# Large-scale structure and galaxy formation

Review lecture

## Exam

**Date**: Friday June 13, 2013
**Time**: 14:00-17:00
**Place**: 414

No books/notes etc.
Calculators allowed, a cheat sheet will be provided with useful equations/constants.

# Our menu today

- Perturbation growth inside and outside the horizon.
- The transition from radiation domination to matter domination - when structure can start forming.
- Recombination - the neutral and dark Universe.
- The first objects - reionization.
- Growth of structure - window functions, power spectra and the (extended) Press-Schechter formalism
- The formation of gaseous halos - virial relations.
- Cooling and heating of gas.
- The formation of spiral galaxies.
- The intervening Universe - the structure of the IGM.

# Cosmology reminders

The Friedmann equations:

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{kc^2}{a^2} = \frac{8\pi G}{3}\rho,$$

$$\dot{\rho} = -3\frac{\dot{a}}{a}\left(\rho + \frac{P}{c^2}\right)$$

We often use it in the form: $(H(z) = \dot{a}/a)$

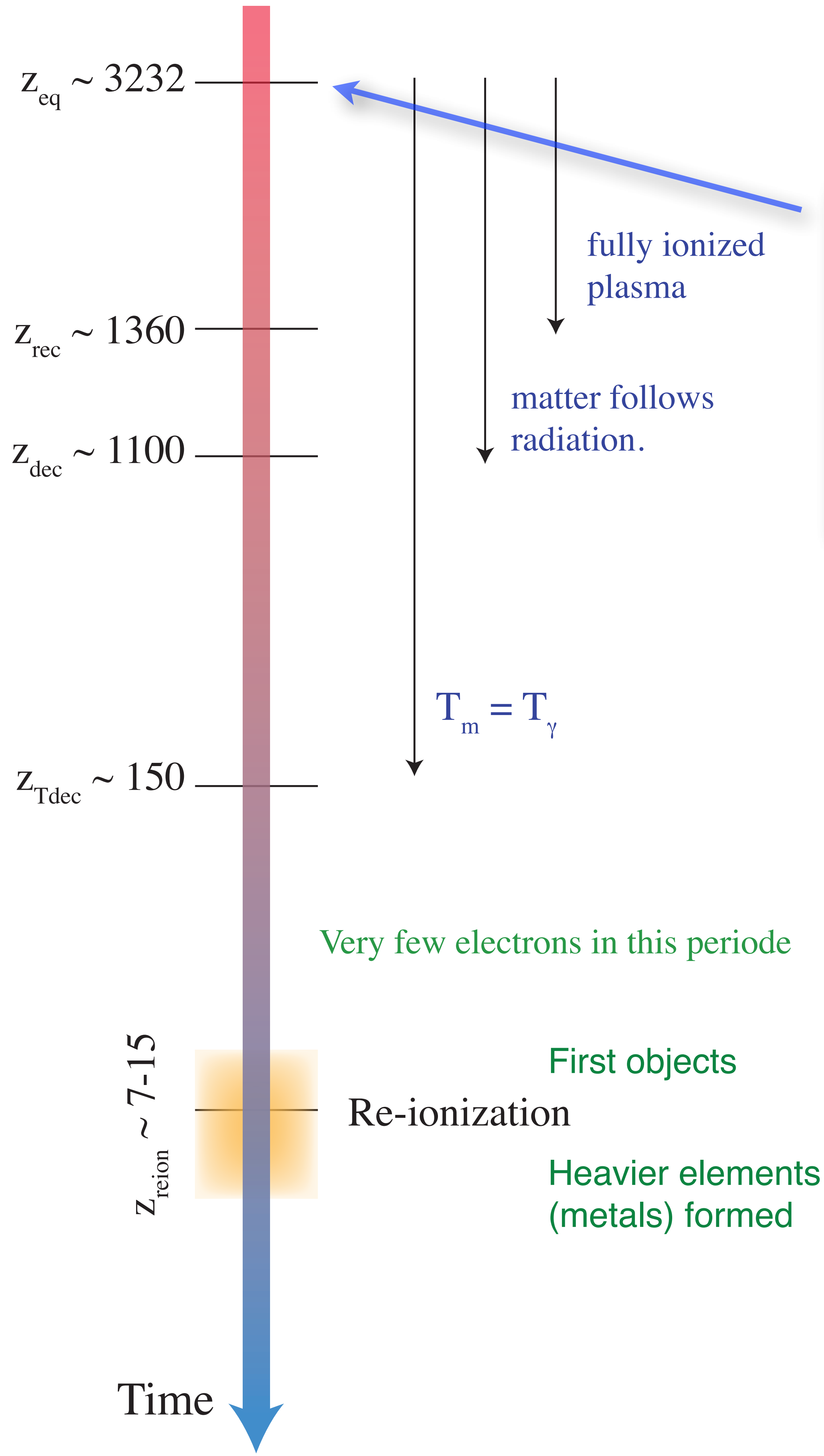
$$H(z)^2 = H_0^2\left[\Omega_\Lambda + (1-\Omega_{\rm tot})(1+z)^2 + \Omega_{\rm NR}(1+z)^3 + \Omega_{\rm R}(1+z)^4\right]$$

From the first Friedmann equation we see there is a critical density corresponding to k=0:

$$\rho_{\rm crit,0} = \frac{3H_0^2}{8\pi G}$$

$$\begin{aligned}\rho_{\rm crit,0} &\approx 3\times 10^{11}\,\frac{h^{-1}\,M_\odot}{(h^{-1}\,{\rm Mpc})^3}\\ &\approx 1.879\times 10^{-29}h^2\,{\rm g/cm^3}.\end{aligned}$$

# Transitions

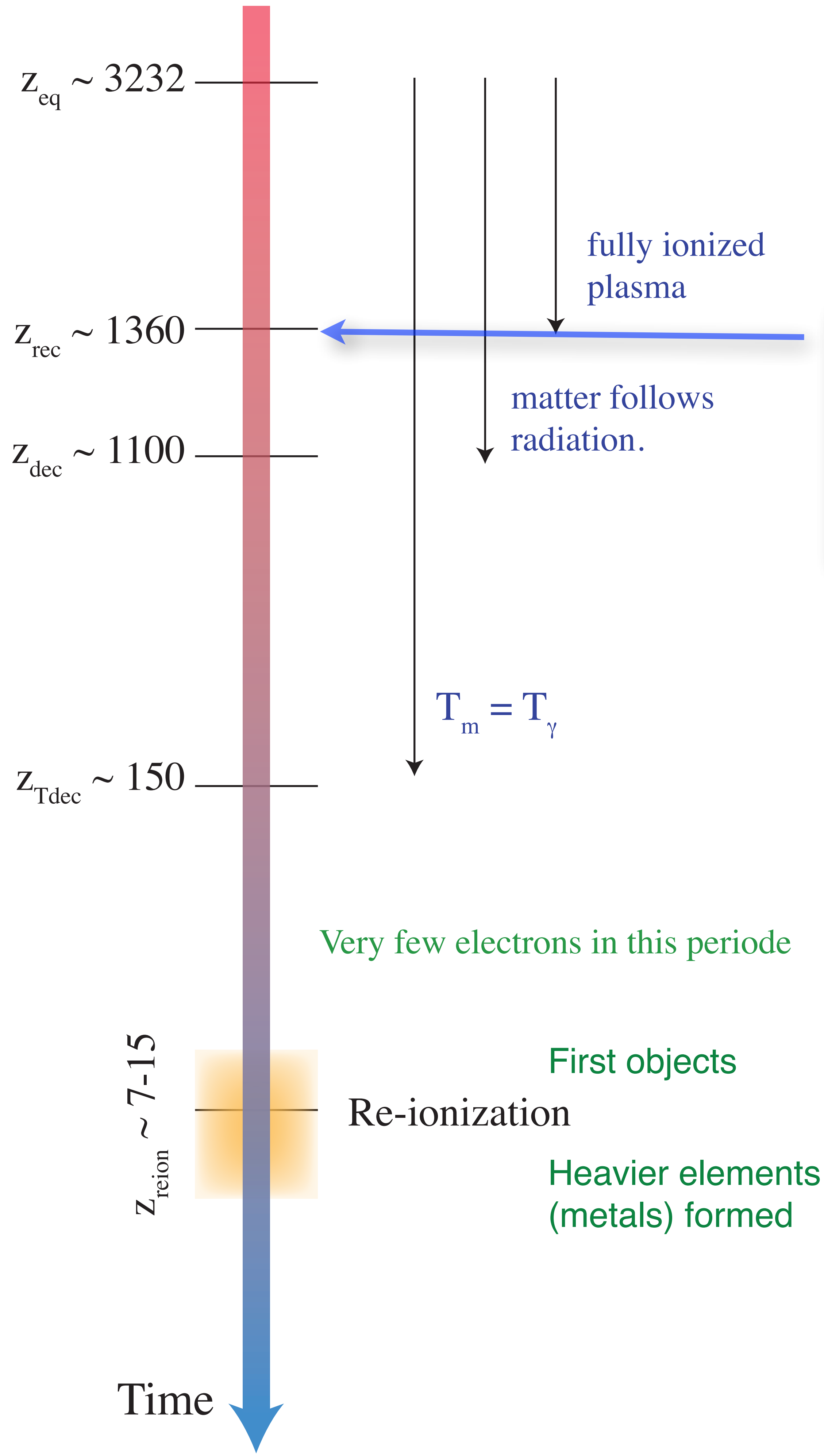



$$\rho_{\rm NR} \propto a^3 \qquad \rho_{\rm R} \propto a^4$$

&

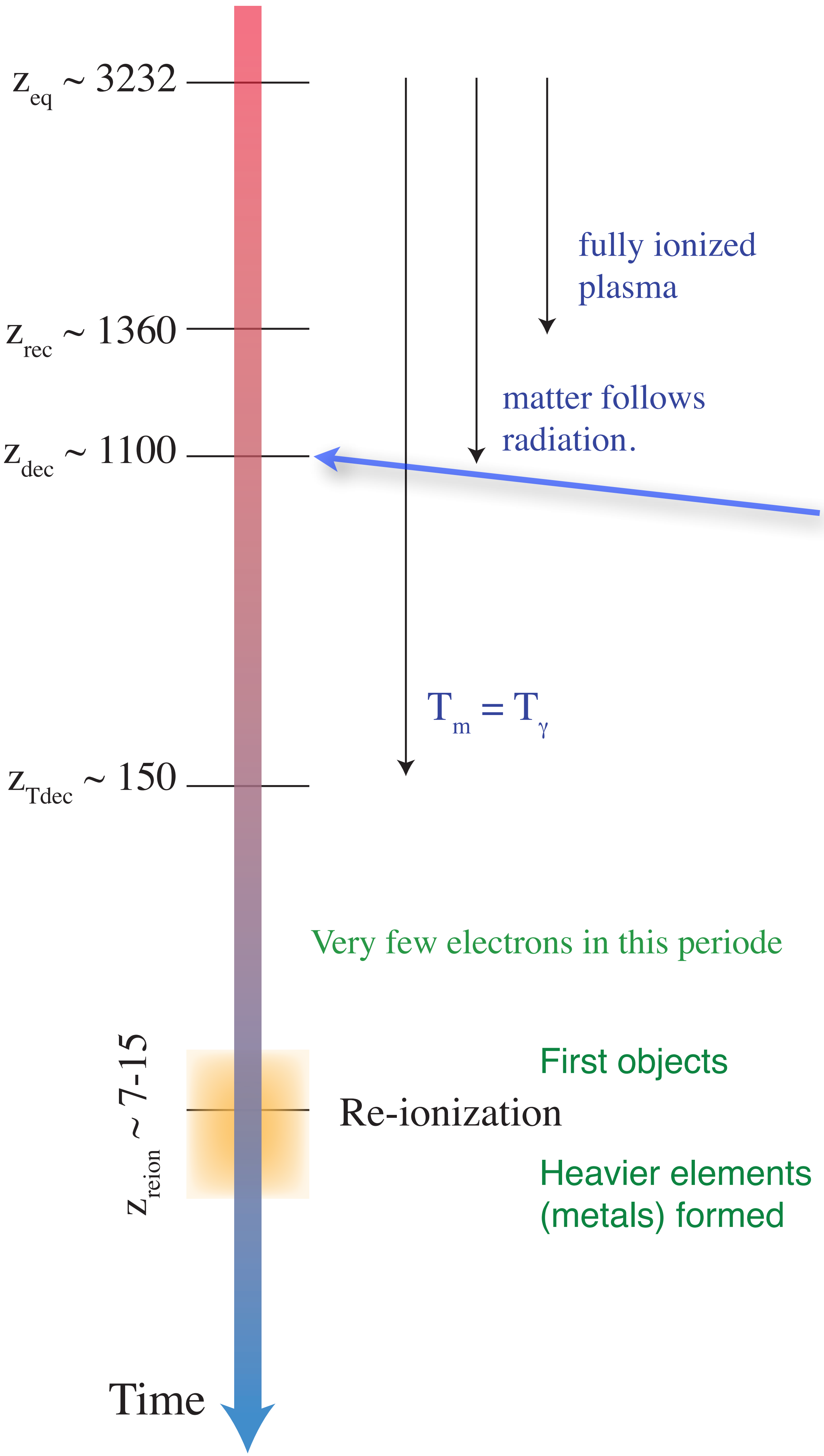
$$\rho_{\rm NR}(t = t_{\rm today}) > \rho_{\rm R}(t = t_{\rm today})$$

