The formation of galaxies and large-scale structure
Small problem set 3

February 25, 2013

The problems here are meant to gain some experience with the properties perturbation growth and non-linear scaling laws.

Problem 1 Scale of perturbations at horizon entry

Show that in a standard CDM model with a Harrison-Zeldovich spectrum ($n = 1$), the variance of the density contrast $\sigma^2 \propto k^3 P(k) \propto k^3 \delta_k^2$, is independent of the scale of the perturbations at the time the fluctuations enter the horizon

Problem 2 Perturbation growth

Assume a cosmology with $\Omega_m = 0.258$, $\Omega_\Lambda = 1 - \Omega_m$, $\Omega_B = 0.0441$, $h = 0.719$, the energy density in photons relative to critical: $\Omega_\gamma = 4.81 \times 10^{-5}$, and that there are three families of neutrinos, each of which has a energy density $(7/8)(4/11)^{4/3}$ that of the photons.

(a) For the cosmology given, sketch the growth of a dark matter perturbation with mass $M = 10^{15} M_\odot$ as a function of scale factor. It is acceptable to make rough approximations here, just spell them out clearly.

(b) If we write

$$\sigma^2(M) = \int \Delta^2(k) \left|\hat{W}(k;R)\right|^2 \frac{dk}{k} \tag{1}$$

Show that $\sigma^2(M) \propto M^{-(n+3)/3}$ for a Gaussian window function (see lecture notes) when the power spectrum can be written $P(k) \propto k^n$. *Hint: No need to carry out complicated integrals..*

Problem 3 Non-linear scaling laws and the fundamental plane

Massive spheroidal galaxies are found to lie on the so-called Fundamental Plane:

$$\log_{10} r_e = a \log_{10} \sigma - b \log \mu_e + \gamma, \tag{2}$$

where r_e is the characteristic radius of the galaxy, σ is the velocity dispersion of the stars in the galaxy, μ is the luminosity per area and γ is a constant.

(a) Base yourself on the non-linear scaling arguments made in the lecture to show that

$$b = \frac{10 - a + 2n + an}{4(a + n)}. \tag{3}$$

The slope of the power spectrum on galaxy scales in LCDM is approximately -2.35. How does your solution compare to the observed fundamental plane of Jørgensen et al (1996) who found $a = 1.24$, $b = 0.82$?

(b) A common "explanation" for the Fundamental Plane is that it reflects the virial theorem, $M^2/R \sim MV^2$, does this work equally well for explaining the relationship between a and b? If yes, why?, if not, why not? [within the approximations used in the lecture].