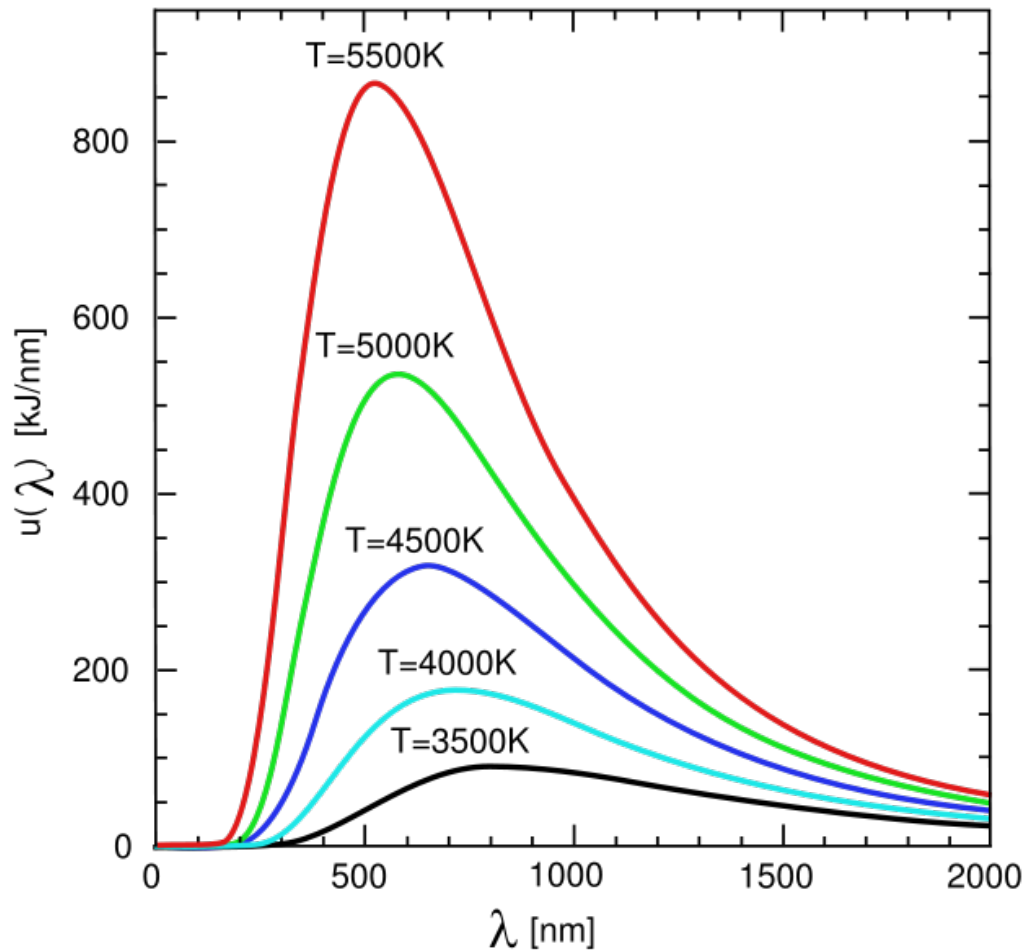


Astronomische Waarneemtechnieken (Astronomical Observing Techniques)

based on lectures by Bernhard Brandl



Lecture 1:

- Black body radiation
- Astronomical magnitudes
- Point \leftrightarrow extended sources

Black Body

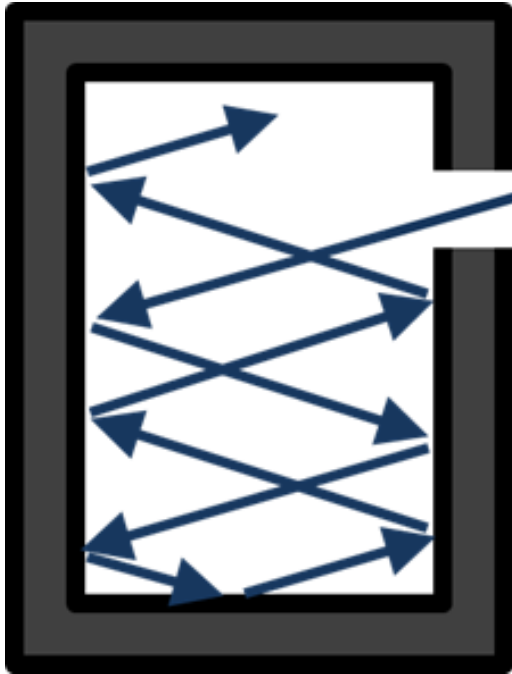
Radiation

Introduction

(from Wikipedia)

Kirchhoff (1860): "...imagine that bodies (...) completely absorb all incident rays, and neither reflect nor transmit any. I shall call such bodies perfectly black, or, more briefly, black bodies."

This shall be true of radiation for all wavelengths and for all angles of incidence.



Cavity at fixed temperature T in thermal equilibrium

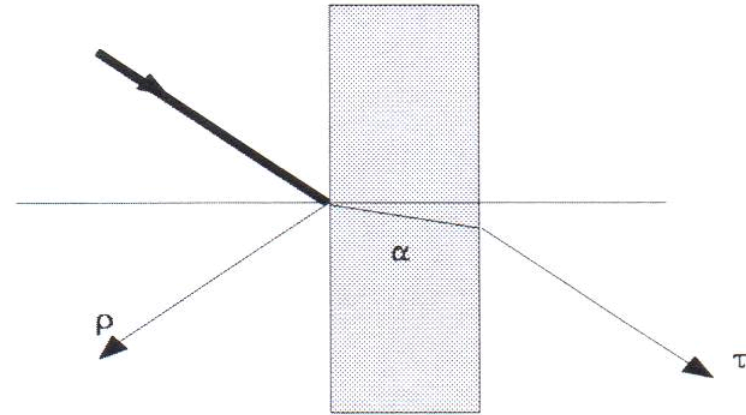
Radiation entering cavity will be "thermalized" by continuous absorption and re-emission of radiation by material in cavity or its walls.

Small hole \rightarrow escaping radiation will approximate black-body radiation independent of properties of cavity or hole.