# Transitions



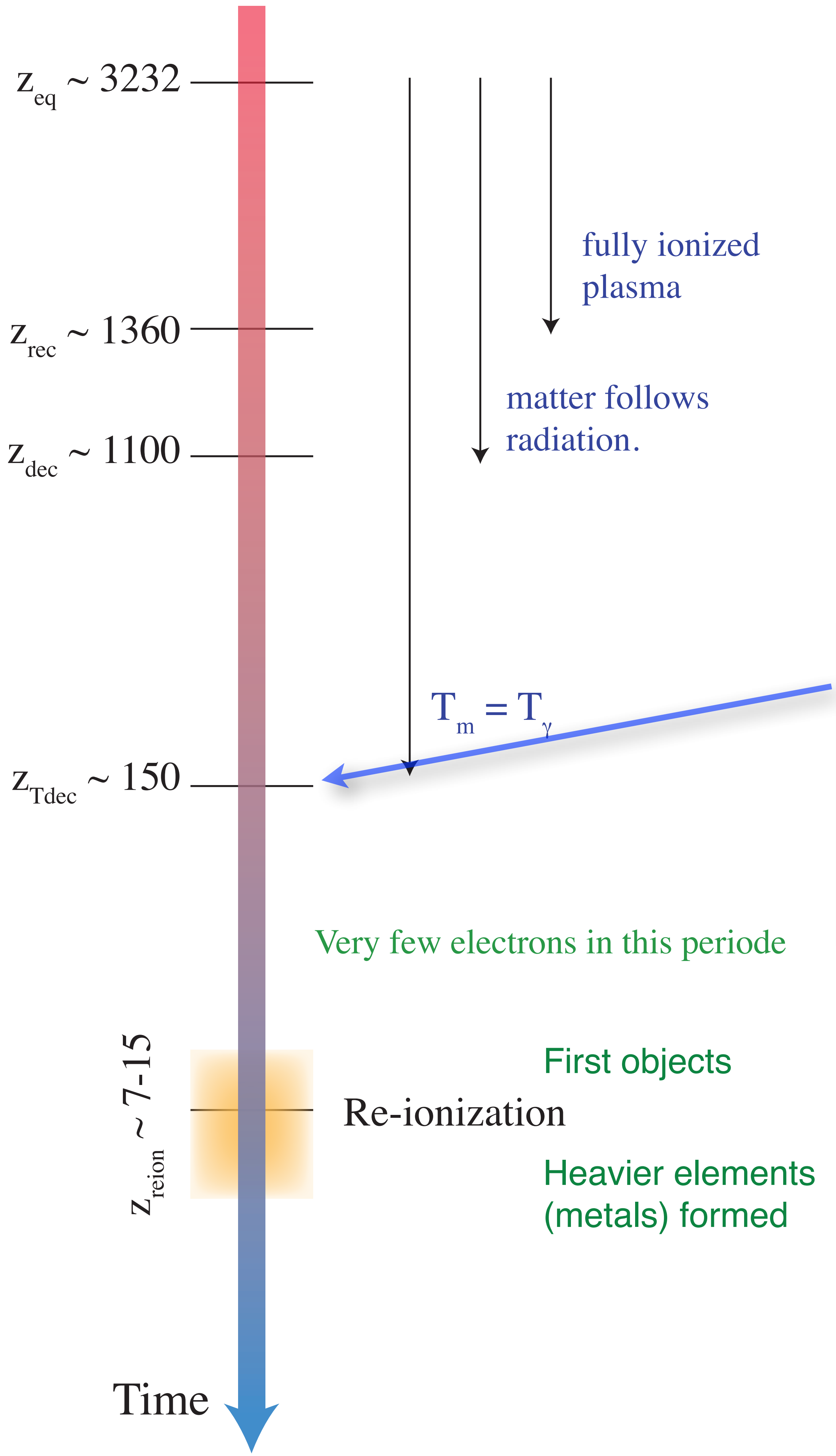

~50% of the hydrogen is neutral

# Transitions



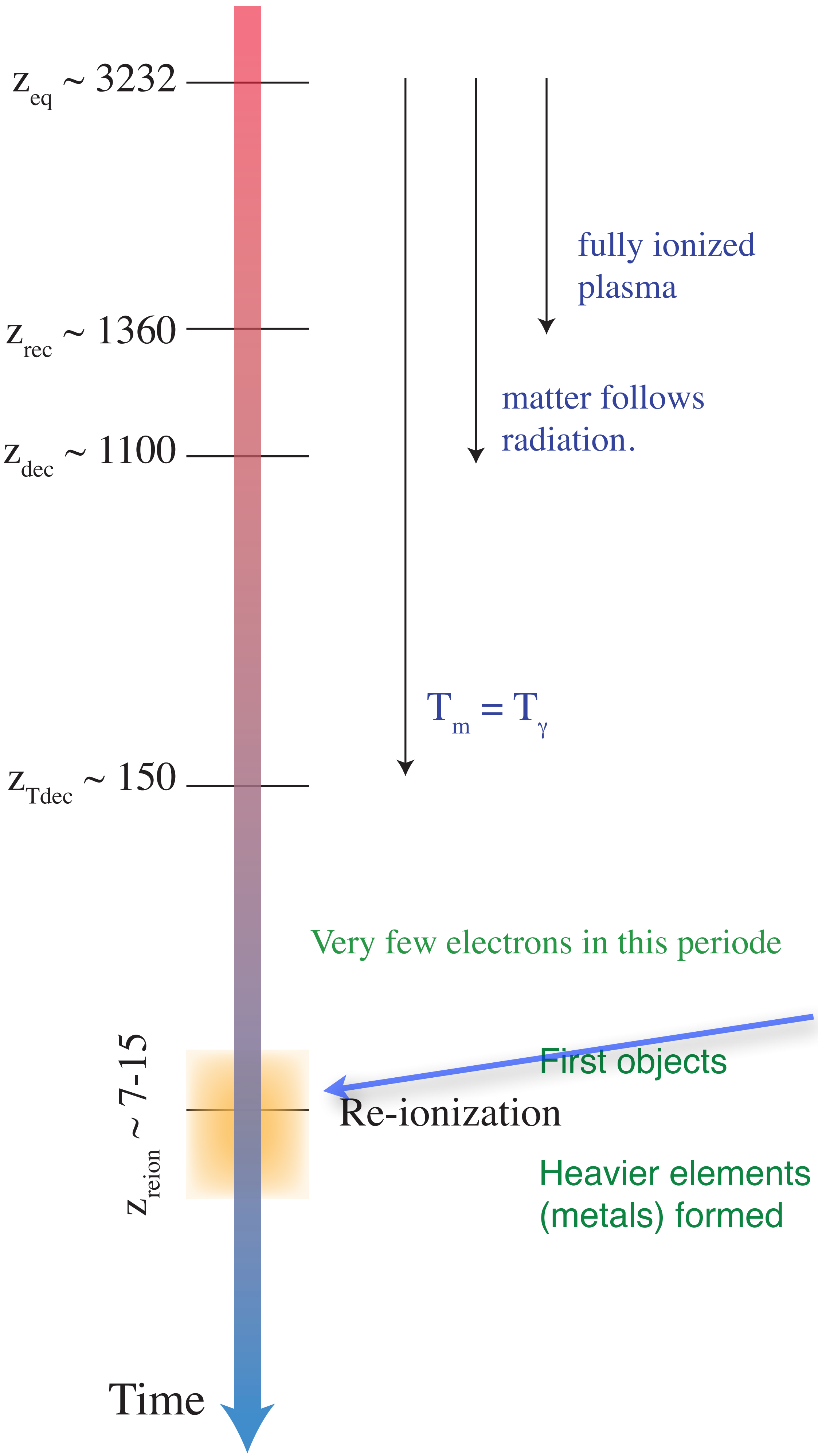

The coupling between electrons and radiation becomes unimportant

# Transitions




The temperature of matter and radiation is no longer kept equal

# Transitions




We can delimit this period roughly using absorption in QSO spectra and polarization of the CMB.

# Perturbation growth (Lecture 2)

**Inside the horizon:** $\mathrm{Recall}:\delta = \dfrac{\rho - \bar{\rho}}{\bar{\rho}}$

$$\frac{\partial^2 \delta}{\partial t^2} + 2\frac{\dot{a}}{a}\frac{\partial \delta}{\partial t} = 4\pi G \rho_b \delta + \frac{c_s^2}{a^2}\nabla_{\vec{x}}^2 \delta.$$

Or, in Fourier space:

$$\frac{\partial^2 \delta_k}{\partial t^2} + 2\frac{\dot{a}}{a}\frac{\partial \delta_k}{\partial t} = 4\pi G \rho_b \delta_k - \frac{c_s^2}{a^2} k^2 \delta_k,$$

Where: $k = 2\pi/\lambda$

To solve this equation we need to solve the Friedmann equation to find a(t) and insert this solution into the above equation.

# Perturbation growth (lecture 2&3)

Inside the horizon: $\mathrm{Recall}: \delta = \frac{\rho - \bar{\rho}}{\bar{\rho}}$

$$\delta \sim \mathrm{const} \quad \text{radiation dominated}$$

$$\delta \propto a \quad \text{matter dominated}$$

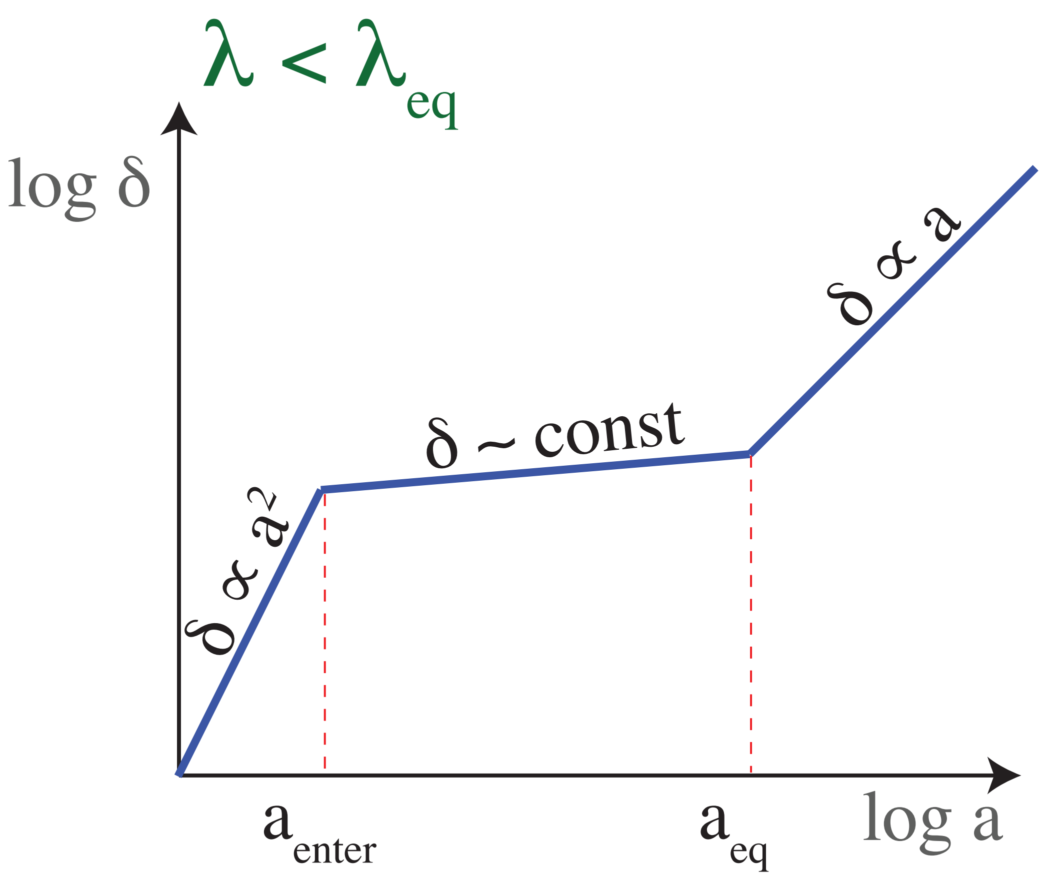
Outside the horizon:

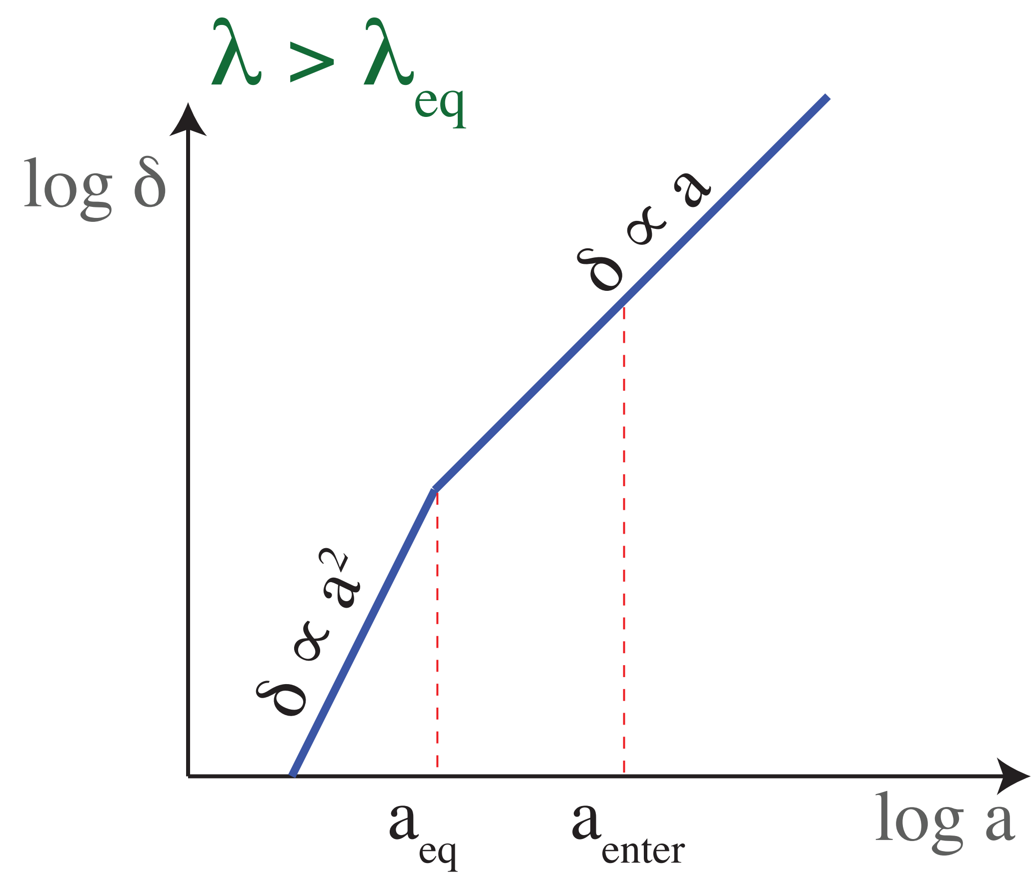
$$\delta \propto a^2 \quad \text{radiation dominated}$$

$$\delta \propto a \quad \text{matter dominated}$$

# Perturbation growth (lecture 2&3)

Inside the horizon: $\text{Recall}: \delta = \frac{\rho - \bar{\rho}}{\bar{\rho}}$

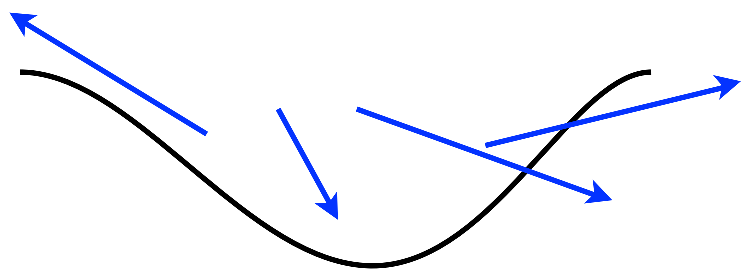



Note that we focus on dark matter perturbations because baryons fall into the dark matter perturbations soon after decoupling.

# Damping of perturbations

## Free streaming:



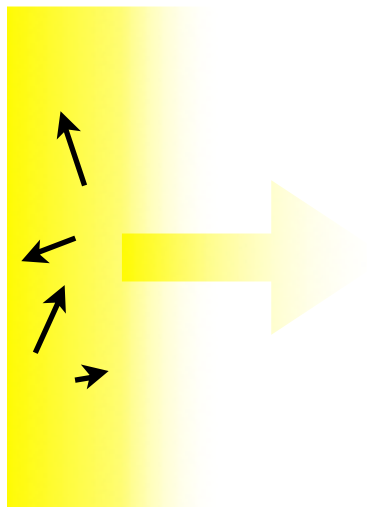
If particles can move, then they can stream out of the perturbations, washing them out.

$$\lambda_{\rm FS} = \int_0^t \frac{v(\tau)}{a(\tau)}\, d\tau.$$

While relativistic, v~c, after going non-relativistic, v~1/a(t).

$$\lambda_{\rm FS} = \frac{2ct_{\rm NR}}{a_{\rm NR}} \left[1 + \ln \frac{a_{\rm eq}}{a_{\rm NR}}\right],$$

# Damping of perturbations



## Silk damping:

Early in the Universe, photons and baryons are tightly coupled (through Thompson scattering) so move together. But later on, photons can start to diffuse relative to baryons, washing out perturbations, making it very difficult to create structure in a baryonic Universe.

$$\lambda_S^2 = \int_0^{t_{\mathrm{dec}}} \frac{c\, l_{\mathrm{mfp}}}{a^2}\, dt.$$

# Statistics of the density field (lecture 3-4)

By calculating statistics of the density field we can encapsulate a lot of its structure and since the initial density perturbations appear to be random, this is a natural approach

Expanding in Fourier modes:

$$\delta(\vec{x}) = \int \frac{d^3\vec{k}}{(2\pi)^3} \hat{\delta}(\vec{k})\, e^{-i\vec{k}\cdot\vec{x}}$$

We can define the power spectrum:

$$P(k) \propto \langle |\hat{\delta}(k)|^2 \rangle$$

and the power per logarithmic interval:

$$\Delta^2(k) = \frac{1}{2\pi^2} k^3 P(k)$$

And the correlation function is the Fourier transform of P(k):

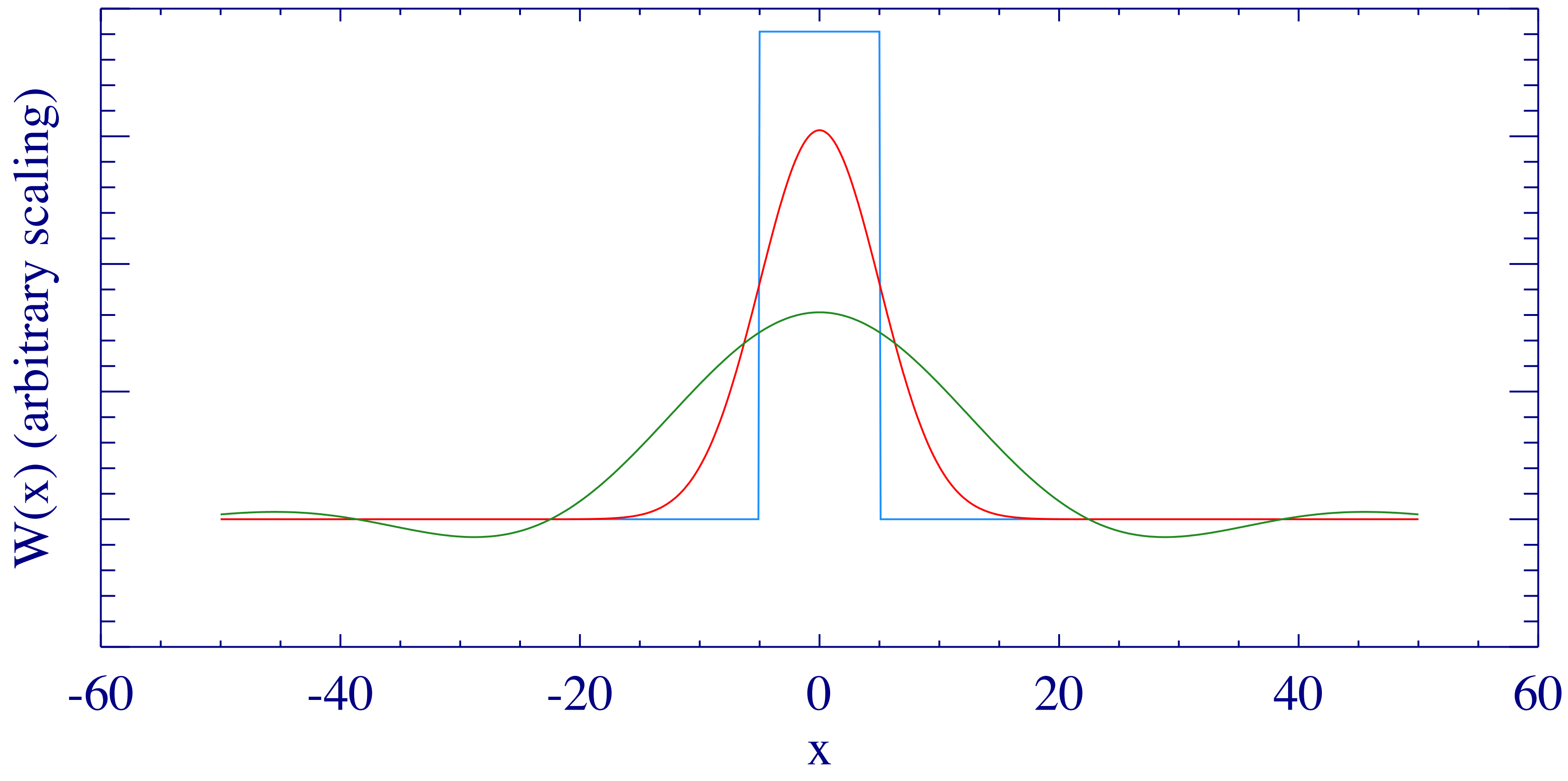
$$\xi(|\vec{x}_1 - \vec{x}_2|) = \xi(r) = \int \frac{d^3\vec{k}}{(2\pi)^3} P(k) e^{-i\vec{k}\cdot\vec{r}},$$

# Statistics of the density field - window functions

To calculate a density in practice, we need a volume:

