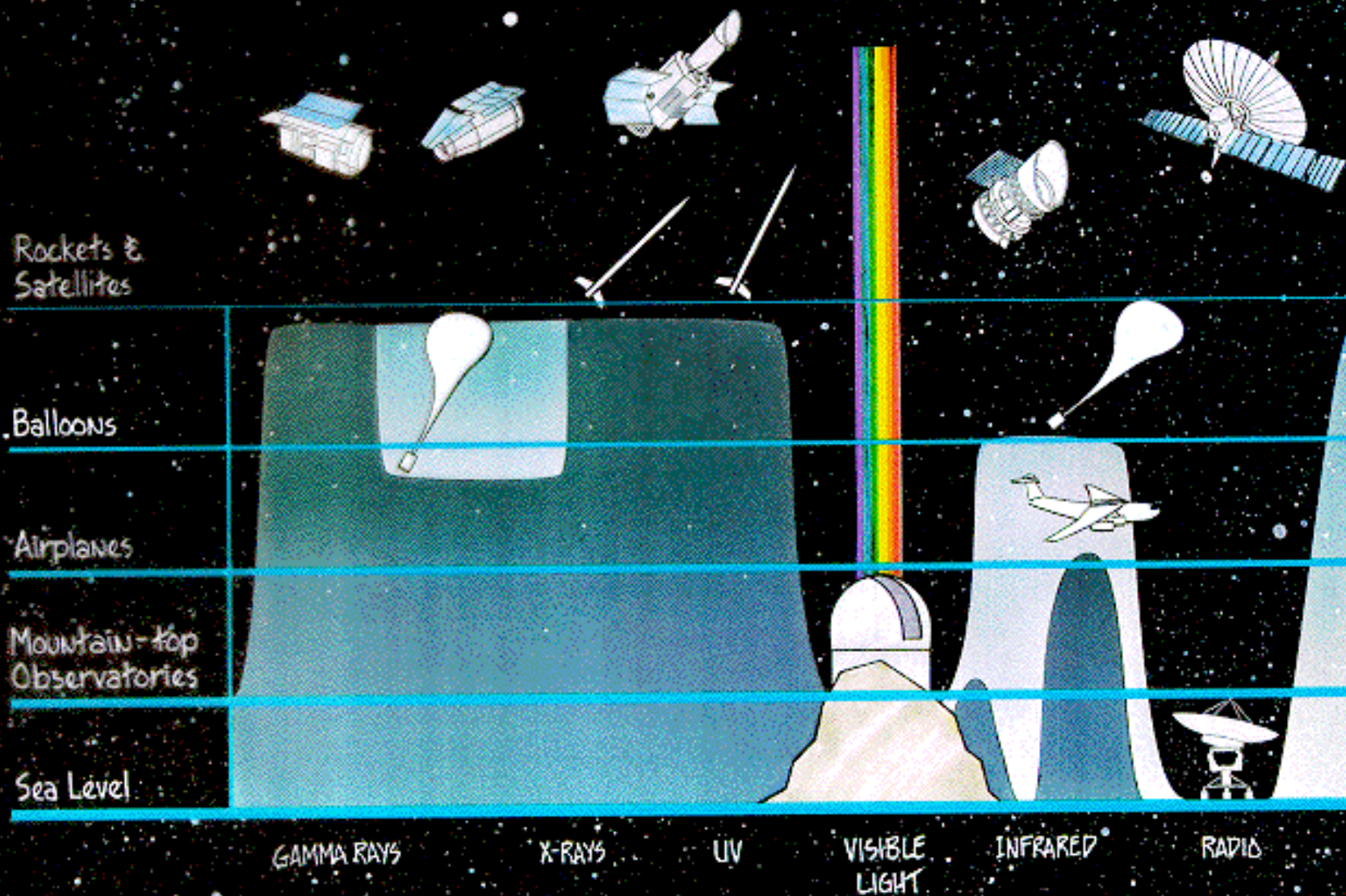
*\*Telluric = related to the Earth; of terrestrial origin*

Atmospheric opacity defines **atmospheric transmission bands**  
(wavelengths accessible to ground-based observations)



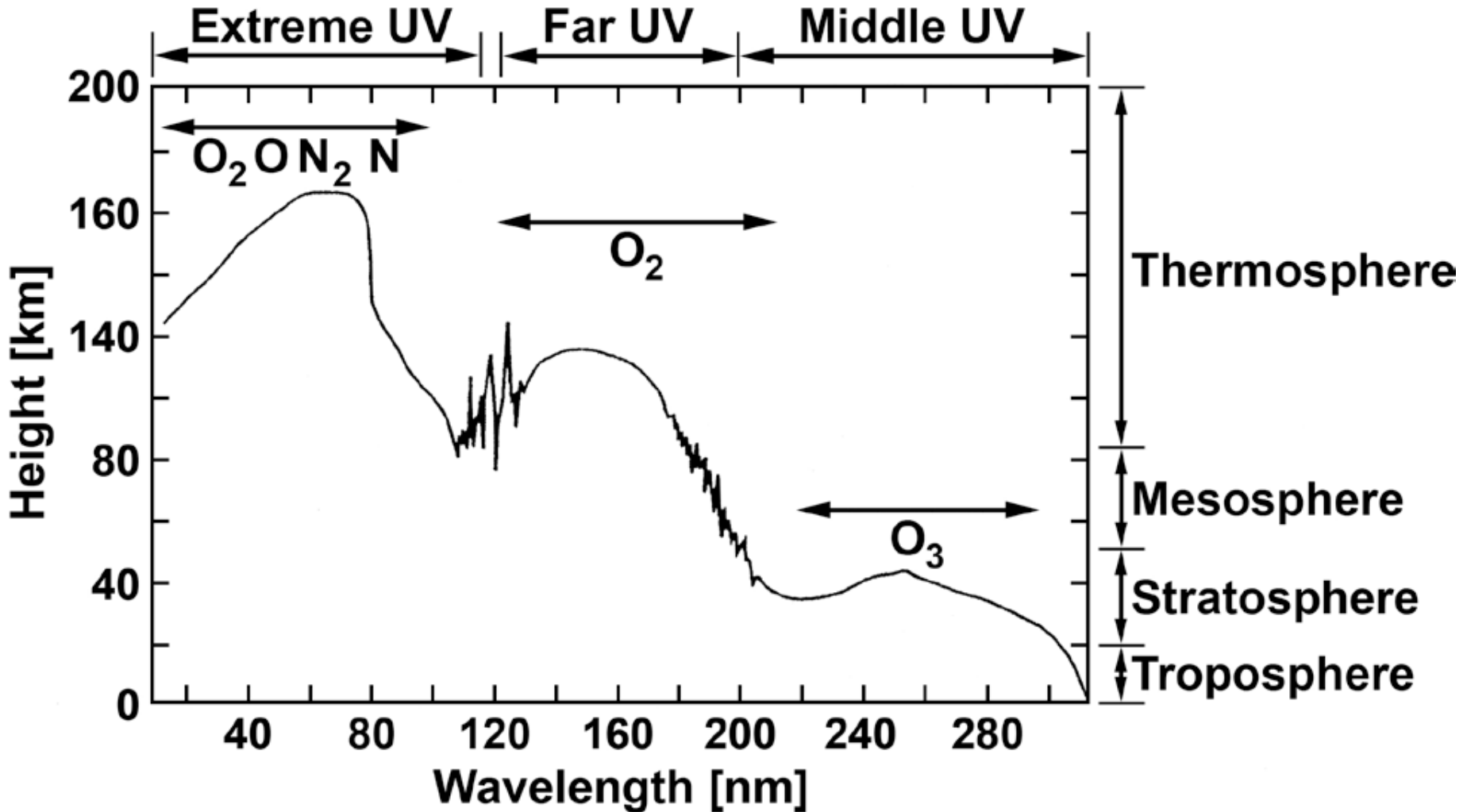
**Ground based** astronomy is limited to visible, near/mid-IR and radio wavelengths.

**Space astronomy** provides access to  $\gamma$ -rays, X-rays, UV, FIR, sub-mm





# UV Absorption





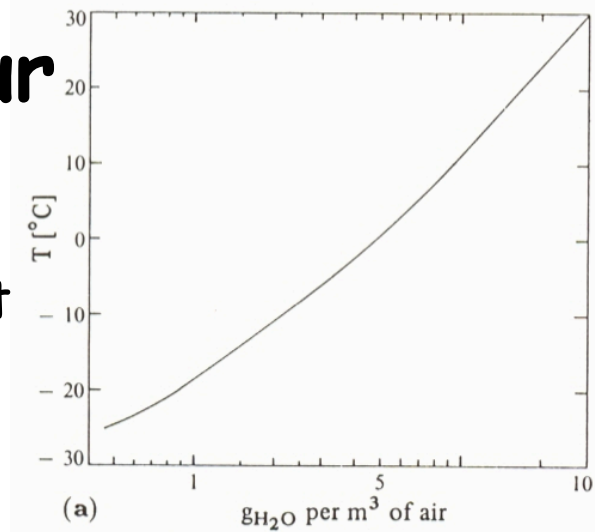
# Absorption by Water Vapour

Water vapor is strong function of  $T$  and  $z$ .

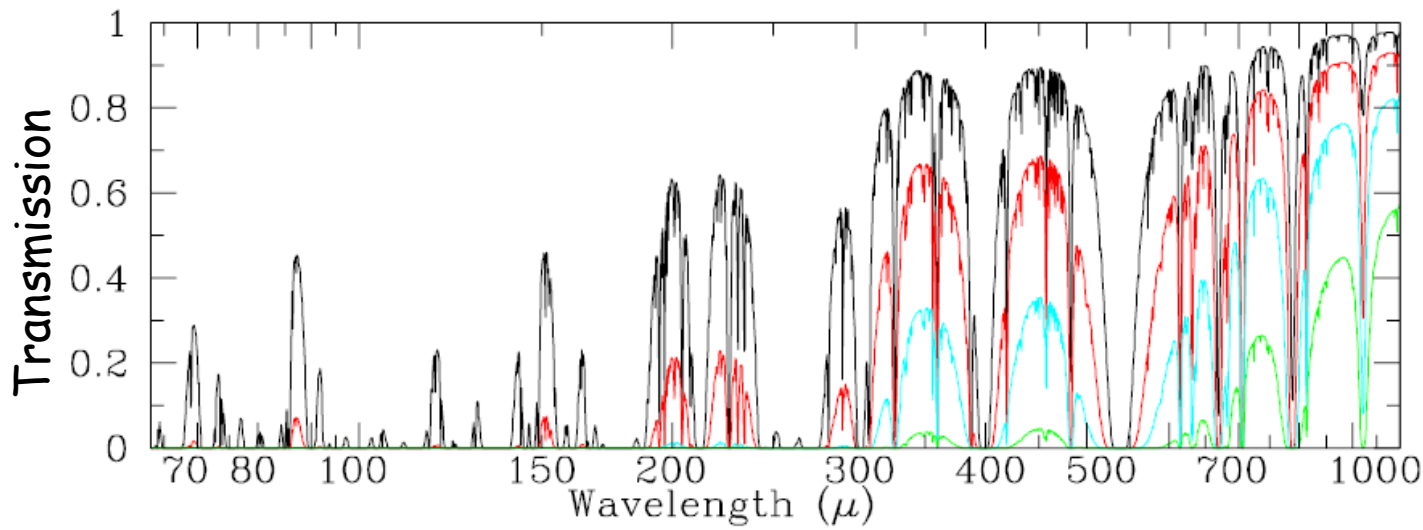
Precipitable water vapor (PWV) is depth of amount of water in column of atmosphere if all water in that column were precipitated as rain.