Kirchhoff's Law

Conservation of power requires:

$$\alpha + \rho + \tau = 1$$



with α = absorptivity, ρ = reflectivity, τ = transmissivity

cavity in thermal equilibrium with completely opaque sides:

$$\left. \begin{array}{l} \varepsilon = 1 - \rho \\ \alpha + \rho + \tau = 1 \\ \tau = 0 \end{array} \right\} \alpha = \varepsilon \quad \varepsilon = \text{emissivity}$$

Kirchhoff's law, applies to **perfect black body**

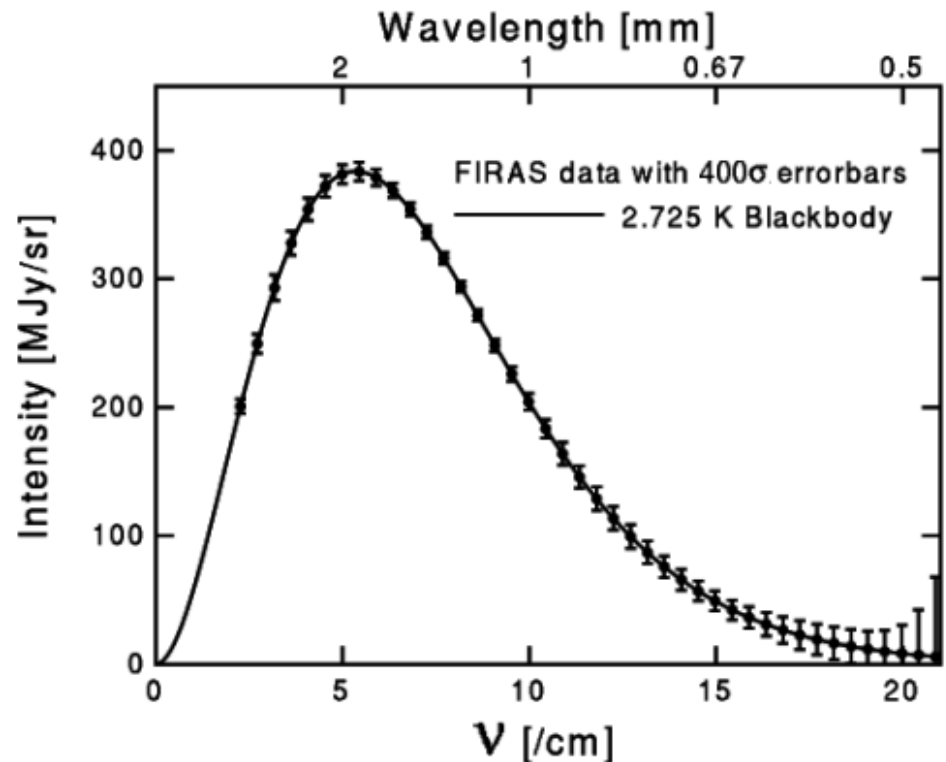
Radiator with $\varepsilon = \varepsilon(\lambda) < 1$ often called **grey body**

Definition of a Black Body

- Black body (BB) is idealized object that absorbs all EM radiation
- Cold ($T \sim 0\text{K}$) BBs are black (no emitted or reflected light)
- At $T > 0\text{K}$ BBs absorb and re-emit characteristic EM spectrum

Many astronomical sources emit close to a **black body**.

Example: COBE measurement of the cosmic microwave background



Black Body Emission

Specific intensity I_ν of blackbody given by **Planck's law**:

$$I_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{\exp\left(\frac{h\nu}{kT}\right) - 1}$$

in units of $[\text{W m}^{-2} \text{sr}^{-1} \text{Hz}^{-1}]$
(see course on Radiative Processes)

In **wavelength units**:

$$I_\lambda(T) = \frac{2hc^2}{\lambda^5} \frac{1}{\exp\left(\frac{hc}{\lambda kT}\right) - 1}$$

in units of $[\text{W m}^{-3} \text{sr}^{-1}]$

Conversion of frequency \Leftrightarrow wavelength units:

$$d\nu = \frac{c}{\lambda^2} d\lambda \quad \text{or} \quad d\lambda = \frac{c}{\nu^2} d\nu$$

Useful Approximations

$$I_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{\exp\left(\frac{h\nu}{kT}\right) - 1}$$

High frequencies ($h\nu \gg kT$) → **Wien's** approximation:

$$I_\nu(T) = \frac{2h\nu^3}{c^2} \exp\left(-\frac{h\nu}{kT}\right)$$

Low frequencies ($h\nu \ll kT$) → **Rayleigh-Jeans'** approximation:

$$I_\nu(T) \approx \frac{2\nu^2}{c^2} kT = \frac{2kT}{\lambda^2}$$

Emission \Leftrightarrow Power \Leftrightarrow Temperature

Total radiated power per unit surface (radiant exitance) is proportional to **fourth power of temperature** T:

$$\int \int_{\Omega \nu} I_{\nu}(T) d\nu d\Omega = M = \sigma T^4$$

$$\sigma = 5.67 \cdot 10^{-8} \text{ W m}^{-2} \text{ K}^{-4} \text{ (Stefan-Boltzmann constant)}$$

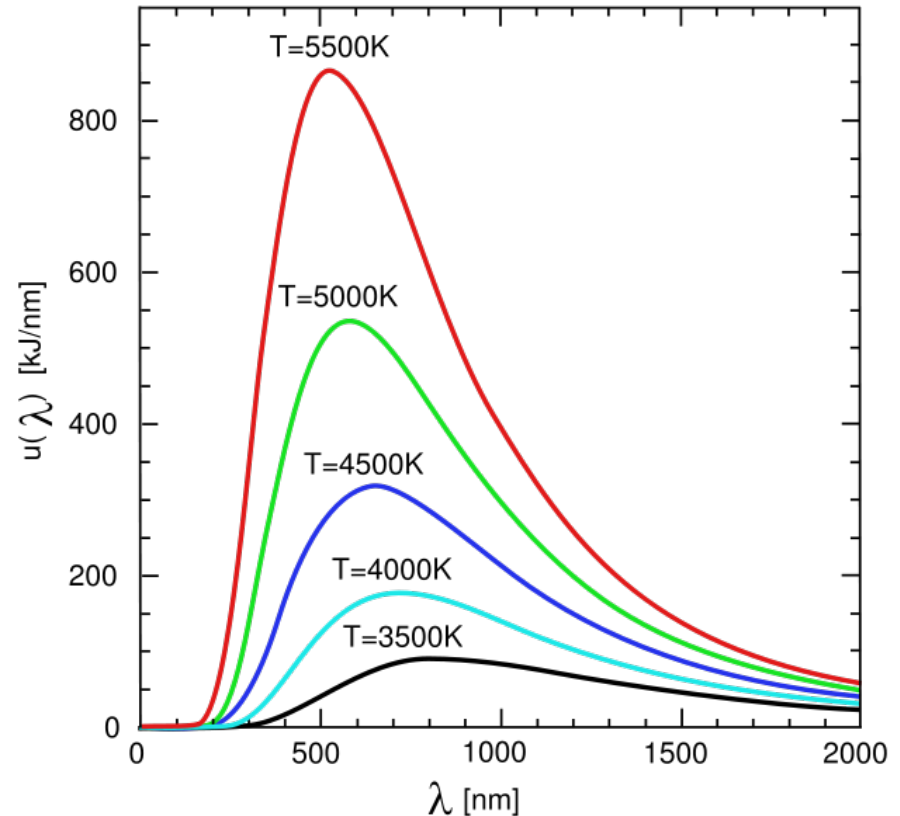
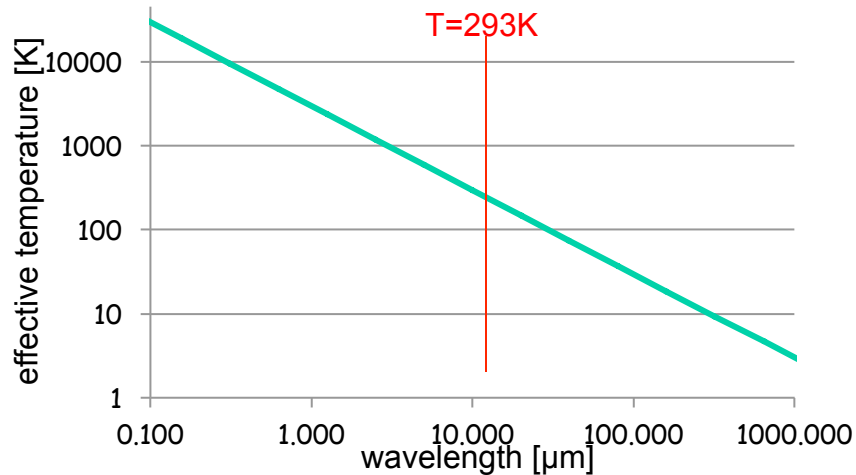
Assuming BB radiation, astronomers often specify the emission from objects via their effective temperature.

