The formation of galaxies and large-scale structure
Problem set 6

Week of April 29, 2014

Cosmological parameters

Some cosmological parameters for reference — if you need cosmological parameters in the problems below, use these.

$$
\begin{aligned}
\Omega_m &= 0.258 \\
\Omega_\Lambda &= 1 - \Omega_m \\
\Omega_b &= 0.0441 \\
\Omega_R &= 2.488 \times 10^{-5} h^{-2} = 4.81 \times 10^{-5} \\
h &= 0.719 \\
H_0 &= 100\, h \,\text{km/s/Mpc} \\
T_{\gamma,0} &= 2.73\,\text{K} \\
\sigma_8 &= 0.796 \quad \text{The mass in a sphere with radius } 8h^{-1}\text{Mpc} \\
\rho_{\text{crit},0} &= 1.879 \times 10^{-29}\, h^2
\end{aligned}
$$

Problem 1 Disk galaxy sizes

In the lecture, we showed that the disk scale length is given by

$$R_d = \frac{1}{\sqrt{2}} \frac{J_d/M_d}{J/M} \lambda R_{\text{vir}}, \tag{1}$$

and that the mask of an exponential disk can be written

$$M_d = 2\pi \Sigma_0 R_d^2. \tag{2}$$

(a) Assuming detailed angular momentum conservation (i.e. no angular momentum transport) during the collapse of the disk, what is the disk scale length of a galaxy with $M = 10^{12} h^{-1} M_\odot$, spin parameter $\lambda = 0.05$ and collapse redshift $z_c = 1$?

(b) Show that the central surface density is given by

$$\Sigma_0 = \frac{1}{\pi} \frac{M_d}{M} \left(\frac{J_d/M_d}{J/M} \right)^{-2} \lambda^{-2} r_{\text{vir}}^{-2} M. \tag{3}$$

What is Σ_0 for a galaxy with $(M, \lambda, z_c) = (10^{12} h^{-1} M_\odot, 0.05, 1)$?.

Problem 2 Lyman-alpha absorbers

In this problem you will generalise and fill in the details of the derivations in the lecture.

(a) Define the dynamical time as

$$t_{\rm dyn} = \frac{1}{\sqrt{G\rho}}, \tag{4}$$

and the sound crossing time of a structure of size L as

$$t_{\rm sc} = \frac{L}{c_s}. \tag{5}$$

Show that $t_{\rm dyn} = t_{\rm sc}$ gives a scale

$$L_J = \left(\frac{\gamma k_B}{\mu m_H^2 G}\right)^{1/2} (1-Y)^{1/2} f_g^{1/2} n_{\rm H}^{-1/2} T^{1/2}, \tag{6}$$

where γ is the ratio of specific heats, Y is the Helium mass fraction and f_g is the baryon fraction.

(b) The Jeans column density of hydrogen is defined to be $n_{\rm H} L_J$, show that the column density of neutral hydrogen can be written

$$N_{\rm HI} = \left(\frac{\gamma k_B}{\mu m_H^2 G}\right)^{1/2} \frac{1-Y/2}{(1-Y)^{1/2}} f_g^{1/2} \alpha_r \Gamma^{-1} n_{\rm H}^{3/2} T^{1/2}, \tag{7}$$

where α_r is the Case A recombination coefficient for hydrogen, Γ is the incident radiation field and the other quantities have the meanings given above.

(c) In the lectures we showed that written in terms of the column density of neutral hydrogen, the Jeans length could be written

$$L_J \sim 10^2 {\rm kpc} \left(\frac{N_{\rm HI}}{10^{14}\ {\rm cm}^{-2}}\right)^{-1/3} T_4^{0.41} \Gamma_{-12}^{-1/3} \left(\frac{f_g}{0.16}\right)^{2/3}. \tag{8}$$

Derive an expression for L_J by combining the equations above.

(d) What is the mass of a Ly-α absorber that is just turning around at $z = 3$?

(e) There is also a He II forest, how can you adapt the formalism for the H I forest to be applicable to this forest?

(f) Assume that the number of absorbers with column density $N_{\rm HI}$ is given by $f(N_{\rm HI}) = 5.3\times10^7\, N_{\rm HI}^{1.46}$ and derive the contribution of absorbers to the mass density of the Universe relative to the critical density, ie. calculate $\Omega_{\rm absorber}$.