Outline

1. Stochastic Processes
2. Autocorrelation and Autocovariance
3. Wide-Sense Stationary and Ergodic Signals
4. Power Spectra
5. Stochastic Nature of Radiation Beam
6. Bose-Einstein Statistics
7. Fermi-Dirac Statistics
8. Quantum Noise and Thermal Noise

Introduction

Statistics

- statistics is fundamental for astronomical data analysis
- radiation is inherently statistical
- need clear understanding of mathematical methods to analyze statistical aspects of data
- here: brief review of statistics and application to photons
- emphasis on physics and less on mathematics
- excellent book: *Statistical Optics* by Joseph W. Goodman

Stochastic Processes

Stochastic Variables

- *random experiment:* cannot predict outcome in advance
- probability P of outcome A must obey:
  - $\mathbf{P}(A) \geq 0$
  - if outcome must be S, then $\mathbf{P}(S) = 1$
  - if A_1, A_2 are mutually exclusive outcomes: $\mathbf{P}(A_1 \textit{ or } A_2) = \mathbf{P}(A_1) + \mathbf{P}(A_2)$
- assign (complex) number $x(A)$ to every possible outcome A
- *stochastic or random variable* X consists of all possible $x(A)$
- *stochastic process:* infinite series of stochastic variables, one for each value of time t
- for specific t, stochastic variable $X(t)$ has certain probability density distribution
- numerical value of stochastic variable $X(t)$ at time t corresponds to particular outcome) from probability distribution at time t

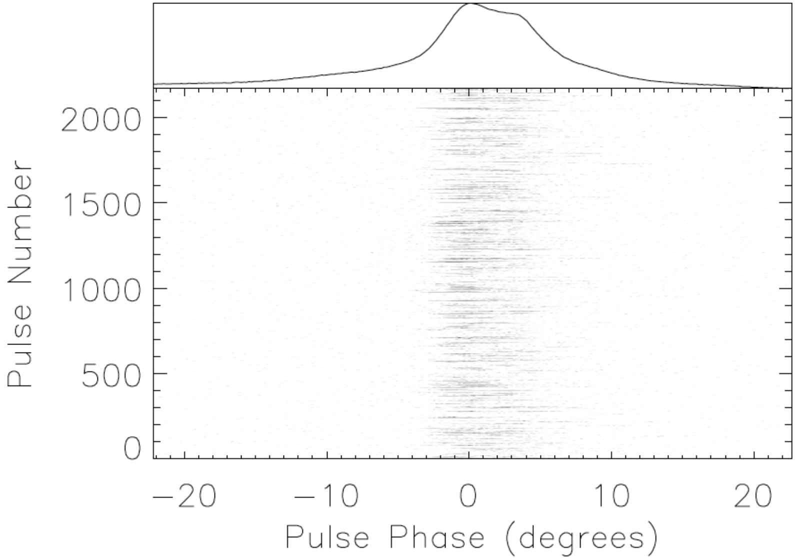
Outcome and Stochastic Variable

- $x(A)$ describes relation between possible outcomes A and stochastic/random variable x
- dice throwing: outcome A_1 is face 1 of dice, etc.
- x is the gain in a game of dice:

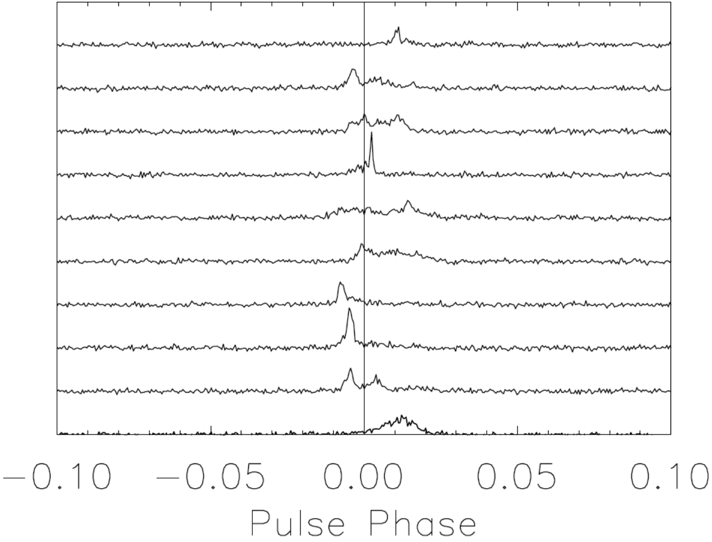
$$\begin{aligned} x(A_1) &= 0 \\ x(A_2) = x(A_3) &= 10 \\ x(A_4) = x(A_5) &= 100 \\ x(A_6) &= 1000 \end{aligned}$$

- example from book of Lena, appendix b
- astronomical: digital output of photometer of a constant source

PSR J0437-4715 (Jenet et al. 1998, Figure 2)