Effective Temperatures

Temperature corresponding to maximum specific intensity given by **Wien's displacement law**:

$$\frac{c}{\nu_{\max}} T = 5.096 \cdot 10^{-3} \text{ mK} \quad \text{or} \quad \lambda_{\max} T = 2.98 \cdot 10^{-3} \text{ mK}$$

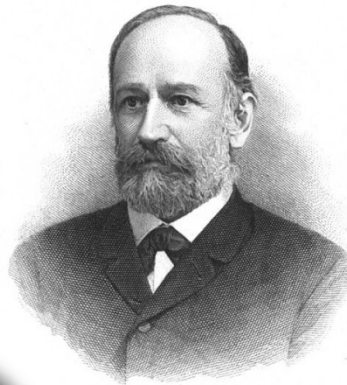
Cooler BBs have peak emission (**effective temperatures**) at longer wavelengths and at lower intensities:





$$\left. \begin{aligned} \varepsilon &= 1 - \rho \\ \alpha + \rho + \tau &= 1 \\ \tau &= 0 \end{aligned} \right\} \alpha = \varepsilon$$

Gustav Kirchhoff (1824 – 1887)



$$M = \sigma T^4$$

Josef Stefan (1835 – 1893) Ludwig Eduard Boltzmann (1844 – 1906)



$$I_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{\exp\left(\frac{h\nu}{kT}\right) - 1}$$

Max Planck (1858 – 1947)

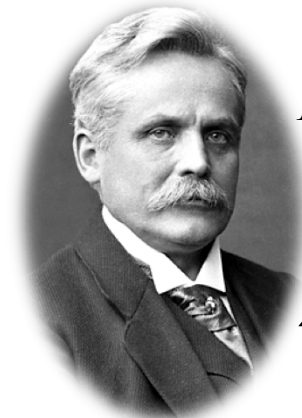


$$I_\nu(T) \approx \frac{2\nu^2}{c^2} kT$$

John William Strutt,
3rd Baron Rayleigh (1842 – 1919)