**Window functions**

$$\delta(\vec{x};R) = \int \delta(\vec{x'})W(\vec{x}-\vec{x'};R)d\vec{x'}$$



# The Gaussian field - linear evolution

In the linear regime, each scale evolves independently

$$\frac{\partial^2 \delta_k}{\partial t^2} + 2\frac{\dot{a}}{a}\frac{\partial \delta_k}{\partial t} = 4\pi G \rho_b \delta_k - \frac{c_s^2}{a^2} k^2 \delta_k,$$

The early Universe appears to give rise to Gaussian perturbations and in that case we find that δ evolves as a Gaussian:

$$P\left(\delta(\vec{x}; R)\right) d\delta = \frac{1}{\sqrt{2\pi}\sigma(R)} e^{-\delta^2/2\sigma^2(R)} d\delta$$

with:

$$\sigma^2(R) = \int \Delta^2(k) \hat{W}(k; R)^2 \frac{dk}{k}$$

# The variance of the field.

$$\sigma^2(M) = \left\langle \left( \frac{M(\vec{x}, R) - \bar{M}}{\bar{M}} \right)^2 \right\rangle = \int \Delta^2(k) \hat{W}(k; R)^2 \frac{dk}{k}$$

For a power-law spectrum and sharp k-space filter:

$$\sigma^2(M) \propto k^{n+3} \propto M^{-(n+3)/3}$$

Also useful (during matter domination):

$$\delta \propto a \propto t^{2/3}$$

$$\xi \propto P(k) \propto \delta^2 \propto a^2 \propto t^{4/3}$$

$$\sigma(M) \propto a \propto t^{2/3}$$

# Statistics of the density field - evolution

If we assume the initial power spectrum is $P(t_i) \propto k^n$

$$P(k;t) = A \begin{cases} 0 & \lambda < \lambda_{\rm FS} \\ k^{n-4} & \lambda_{\rm FS} < \lambda < \lambda_{\rm eq} \\ k^n & \lambda > \lambda_{\rm eq} \end{cases}$$

Note that the epoch of matter-radiation equality introduces a characteristic scale in P(k).

$$\sigma(M)^2 \propto \begin{cases} 0 & M < M_{\rm FS} \\ M^{-(n-1)/3} & M_{\rm FS} < M < M_{\rm eq} \\ M^{-(n+3)/3} & M > M_{\rm eq} \end{cases}$$

# When do structures go non-linear? (Lecture 4)

$$\sigma(M) \sim 1 \quad \Leftrightarrow \quad \delta \sim 1$$

Using that $\delta \propto a$ in the matter dominated epoch:

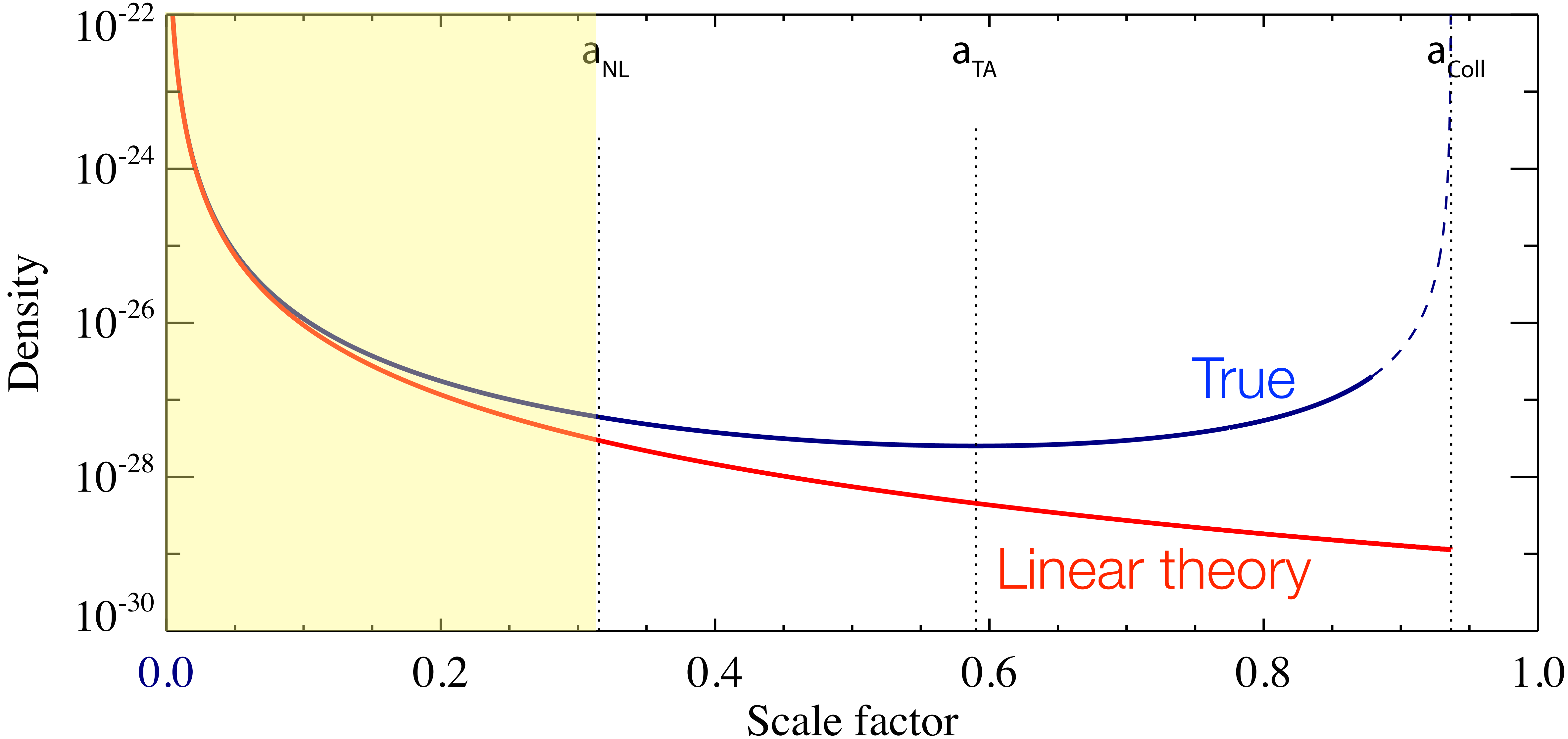
$$\sigma_M(t_{\rm NL}) = \sigma_M(t_i)\left(\frac{t_{\rm NL}}{t_i}\right)^{2/3} = 1$$

$$\Downarrow$$

$$t_{\rm NL} \propto \sigma_M(t_i)^{-3/2}$$

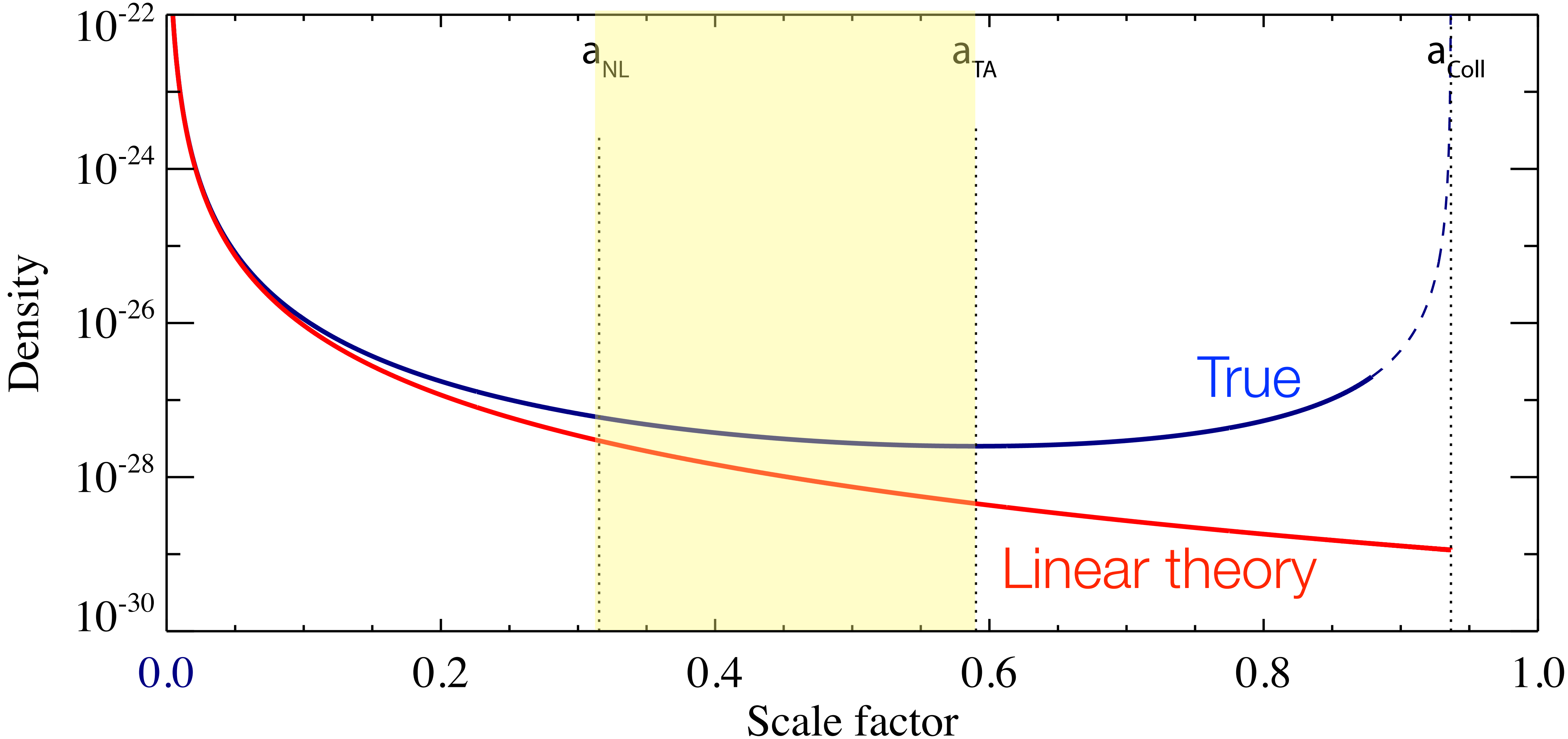
$$t_{\rm NL} \propto M^{(n+3)/4}.$$

# Spherical collapse (lecture 5 & problem set)



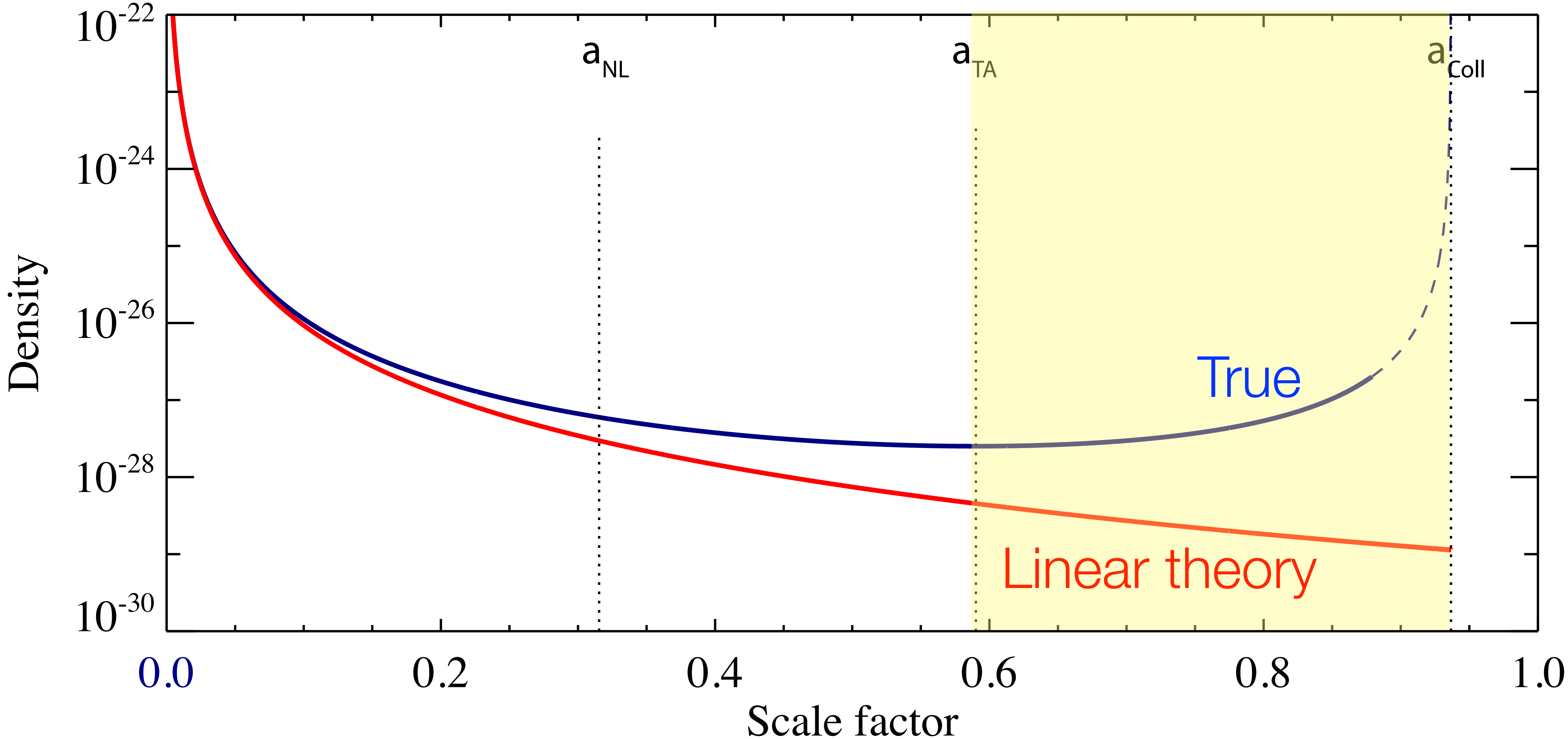

Initial growth follows the general expansion of the Universe

# Spherical collapse




After going non-linear the expansion of the perturbation slows down until the time of turn-around when it stops.

# Spherical collapse




The perturbation then collapses and formally reaches infinite density at $t \sim 2\, t_{TA}$ when the virial theorem says the radius $r \sim r_{TA}/2$

# Key numbers from the spherical collapse model

At turn-around we have (flat Universe):

$$\rho_{\mathrm{TA}} = \frac{9\pi^2}{16}\bar{\rho}(t_{\mathrm{TA}}) \approx 5.5\bar{\rho}(t_{\mathrm{TA}})$$

(Exact)

$$\delta_{\mathrm{Lin,TA}} \approx 1.067$$

(linear theory)

At collapse we have (flat Universe):

$$\rho_{\mathrm{coll}} = 18\pi^2\bar{\rho}(t_{\mathrm{coll}}) \approx 178\bar{\rho}(t_{\mathrm{coll}})$$

$$\delta_{\mathrm{Lin,coll}} \approx 1.69$$

Key point: This allows us to use linear theory (=easy) to estimate when a structure has collapsed (=virialised). Very useful!

$$\delta(\vec{x}, t) > \delta_c \approx 1.69 \quad \Leftrightarrow \quad \delta_0(\vec{x}) > \frac{\delta_c}{D(t)} \stackrel{\mathrm{def}}{=} \delta_c(t)$$

# Redshift space distortions (lecture 5)

Real space    Redshift space

$$n(\vec{r})d^3\vec{r} = n^{(s)}(\vec{s})d^3\vec{s}$$

Jacobian: $d^3\vec{s} = \left(1 + \frac{v_r}{Hr}\right)^2 \left(1 + \frac{1}{H}\frac{\partial v_r}{\partial r}\right) d^3\vec{r}$