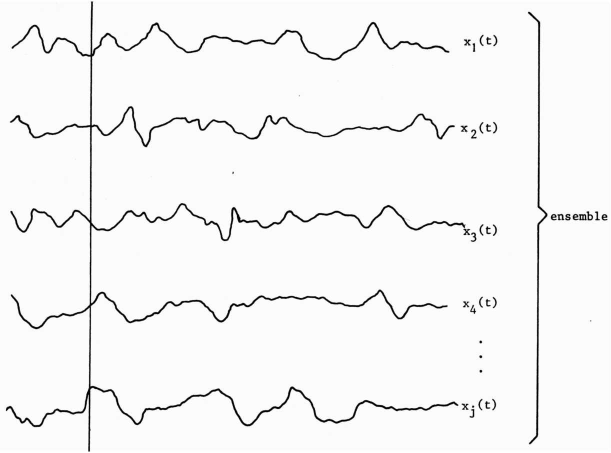


PSR J0437-4715 (Jenet et al. 1998, Figure 4)



Realisation and Ensemble

- time series of outcomes represents a single function in time, called a *realisation* of stochastic process
- full set of all realisations is *ensemble of time functions*



Distribution Functions

- for every t, $X(t)$ is distributed according to momentary cumulative distribution, the *first-order distribution*

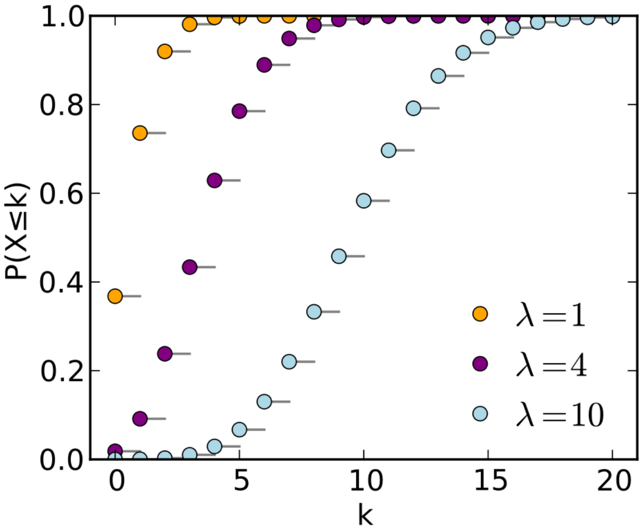
$$F(x;t) = \mathbf{P}\{X(t) \leq x\}$$

- indicates probability that outcome at t will not exceed x
- *probability density function (PDF)* (or first-order density) of $X(t)$ defined by

$$f(x;t) \equiv \frac{\partial F(x;t)}{\partial x}$$

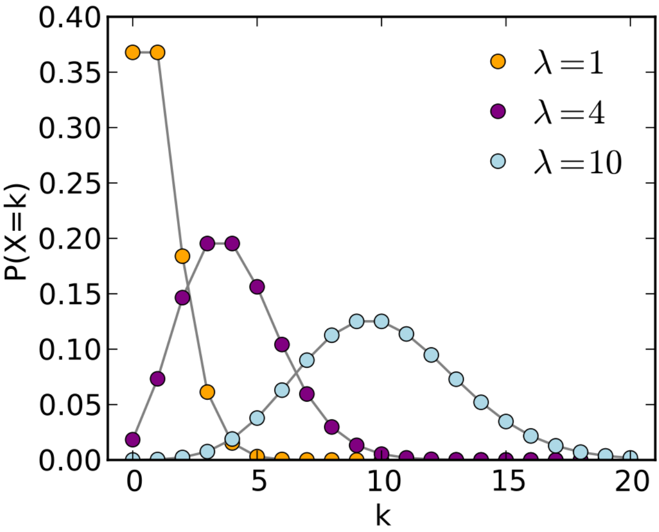
- probability function often
  - binomial: $\binom{n}{k} p^k (1-p)^{n-k}$
  - Poisson: $f(k;\lambda) = \frac{\lambda^k e^{-\lambda}}{k!}$
  - Gaussian (normal): $\frac{1}{\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