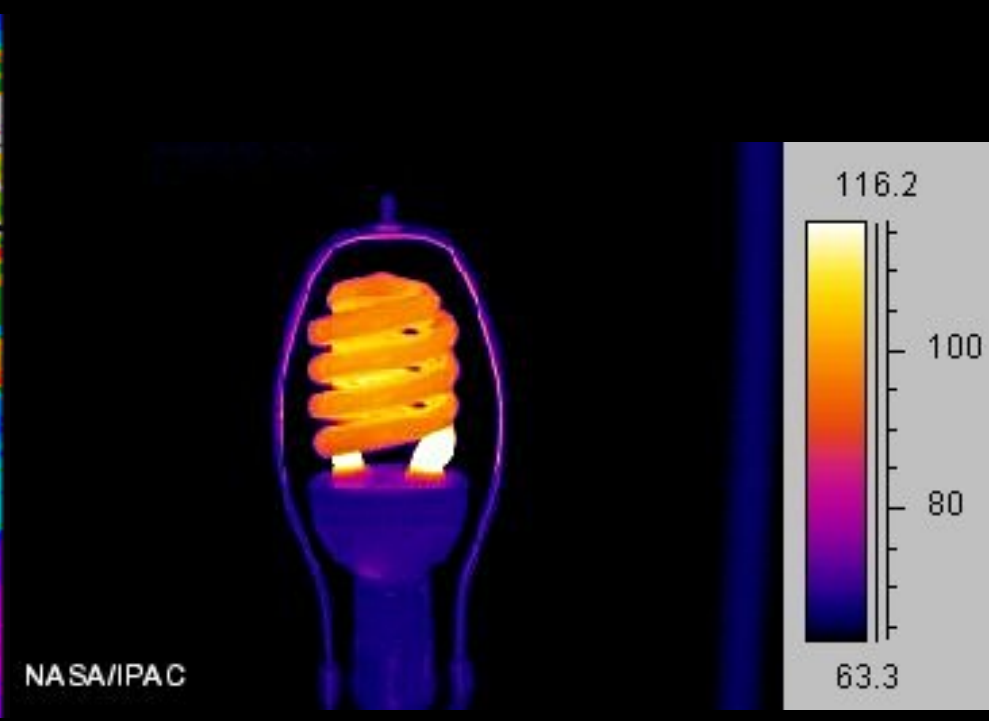
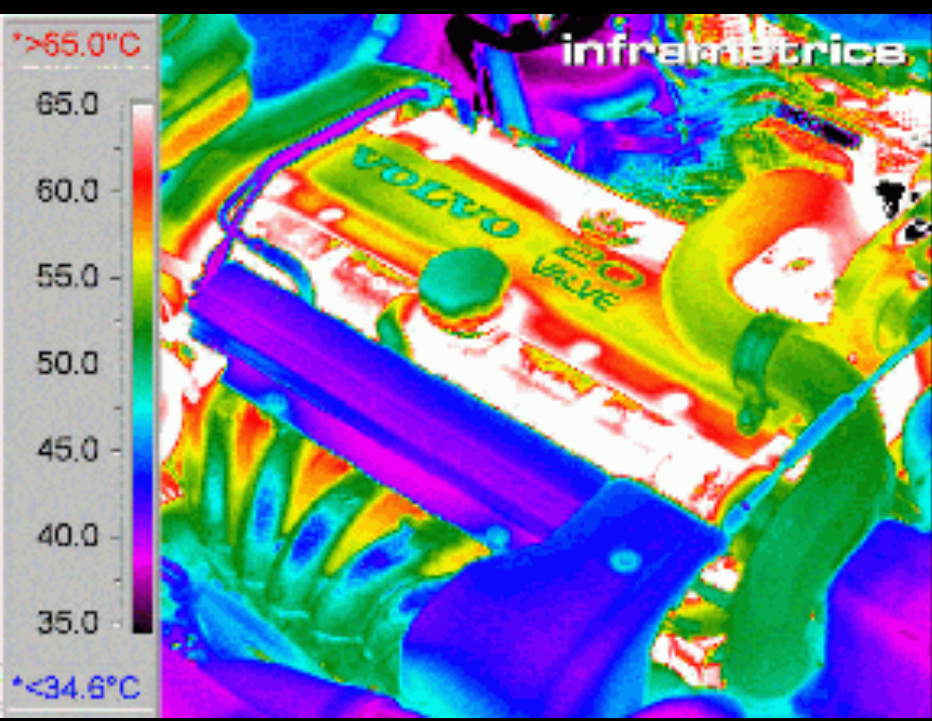
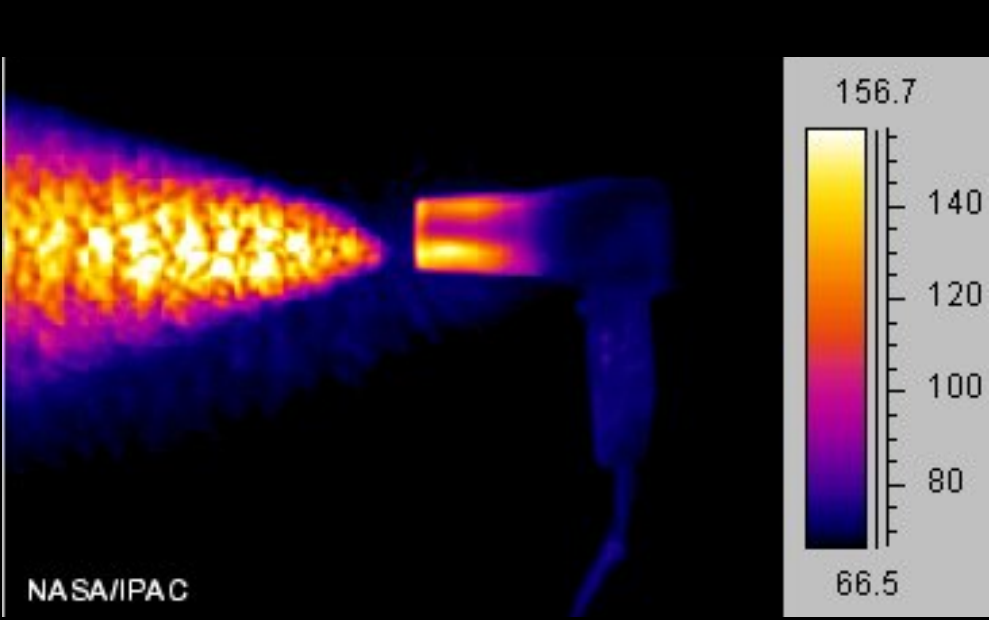
Sir James Hopwood Jeans
(1877 – 1946)



$$I_\nu(T) = \frac{2h\nu^3}{c^2} \exp\left(-\frac{h\nu}{kT}\right)$$

$$\lambda_{\max} = \frac{2.98 \cdot 10^{-3} \text{ mK}}{T}$$

Wilhelm Wien (1864 – 1928)



NASA/IPAC

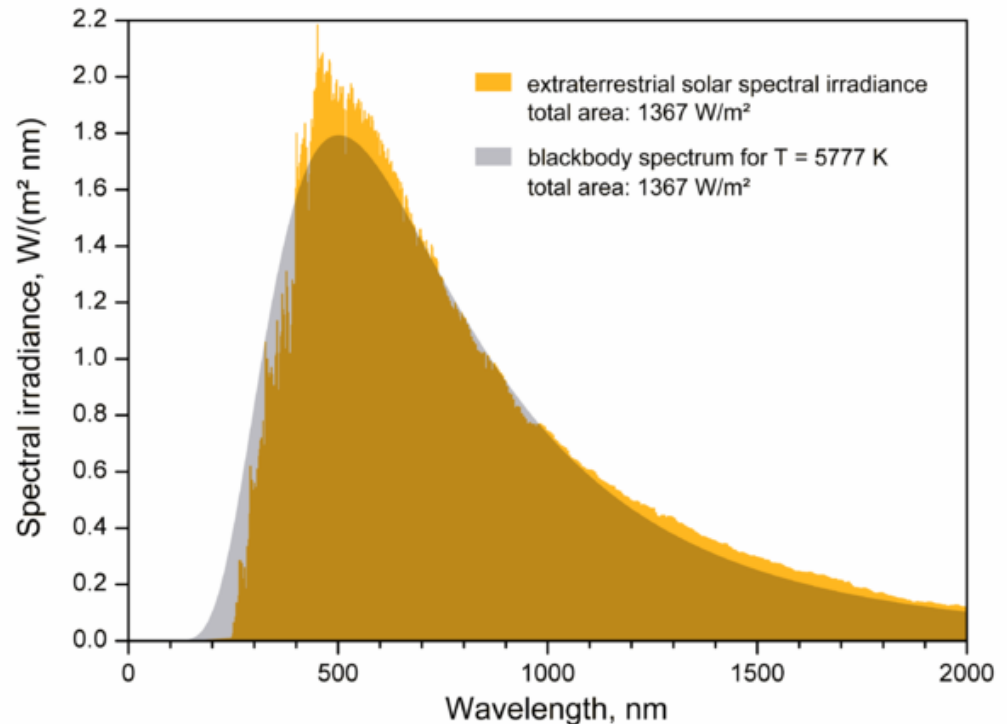
NASA/IPAC

Grey Bodies

Many emitters close to but not perfect black bodies.
With wavelength-dependent emissivity $\epsilon < 1$:

$$I_{\lambda}(T) = \epsilon(\lambda) \cdot \frac{2hc^2}{\lambda^5} \frac{1}{\exp\left(\frac{hc}{\lambda kT}\right) - 1}$$

Example: the Sun
(like many stars)



Brightness Temperature

Brightness temperature is temperature a perfect black body would have to be at to duplicate the observed intensity of grey body object at frequency ν .