Linear continuity equation

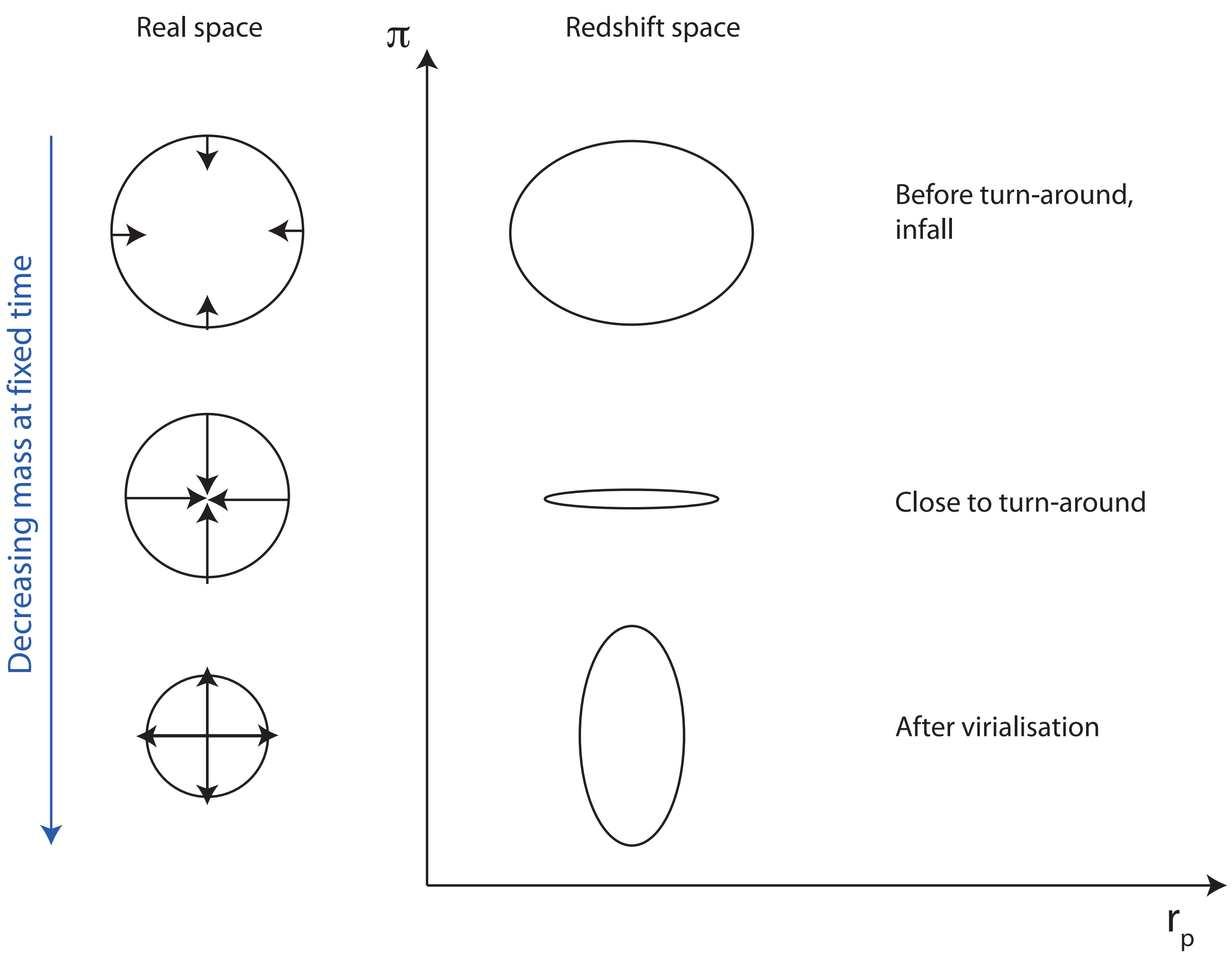
Cos angle to line of sight

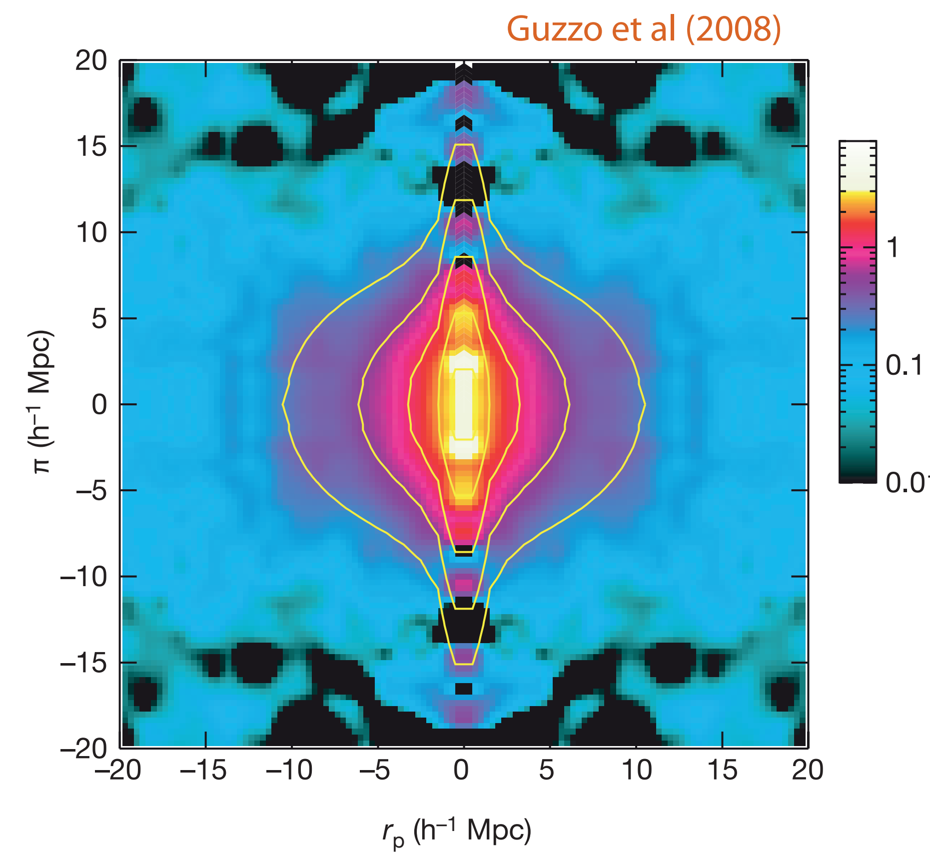
$$\delta^{(s)}_{\vec{k}} = \delta_{\vec{k}}(1 + f(\Omega_m)\mu^2)$$

$\approx \Omega_m^{0.6}$

In the linear regime (large scales)

# Redshift space distortions (lecture 5)



# Mass-functions (lecture 5)

## The Press-Schechter ansatz:

*The fraction of mass elements in halos with mass >M is equal to the probability of having $\delta_S > \delta_c(t)$: $P(\delta_S > \delta_c(t))$, where $\delta_S$ is the density field smoothed on scale M.*

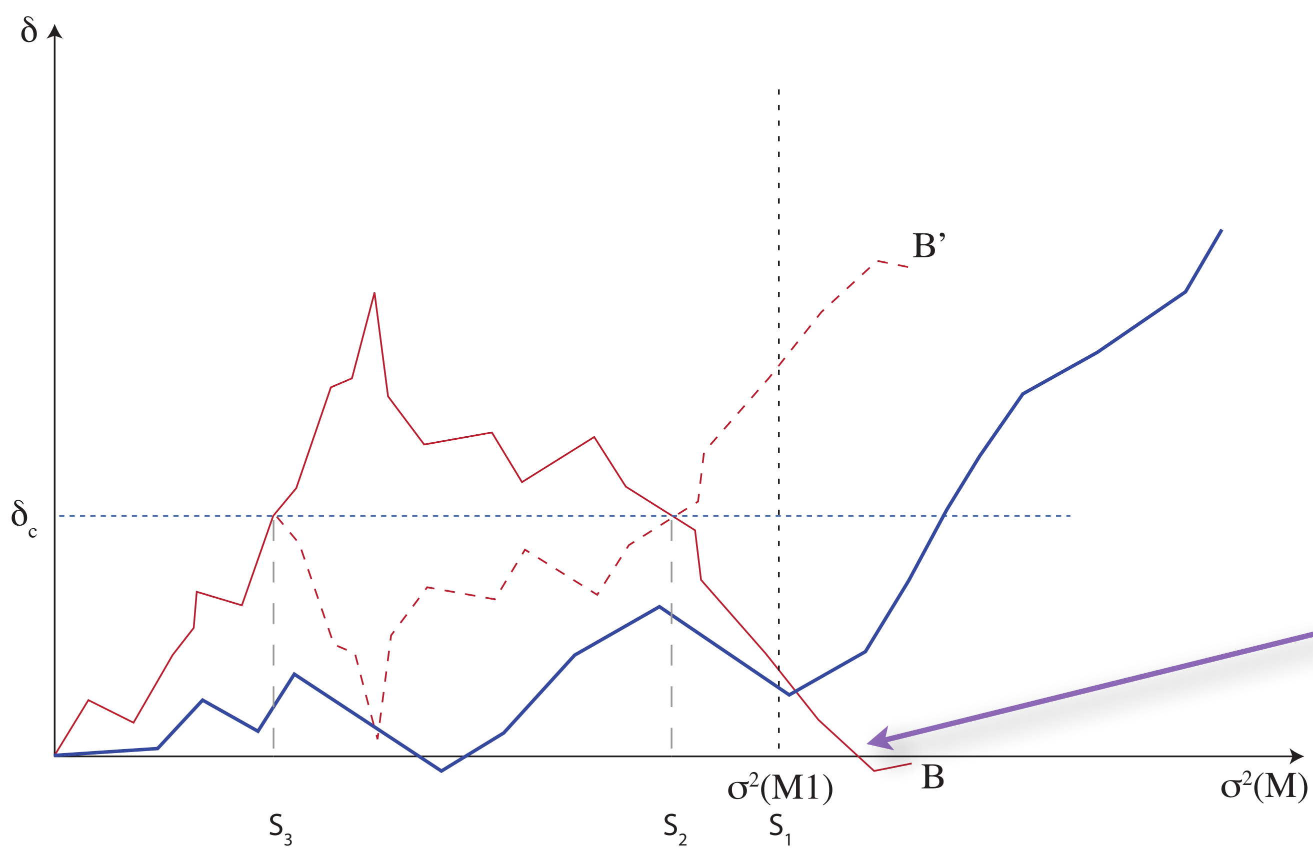
$$P(\delta_S > \delta_c(t)) = \frac{1}{\sqrt{2\pi}\sigma(M)} \int_{\delta_c(t)}^{\infty} e^{-\delta_S^2/2\sigma^2(M)} d\delta$$
$$= \frac{1}{2}\,\mathrm{erfc}\left[\frac{\delta_c(t)}{\sqrt{2}\sigma(M)}\right],$$

# Mass-functions (lecture 5 & 6)

From the P-S ansatz we find the number density function of dark halos through:

$$F(>M) = 2P[\delta_S > \delta_c(t)].$$

Where the factor 2 comes because we need to include cases where a system might be underdense on a scale S, but overdense on a scale > S.




We must not forget these

# Mass-functions (lecture 5 & 6)

We then get:

$$n(M,t)dM = \frac{\bar{\rho}}{M}\frac{\partial F(>M)}{\partial M}dM$$

$$n(M,t)dM = \sqrt{\frac{2}{\pi}}\frac{\bar{\rho}}{M^2}\frac{\delta_c(t)}{\sigma(M)}e^{-\delta_c^2(t)/2\sigma^2(M)}\left|\frac{d\ln\sigma(M)}{d\ln M}\right|dM$$

This is in pretty good overall agreement with numerical simulations (although you need to make some corrections to do really well), but the resulting function has some puzzling features:

# Mass-functions (lecture 5)

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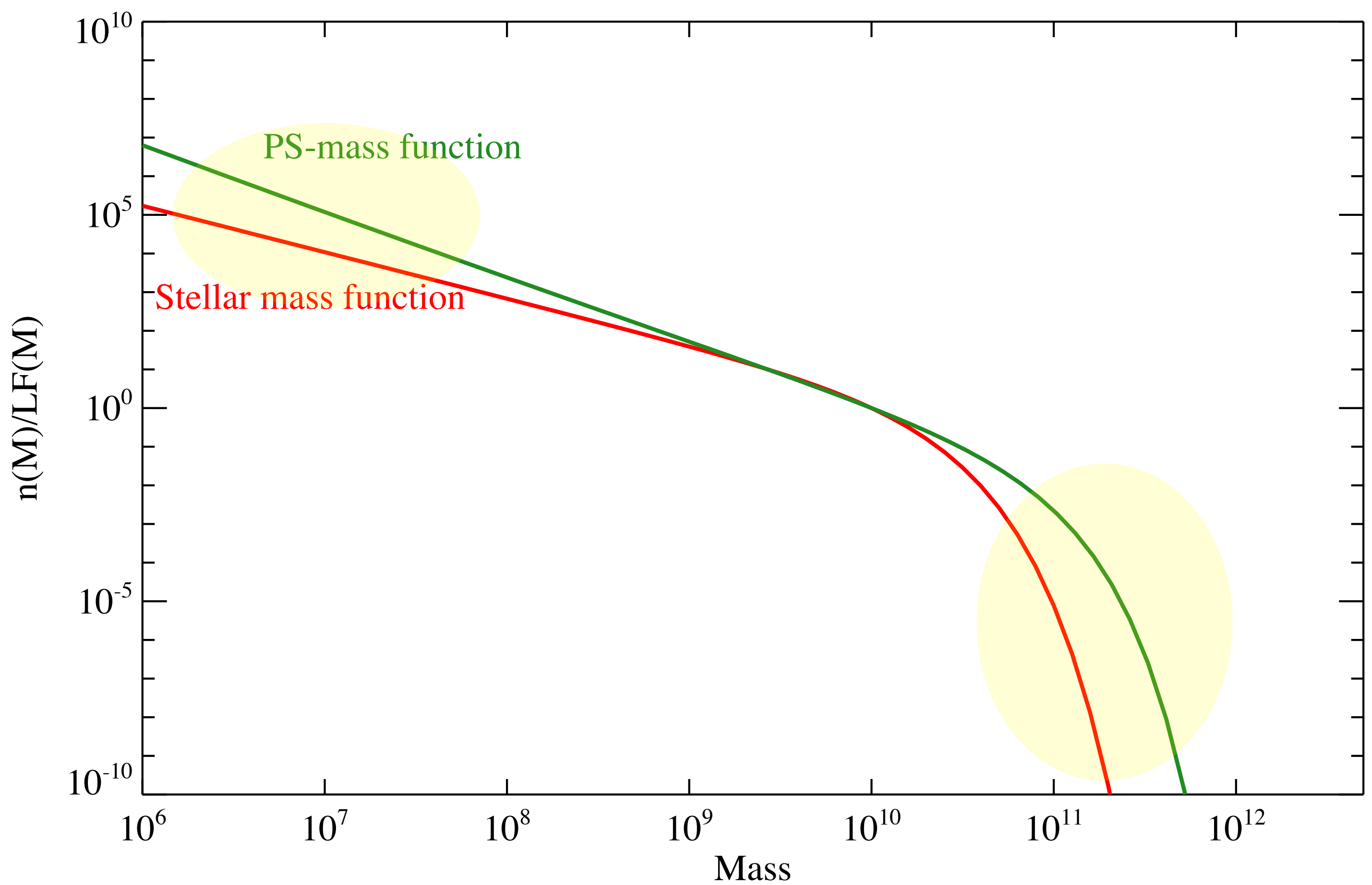



Feedback?

# The Navarro-Frenk-White profile (Lecture 6)

Numerical simulations indicate that dark matter profiles have a near **universal shape**. The most popular fitting function is the NFW profile:

$$\rho(r) = \rho_{\rm crit} \frac{\delta_{\rm char}}{(r/r_s)(1 + r/r_s)^2}$$

$$\delta_{\rm char} = \frac{\Delta_c}{3} \frac{c^3}{\ln(1+c) - c/(1+c)}$$

$$c = r_{\rm vir}/r_s$$ (the concentration parameter)

# Virial relations (lecture 6)

Isothermal sphere: $\rho(r) = \frac{V_c^2}{4\pi G r^2}$

$$\rho_{\rm collapse} = 18\pi^2\bar{\rho} \equiv \Delta_c\bar{\rho} \qquad \delta_{\rm lin} = 1.69,$$

This can be used to find the virial radius:

$$\rho(z) \quad \frac{1}{\bar{\rho}(z)} = \Delta_c \qquad \bar{\rho}(z) = \Omega_m \rho_{\rm crit}(z)$$

$$\frac{3M}{4\pi r_{\rm vir}^3} \quad \frac{8\pi G}{\Omega_m 3H(z)^2} = \Delta_c$$

Which gives:

$$r_{\rm vir} = \left(\frac{2G}{H_0^2\Delta_c}\right)^{1/3} M^{1/3}(1+z)^{-1}\Omega_m^{-1/3}$$

(note that there is a lot of variation as to what people adopt for $\Delta_c$)

# Virial temperature & velocity

The virial theorem:

$$\frac{1}{2}\frac{d^2 I}{dt^2} = 2K + W + \Sigma$$

$N_{\mathrm{part}}$

$$K = \frac{3}{2}\frac{k_B T M_{\mathrm{gas}}}{\mu m_P} \qquad W = f_S \frac{G M M_{\mathrm{gas}}}{r_{\mathrm{vir}}}$$

Ignoring the surface pressure and change in inertial moment:

$$T_{\mathrm{vir}} = \frac{f_S}{3}\frac{G M \mu m_P}{r_{\mathrm{vir}} k_B}$$

And the circular velocity is defined through:

$$V_c^2 = \frac{GM}{r_{\mathrm{vir}}}$$

# Virial radii & dark matter halos

What happens when baryonic matter falls into a dark matter halo?

Simplistically, the matter will be shocked at the virial radius or near it to a temperature close to the virial temperature of the halo.

It will then try to cool, and if it can do so efficiently, it will start to collapse towards the center of the halo. It will probably mostly conserve its specific angular momentum leading to the formation of a disk.