Poisson Cumulative Distribution



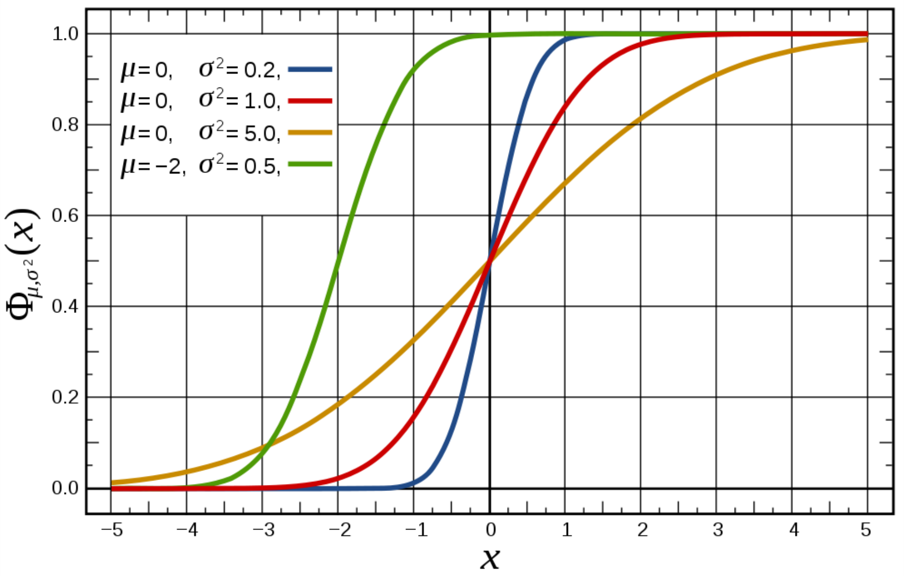
en.wikipedia.org/wiki/Poisson_distribution

Poisson Probability Density Function



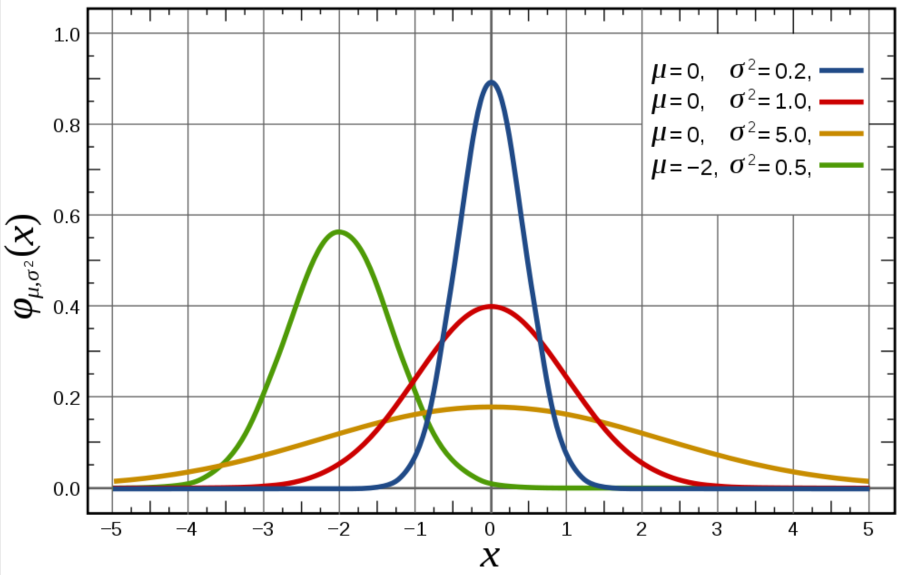
en.wikipedia.org/wiki/Poisson_distribution

Gaussian Cumulative Distribution



en.wikipedia.org/wiki/Normal_distribution

Gaussian Probability Density Function



en.wikipedia.org/wiki/Normal_distribution

Mean and Variance

- properties of distributions often described by a few parameters, often *moments* of distribution
- *mean* or *average* $\mu(t)$ of $X(t)$ is expected value of $X(t)$

$$\mu(t) = \mathbf{E}\{X(t)\} = \int_{-\infty}^{+\infty} x\, f(x; t)\, dx$$

- *variance* of $X(t)$ is the expected value of the square of the difference of $X(t)$ and $\mu(t)$

$$\sigma^2(t) = \mathbf{E}\{(X(t) - \mu(t))^2\} = \mathbf{E}\{X^2(t)\} - \mu^2(t)$$

- variance is square of standard deviation

Higher-Order Distributions

- generalization to higher-order distributions
- second order distribution of process $X(t)$ is joint distribution

$$G(x_1, x_2; t_1, t_2) = \mathbf{P}\{X(t_1) \leq x_1, X(t_2) \leq x_2\}$$

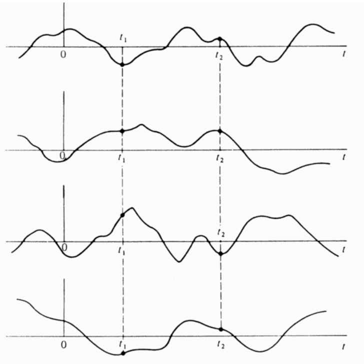
- corresponding derivative is

$$g(x_1, x_2; t_1, t_2) \equiv \frac{\partial^2 G(x_1, x_2; t_1, t_2)}{\partial x_1 \, \partial x_2}$$