The amount of PWV above an altitude  $z_0$  is:

$$w(z_0) = \int_{z_0}^{\infty} N_{H_2O} dz, \quad \text{where} \quad N_{H_2O} [\text{m}^{-3}] = 4.3 \times 10^{25} \frac{P}{P_0} \frac{T}{T_0} r(z)$$

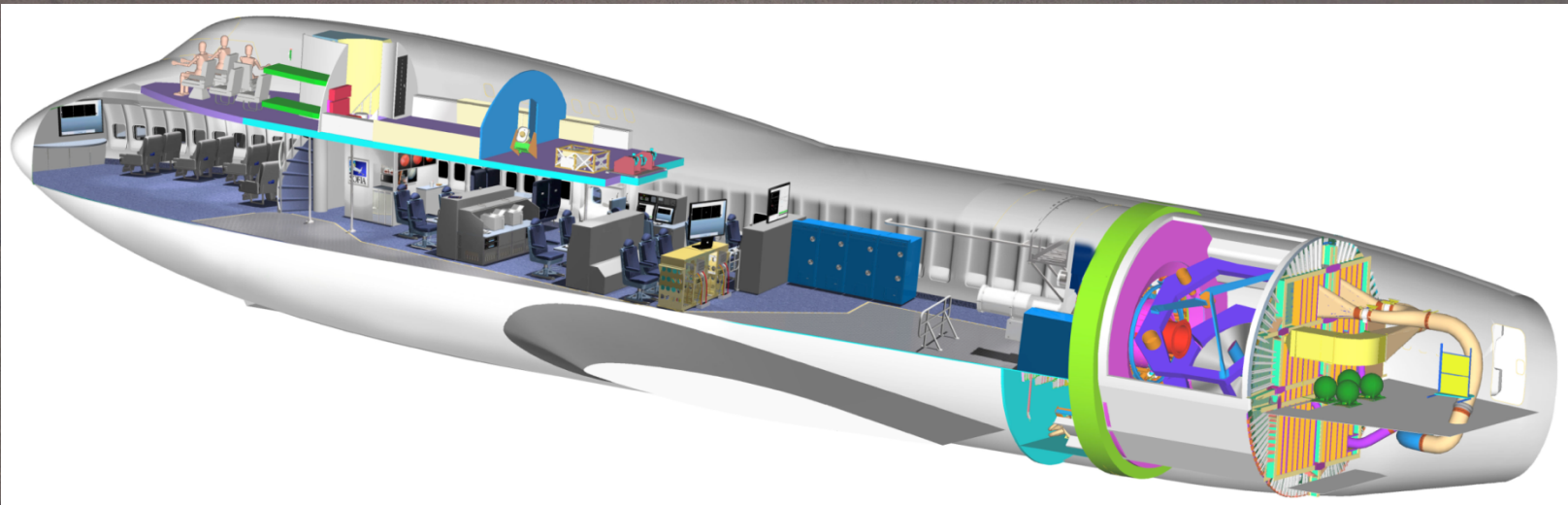


Scale height for PWV is only  $\sim 3$  km  $\rightarrow$  observatories at high altitudes



0.1 mm PWV  
 0.4 mm PWV  
 1.0 mm PWV  
 3.0 mm PWV

FIR/sub-mm astronomy is also possible from airplanes, e.g. the **Stratospheric Observatory for Infrared Astronomy (SOFIA)**



# 3. Atmospheric Emission

## A. Fluorescent Emission

**Fluorescence** = recombination of electrons with ions.

Recombination probability low; takes several hours → night time

- Produces both continuum + line emission = **airglow**
- Occurs mainly at ~ 100 km height
- Main sources of emission: O I, Na I, O<sub>2</sub>, OH (←NIR), H

Emission intensity measured in **Rayleigh**:

$$1 \text{ Rayleigh} = 10^6 \text{ photons cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1} = \frac{1.58 \cdot 10^{-11}}{\lambda[\text{nm}]} \text{ W cm}^{-2} \text{ sr}^{-1}$$

# B. Thermal Emission

Up to 60 km atmosphere in **local thermodynamic equilibrium (LTE)**, i.e., the excitation levels are thermally populated.

Calculating specific energy received requires **full radiative transfer calculation** (see below), but for  $\tau \ll 1$  one can use the **approximation**:

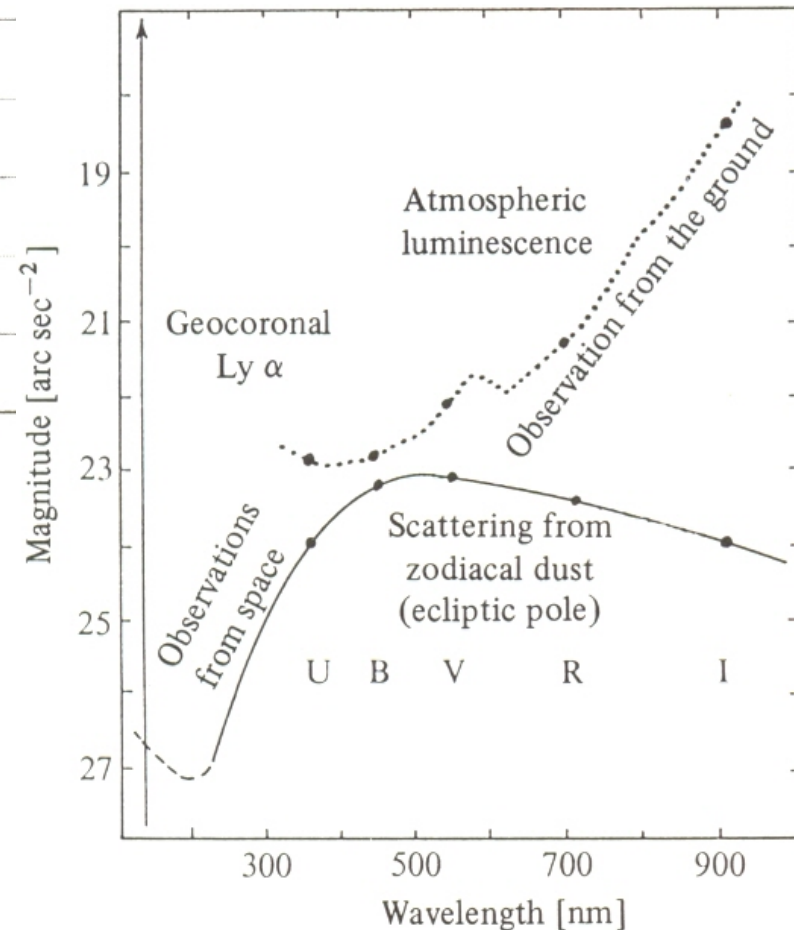
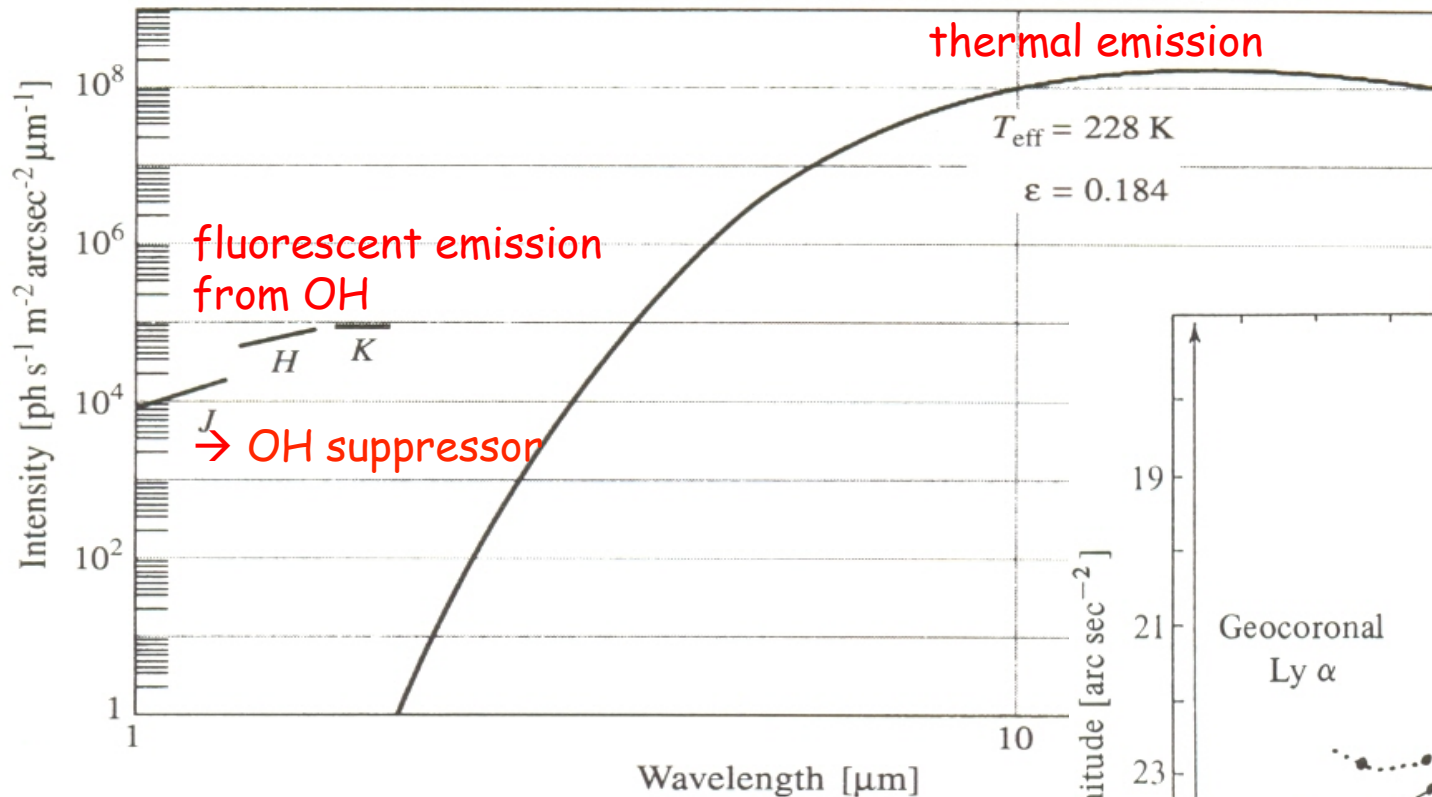
$$I_{\lambda}(z) = \tau_{\lambda} B_{\lambda}(\bar{T}) \frac{1}{\cos \theta}$$

where  $B(\underline{T})$  is the Planck function at mean temperature of atmosphere.