# Time-scales of collapse (lecture 8)

Expansion time of the Universe:

$$t_H \sim H(z)^{-1}$$

Free-fall time-scale:

$$t_{\rm dyn} \sim (G\rho)^{-1/2}$$

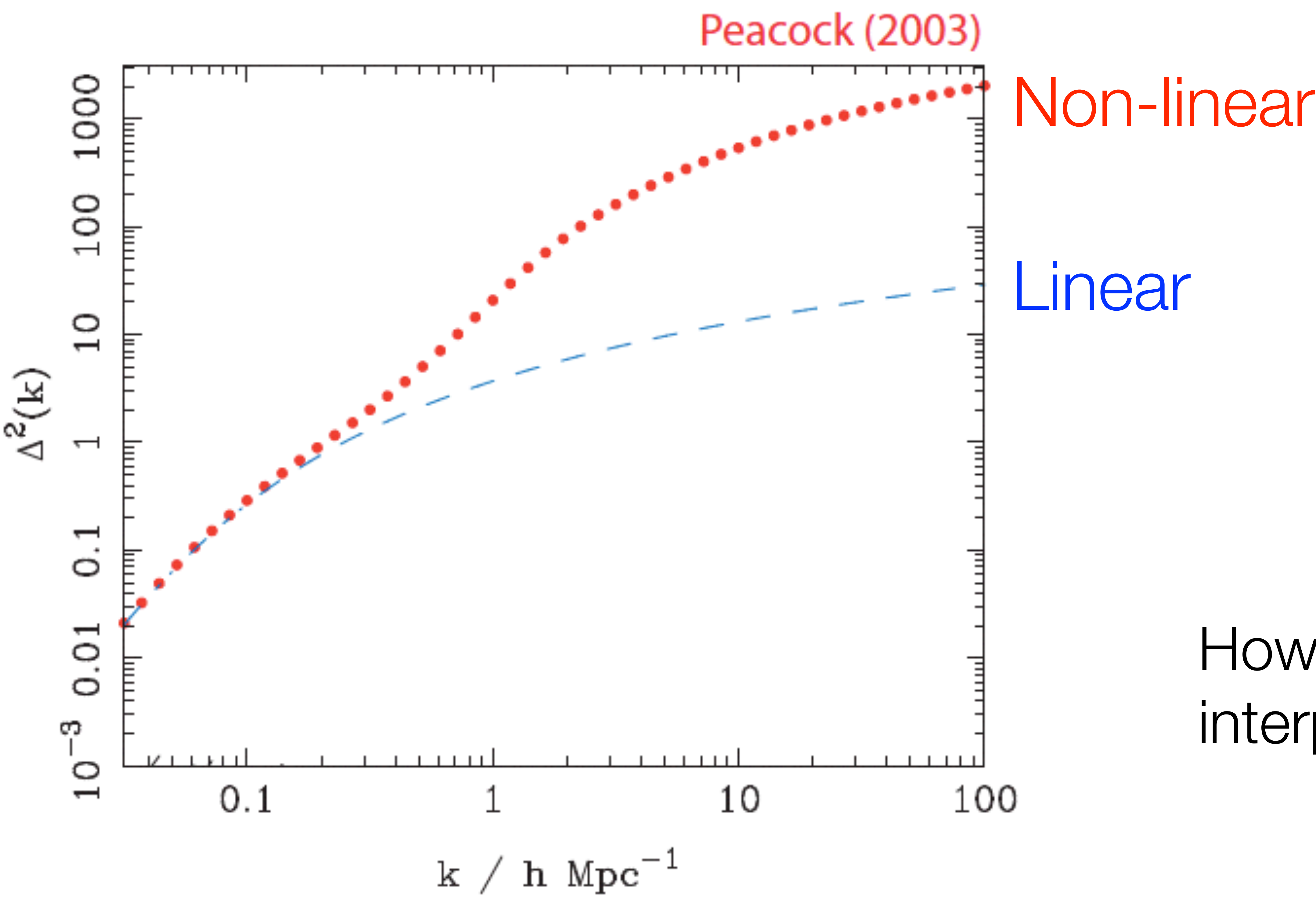
$$t_{\rm dyn} = \left(\frac{3\pi}{32G\rho}\right)^{1/2}$$

Cooling time:

$$t_{\rm cool} = \frac{E}{\dot{E}} = \frac{E}{n_H^2\Lambda} = \frac{\frac{3}{2}nkT}{\left(\frac{4}{9}\right)^2 n^2\Lambda}$$

Fully ionized monatomic gas with X=3/4 Y=1/4

# The halo-model




How can we interpret this?

# The halo-model

Simplification: All (dark) matter is in virialised halos

$$\rho_h(\vec{x}) = M_h \, u(\vec{x}|M_h)$$

Universal density profile

The correlation function/P(k) can then be written:

$$\xi(r) = \xi^{1h}(r) + \xi^{2h}(r)$$

Within halo

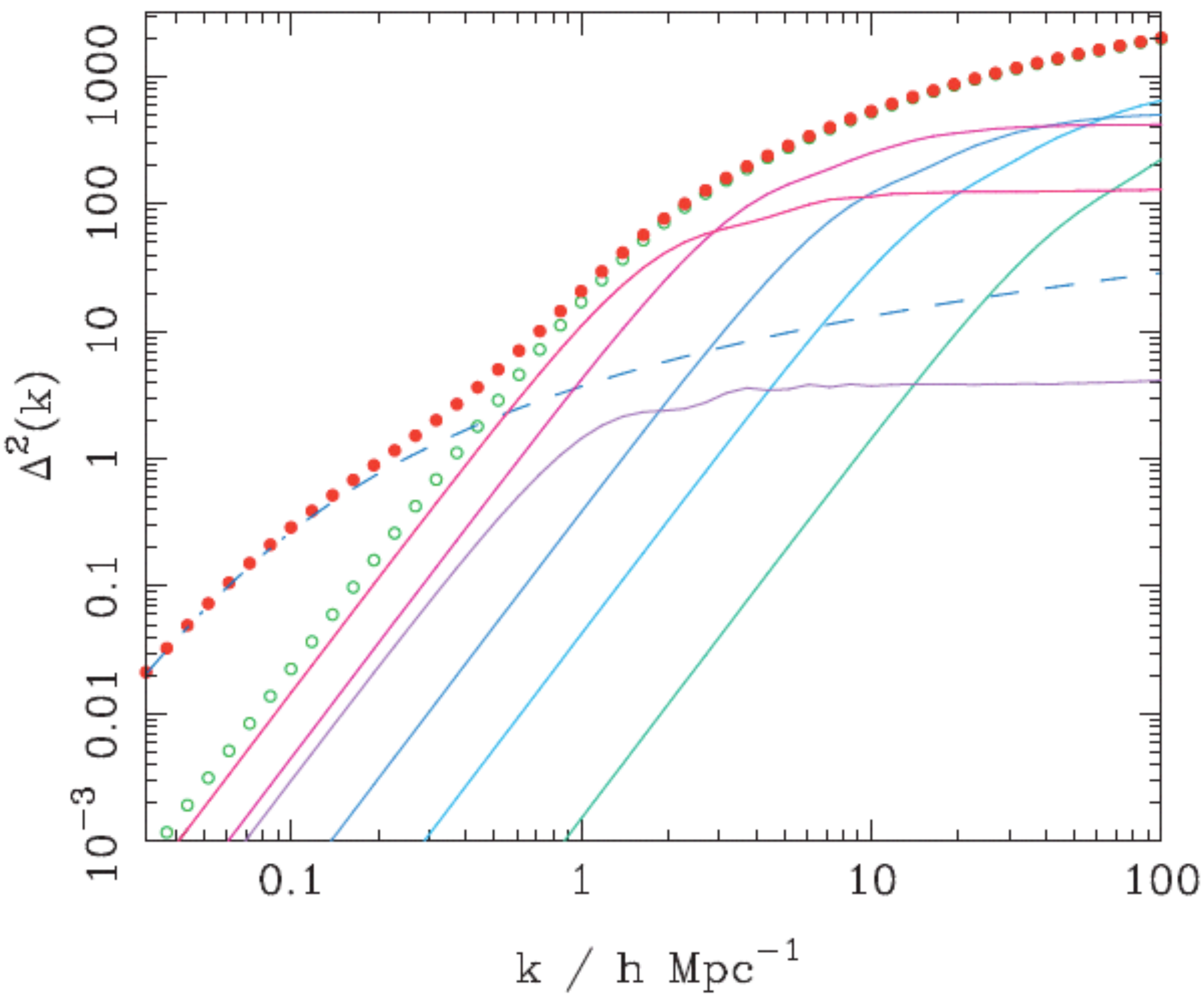
Between haloes

And: $$\xi^{2h}(r) \approx \xi^{\rm lin}_{mm}(r)$$

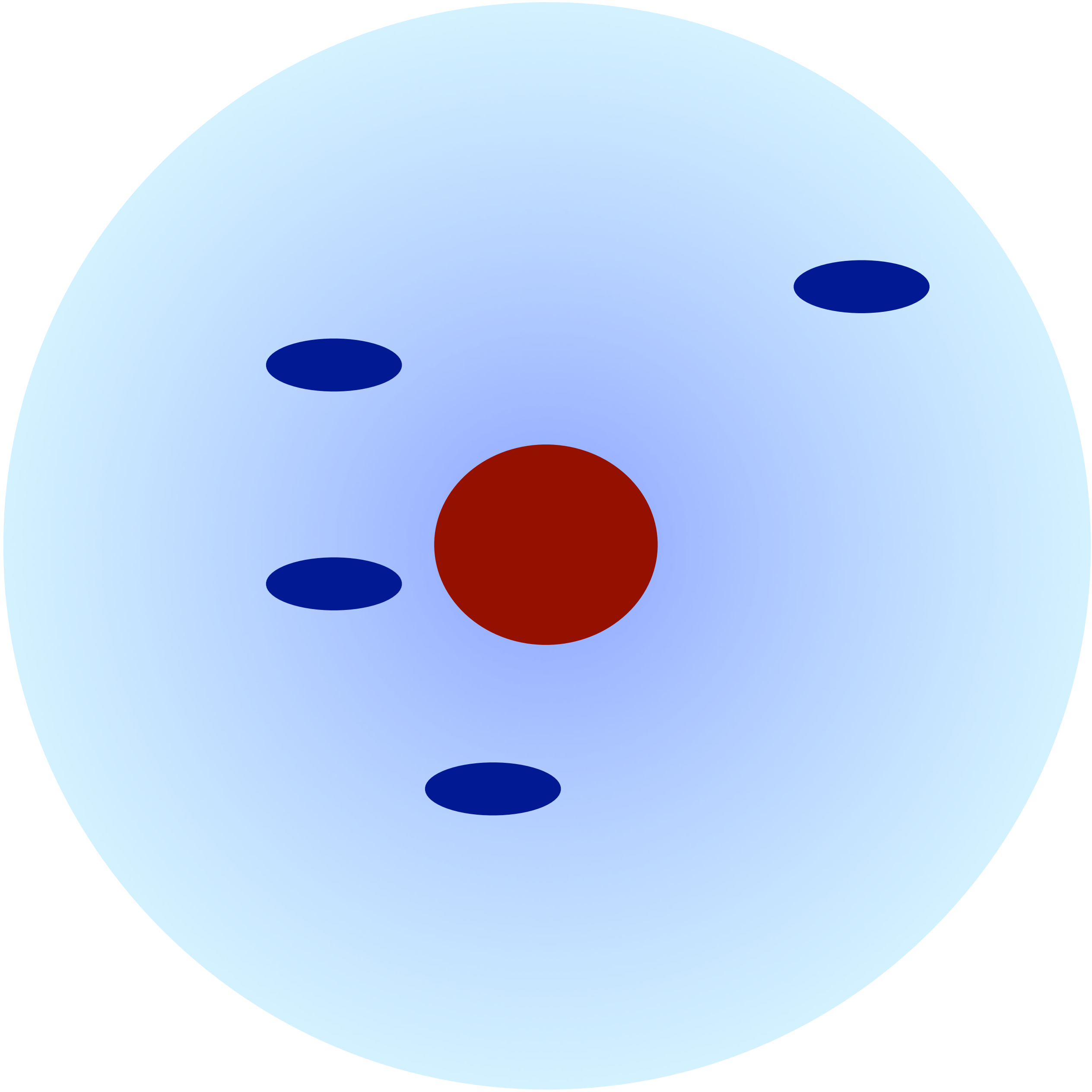
# The halo-model

$$\xi(r) = \xi^{1h}(r) + \xi^{2h}(r) \quad \& \quad \xi^{2h}(r) \approx \xi^{\mathrm{lin}}_{mm}(r)$$

Peacock (2003)



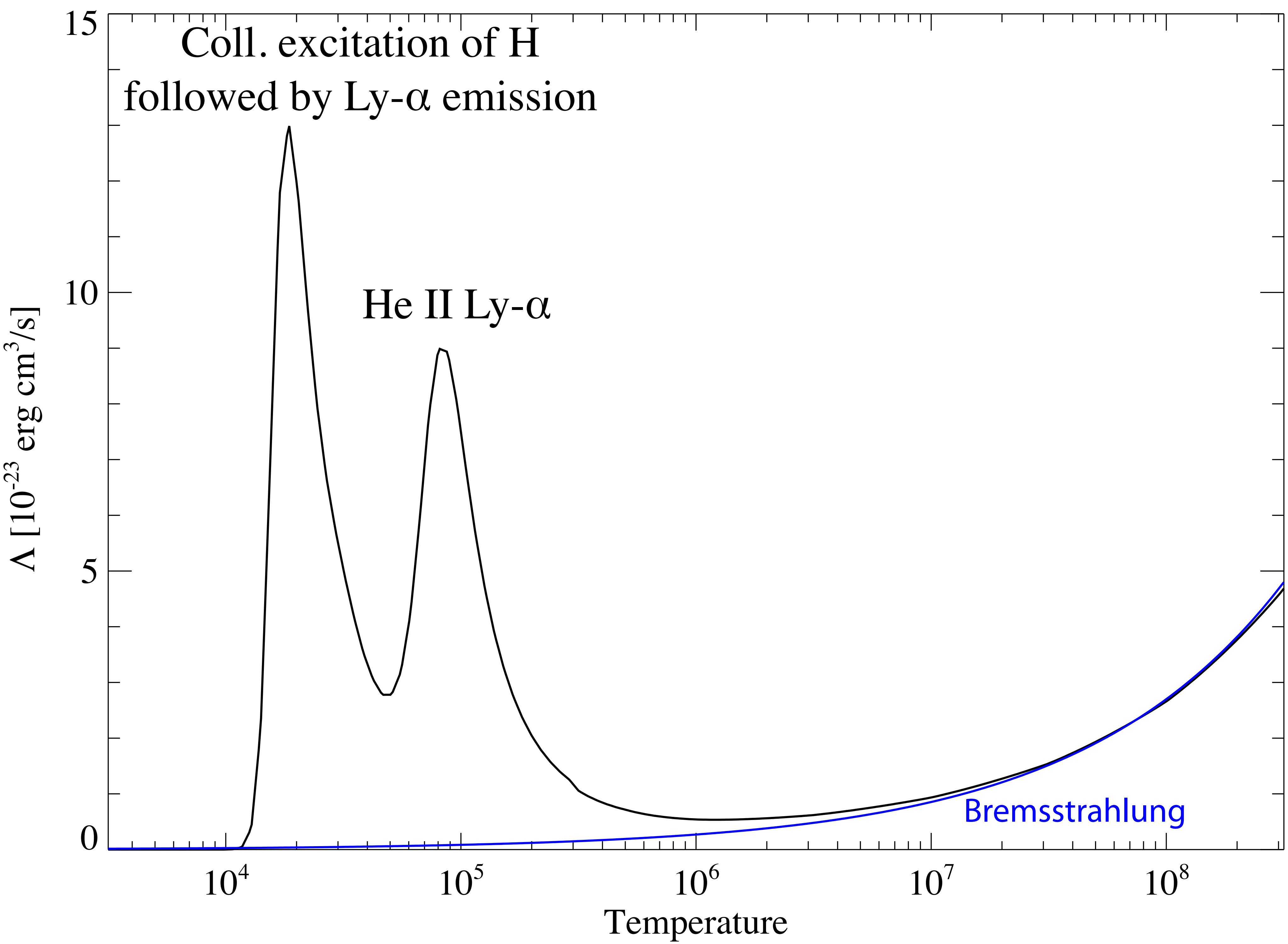
# Satellites & centrals



Simulations and observations: Useful to split galaxy population into centrals and satellites.

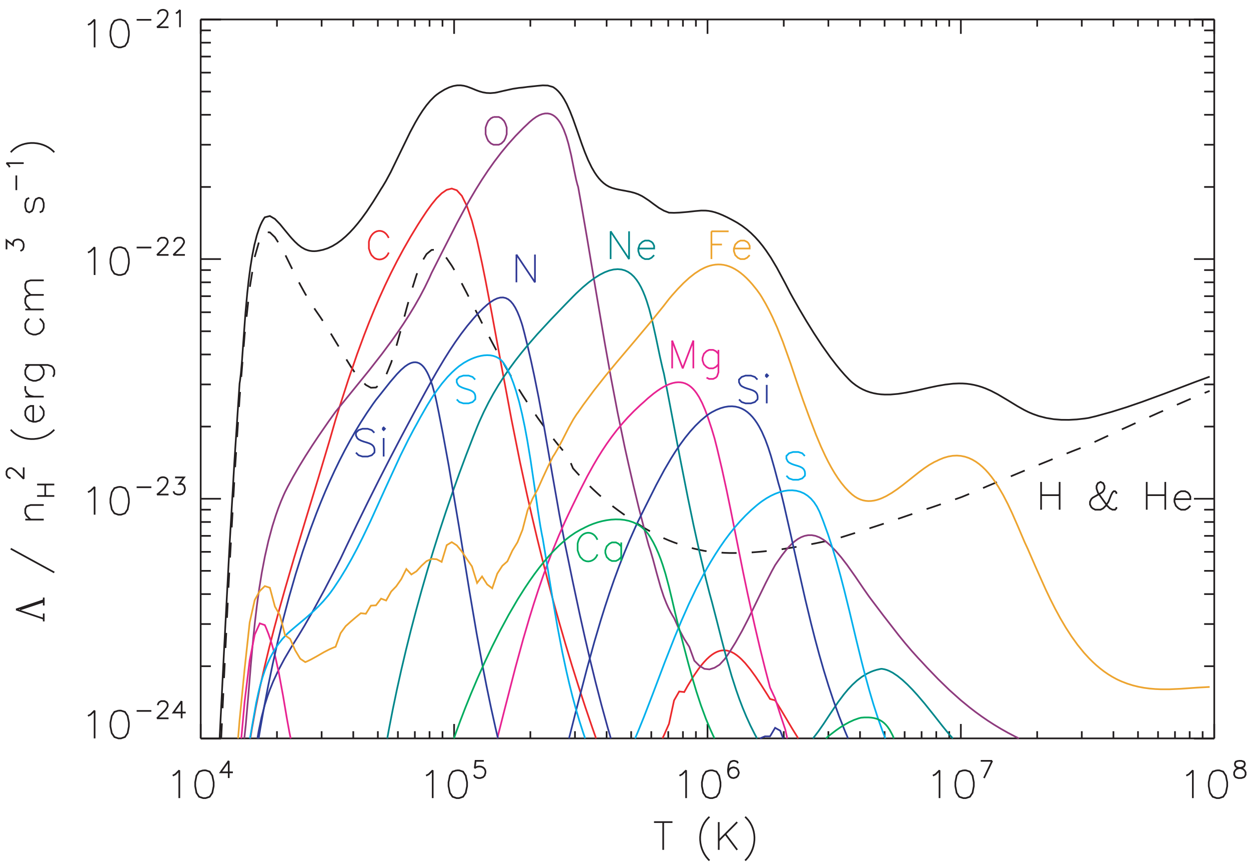
Satellites are affected by the halo of the central:
- Stripping
- Strangulation
- Harassment
etc.