Autocorrelation

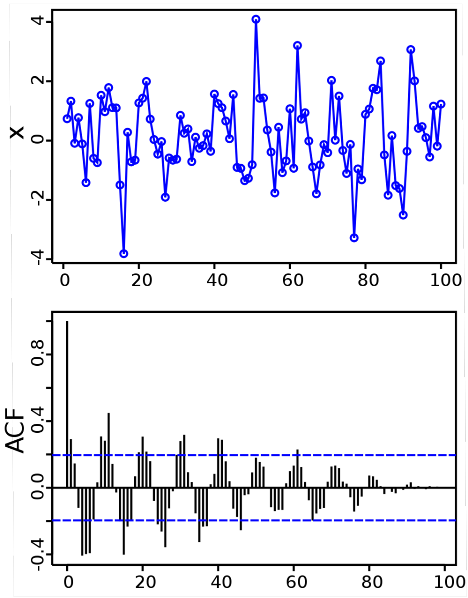
- *autocorrelation* $R(t_1, t_2)$ of $X(t)$ is expected value of $X(t_1) \cdot X^*(t_2)$ (* is complex conjugate)

$$R(t_1, t_2) = \mathbf{E}\{X(t_1) \cdot X^*(t_2)\} = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} x_1\ x_2^*\ g(x_1, x_2^*; t_1, t_2)\ dx_1 dx_2$$



X, Y two independent random variables with probability distributions f, $g \Rightarrow$ probability distribution of difference $Y - X$ given by the cross-correlation $R(f, g)$. convolution $f * g$ gives probability distribution of sum $X + Y$.

Autocorrelation Finds Hidden Sinusoidal Variation



en.wikipedia.org/wiki/Autocorrelation

Power and Autocovariance

- for $t_1 = t_2 = t$, $R(t_1, t_2)$ is average power of signal at time t:

$$R(t) = R(t,t) = \mathbf{E}\{X^2(t)\} = \mathbf{E}\{\,|X(t)|^2\,\}$$

- average of $X(t)$ not generally zero
- *autocovariance* $C(t_1, t_2)$, centered around averages $\mu(t_1)$ and $\mu(t_2)$:

$$C(t_1, t_2) = \mathbf{E}\{(X(t_1) - \mu(t_1)) \cdot (X(t_2) - \mu(t_2))^*\}$$

- For $t_1 = t_2 = t$

$$C(t) = C(t,t) = R(t,t) - |\mu(t)|^2 = \sigma^2(t)$$

- $C(t)$ is average power contained in fluctuations of signal around its mean at time t

Wide-Sense Stationary and Ergodic signals

Wide-Sense Stationary Signals

- *wide-sense stationary (wss)* signal:
  - average does not depend on time
  - autocorrelation only depends on time difference $\tau \equiv t_2 - t_1$
- following relations hold:

$$\begin{aligned}
\textit{signal average} \quad \mu(t) &= \mu = \textit{constant} \\
\textit{autocorrelation} \quad R(t_1, t_2) &= R(\tau) \\
\textit{autocovariance} \quad C(t_1, t_2) &= C(\tau) = R(\tau) - \mu^2
\end{aligned}$$

- for $\tau = 0$: $R(0) = \mu^2 + C(0) = \mu^2 + \sigma^2$
- total power in signal equals power in average signal plus power in fluctuations around average
- autocorrelation and autocovariance are even functions, i.e. $R(-\tau) = R(\tau)$, $C(-\tau) = C(\tau)$

Ergodic Signals

- wss-stochastic process $X(t)$ is considered *ergodic* when *momentaneous average* over $X(t)$ can be interchanged with its *time average*, i.e. when

$$\mu = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{1}{2}T}^{+\frac{1}{2}T} X(t)\, dt = \mathbf{E}\{X(t)\}$$

$$R(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{1}{2}T}^{+\frac{1}{2}T} X^*(t) \cdot X(t+\tau)\, dt = \mathbf{E}\{X^*(t) \cdot X(t+\tau)\}$$

Power Spectral Density

Introduction

- autocorrelation function $R(\tau)$ has Fourier transform

$$S(s) = \int_{-\infty}^{+\infty} R(\tau) \cdot e^{-2\pi i s \tau} \, d\tau$$

- $R(\tau)$ has dimension of signal power (e.g. Watt)
- dimension of $S(s)$ is [power $\times$ time] or power per unit frequency, e.g. Watt$\cdot$Hz^{-1}.
- $S(s)$ is the *power spectral density* (PSD)
- represents double-sided (frequency $-\infty \rightarrow +\infty$) power density of stochastic variable $X(t)$

Wiener-Khinchine Relation

- Fourier pair $R(\tau) \Leftrightarrow S(s)$ is *Wiener-Khinchine relation*
- $\tau = 0$:

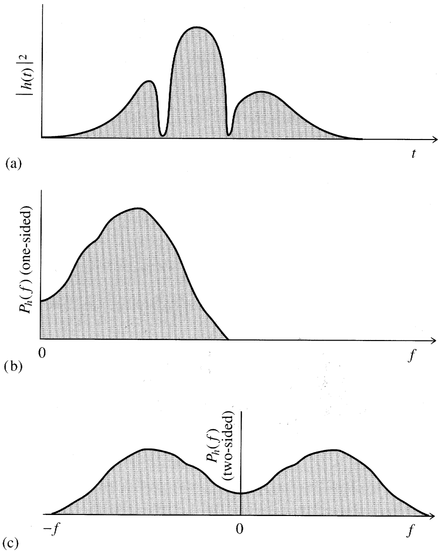
$$R(0) = \mathbf{E}\left\{ |X(t)|^2 \right\} = \int_{-\infty}^{+\infty} S(s)\, ds$$