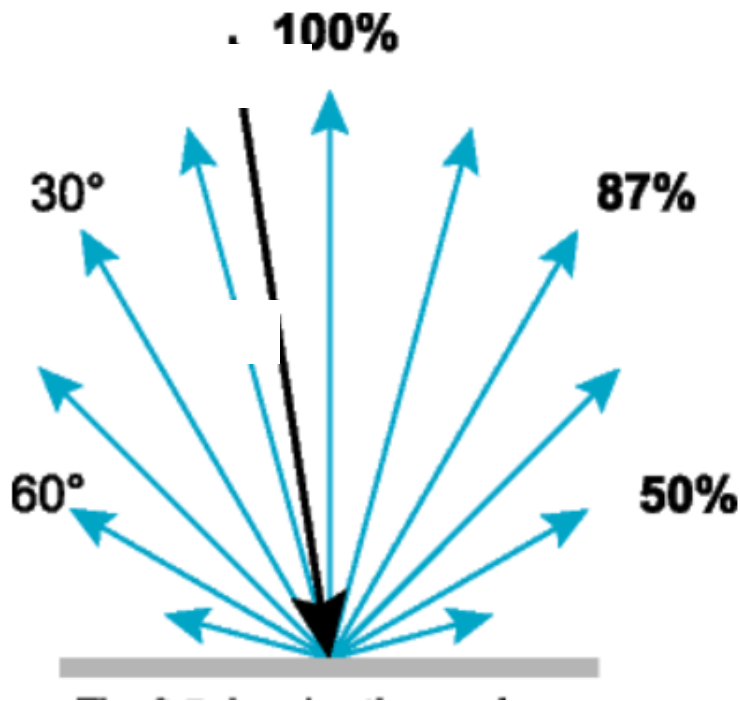
For low frequencies ($h\nu \ll kT$):

$$T_b = \varepsilon(\nu) \cdot T \stackrel{\text{Rayleigh-}}{=} \underset{\text{Jeans}}{\varepsilon(\nu)} \cdot \frac{c^2}{2k\nu^2} I_\nu$$

Only for perfect BBs is T_b the same for all frequencies.

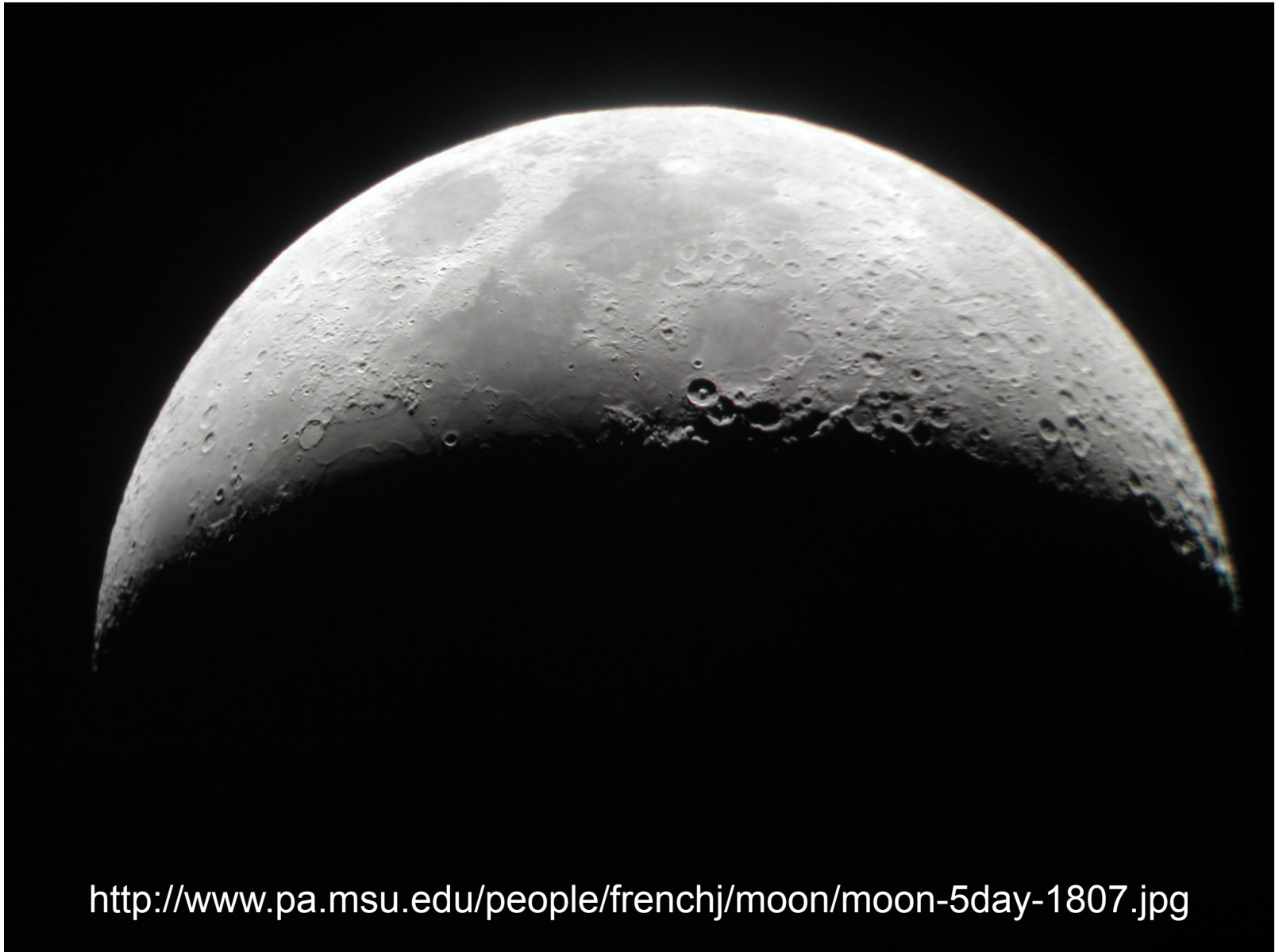
Lambert's Cosine Law

(Wikipedia:) **Lambert's cosine law** states that the radiant intensity from an ideal **diffusively reflecting surface** is directly proportional to the cosine of the angle θ between the surface normal and the observer.