For  $\bar{T} = 250$  K and  $\theta = 0$ :

Spectral band	<i>L</i>	<i>M</i>	<i>N</i>	<i>Q</i>
Mean wavelength [ $\mu\text{m}$ ]	3.4	5.0	10.2	21.0
Mean optical depth $\tau$	0.15	0.3	0.08	0.3
Magnitude [arcsec <sup>-2</sup> ]	8.1	2.0	-2.1	-5.8
Intensity [Jy arcsec <sup>-2</sup> ] <sup>a</sup>	0.16	22.5	250	2 100

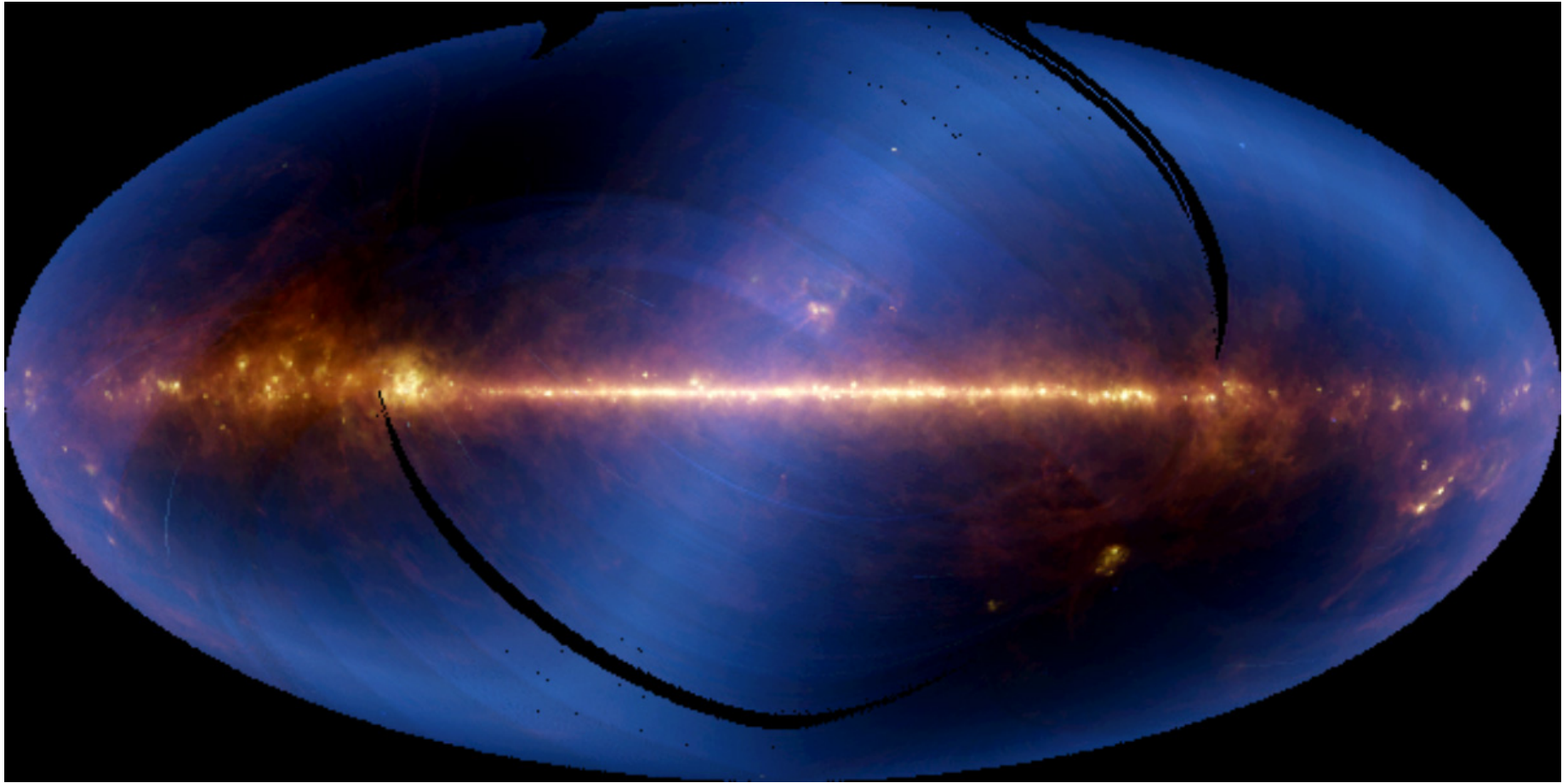
# Fluorescent and Thermal Emission



Sky surface brightness.

Important as even an unresolved point source has a finite angular diameter when viewed through a telescope.

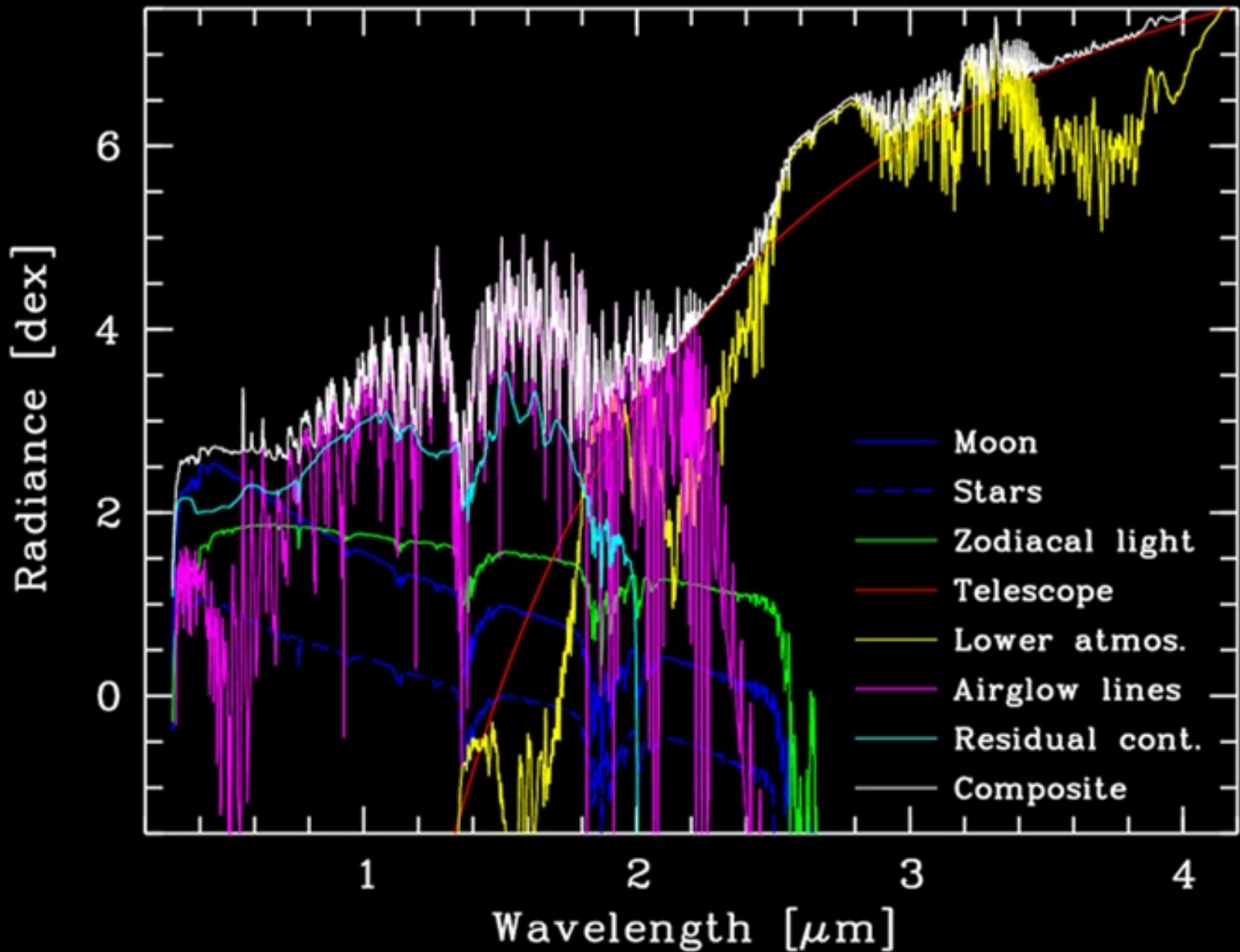
# Emission from Space



[www.ipac.caltech.edu/Outreach/Gallery/IRAS/allsky.html](http://www.ipac.caltech.edu/Outreach/Gallery/IRAS/allsky.html)



# Total Emission in near-infrared





# 4. Scattering, Refraction and Dispersion

## A. Scattering by Air Molecules

Molecular scattering in visible and NIR is Rayleigh scattering; scattering cross-section given by:

$$\sigma_R(\lambda) = \frac{8\pi^3}{3} \frac{(n^2 - 1)^2}{N^2 \lambda^4}$$

where  $N$  is the number of molecules per unit volume and  $n$  is the refractive index of air ( $n-1 \sim 8 \cdot 10^{-5} P/T$ ).

Remember, Rayleigh scattering is not isotropic:  $I_{scattered} = I_0 \frac{3}{16\pi} \sigma_R (1 + \cos^2 \theta) d\omega$

# B. Aerosol Scattering

Aerosols (sea salt, hydrocarbons, volcanic dust) are much bigger than air molecules  $\rightarrow$  Rayleigh scattering does *not* apply.

Instead, scattering is described by **Mie's theory** (from classical electrodynamics, using a "scattering efficiency factor"  $Q$ ):

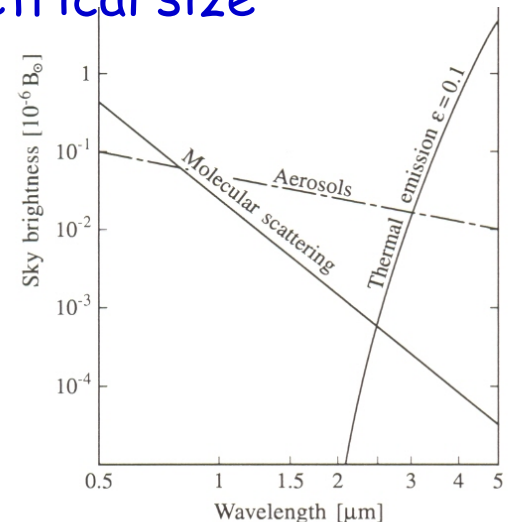
$$Q_{\text{scattering}} = \frac{\sigma_M}{\pi a^2} = \frac{\text{scattering cross section}}{\text{geometrical cross section}}$$

If  $a \gg \lambda$  then  $Q_{\text{scattering}} \sim Q_{\text{absorption}}$  and:

- the scattered power is equal to the absorbed power
- the **effective cross section is twice the geometrical size**

If  $a \sim \lambda$  then  $Q_s \sim 1/\lambda$  (for dielectric spheres):