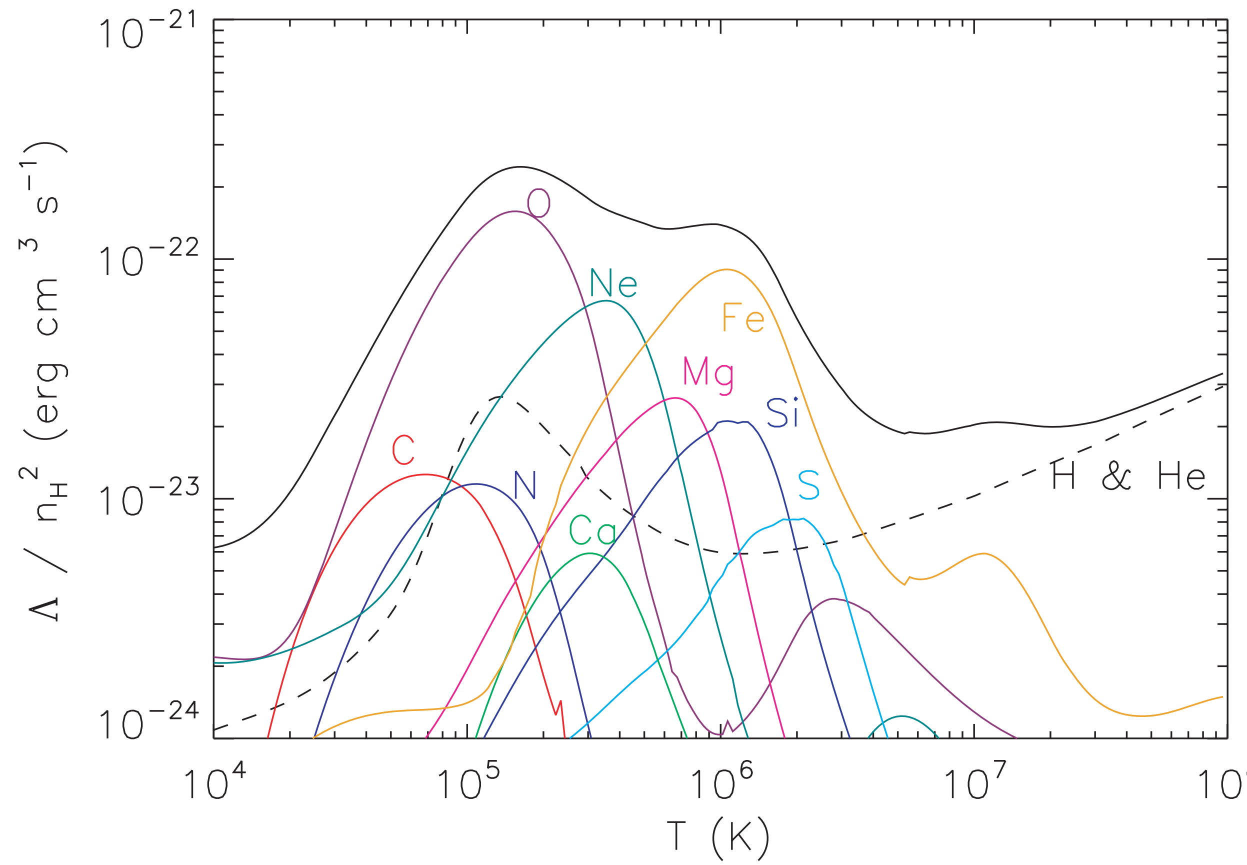
# Cooling function for Z=0 (lecture 8 & 9)



# Cooling functions - Z>0 & photo-ionization (lecture 8,9)

Wiersma, Schaye & Smith 2009, MNRAS, 393, 99




**Z>0**: More cooling because more bound levels, and extending to higher temperature because (some) bound levels are more tightly bound.

**Photo-ionization**: Removes bound levels: Reduces cooling, particularly at T ~ $10^4$ K, but might increase it at lower temperatures by leaving free electrons.

# Time-scales of collapse

$t_{\rm cool} > t_H$

Cannot cool efficiently. No collapse

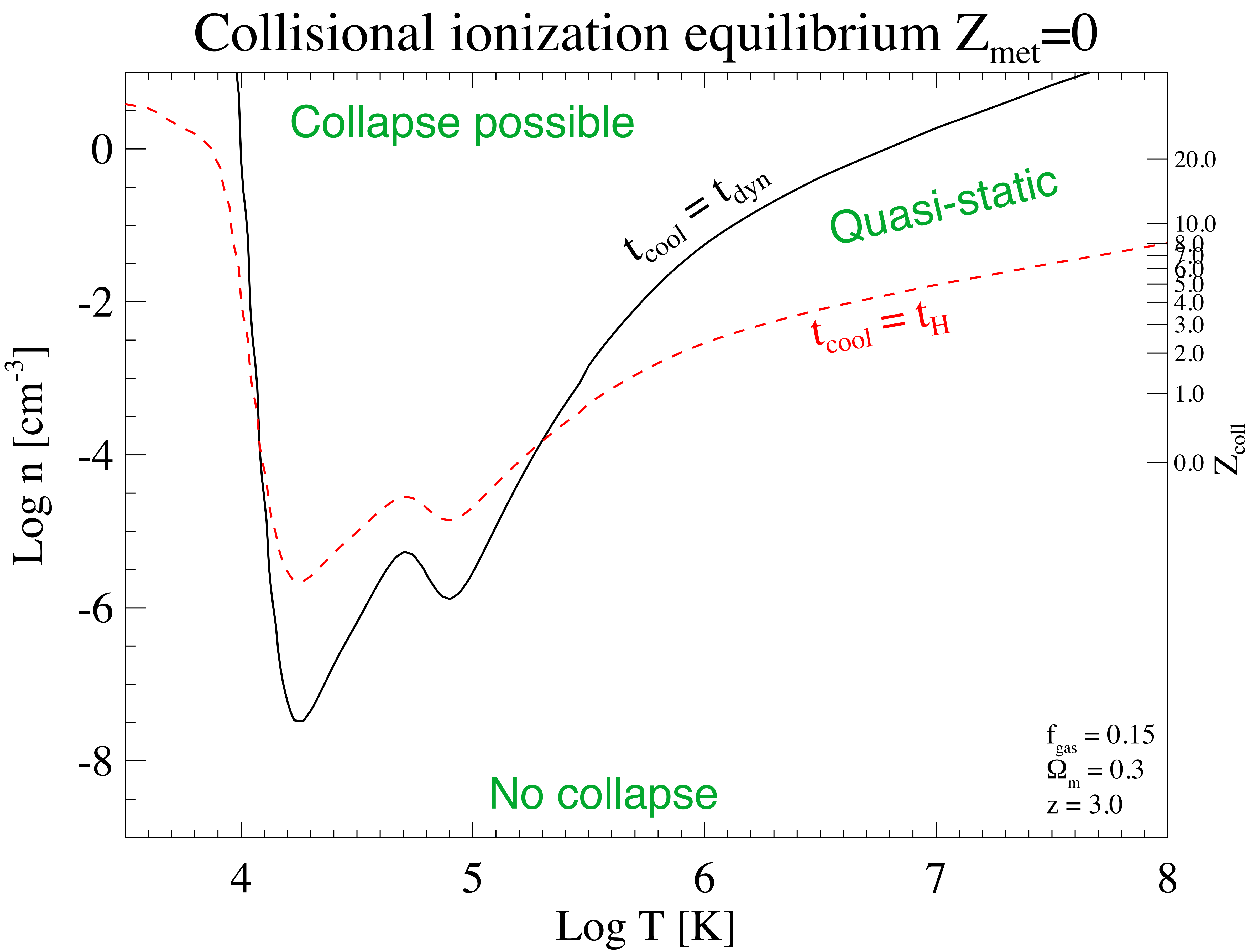
$t_{\rm dyn} < t_{\rm cool} < t_H$

Slow collapse set by cooling time, system moving along tracks of constant Jeans mass

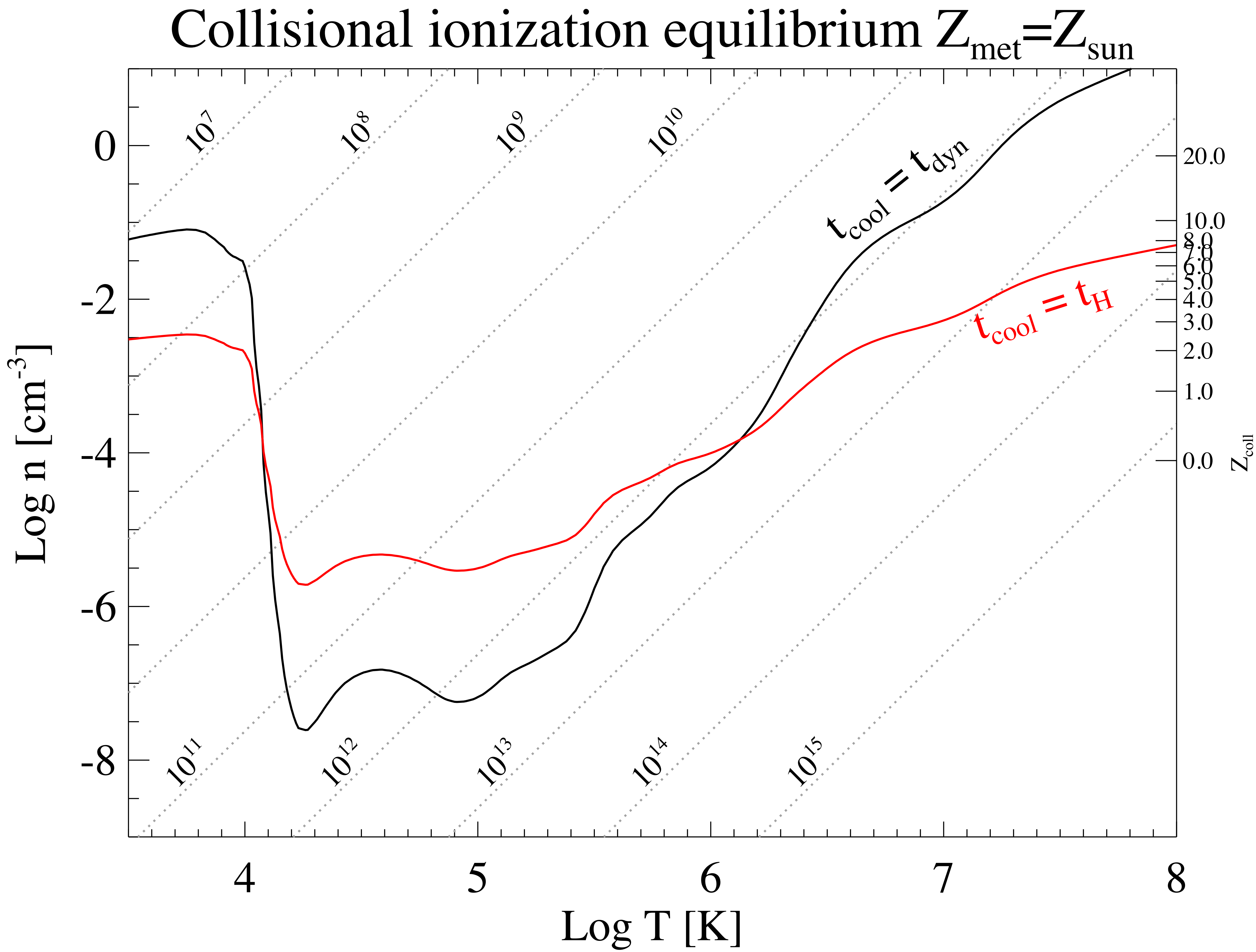
$t_{\rm cool} < t_{\rm dyn}$

Efficient collapse on free-fall time-scale

$t_H \sim H(z)^{-1}$ $\quad$ $t_{\rm dyn} \sim (G\rho)^{-1/2}$ $\quad$ $t_{\rm cool} = \dfrac{E}{n_H^2 \Lambda}$



Collisional ionization equilibrium $Z_{met}$=0
Collapse possible
Quasi-static
No collapse
$t_{cool} = t_{dyn}$
$t_{cool} = t_H$
Log n [$cm^{-3}$]
Log T [K]
$Z_{coll}$
0
-2
-4
-6
-8
4
5
6
7
8
20.0
10.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
$f_{gas}$ = 0.15
$\Omega_m$ = 0.3
z = 3.0



Collisional ionization equilibrium $Z_{met}=Z_{sun}$
Log n [cm$^{-3}$]
Log T [K]
$t_{cool} = t_{dyn}$
$t_{cool} = t_H$
$10^7$
$10^8$
$10^9$
$10^{10}$
$10^{11}$
$10^{12}$
$10^{13}$
$10^{14}$
$10^{15}$
$z_{coll}$
20.0
10.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
0
-2
-4
-6
-8
4
5
6
7
8

# Inverse Compton cooling (lecture 9)

$$\frac{du_\gamma}{dt} = \frac{4k_B}{m_e c}\sigma_T n_e u_\gamma (T_e - T_\gamma)$$

Energy loss/gain from inverse Compton scattering off CMB photons

$$t_{\mathrm{cool}} = \frac{E}{\dot{E}} \propto \frac{kT_e n_e}{n_e T_e (1+z)^4} \propto (1+z)^{-4}$$

Most cooling processes are sensitive to the density squared so the time scale  depends on the density, but the inverse Compton cooling time-scale is independent of density (as long as there are electrons present!)

This therefore dominates in the low-density gas and (because of the redshift dependence) at high redshift.

# Cooling flows (lecture 9)

Galaxy clusters often show cooling flows. This is because

$$t_{\rm cool} = \frac{\frac{3}{2} n(r)\, k_B T(r)}{n_H(r)^2 \Lambda[T(r)]}$$

Typically decreases towards the center (higher density) and hence there is a drop in pressure and above lying gas will drop to the center.

Why then is there not enormous amounts of star formation in the center of galaxy clusters?

One possible answer: Feedback from AGN.

# Getting gas into halos

Starting with the extended Press-Schechter the mass growth of halos can be written as (where q must be calibrated from simulations):

$$\frac{d\ln M_h}{d\omega} = -\left(\frac{2}{\pi}\right)^{1/2}\left[\sigma^2(M_h/q) - \sigma^2(M_h)\right]^{-1/2}$$

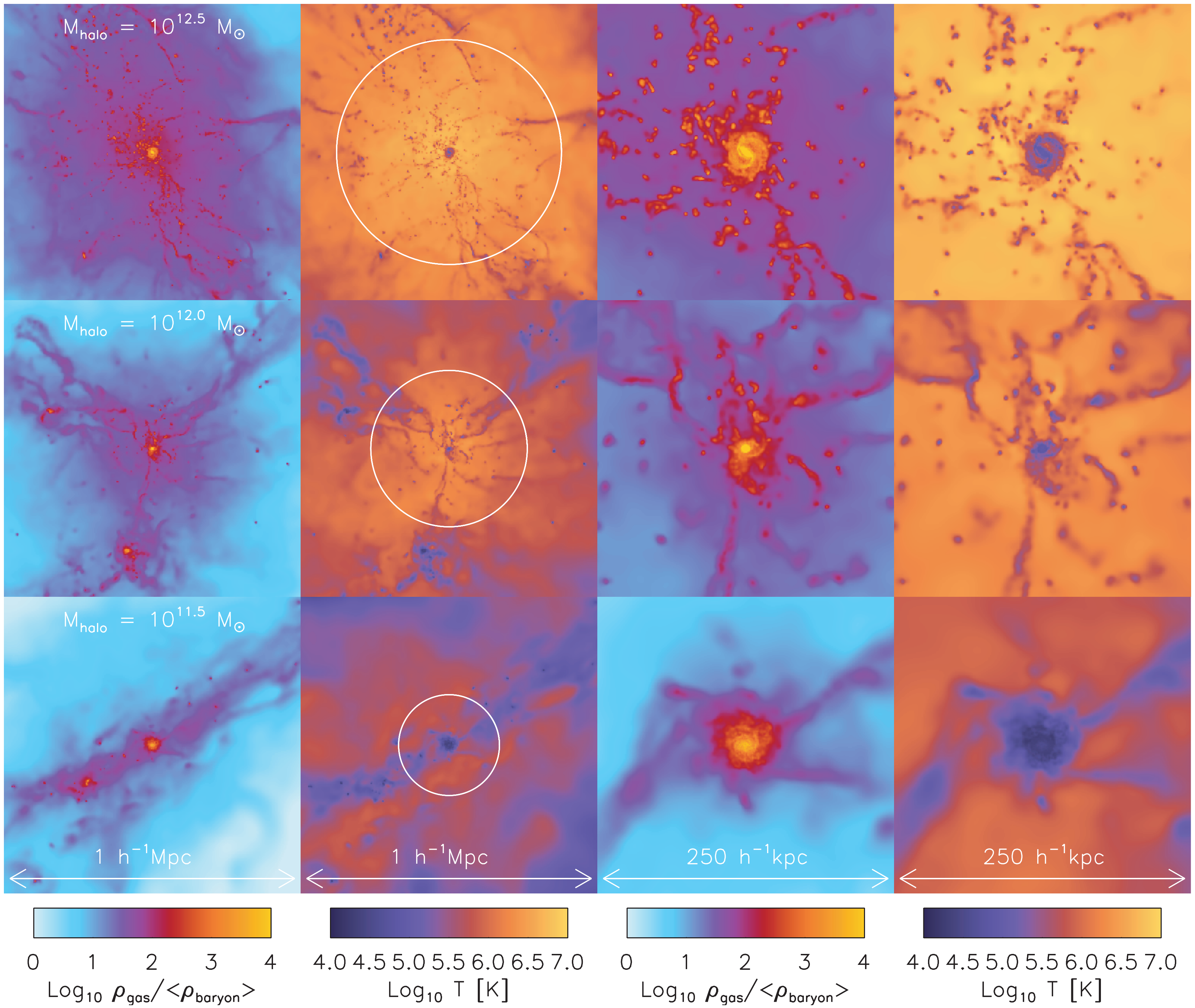
This gave us a growth rate of halos:

$$\frac{dM_h}{dt} \approx 230\left(\frac{M_h}{10^{12}h^{-1}M_\odot}\right)^{1.1}\left(\frac{\sigma_8}{0.8}\right)^{-1}(1+z)^{5/2}\ M_\odot/\mathrm{yr}$$

which is in good agreement with simulations. But this does not give us the accretion rate onto galaxies - at least without modifications!