- $\mu = 0$:

$$R(0) = C(0) = \sigma^2 = \int_{-\infty}^{+\infty} S(s)\, ds$$

- power contained in fluctuations of signal centered around zero mean is equivalent to integration of spectral power density over entire frequency band

Example



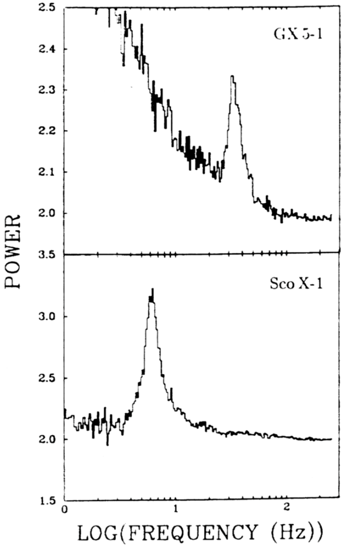
- area under square of function $h(t)$ (a) equals area under one-sided power spectrum at positive frequencies (b) and area under double-sided spectrum (c)
- note factor 2 in amplitude between one-sided and double-sided spectrum

One-Sided Power Spectral Density

- physical frequencies run from zero (i.e. DC) to $+\infty$, mostly use one-sided power spectral density $S^{OS}(s) = 2S(s)$
- total signal power follows from integration over physical frequency domain:

$$R(0) = \int_0^\infty S^{OS}(s)\, ds$$

Example



- PSDs of light curves of Low Mass X-Ray Binaries
- GX5-1 shows “red-noise” excess, i.e. power law dependence ($s^{-\beta}$)
- broad peak are quasi-periodic oscillations (QPOs), indicating temporarily coherent oscillations in luminosity.
- Sco X-1, shows QPOs without stochastic variability

Finite Time Series

- autocorrelation function of stochastic process is a time average
- replace $X(t)$ with general function $f(u)$:

$$R(x) = \int_{-\infty}^{+\infty} f^*(u) \cdot f(u+x)\, du = \int_{-\infty}^{+\infty} f^*(u) \cdot f(u-x)\, du$$

- Fourier transform

$$\Phi(s) = \int_{-\infty}^{+\infty} R(x) \cdot e^{-2\pi isx}\, dx = F(s) \cdot F^*(s) = |F(s)|^2$$

- $f(u) \Leftrightarrow F(s)$
- *Wiener-Khinchin theorem for finite function $f(u)$*
- only non-zero over limited interval of coordinate u

Parseval's Theorem

- $x = 0$:

$$R(0) = \int_{-\infty}^{+\infty} |f(u)|^2 \, du = \int_{-\infty}^{+\infty} \Phi(s) \, ds = \int_{-\infty}^{+\infty} |F(s)|^2 \, ds$$

- *Parseval's theorem:*

$$\int_{-\infty}^{+\infty} |f(u)|^2 \, du = \int_{-\infty}^{+\infty} |F(s)|^2 \, ds$$

- each integral represents amount of “energy” in system
- one integral over all values of a coordinate (e.g. angular coordinate, wavelength, time)
- other over all spectral components in Fourier domain
- $\Phi(s)$ has dimension of an *energy density*
- sometimes wrongly referred to as power density

Stochastic Nature of Radiation Beam

Introduction

- electromagnetic or particle radiation from astronomical source fluctuates because of incoherent emission process
- obvious for particles, less obvious for wave description of electromagnetic radiation
- Bose-Einstein statistics to derive magnitude of intrinsic fluctuations of blackbody radiation source
- photons are *bosons*
- not subject to Pauli exclusion principle
- many bosons may occupy the same quantum state

Bose-Einstein Statistics

- particles in unit volume of space are distributed in momentum-space in boxes with volumes proportional to h^3 ($h =$ Planck's constant)
- for each energy finite number Z of boxes
- $Z \propto 4\pi p^2 dp$ with p the particle momentum
- n_i particles with energies ϵ_i
- number of boxes available at that energy Z_i
- bosons can share a box

Bose-Einstein Statistics (continued)

- number of ways $W(n_i)$ in which n_i bosons can be distributed over Z_i boxes:
$$W(n_i) = \frac{(n_i + Z_i - 1)!}{n_i!(Z_i - 1)!}$$
- equivalent to laying n_i particles and $Z_i - 1$ boundaries in a row
- number of permutations is $(n_i + Z_i - 1)!$
- particles and boundaries can be interchanged
- put n_j bosons into Z_j boxes
- number of ways in which $N = \sum_{i=1}^{\infty} n_i$ bosons can be distributed over boxes in momentum space: $W = \Pi_{i=1}^{\infty} W(n_i)$