Johann Heinrich Lambert
(1728 – 1777)

The Moon: Lambertian Scatterer?



<http://www.pa.msu.edu/people/frenchj/moon/moon-5day-1807.jpg>

Lambertian Emitters

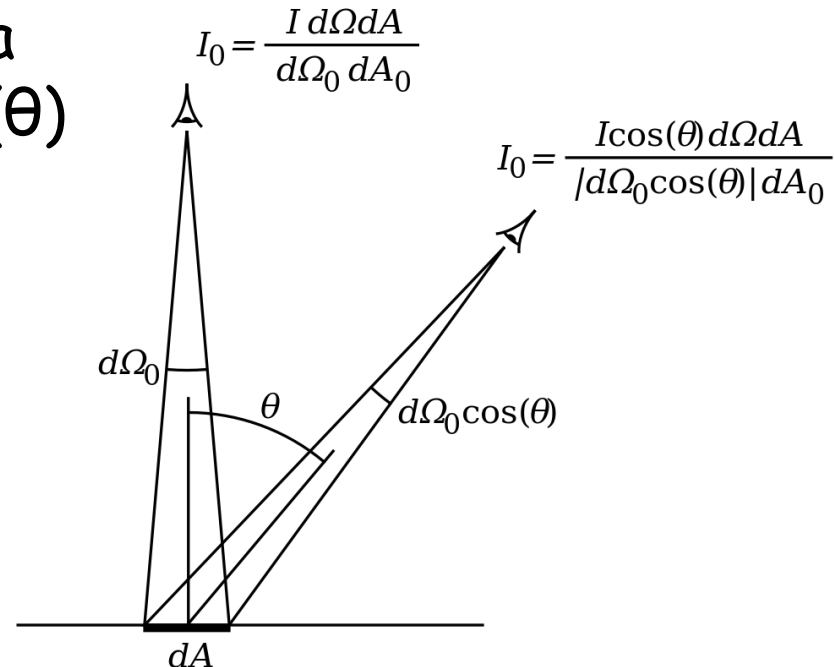
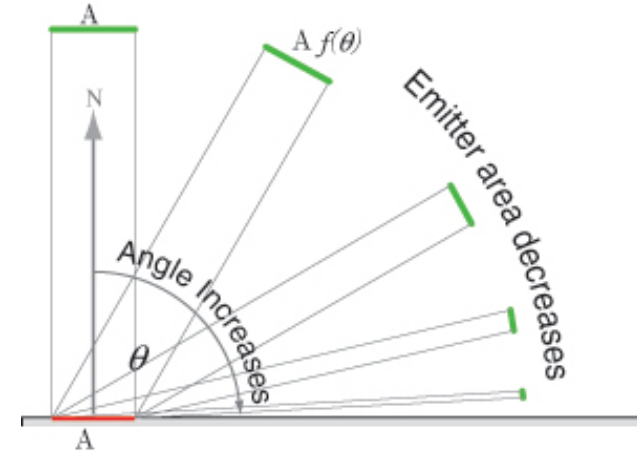
Radiance of Lambertian emitters is independent of direction θ of observation (i.e., isotropic).

Two effects:

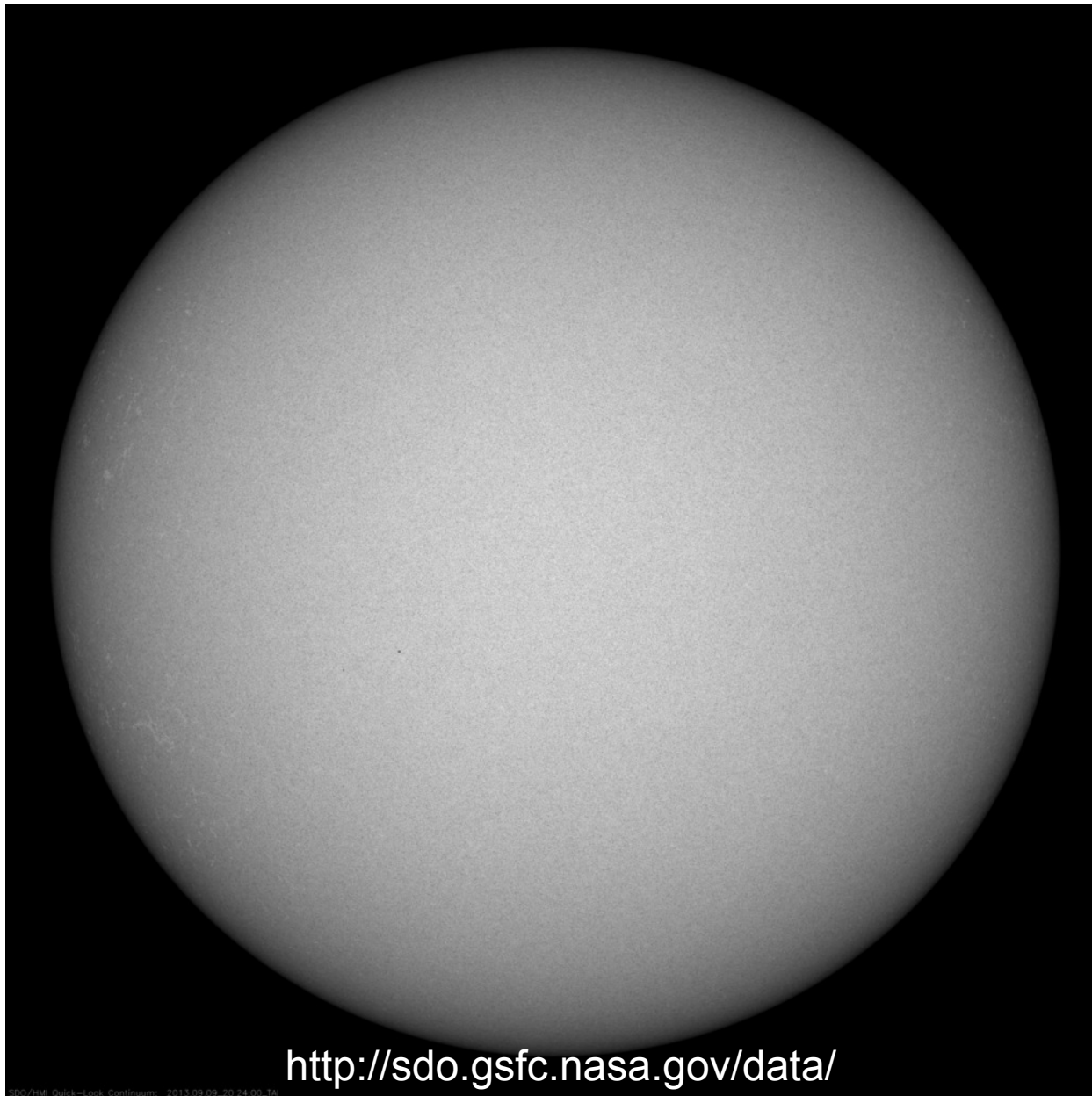
1. Lambert's cosine law \rightarrow radiant intensity and $d\Omega$ are reduced by $\cos(\theta)$
2. Emitting surface area dA for a given $d\Omega$ is increased by $\cos^{-1}(\theta)$

\rightarrow Two effects cancel

Perfect black bodies are Lambertian emitters!