This accretion can be approximately assumed to be spherical when considering a sample of halos. And when it comes into the halo a shock may form

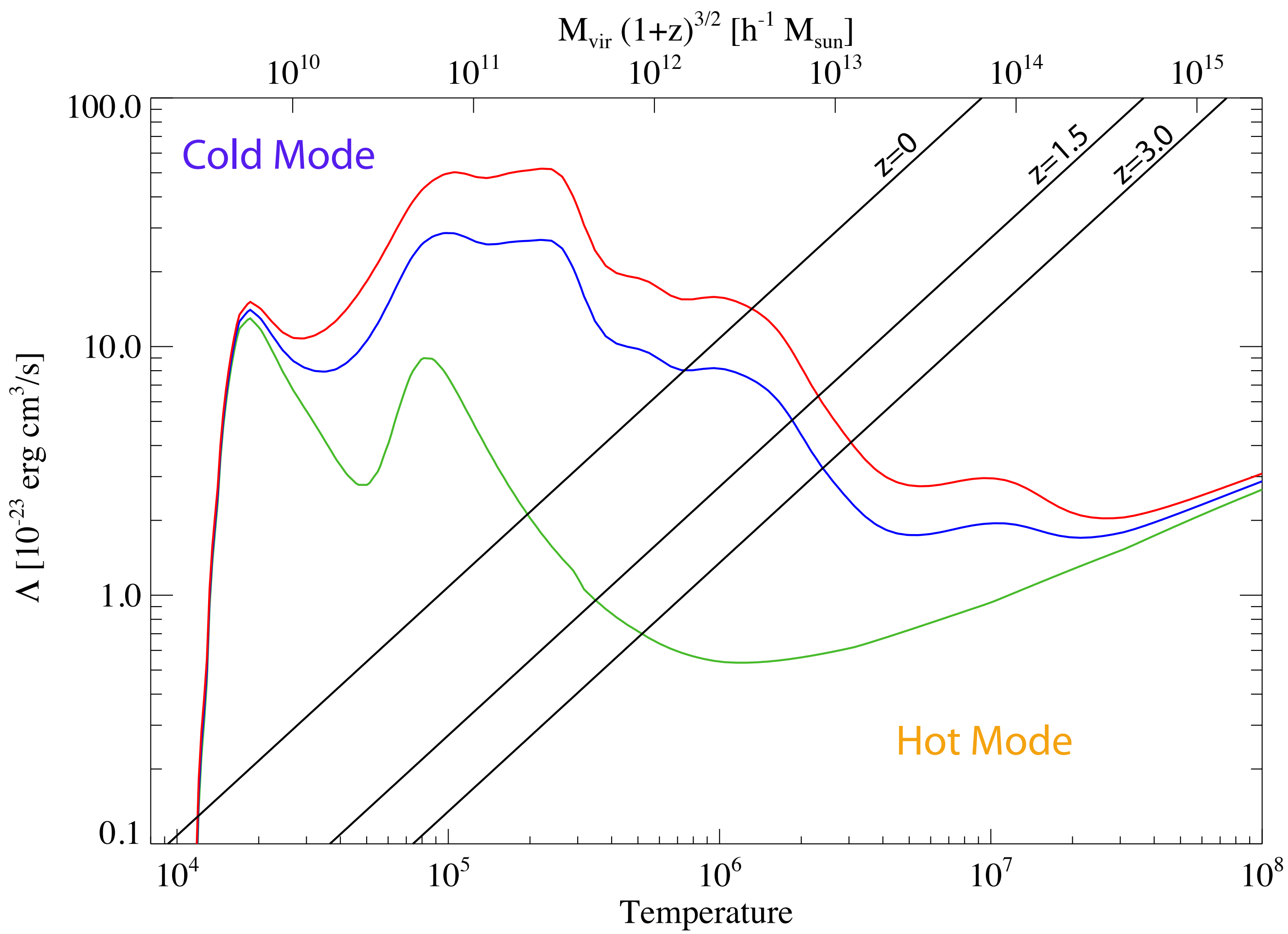
# Cold & Hot accretion (Lecture 11)



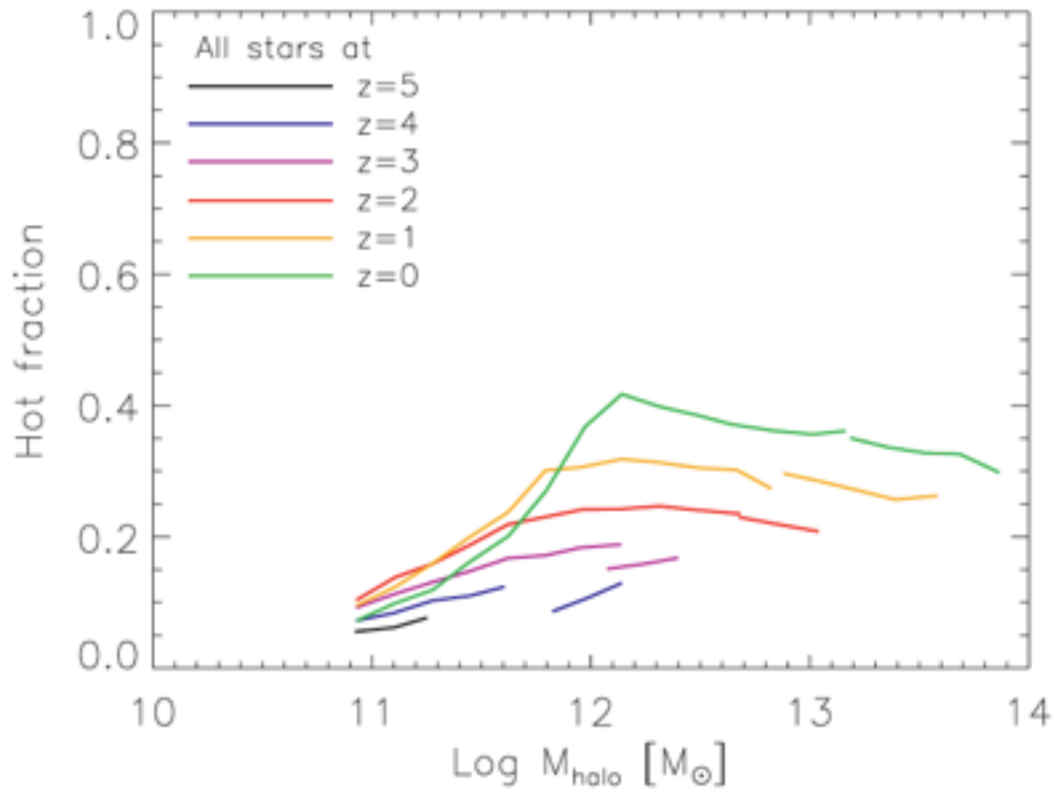

High M: Virial shock

Low M: No stable shock & cold streams enter halo.

# Cold & Hot accretion (Lecture 11)




vd Voort et al 2012



# Feedback - from stars:

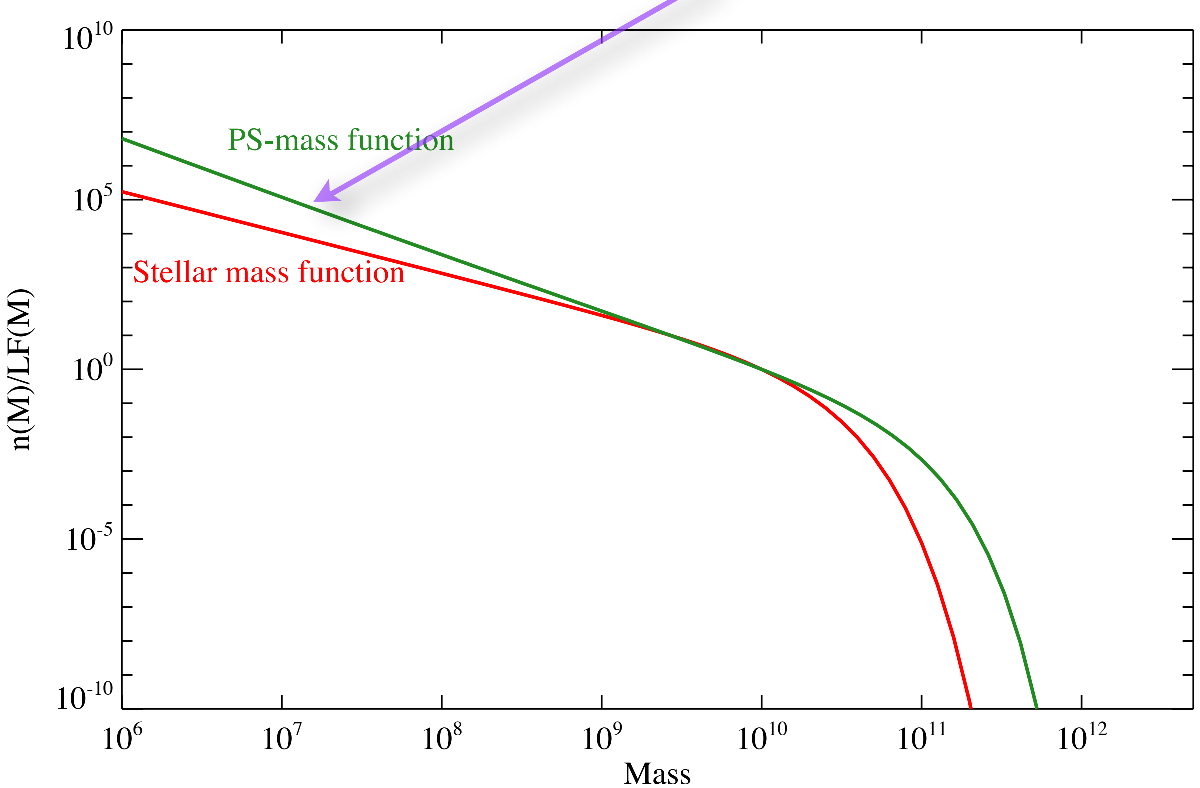
Binding energy in a halo of mass M: $E_b = f_S \frac{G M M_{\rm gas}}{r_{\rm vir}}$

Supernova feedback: $E_{\rm SN} = \epsilon_{\rm SN} M_* c^2$

$$\epsilon_{\rm SN} \approx \frac{10^{51}\rm erg}{100 M_\odot c^2} \approx 5.5 \times 10^{-6}$$

$$\frac{E_{\rm SN}}{E_b} \approx 10 \left(\frac{M_*}{M_{\rm gas}}\right) \left(\frac{V_c}{300\rm km/s}\right)^{-2}$$

Helps here?




Thus supernova feedback is effective in small halos ($V_C$ small), and/or if a lot of the gas is turned into stars.

Bit is unlikely to be the full story because it would have to have a minimum efficiency at some mass.

# Feedback

Binding energy in a halo of mass M: $$E_b = f_S \frac{GMM_{\rm gas}}{r_{\rm vir}}$$

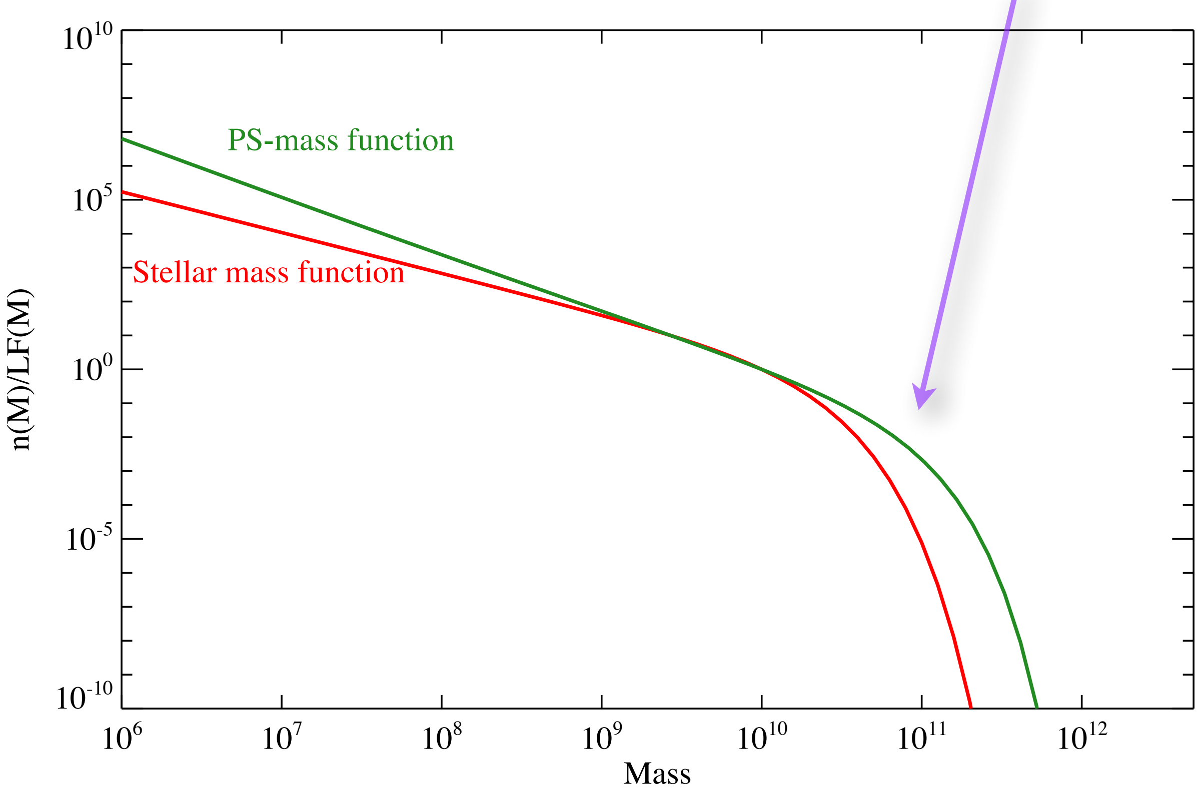
AGN feedback $$E_{\rm AGN} = \epsilon_{\rm AGN} M_{\rm BH} c^2$$

$\epsilon_{\rm AGN} \approx 0.1$

$$\frac{E_{\rm AGN}}{E_b} \approx 200 \left(\frac{M_{\rm bulge}}{M_{\rm gas}}\right) \left(\frac{V_c}{300{\rm km/s}}\right)^{-2} \left(\frac{\epsilon_{\rm AGN}}{0.1}\right)$$

Helps here?

Thus AGN feedback is effective at all masses but particularly when there is a sizeable bulge component (= massive BH).



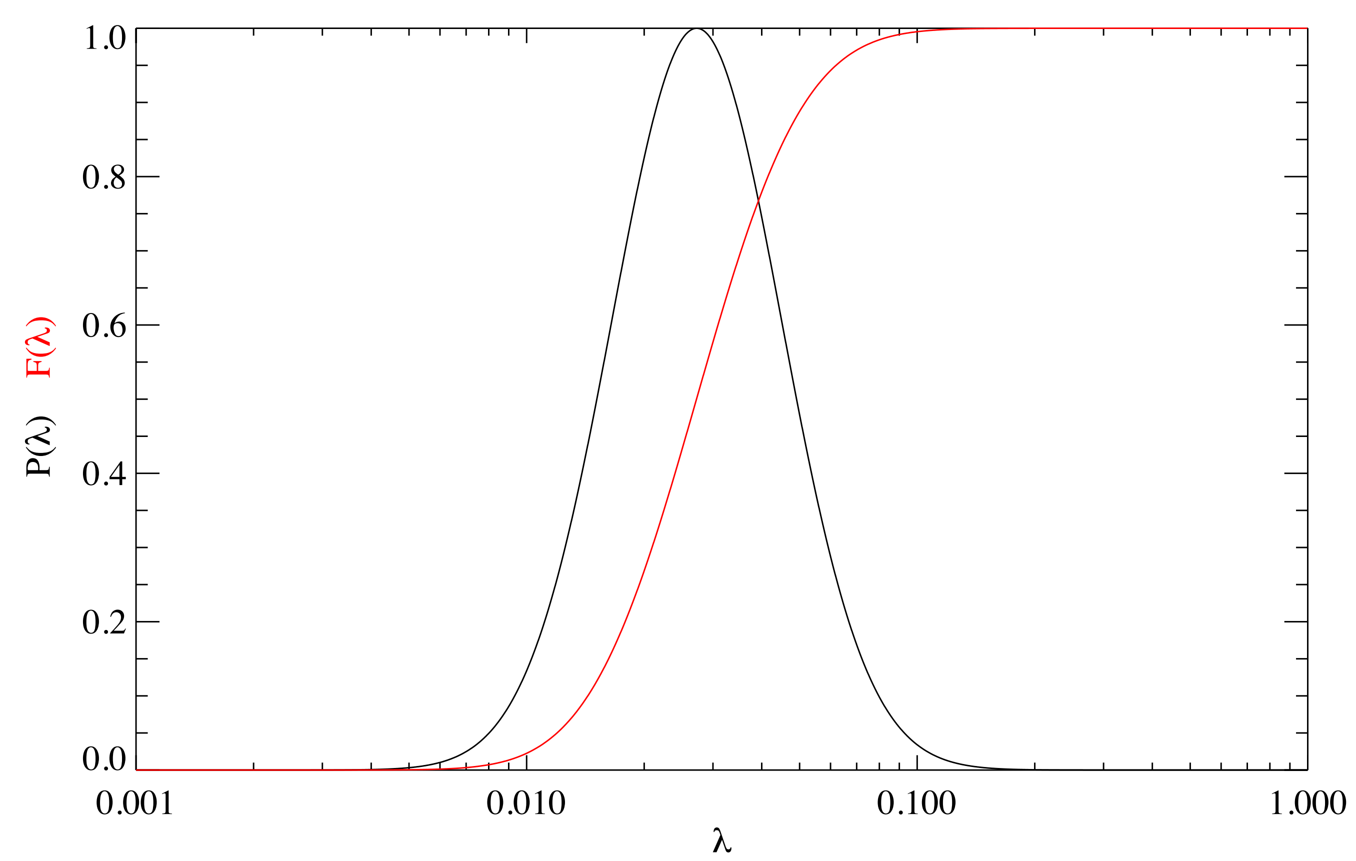
# Angular momentum (lecture 11)

The spin parameter

$$\lambda = \frac{J}{J_{\rm circ}} = \frac{J|E|^{1/2}}{GM^{5/2}}$$

measures the importance of circular motion.