Most Likely Particle Distribution

- statistical physics assumption: probability of distribution is proportional to number of ways in which this distribution can be obtained
- collisions between particles re-distribute particles over boxes
- most likely particle distribution is the one which can be reached in most different ways
- find maximum of W by determining maximum of $\ln W$

$$\ln W = \sum_{i=1}^{\infty} \ln W(n_i) \Rightarrow \Delta \ln W = \sum_{i=1}^{\infty} \frac{\partial \ln W(n_i)}{\partial n_i} \Delta n_i = 0$$

- Using Stirlings approximation ($\ln x! \simeq x \ln x - x$) for large x:

$$\begin{aligned} \ln W(n_i) &= (n_i + Z_i - 1)\ln(n_i + Z_i - 1) - (n_i + Z_i - 1) \\ &\quad - n_i \ln n_i + n_i - (Z_i - 1)\ln(Z_i - 1) + (Z_i - 1) \\ &= (n_i + Z_i - 1)\ln(n_i + Z_i - 1) \\ &\quad - n_i \ln n_i - (Z_i - 1)\ln(Z_i - 1) \end{aligned}$$

Bose-Einstein Statistics

- for nearby number $n_i + \Delta n_i$ and to first order in Δn_i:

$$\begin{aligned} \Delta \ln W(n_i) &\equiv \ln W(n_i + \Delta n_i) - \ln W(n_i) \simeq \Delta n_i \frac{\partial \ln W(n_i)}{\partial n_i} \\ &= \Delta n_i \left[\ln(n_i + Z_i - 1) - \ln n_i \right] \end{aligned}$$

- For equilibrium, i.e. the most likely particle distribution

$$\Delta \ln W = \sum_{i=1}^{\infty} \Delta n_i \left[\ln(n_i + Z_i - 1) - \ln n_i \right] = 0$$

Thermodynamic Equilibrium

- system in thermodynamic equilibrium:
  - number of particles $N = \sum n_i$ per unit volume is constant
  - energy $E = \sum n_i \epsilon_i$ per unit volume is constant
- variations in n_i must conserve N and E:

$$\begin{aligned}\Delta N &= \sum_{i=1}^{\infty} \Delta n_i = 0 \\ \Delta E &= \sum_{i=1}^{\infty} \epsilon_i \Delta n_i = 0\end{aligned}$$

- Therefore

$$\begin{aligned}\Delta \ln W - \alpha \Delta N - \beta \Delta E &= \\ \sum_{i=1}^{\infty} \Delta n_i \left[\ln(n_i + Z_i - 1) - \ln n_i - \alpha - \beta\epsilon_i\right] &= 0\end{aligned}$$

Bose-Einstein Distribution

- $\sum_{i=1}^{\infty} \Delta n_i \left[\ln(n_i + Z_i - 1) - \ln n_i - \alpha - \beta\epsilon_i\right] \; = \; 0$ for arbitrary variations Δn_i if for each i

$$\ln(\overline{n}_i + Z_i - 1) - \ln \overline{n}_i - \alpha - \beta\epsilon_i = 0 \Rightarrow \frac{\overline{n}_i}{Z_i - 1} = \frac{1}{e^{\alpha+\beta\epsilon_i} - 1}$$

- this is Bose-Einstein distribution
- $Z_i \gg 1$: $\overline{n}_i/(Z_i - 1) \Rightarrow \overline{n}_i/Z_i$, which represents the average occupation at energy level ϵ_i (occupation number).
- values of α and β depend on total number of particles and total energy
- can be determined by substituting n_i in $N = \sum_{i=1}^{\infty} n_i$ and in $E = \sum_{i=1}^{\infty} n_i\epsilon_i$

Planck Function

- Planck function does not require that number of photons be conserved
- atom can absorb one photon and then emit 2 photons
- obtain the Planck function by dropping the α term

$$\frac{\overline{n}_i}{Z_i - 1} = \frac{1}{e^{\beta\epsilon_i} - 1} = \overline{n}_{\nu_k}$$

- $\overline{n}_{\nu_k}$ is average occupation number of photons at frequency ν_k
- photons do not collide directly with one another, but reach equilibrium only via interaction with atoms

Connection to Thermodynamics

- connection to thermodynamics via

$$S \equiv k \ln W \Rightarrow \Delta S = k \Delta \ln W$$

- therefore we get

$$\Delta S = k\alpha \Delta N + k\beta \Delta E$$

- $T\Delta S = -\zeta \Delta N + \Delta E$ we find that $\beta = 1/(kT)$
- $\zeta \equiv -\alpha/\beta$ is thermodynamical potential per particle