The Sun: Lambertian Emitter?



<http://sdo.gsfc.nasa.gov/data/>

Astronomical Magnitudes

Summary of Radiometric Quantities

(see course on Radiative Processes!)

Name	Symbol	Unit	Definition	Equation
Spectral radiance or specific intensity	L_ν , I_ν	$W m^{-2} Hz^{-1} sr^{-1}$	Power leaving unit projected surface area into unit solid angle and unit $\Delta\nu$	
Spectral radiance or specific intensity	L_λ , I_λ	$W m^{-3} sr^{-1}$	Power leaving unit projected surface area into unit solid angle and unit $\Delta\lambda$	
Radiance or Intensity	L , I	$W m^{-2} sr^{-1}$	Spectral radiance integrated over spectral bandwidth	$L = \int L_\nu d\nu$
Radiant <u>exitance</u>	M	$W m^{-2}$	Total power emitted per unit surface area	$M = \int L(\theta) d\Omega$
Flux or luminosity	Φ , L	W	Total power emitted by a source of surface area A	$\Phi = \int M dA$
Spectral irradiance or flux density	L_ν, F_ν, I_ν	$W m^{-2} Hz^{-1}$ *	Power received at a unit surface element per unit $\Delta\nu$	
Spectral irradiance or flux density	$L_\lambda, F_\lambda, I_\lambda$	$W m^{-3}$ *	Power received at a unit surface element per unit $\Delta\lambda$	
Irradiance	E	$W m^{-2}$	Power received at a unit surface element	$E = \frac{\int M dA}{4\pi r^2}$

Karl Guthe Jansky (1905 – 1950)



* $10^{-26} W m^{-2} Hz^{-1} = 10^{-23} erg s^{-1} cm^{-2} Hz^{-1}$ is called 1 **Jansky**

Optical Astronomers use 'Magnitudes'

Origins in Greek classification of stars according to their visual brightness. Brightest stars were $m = 1$, faintest detected with bare eye were $m = 6$.

Later formalized by Pogson (1856): $1^{\text{st}} \text{ mag} \sim 100 \times 6^{\text{th}} \text{ mag}$

Magnitude	Example	#stars brighter
-27	Sun	
-13	Full moon	
-5	Venus	
0	Vega	4
2	Polaris	48
3.4	Andromeda	250
6	Limit of naked eye	4800
10	Limit of good binoculars	
14	Pluto	
27	Visible light limit of 8m telescopes	

Apparent Magnitude

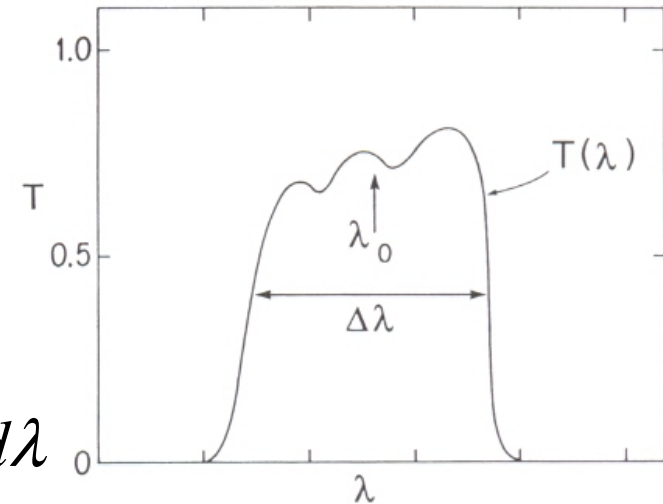
Apparent magnitude is relative measure of monochromatic flux density F_λ of a source:

$$m_\lambda - M_0 = -2.5 \cdot \log\left(\frac{F_\lambda}{F_0}\right)$$

M_0 defines reference point (usually magnitude zero).

In practice, measurements through transmission filter $T(\lambda)$ that defines bandwidth:

$$m_\lambda - M = -2.5 \log \int_0^\infty T(\lambda) F_\lambda d\lambda + 2.5 \log \int_0^\infty T(\lambda) d\lambda$$



Photometric Systems

Filters usually matched to atmospheric transmission

→ different observatories = different filters

→ many photometric systems:

- Johnson UBV system →
- Gunn griz
- USNO
- SDSS
- 2MASS JHK
- HST filter system (STMAG)
- AB magnitude system
- ...

Name	λ_0 [μm]	$\Delta\lambda_0$ [μm]
U	0.36	0.068
B	0.44	0.098
V	0.55	0.089
R	0.70	0.22
I	0.90	0.24
J	1.25	0.30
H	1.65	0.35
K	2.20	0.40
L	3.40	0.55
M	5.0	0.3
N	10.2	5
Q	21.0	8

AB and STMAG Systems

For given flux density F_ν , **AB magnitude** defined as:

$$m(AB) = -2.5 \cdot \log F_\nu - 48.60$$

- object with constant flux per unit **frequency** interval has zero color.
- zero points defined to match zero points of Johnson V-band
- used by SDSS and GALEX
- F_ν in units of $[\text{erg s}^{-1} \text{ cm}^2 \text{ Hz}^{-1}]$

STMAG system defined such that object with constant flux per unit **wavelength** interval has zero color.

- STMAGs are used by the HST photometry packages.

Color Indices

Color index = difference of magnitudes at different wavebands = ratio of fluxes at different wavelengths.