- the **scattered intensity goes with  $1/\lambda$**



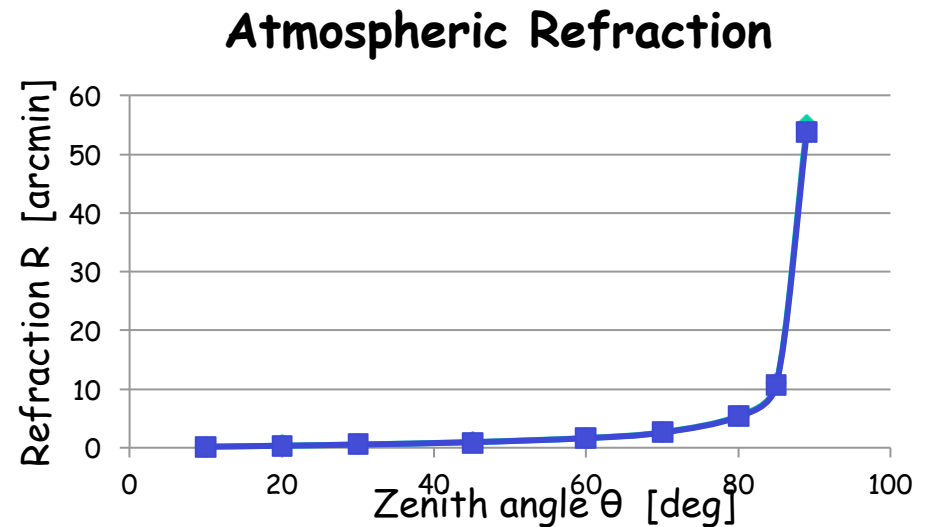
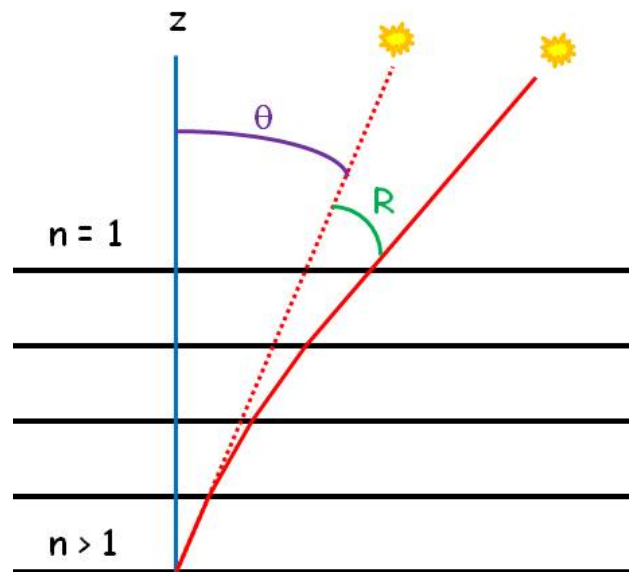
# Refraction



# Atmospheric Refraction

Due to **atmospheric refraction**, the *apparent* location of a source is significantly altered (up to half a degree near the horizon)  
 → telescope pointing model.

$$\text{Refraction } R = (n(\lambda) - 1) \tan \theta$$



Note that the refractive index of air depends on the wavelength  $\lambda$ :

$$[n(\lambda) - 1] \times 10^6 = 64.328 + \frac{29498.1}{146 - \frac{1}{\lambda_0^2}} + \frac{255.4}{41 - \frac{1}{\lambda_0^2}}$$

(valid for dry air, 1 atm pressure,  $T \sim 290\text{K}$  and  $\lambda_0$  in  $[\mu\text{m}]$ ).

# Green Flash



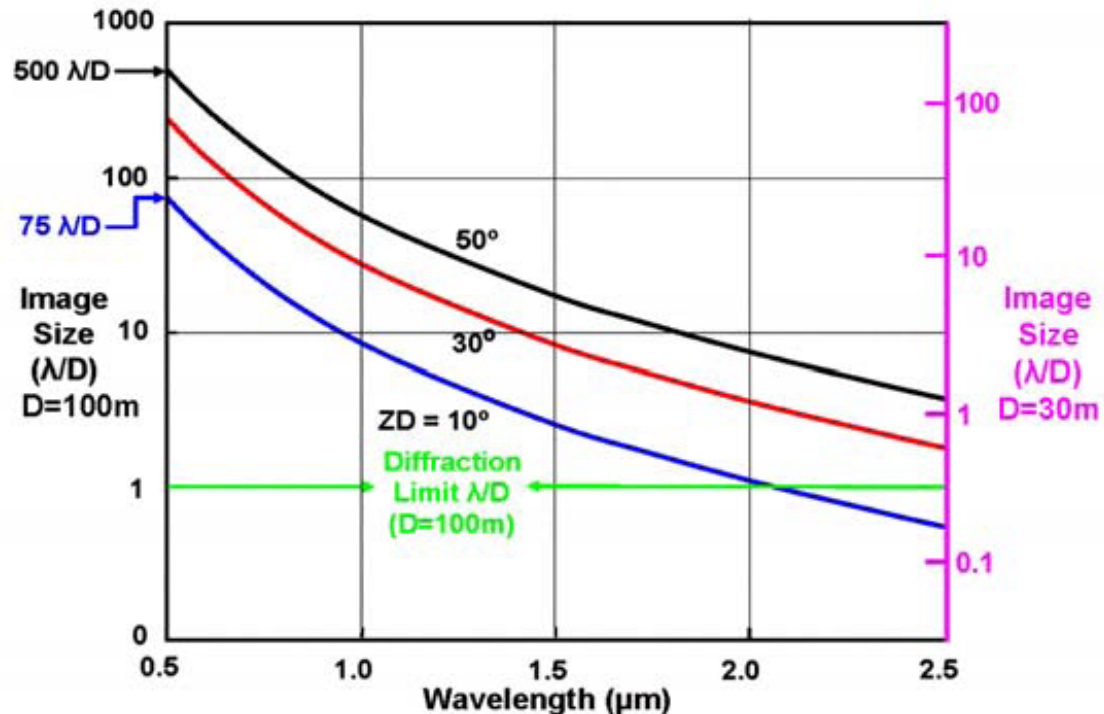
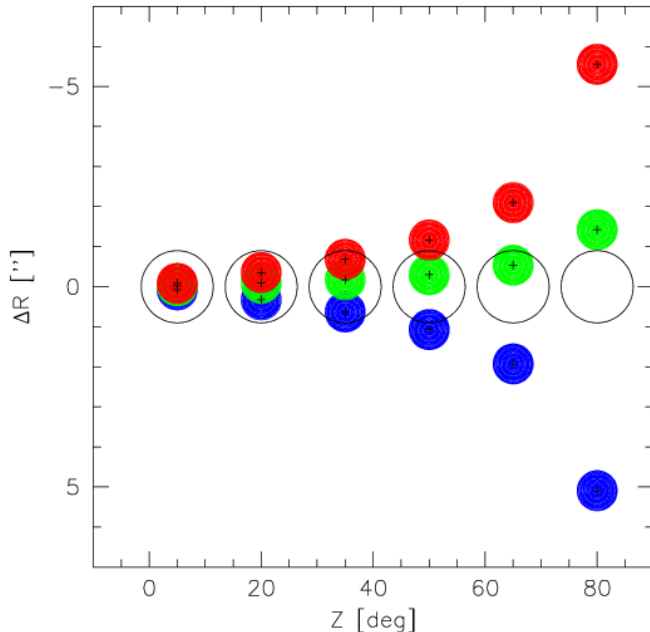
# Atmospheric Dispersion

**Dispersion:** The elongation of points in broadband filters due to  $n(\lambda)$  [→ "rainbow"].

The magnitude of the dispersion is a strong function of airmass and wavelength.

No problem is dispersion  $< \lambda/D \leftarrow$  o.k. for small or seeing limited telescopes, but big problem for large, diffraction limited telescopes

[H=30% (= > Pw=368.0849304 Po)] [P=77500 Po] [T=283.1499939 K] [ $\lambda_w=450$  nm] [D(10)=1.799999952 °] [1" FWHM]

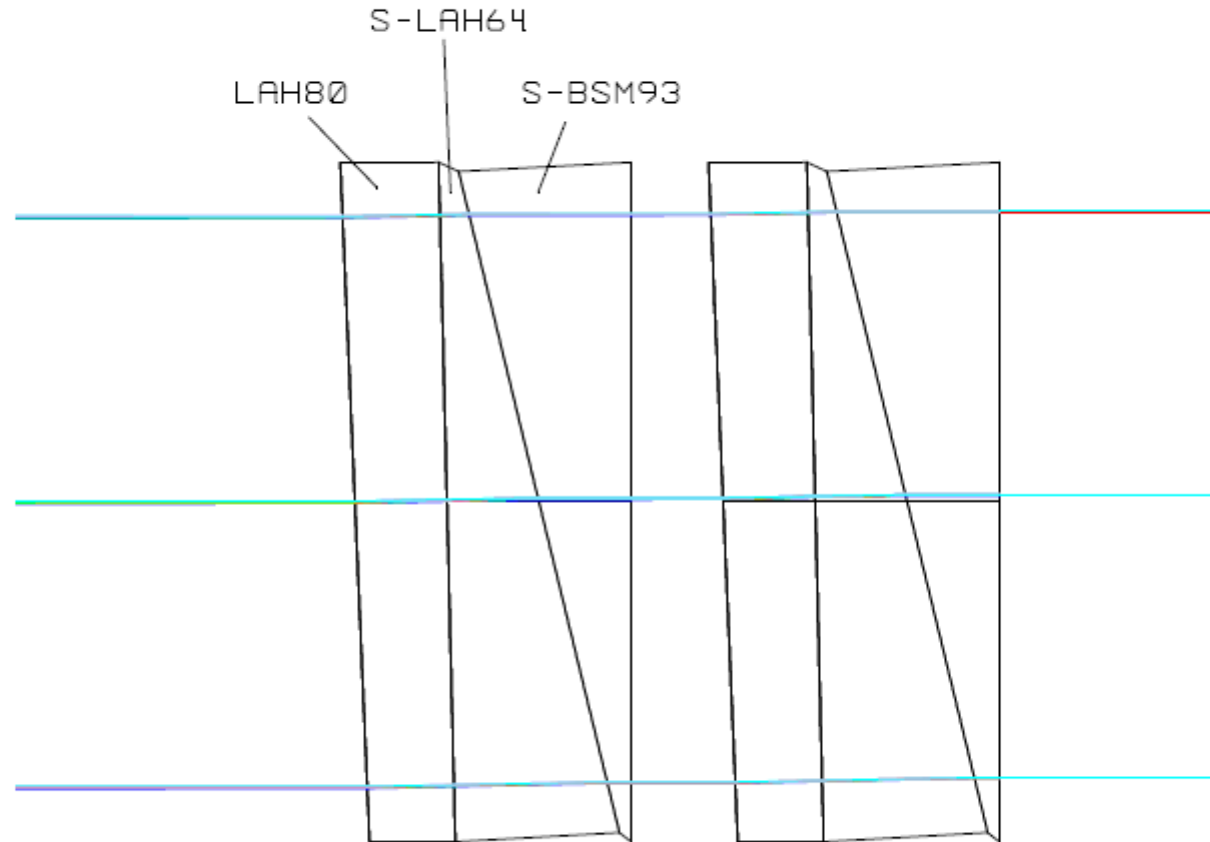
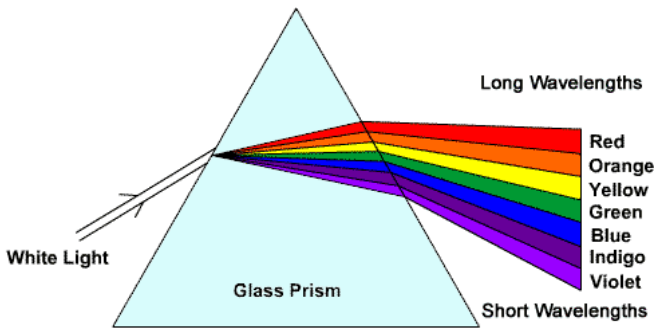