Fluctuations Around Equilibrium

- number of ways in which $n_i + \Delta n_i$ particles can be distributed as compared to that for n_i particles (to second order):

$$\ln W(n_i + \Delta n_i) = \ln W(n_i) + \Delta n_i \frac{\partial \ln W(n_i)}{\partial n_i} + \frac{\Delta n_i^2}{2} \frac{\partial^2 \ln W(n_i)}{\partial n_i^2}$$

- in equilibrium term proportional to Δn_i is zero
- rewrite as

$$W(n_i + \Delta n_i) = W(n_i) \mathrm{e}^{-\frac{W''(n_i)}{2} \Delta n_i^2} \ \textit{where}\ W''(n_i) \equiv -\frac{\partial^2 \ln W(n_i)}{\partial n_i^2}$$

- probability of deviation Δn_i drops exponentially with square of Δn_i
- probability of Δn_i is a Gaussian

Fluctuations Around Equilibrium (continued)

- average value for Δn_i^2 by integrating over all values of Δn_i:

$$\overline{\Delta n_i^2} = \frac{\int_{-\infty}^{\infty} \Delta n_i^2 W(n_i) e^{-\frac{W''(n_i)}{2}\Delta n_i^2} d\Delta n_i}{\int_{-\infty}^{\infty} W(n_i) e^{-\frac{W''(n_i)}{2}\Delta n_i^2} d\Delta n_i} = \frac{1}{W''(n_i)}$$

- $W(n_i)$, $W''(n_i)$ do not depend on $\Delta n_i \Rightarrow$ constants in integrations
- maximum negative deviation has $\Delta n_i = -n_i$
- maximum positive deviation $\Delta n_i = N - n_i$
- integrals to be evaluated between these values
- for large Δn_i integrand drops rapidly to zero
- extend integrals to full range $-\infty$ to $+\infty$ without changing result

Variance

- variance from second derivative of $\ln W(n_i)$ and changing sign:

$$\overline{\Delta n_i^2} = \left[W''(n_i)\right]^{-1} = \frac{\overline{n}_i(\overline{n}_i + Z_i - 1)}{Z_i - 1} = \overline{n}_i \left[1 + \frac{1}{e^{\alpha+\beta\epsilon_i} - 1}\right]$$

- drop the α for Planck function:

$$\overline{\Delta n_i^2} = \overline{n}_i \left[1 + \frac{1}{e^{\beta\epsilon_i} - 1}\right] = \overline{n_i}(1 + \overline{n}_{\nu_k})$$

- fluctuation in average occupation number

$$\overline{\Delta n_{\nu_k}}^2 = \frac{\overline{\Delta n_i^2}}{Z_i} = \overline{n}_{\nu_k}(1 + \overline{n}_{\nu_k})$$

Fermi-Dirac Statistics

Distribution

- particles not allowed to share a box
- number of ways $W(n_i)$ in which n_i particles can be distributed over Z_i boxes with energies ϵ_i:

$$W(n_i) = \frac{Z_i!}{n_i!(Z_i - n_i)!}$$

- difference in $\ln W(n_i)$ between nearby numbers to first order in Δn_i:

$$\ln W(n_i + \Delta n_i) - \ln W(n_i) = -\Delta n_i \left[\ln n_i - \ln(Z_i - n_i)\right]$$

- equilibrium $\Rightarrow$ *Fermi Dirac distribution*:

$$\frac{\overline{n}_i}{Z_i} = \frac{1}{e^{\alpha+\beta\epsilon_i} + 1} = \overline{n}_k$$

Fluctuations

- average value of square of deviation

$$\overline{\Delta n_i^2} = \frac{\overline{n}_i(Z_i - \overline{n}_i)}{Z_i} = \overline{n}_i\left[1 - \frac{1}{e^{\alpha+\beta\epsilon_i}+1}\right] = \overline{n}_i(1 - \overline{n}_k)$$

- The fluctuation in the average occupation number

$$\overline{\Delta n_k}^2 = \frac{\overline{\Delta n_i^2}}{Z_i} = \overline{n}_k(1 - \overline{n}_k)$$

Introduction

- volume density of photons in blackbody Bose gas between frequencies ν, $\nu + d\nu$ from

$$\bar{N}(\nu)d\nu = g(\nu)\bar{n}_\nu d\nu$$

- $g(\nu_k)$ is volume density of quantum states per unit frequency at ν_k
- stochastic variables n_{ν_k} independent $\Rightarrow$ Bose-fluctuations

$$\overline{\Delta N^2}(\nu) = \bar{N}(\nu)\left(1 + \frac{1}{\exp(h\nu/kT) - 1}\right)$$

- $\bar{N}(\nu)$ follows from specific energy density $\bar{\rho}(\nu) = \rho(\nu)^{equilibrium}$:

$$\bar{\rho}(\nu)d\nu = \frac{8\pi h}{c^3}\frac{\nu^3}{\exp\left(\frac{h\nu}{kT}\right) - 1}d\nu$$