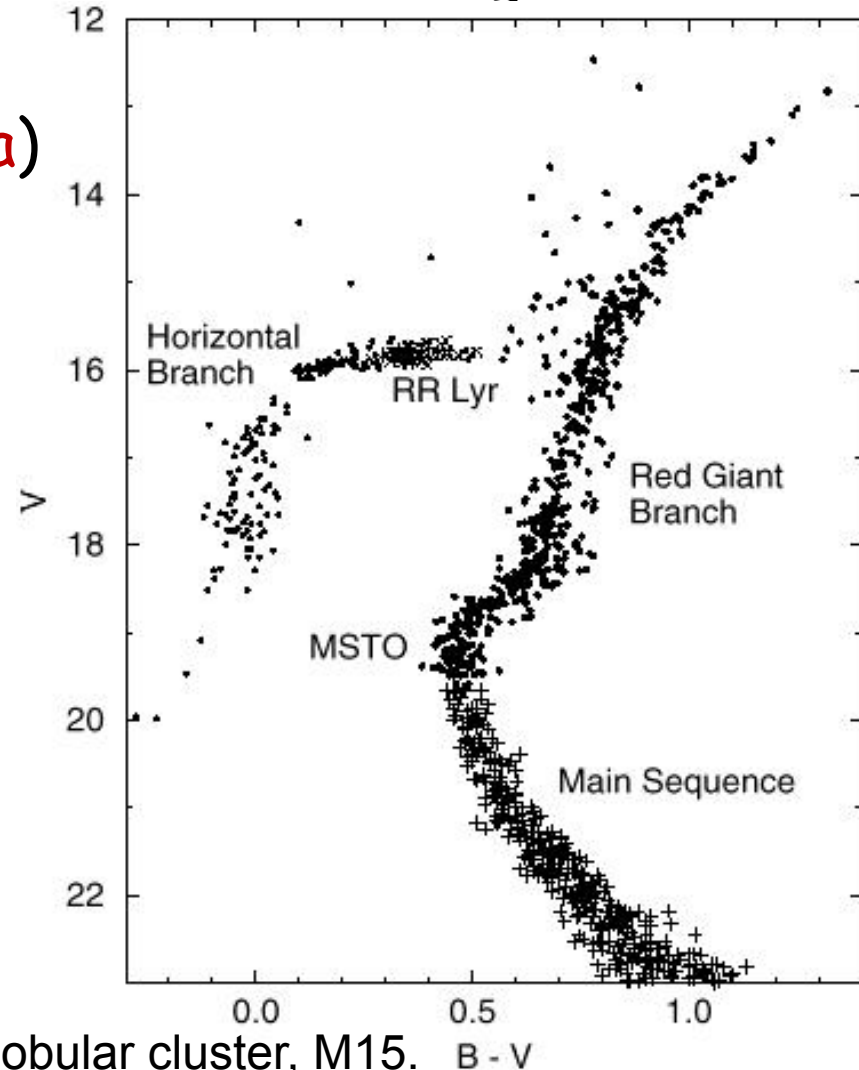
- Color indices of A0V star (**Vega**) about zero longward of V

- Color indices of blackbody in **Rayleigh-Jeans tail** are:

$$B-V = -0.46$$

$$U-B = -1.33$$

$$V-R = V-I = \dots = V-N = 0.0$$



Color-magnitude diagram for a typical globular cluster, M15.

Absolute Magnitude

Absolute magnitude = apparent magnitude of source if it

were **at distance $D = 10$ parsecs**:

$$M = m + 5 - 5 \log D$$

$$M_{\text{Sun}} = 4.83 (V); M_{\text{Milky Way}} = -20.5 \rightarrow \Delta \text{mag} = 25.3 \rightarrow \Delta \text{lumi} = 14 \text{ billion } L_{\odot}$$

However, **interstellar extinction E** or **absorption A** affects the apparent magnitudes

$$E(B - V) = A(B) - A(V) = (B - V)_{\text{observed}} - (B - V)_{\text{intrinsic}}$$

Need to include **absorption** to obtain correct absolute magnitude:

$$M = m + 5 - 5 \log D - A$$

Bolometric Magnitude

Bolometric magnitude is luminosity expressed in magnitude units = integral of monochromatic flux over all wavelengths:

$$M_{bol} = -2.5 \cdot \log \frac{\int_0^{\infty} F(\lambda) d\lambda}{F_{bol}} \quad ; F_{bol} = 2.52 \cdot 10^{-8} \frac{\text{W}}{\text{m}^2}$$

If source radiates isotropically:

$$M_{bol} = -0.25 + 5 \cdot \log D - 2.5 \cdot \log \frac{L}{L_{\odot}} \quad ; L_{\odot} = 3.827 \cdot 10^{26} \text{ W}$$

Bolometric magnitude can also be derived from visual magnitude plus a bolometric correction BC:

$$M_{bol} = M_V + BC$$

BC is large for stars that have a peak emission very different from the Sun's.

Photometric Systems and Conversions

Name	λ_0 [μm]	$\Delta\lambda_0$ [μm]	F_λ [$\text{W m}^{-2} \mu\text{m}^{-1}$]	F [Jy]	
U	0.36	0.068	4.35×10^{-8}	$1\ 880$	Ultraviolet
B	0.44	0.098	7.20×10^{-8}	$4\ 650$	Blue
V	0.55	0.089	3.92×10^{-8}	$3\ 950$	Visible
R	0.70	0.22	1.76×10^{-8}	$2\ 870$	Red
I	0.90	0.24	8.3×10^{-9}	$2\ 240$	Infrared
J	1.25	0.30	3.4×10^{-9}	$1\ 770$	Infrared
H	1.65	0.35	7×10^{-10}	636	Infrared
K	2.20	0.40	3.9×10^{-10}	629	Infrared
L	3.40	0.55	8.1×10^{-11}	312	Infrared
M	5.0	0.3	2.2×10^{-11}	183	Infrared
N	10.2	5	1.23×10^{-12}	43	Infrared
Q	21.0	8	6.8×10^{-14}	10	Infrared

1 Jy = 10^{-26} W m⁻² Hz⁻¹.

Point Sources and Surface Brightness

Point Sources and Extended Sources



Point sources = spatially unresolved

Brightness $\sim 1 / \text{distance}^2$

Size given by observation

Extended sources = well resolved

Surface brightness $\sim \text{const}(\text{distance})$

Brightness $\sim 1/d^2$ and size $\sim 1/d^2$

Surface brightness [mag/arcsec²] is constant with distance!

Calculating Surface Brightness

To describe the **surface brightness** of extended objects one uses units of **mag/sr** or **mag/arcsec²**.

Magnitudes are logarithmic units; to get the surface brightness of an area A :

$$S = m + 2.5 \cdot \log_{10} A$$

The observed surface brightness [mag/arcsec²] can be converted into physical surface brightness units via

$$S[\text{mag/arcsec}^2] = M_{\odot} + 21.572 - 2.5 \cdot \log_{10} S[L_{\odot}/\text{pc}^2]$$

with $L_{\odot} = 3.839 \times 10^{26} \text{ W} = 3.839 \times 10^{33} \text{ erg s}^{-1}$