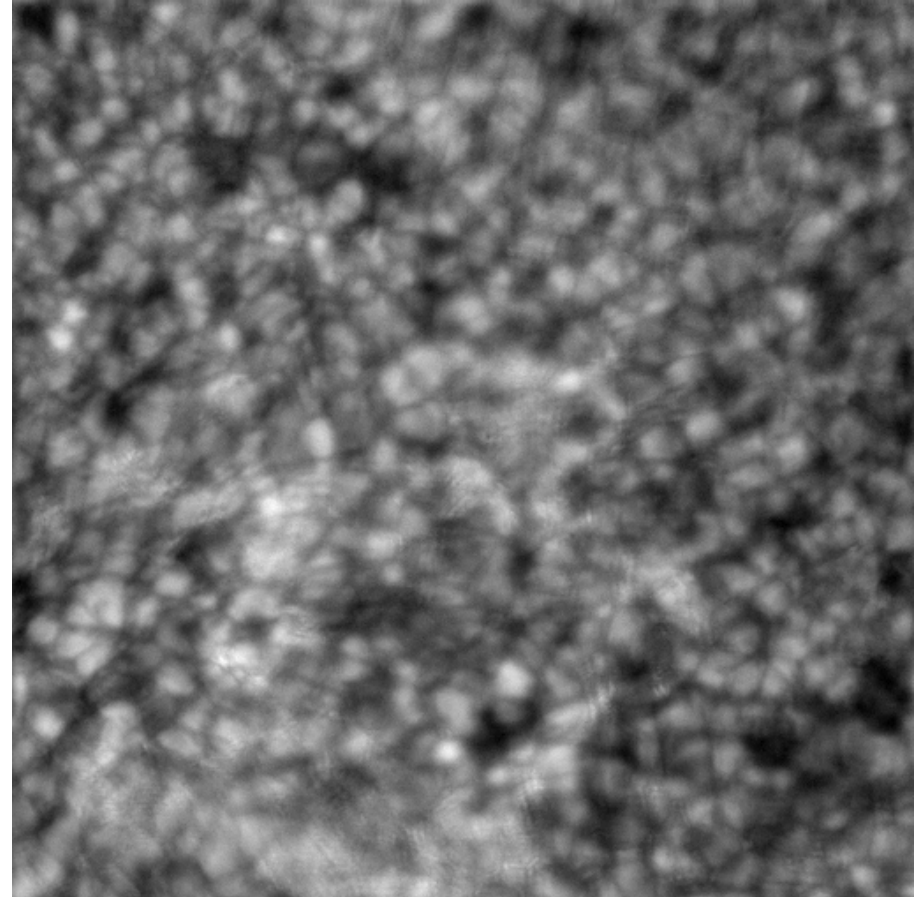
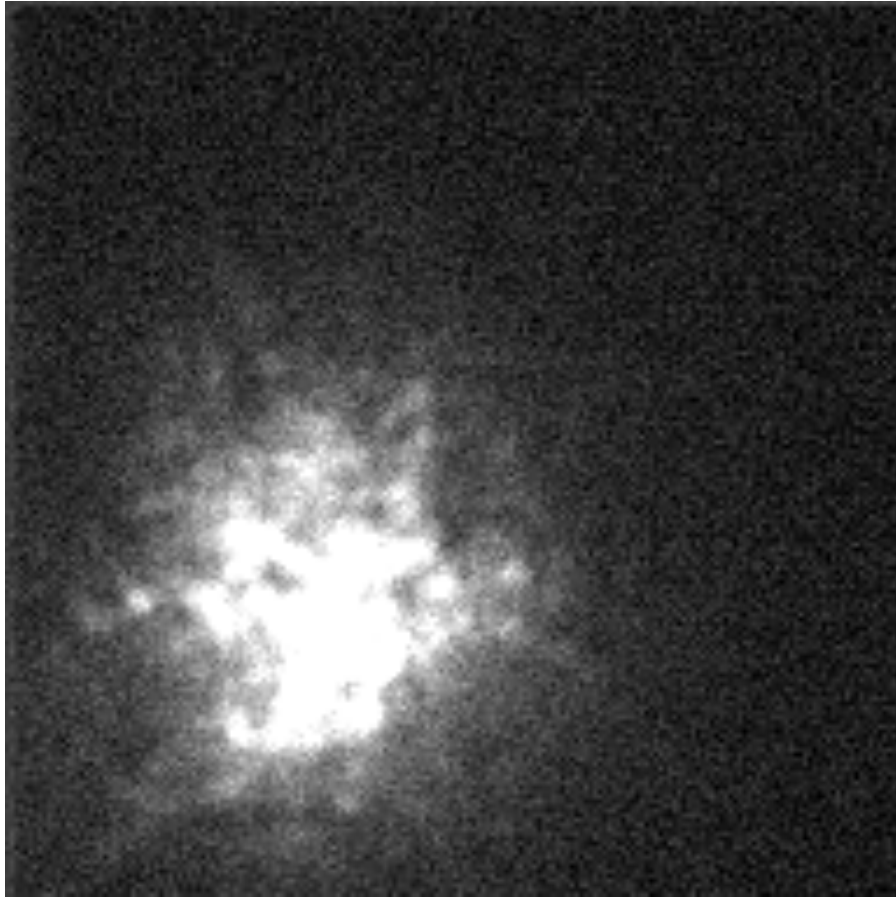
# Atmospheric Dispersion Corrector

To counterbalance atmospheric dispersion use:

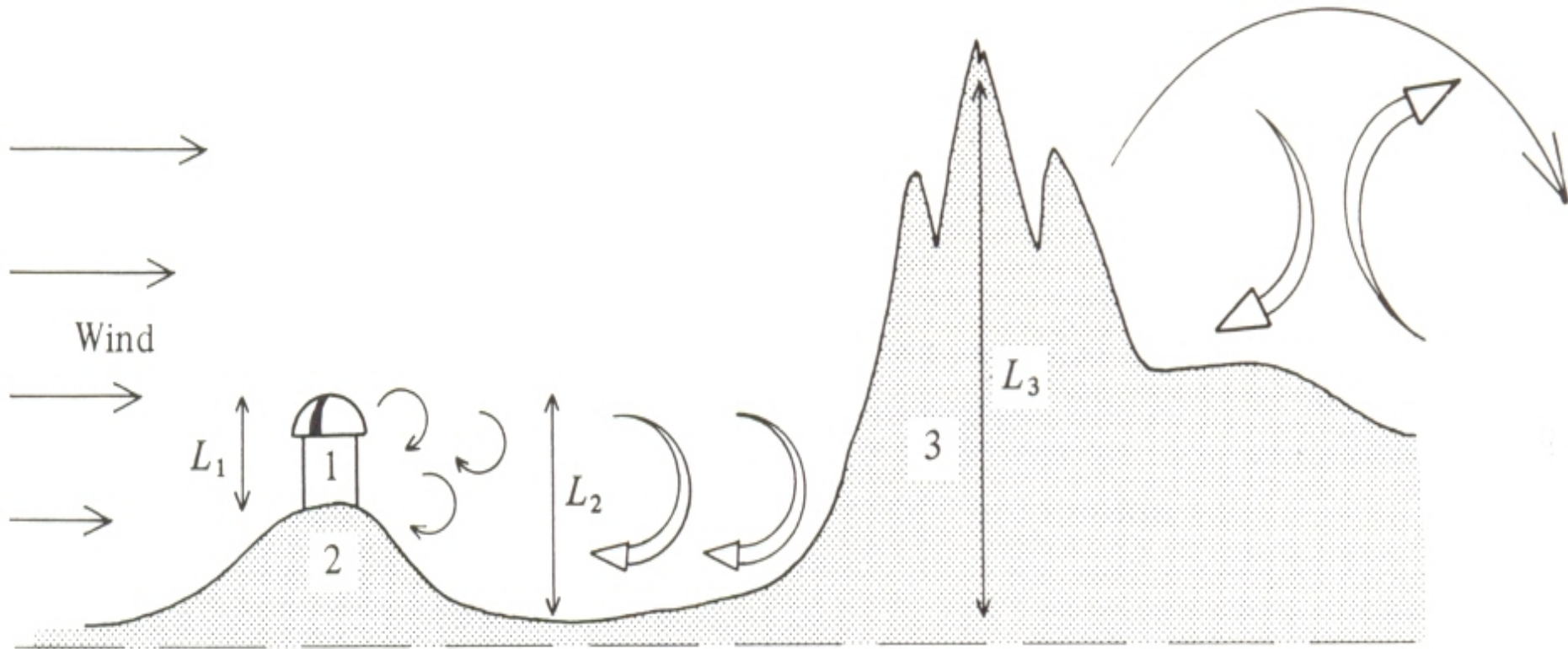
1. a refractive element (e.g., prism)
2. a second prism (different material with different dispersion) to maintain the optical axis
3. use a second (identical) double prism assembly to adjust the strength of the correction for different zenith angles.



# Seeing



# 5. Atmospheric Turbulence



*The scales  $L_1$ ,  $L_2$ ,  $L_3$  are characteristic of the outer (external) scales of turbulence caused by the wind around the obstacles 1, 2, 3.*

# Reynolds Number

Turbulence develops in a fluid when the Reynolds number  $Re$

$$Re = \frac{\rho VL}{\mu} = \frac{VL}{\nu}$$

exceeds a critical value.

$V$  is the flow velocity

$\mu$  is the dynamic viscosity

$\nu$  the kinematic viscosity of the fluid ( $\nu_{\text{air}} = 1.5 \cdot 10^{-5} \text{ m}^2 \text{ s}^{-1}$ )

$L$  the characteristic length, e.g. a pipe diameter.

At  $Re \sim 2200$  the transition from laminar to turbulent flow occurs.

*Example:* wind speed  $\sim 1 \text{ m/s}$ ,  $L = 15\text{m} \rightarrow Re = 10^6 \rightarrow$  turbulent!

# Power Spectrum of Turbulence

The kinetic energy of large scale ( $\sim L$ ) movements is gradually transferred to smaller and smaller scales, down to a minimum scale length  $l_0$ , at which the energy is dissipated by viscous friction.

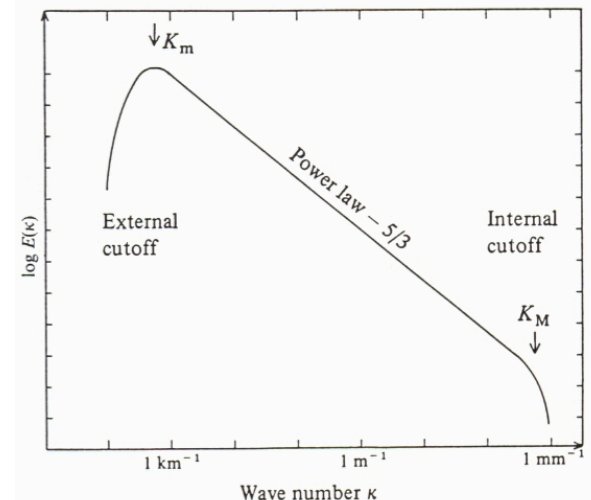
The local velocity field can be decomposed into spatial harmonics of the wave vector  $\kappa$ .

The reciprocal value  $1/\kappa$  represents the scale under consideration.

