Astronomical Observing Techniques 2013:
Exercises on Fourier Transforms
(Due on 10 October 2013 at 14:30)

October 7, 2013

1 Fourier transform properties

The Fourier pairs $f(x)$ and $F(s)$ are defined as follows:
$\int_{-\infty}^{+\infty} f(x)e^{-2\pi ixs}\,dx = F(s)$: $\mathcal{F}\{f(x)\} = F(s)$, the Fourier transform of $f(x)$ and
$\int_{-\infty}^{+\infty} F(s)e^{2\pi ixs}\,ds = f(x)$: $\hat{\mathcal{F}}\{F(s)\} = f(x)$, the inverse Fourier transform of $F(s)$

a) show that: $\mathcal{F}\{a(f(x)) + b(g(x))\} = a\mathcal{F}\{f(x)\} + b\mathcal{F}\{g(x)\}$

b) show that: $\mathcal{F}\{f(x-a)\} = e^{-2\pi ias}F(s)$

c) show that: $\mathcal{F}\{f(ax)\} = \frac{1}{|a|}F(s/a)$

2 Fourier transforms of special functions

Compute the Fourier transforms (definition in Exercise 1) of:

a) $\delta(x)$

b) $\delta(x+a)$

c) $e^{-x^2\pi}$

d) $\frac{1}{2}\{\delta(x+\frac{1}{2}) + \delta(x-\frac{1}{2})\}$

e) $\prod(x), 1 \text{ for } |x| < \frac{1}{2}a, \text{ else } 0$

3 Fourier transform of derivative

Show that $\mathcal{F}\{\frac{df(x)}{dx}\} = 2\pi isF(s)$.

4 Two-dimensional Fourier transforms

The 2D Fourier pairs $f(x,y)$ and $F(u,v)$ are defined as follows:
$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)e^{-2\pi i(xu+vy)}\,dxdy = F(u,v)$: $\mathcal{F}\{f(x,y)\} = F(u,v)$, the Fourier transform of $f(x,y)$ and
$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} F(u,v)e^{2\pi i(xu+vy)}\,dudv = f(x,y)$: $\hat{\mathcal{F}}\{F(u,v)\} = f(x,y)$, the inverse Fourier transform of $F(u,v)$

Compute the 2D Fourier transforms of:

a) $\delta(x,y)$

b) $\delta(x-a, y-b)$

5 Convolution

We have a diffraction limited 0.1 arcsec image of a star from the Hubble Space Telescope, describe what happens if we convolve this image with a Gaussian having a width of about 2 arcsec?