- using $\bar{N}(\nu) = \bar{\rho}(\nu)/h\nu$

Rdiation Detection

- detector inside blackbody radiation field at temperature T
- incident photon flux:

$$\bar{n}(\nu) = \frac{1}{2}\frac{c}{4\pi}\bar{N}(\nu)A_e\Omega$$

- factor $\frac{1}{2}$ refers to one component of polarisation
- A_e is effective area of detector
- Ω is solid angle subtended by detector beam viewing radiation field
- if radiation illuminates extended surface (A_e) with various directions of the wave vector, i.e. an omnidirectional blackbody radiation field, coherence theory states that spatial coherence is limited to $A_e\Omega \approx \lambda^2$, the so-called *extent (etendue) of coherence*.
- same as size $\theta = \lambda/D$ of diffraction-limited beam ($\Omega \approx \theta^2$) for aperture diameter D: $A_e \approx D^2$

Radiation Detection (continued)

- substituting $\bar{N}(\nu)$, specific photon flux $\bar{n}(\nu)$ (in photons s^{-1} Hz^{-1}) becomes:

$$\begin{aligned} \bar{n}(\nu) &= \frac{1}{\exp(h\nu/kT) - 1} \\ \overline{\Delta n^2}(\nu) &= \bar{n}_\nu \left(1 + \frac{1}{\exp(h\nu/kT) - 1}\right) \end{aligned}$$

Poisson Noise

- from before:

$$\bar{n}(\nu) = \frac{1}{\exp(h\nu/kT) - 1}$$

$$\overline{\Delta n^2}(\nu) = \bar{n}_\nu \left(1 + \frac{1}{\exp(h\nu/kT) - 1}\right)$$

- in extreme case $h\nu \gg kT$, second term becomes much smaller than 1:

$$\overline{\Delta n^2}(\nu) = \bar{n}(\nu)$$

- is Poissonian noise in a sample containing $\bar{n}(\nu)$ photons.
- *quantum limit of fluctuations*
- represents minimum value of intrinsic noise present in any radiation beam
- Obviously holds for particle radiation

Thermal Noise

- for photon with $h\nu \ll kT$, noise is expressed in terms of average radiation power $\bar{P}(\nu)$ (e.g. in Watt Hz^{-1})
- by writing $\bar{P}(\nu) = (h\nu)\bar{n}(\nu)$ and $\overline{\Delta P^2}(\nu) = (h\nu)^2\overline{\Delta n^2}(\nu)$:

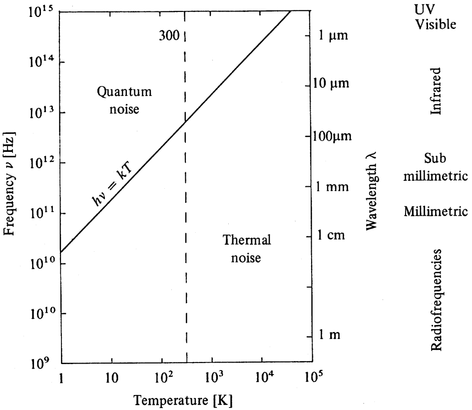
$$\overline{\Delta P^2}(\nu) = \bar{P}(\nu)\left(h\nu + \frac{h\nu}{\exp(h\nu/kT) - 1}\right) = \bar{P}(\nu)(h\nu + \bar{P}(\nu))$$

- $h\nu \ll kT$:

$$\begin{aligned} \overline{\Delta P^2}(\nu) &= \bar{P}^2(\nu) \\ \textit{and} \qquad \bar{P}(\nu) &= kT \end{aligned}$$

- expression for classical thermal noise power per unit frequency bandwidth
- *thermal limit*

Quantum Noise and Thermal Noise



- transition between quantum limit to thermal limit at $h\nu \approx kT$
- $T \approx 300$ K $\Rightarrow \nu \approx 6$ THz, $\lambda \approx 50\ \mu$m

Quantum and Thermal Noise

- radio observations always dominated by the wave character of incoming beam $\Rightarrow$ in thermal limit
- treatment of noise in radio observations differs drastically from measurements at shorter wavelengths
- submillimeter and infrared observations strive to be in the quantum limit
- fluctuations in average power $\bar{P}(\nu)$ for thermal limit can be interpreted as importance of wave packet interference $\Rightarrow$ interference will cause fluctuations to become of same magnitude as signal
- low frequency fluctuations can be thought of as caused by random phase differences and beats of the wavefields

Detector Outside of Blackbody Photon Gas

- expression for fluctuations in blackbody photon gas applies only to detector in interior of blackbody where $\lambda^2 = c^2/\nu^2 = A_e\Omega$
- if not, even in limit $h\nu \ll kT$ quantum noise may dominate
- example: blackbody star at temperature T, observed at frequency ν, where $h\nu \ll kT$, thermal noise shoudl dominate
- star is so far away that radiation is unidirectional and $A_e\Omega \ll \lambda^2$
- photons will arrive well separated in time and quantum noise dominates