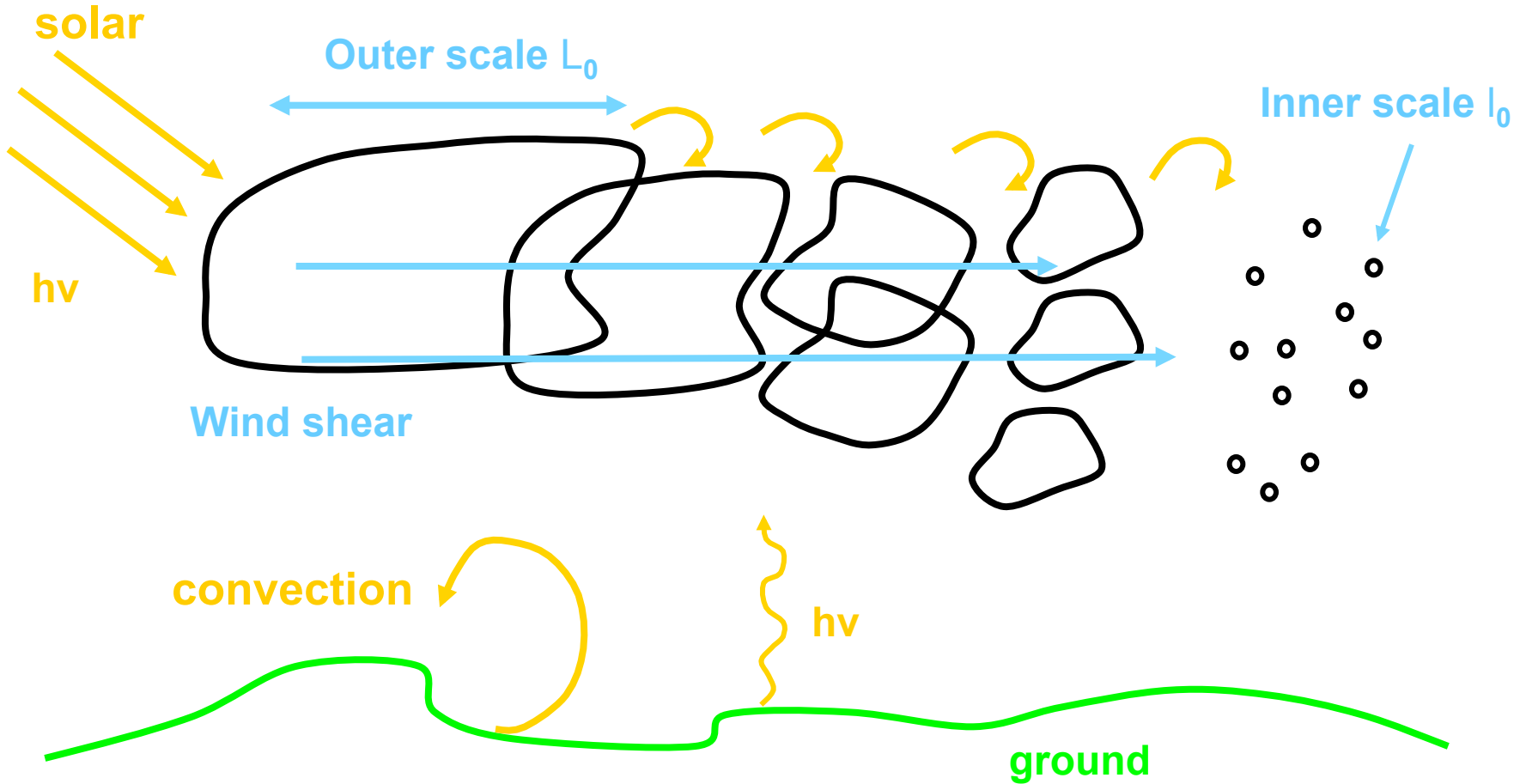
The mean 1D spectrum of the kinetic energy, or **Kolmogorov spectrum**, is:

$$E(\kappa) \propto \kappa^{-5/3}$$

where  $l_0$  is the **inner scale**,  $L_0$  the **outer scale** of the turbulence, and  $L_0^{-1} < \kappa < l_0^{-1}$



# Kolmogorov Turbulence



# Air Refractive Index Fluctuations

Winds mix layers of different temperature  $\rightarrow$  fluctuations of temperature  $T \rightarrow$  fluctuations of density  $\rho \rightarrow$  fluctuations of refractive index  $n$ .

Of interest: difference between  $n(r)$  at point  $r$  and  $n(r+\rho)$  at a nearby point  $r+\rho$ . The **variance** of the two values is given by:

$$D_n(\rho) = \langle |n(r) - n(r + \rho)|^2 \rangle = C_n^2 \rho^{2/3}$$

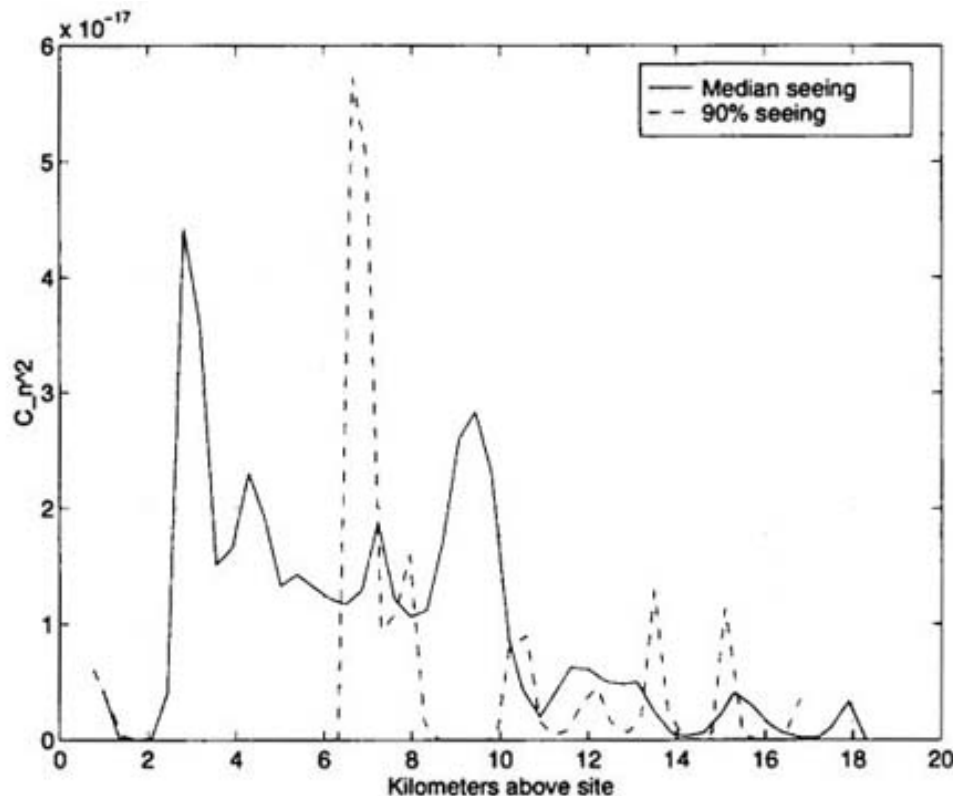
where  $D_n(\rho)$  is the index **structure function** and  $C_n^2$  is the index structure coefficient or **structure constant of the refractive index**.

# Air Refractive Index Fluctuations (2)

Usually, one is only interested in the *integral* of fluctuations along the line of sight:  $C_n^2 \cdot \Delta h$ .

Typical value:  $C_n^2 \cdot \Delta h \sim 4 \cdot 10^{-13} \text{ cm}^{1/3}$  for a 3 km altitude layer

**But:** there are always several **layers of turbulence**



Median seeing conditions on Mauna Kea are taken to be  $r_0 \sim 0.23$  meters at 0.55 microns. The 10% best seeing conditions are taken to be  $r_0 \sim 0.40$  meters. Figure taken from a paper by Ellerbroek and Tyler (1997).



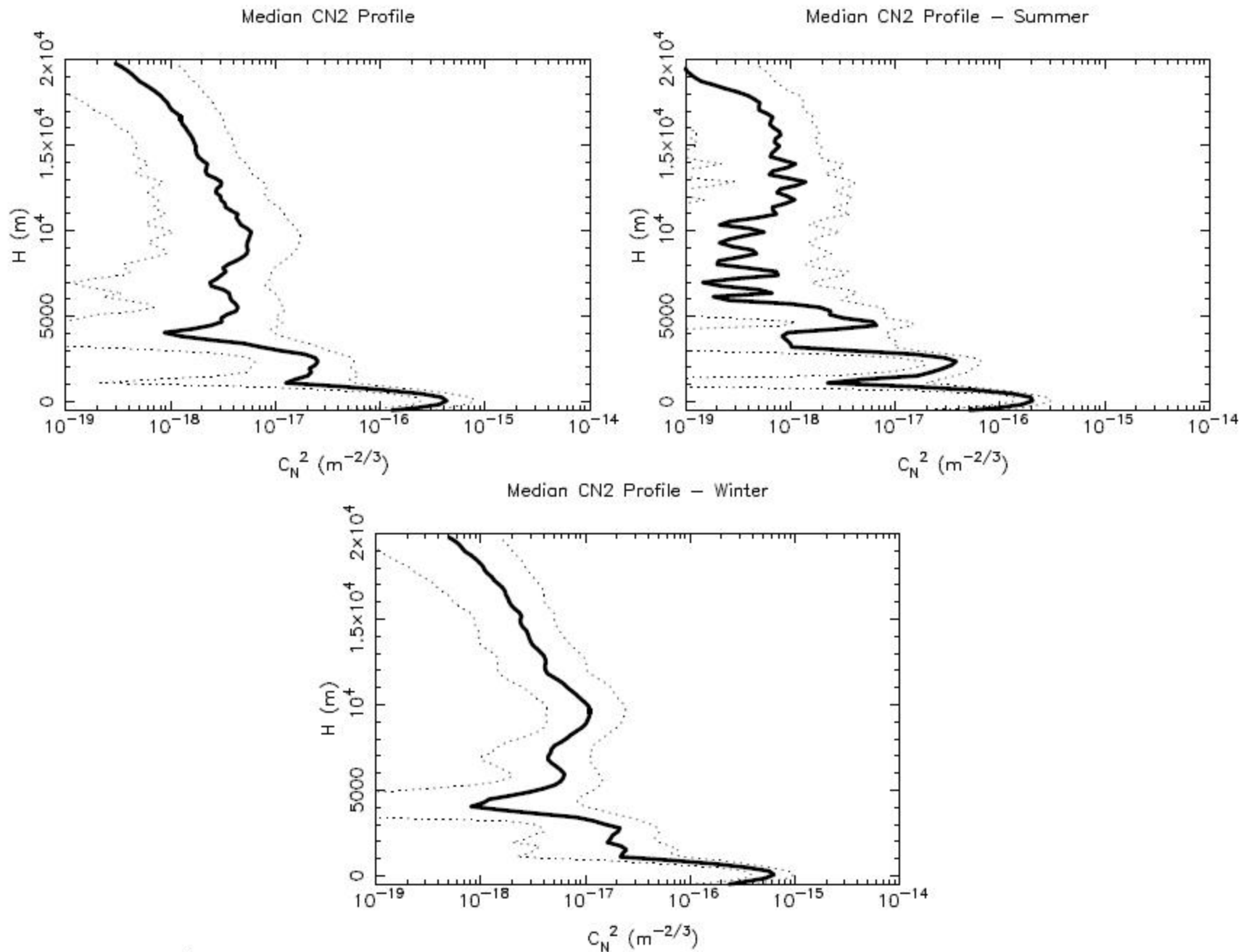
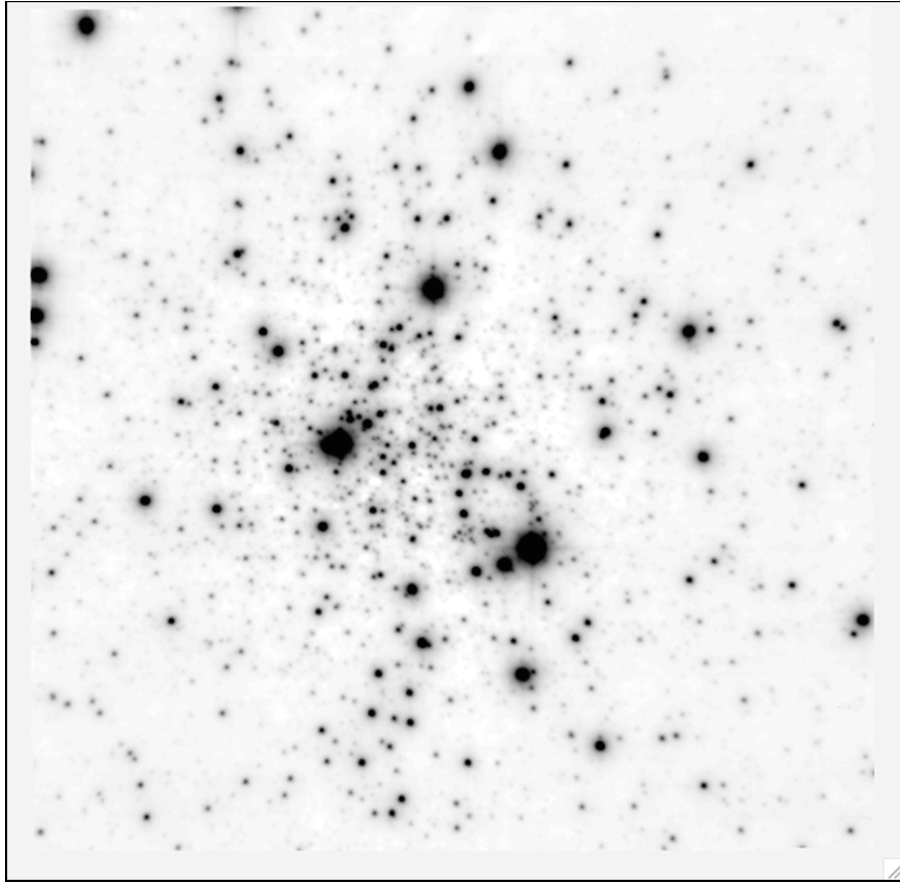
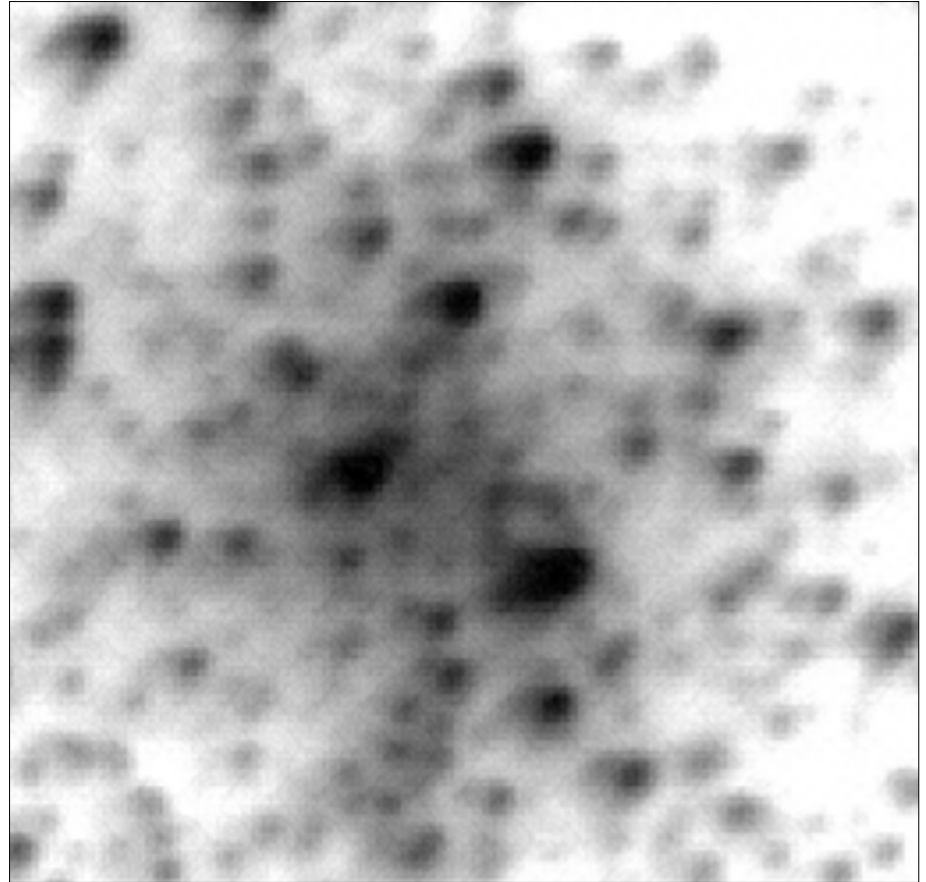


Figure 2. Median  $C_N^2$  profile obtained with the complete sample of 43 nights, the summer [April-June] and winter [October-March] time samples. Results are obtained with the standard GS technique.

# Image Degradation by the Atmosphere ...and why we care ...!

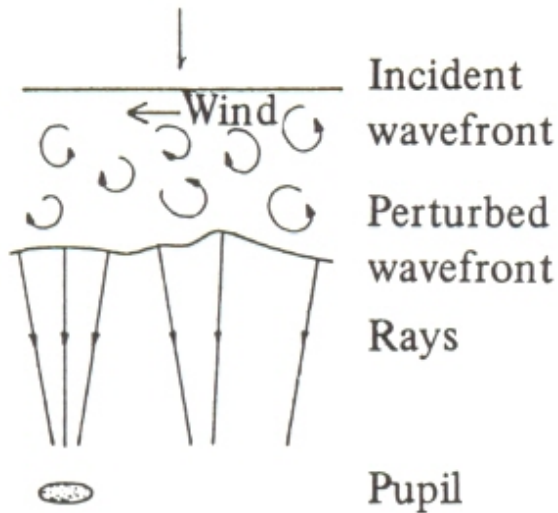
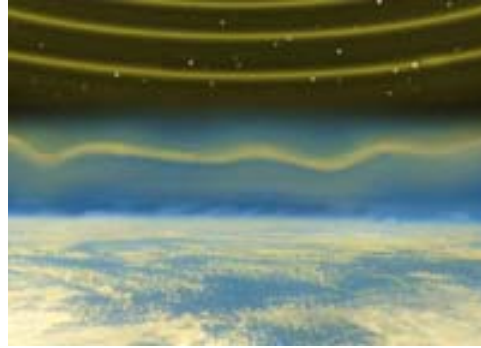


PHARO LGS Ks image  
500s integ., 40" FOV, 150 mas FWHM

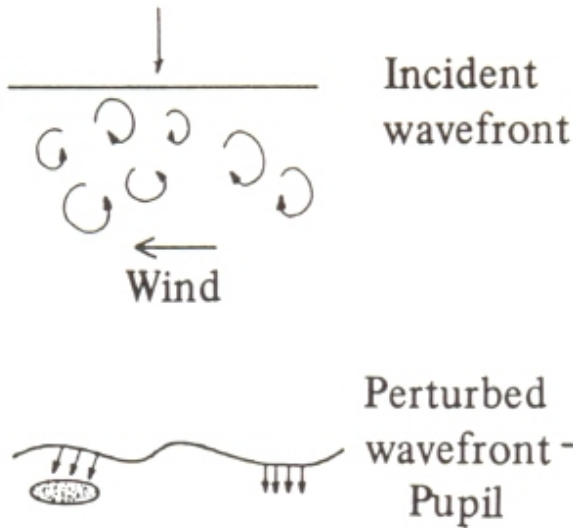


WIRO H image  
Kobulnicky et al. 2005, AJ 129,  
239-250

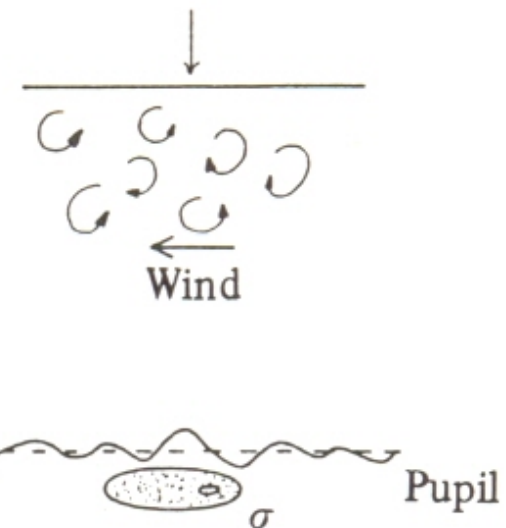
# Aspects of Image Degradation



**Scintillation** - the energy received by the pupil varies in time



**Image motion** - the average slope of the wavefront at the pupil varies ("tip-tilt")



**Image blurring** - the spatial coherence of the wavefront is reduced ("seeing")

# The Fried Parameter $r_0$

The radius of the spatial coherence area is given by the so-called **Fried parameter  $r_0$** :

$$r_0(\lambda) = 0.185\lambda^{6/5} \left[ \int_0^{\infty} C_n^2(z) dz \right]^{-3/5}$$

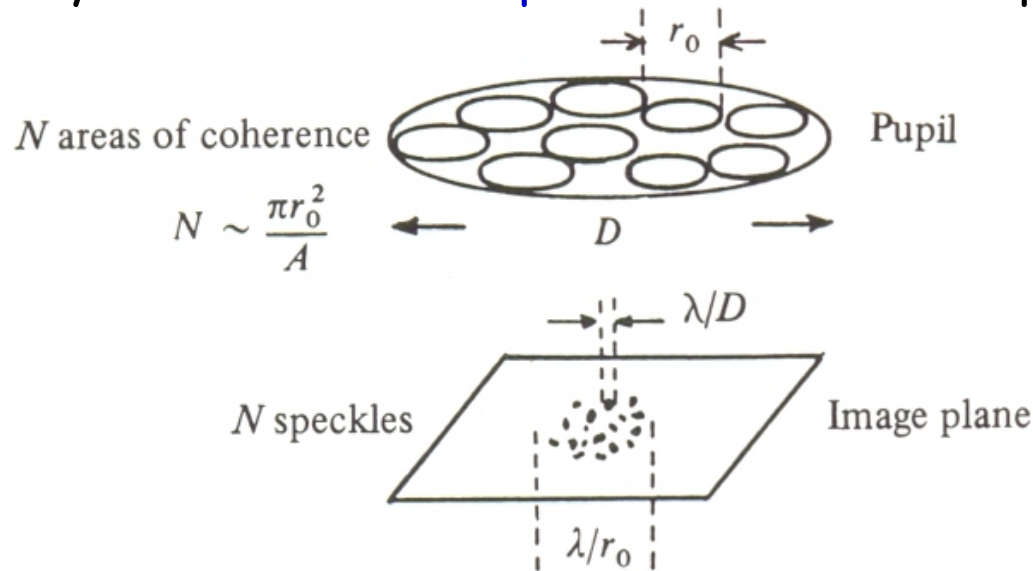
*Note that  $r_0$  increases as the  $6/5$  power of the wavelength and decreases as the  $-3/5$  power of the air mass.*

Another "definition" is that  $r_0$  is the **average turbulent scale over which the RMS optical phase distortion is 1 radian.**

The angle  $\Delta\theta = \frac{\lambda}{r_0}$  is often called the atmospheric **seeing**.

# Short Exposures through Turbulence

Random intensity distribution of **speckles** in the focal plane:



The observed image from some source is given by the convolution of  $I_0$  with the **MTF** or **pupil transfer function**  $T(\theta)$ :

$$I(\theta) = I_0(\theta) * T(\theta) \quad \text{or} \quad \langle |I(\omega)|^2 \rangle = |I_0(\omega)|^2 \langle |T(\omega)|^2 \rangle$$

If a point source is observed as reference through the same  $r_0$  we can

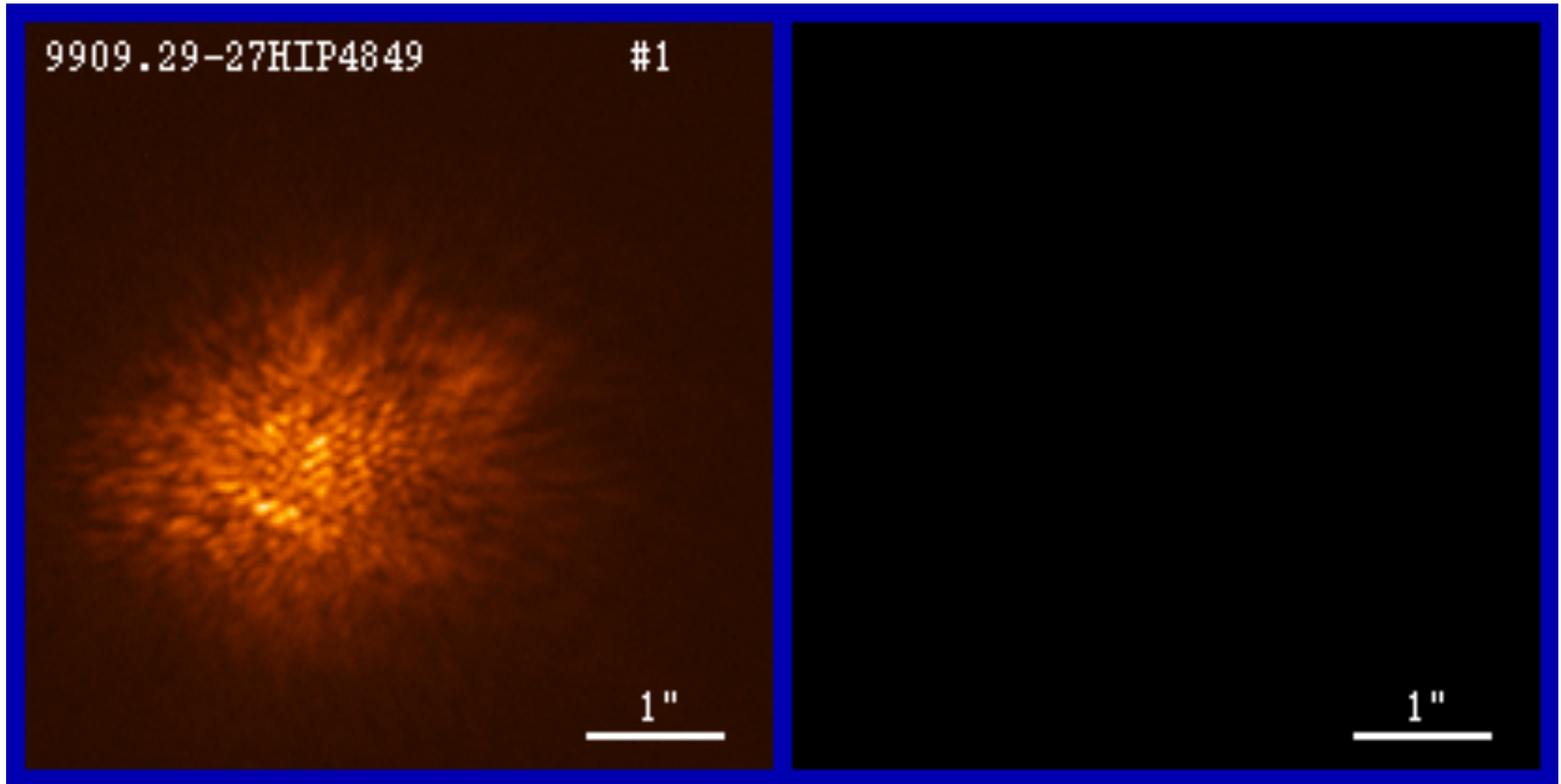
calculate: 
$$|I_0(\omega)| = \left( \frac{\langle |I(\omega)|^2 \rangle_{obs}}{\langle |T(\omega)|^2 \rangle_{obs}} \right)^{1/2}$$

This is called **speckle interferometry**.

# Speckle Interferometry

*Example: Real-time bispectrum speckle interferometry: 76 mas resolution.*

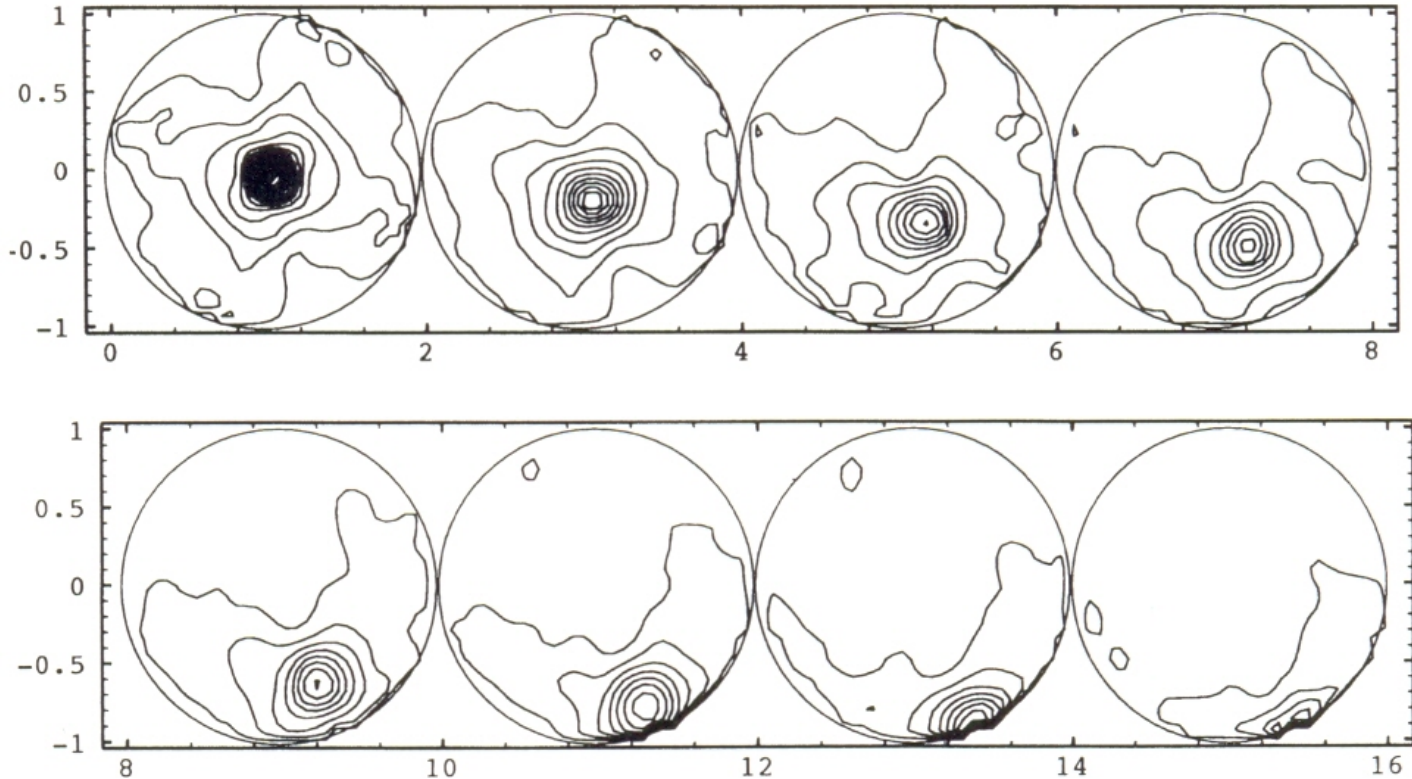
<http://www3.mpifr-bonn.mpg.de/div/ir-interferometry/movie/speckle/specklemovie.html>



Several related techniques do exist, e.g., Shift-and-add, Lucky Imaging, bispectrum analysis, Aperture masking, Triple correlation, ...

# Turbulence Correlation Time $\tau_c$

Time series of a patch of atmosphere above the 3.6m telescope aperture (Gendron 1994)



Two effects:

1. The turbulence does not change arbitrarily fast but with a **correlation time** or **coherence time**  $\tau_c$ .
2. Often, the turbulent time scales  $\gg$  time for the turbulent medium to pass the telescope aperture (wind speed)  $\rightarrow$  "**frozen turbulence**"

# Summary: $r_0$ , seeing, $\tau_0$ , $\theta_0$

The **Fried parameter**  $r_0(\lambda) = 0.185\lambda^{6/5} \left[ \int_0^\infty C_n^2(z) dz \right]^{-3/5}$  is the radius of the spatial coherence area.

It is the **average turbulent scale over which the RMS optical phase distortion is 1 radian**. Note that  $r_0$  increases as  $\lambda^{6/5}$ .

$\Delta\theta = \frac{\lambda}{r_0} \sim \lambda^{-1/5}$  is called the **seeing**. At good sites  $r_0$  (0.5 $\mu\text{m}$ )  $\sim 10 - 30$  cm.

The **atmospheric coherence** (or Greenwood delay) **time** is:  $\tau_0 = 0.314 \frac{r_0}{\bar{v}}$   
It is the maximum time delay for the RMS wavefront error to be less than 1 rad (where  $v$  is the mean propagation velocity).

The **isoplanatic angle**  $\theta_0 = 0.314 \cos \zeta \frac{r_0}{h}$  is the angle over which the RMS wavefront error is smaller than 1 rad.



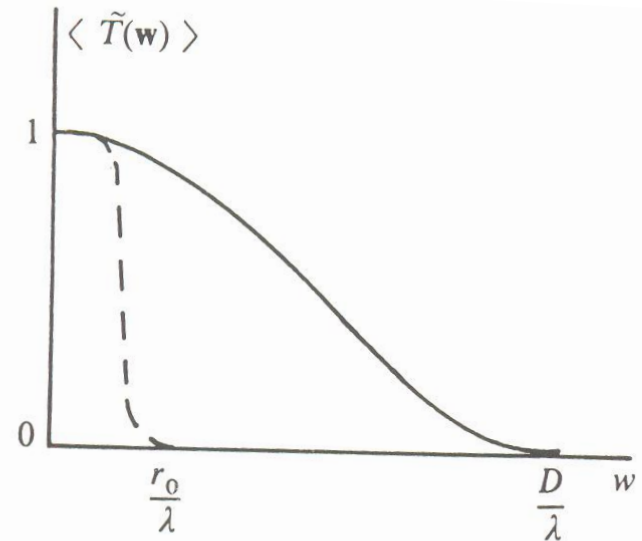
# Long Exposures through Turbulence

When  $t_{\text{int}} \gg \tau_c$  the image is the mean of the instantaneous intensity:

$$I(\theta) = \langle I_0(\theta) * T(\theta, t) \rangle$$

With the mean modulation transfer function (MTF):

$$\langle \tilde{T}(\omega) \rangle \approx \exp\left[-1.45 k^2 C_n^2 \Delta h (\lambda \omega)^{5/3}\right]$$



→ The image is smeared or **spatially filtered** (loss of high spatial frequencies).

The angular dimension now has order of  $\lambda/r_0$  rather than  $\lambda/D$ .

In other words:

As long as  $D > r_0$ , bigger telescopes will *not* provide sharper images.