Dark matter halos have little rotation



# The formation of disk galaxies (lecture 11 & 12)

Starting with a dark matter halo we write

$$M_d = f_d M_{\rm halo} = f_d M_{\rm vir}$$

Assuming an exponential disk $\Sigma(R) = \Sigma_0 e^{-R/R_d}$

we can write the angular momentum

$$J_d = 2\pi\Sigma_0 \int_0^\infty V_c(R) R^2 e^{-R/R_d} dR \approx 2 R_d M_d V_{\rm vir}$$

and if we relate this to the halo angular momentum $J_d = j_d J$

we find $R_d = \frac{1}{\sqrt{2}} \left(\frac{j_d}{f_d}\right) \lambda$

this gives us a decent representation of real disk galaxies, but...

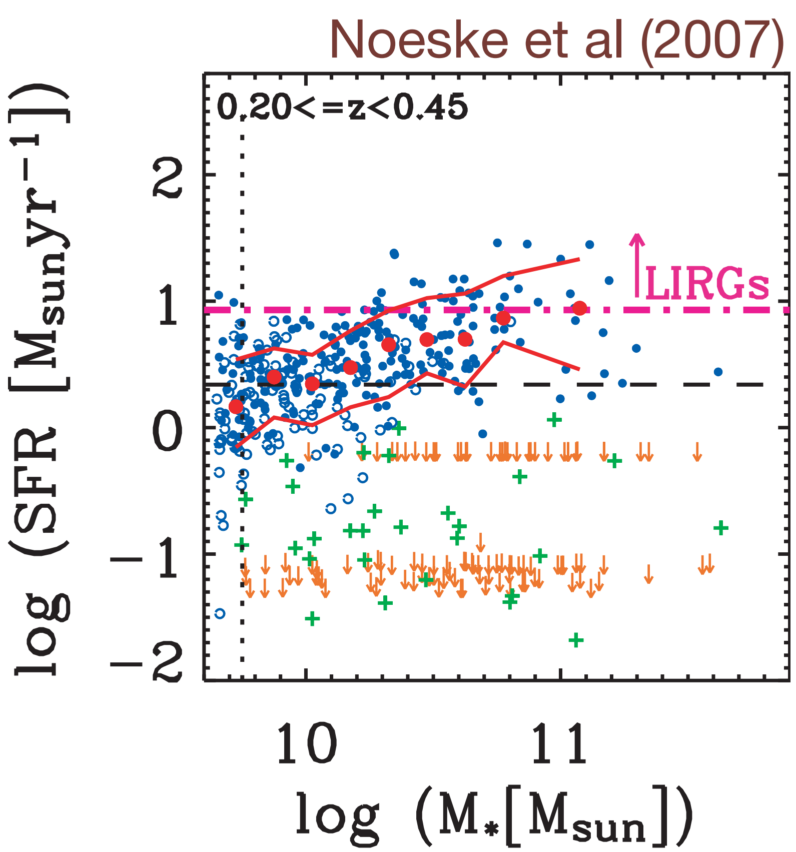
# The formation of disk galaxies

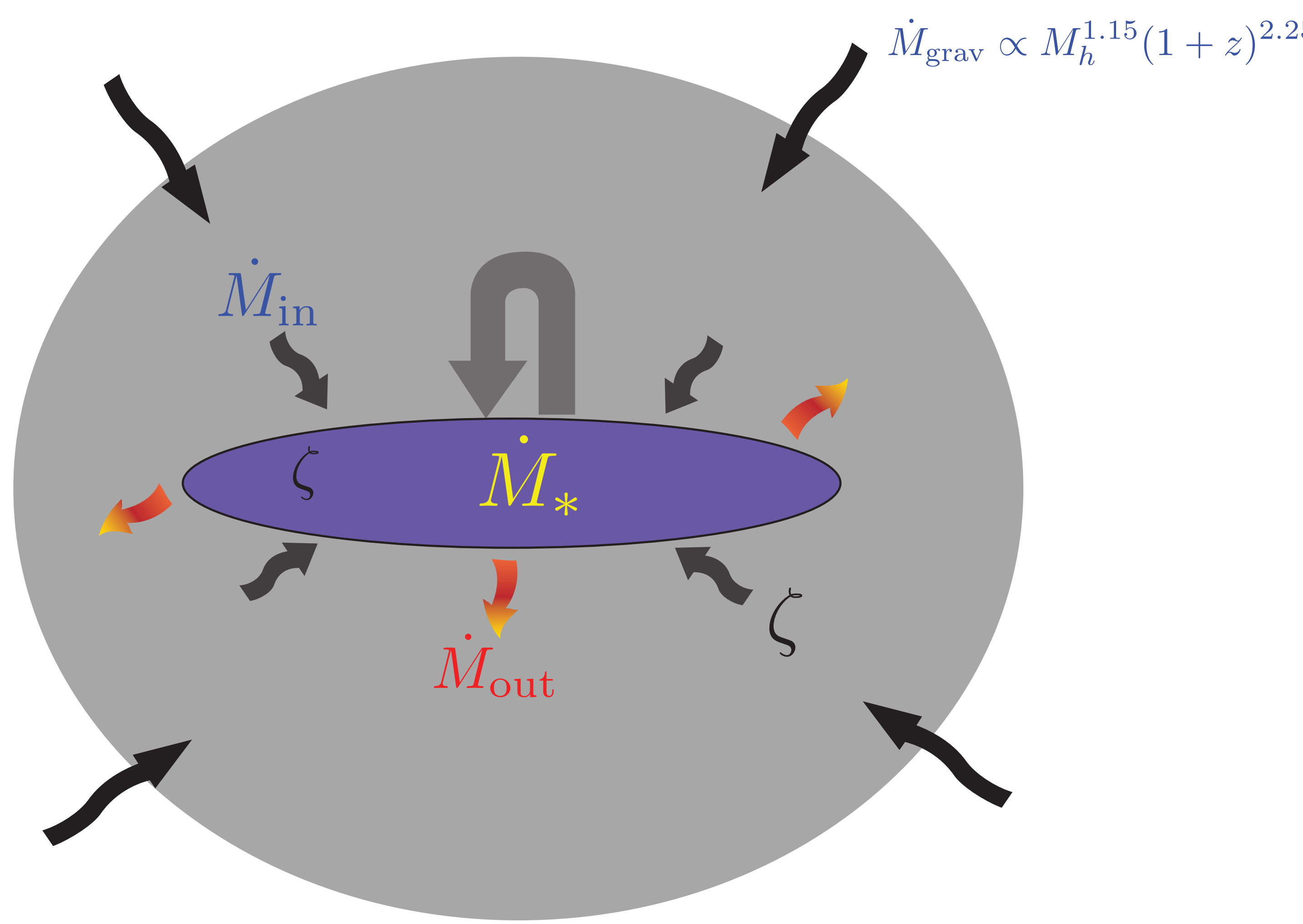
The problem with this formalism as it stands is that it implies that the fraction of the baryons in a halo that ends up in the disk must be very small, $f_d \sim 5\%$.

But if we use a more realistic halo profile (e.g. NFW) & include the self-gravity of the disk, we can get much more sensible numbers.

To create more realistic models, we need to include criteria for stability (e.g. Toomre Q parameter) & star formation rate (e.g. Kennicutt-Schmidt relation)

# Explaining the real Universe - the equilibrium galaxies




Common approximation:

$$\dot{M}_{\rm in} = \dot{M}_{\rm out} + \dot{M}_* \quad \longrightarrow \quad \dot{M}_* = {\rm SFR} = \frac{\dot{M}_{\rm in}}{1+\eta}$$

# Equilibrium models

$$\frac{\dot{M}_{\rm grav}}{\dot{M}_{\rm halo}} \approx 0.47 f_b \left(\frac{M_{\rm halo}}{10^{12}\ M_\odot}\right)^{0.15} \left(\frac{1+z}{3}\right)^2 .25\ {\rm Gyr}^{-1}$$

✚ Preventive feedback

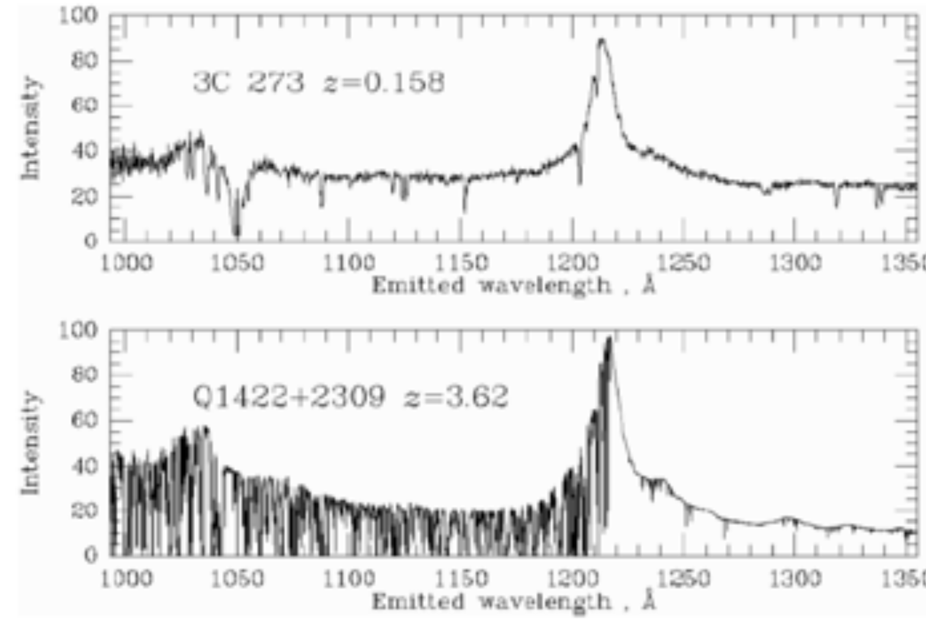
$$\zeta_{\rm shocks} \approx 0.47 \left(\frac{1+z}{4}\right)^{0.38} \left(\frac{M_{\rm halo}}{10^{12}\ M_\odot}\right)^{-0.25},$$

⬇

$$\mathrm{SFR} \propto M_{\rm halo}^{-0.1} (1+z)^{2.63}$$

In decent agreement with the observations (probably a bit fortunately)

# The Lyman-alpha forest




The typical length scales of the systems is the local Jeans length - equality of dynamical and sound-crossing time.

We found that they were big

$$L \sim 100 \text{ kpc} \left( \frac{N_{\mathrm{HI}}}{10^{14} \text{ cm}^{-2}} \right)^{-1/3} T_4^{0.41} \Gamma_{-12}^{-1/3} \left( \frac{f_g}{0.16} \right)^{2/3} .$$

And typically low density

$$N_{\mathrm{HI}} \approx 2.7 \times 10^{13} \text{ cm}^{-2} \, (1+\delta)^{3/2} T_4^{-0.26} \Gamma_{-12}^{-1} (\cdots)$$

# The Gunn-Peterson effect and the IGM (lecture 12/13)

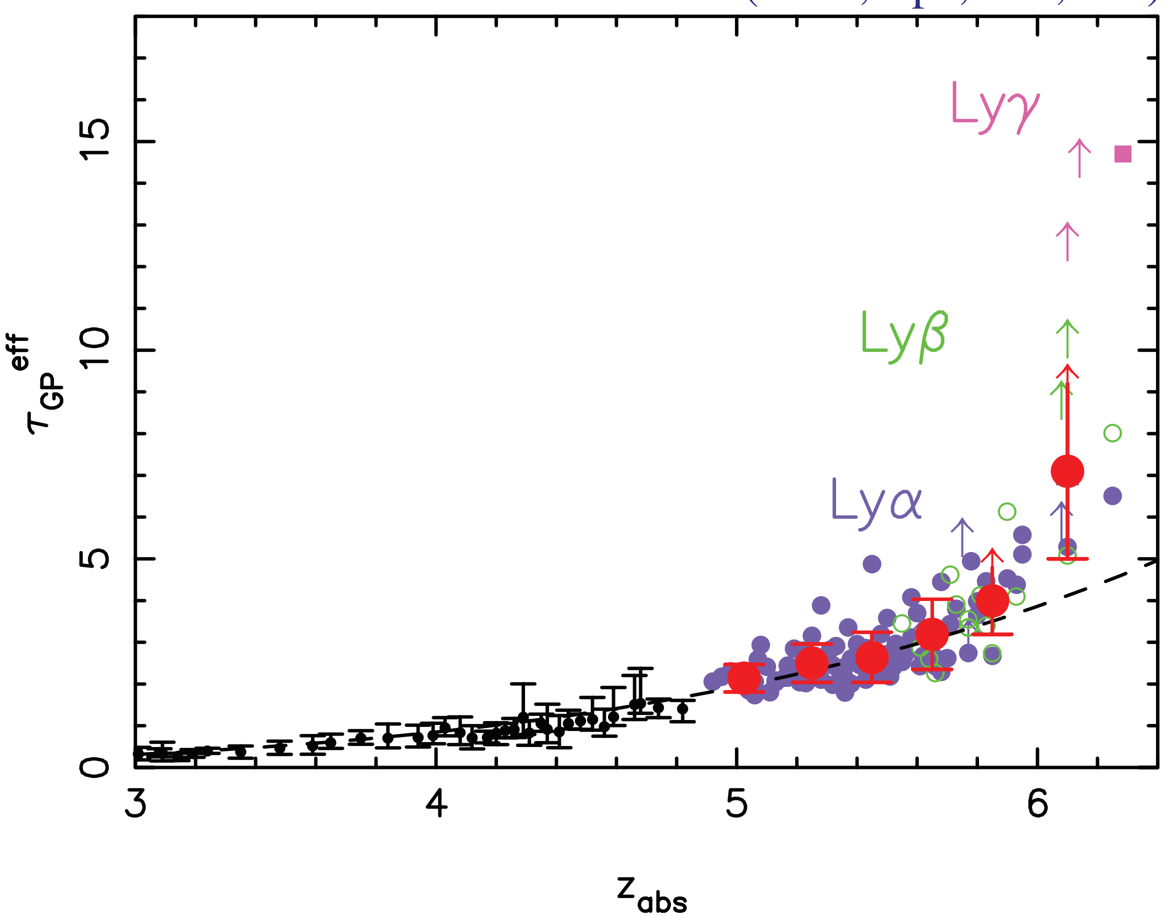
The optical depth to absorption by neutral hydrogen:

$$\tau(\nu_{\rm obs}) = \int_{\nu_{\rm obs}}^{\nu_{\rm obs}(1+z)} n_{\rm HI}(z)\sigma(\nu)\left|\frac{dl}{dz}\right|\frac{d\nu}{\nu_{\rm obs}}$$

Is seen to increase with redshift, but it is still measureable at z~6, so the Universe was not neutral then.

Thus $z_{\rm reionisation} > 6$

Fan et al (2006, ApJ, 132, 117)



# Probing the neutral nature of the IGM

However hydrogen absorption is very efficient:

$$\tau(\nu_{\rm obs}) \approx 1.7 \times 10^5 \, \frac{n_{\rm HI}}{n_{\rm H}} \, \frac{n_{\rm H}}{\bar{n}_{\rm H}} \cdots$$

ionisation fraction over-density

Thus we need other probes - 21 cm/CMB

The CMB results says $z_{\text{reionization}} < 12$ or so.

Besides the reionization question - hydrogen absorption is a very good probe of small over-densities.

# Thermal balance - photoionization equilibrium

$$\frac{n_{\mathrm{HI}}}{t_{\mathrm{ion}}} = \frac{n_{\mathrm{HII}}}{t_{\mathrm{rec}}} \quad \Longrightarrow \quad \frac{n_{\mathrm{HI}}}{n_{\mathrm{H}}} \approx \frac{t_{\mathrm{ion}}}{t_{\mathrm{rec}}}$$

Highly ionized

$$t_{\mathrm{rec}} = \frac{1}{n_e \alpha_r}$$

This can be used to estimate neutral fractions and the Gunn-Peterson optical depth if we combine it with observational constraints. It is also common to introduce $\Gamma \overset{\mathrm{def}}{=} t_{\mathrm{ion}}^{-1}$

$$\frac{n_{\mathrm{HI}}}{n_{\mathrm{H}}} = \frac{t_{\mathrm{ion}}}{t_{\mathrm{rec}}} = 5.8 \times 10^{-6}\, \frac{n_{\mathrm{H}}}{\bar{n}_{\mathrm{H}}} \left(\frac{1+z}{4}\right)^3 T_4^{-0.7} \cdots \Gamma_{-12}^{-1}$$

# Thermal balance

$$dE = dQ - pdV$$

$$\frac{1}{T}\frac{dT}{dt} = \boxed{\frac{1}{\mu}\frac{d\mu}{dt}} + \frac{\Lambda_h - \Lambda_c}{u} - 3(\gamma - 1)H(t)$$

Change in particle number/ionization state

# Thermal balance

$$dE = dQ - pdV$$

$$\frac{1}{T}\frac{dT}{dt} = \frac{1}{\mu}\frac{d\mu}{dt} + \boxed{\frac{\Lambda_h - \Lambda_c}{u}} - 3(\gamma - 1)H(t)$$

Balance of heating
vs cooling

# Thermal balance

$$dE = dQ - pdV$$

$$\frac{1}{T}\frac{dT}{dt} = \frac{1}{\mu}\frac{d\mu}{dt} + \frac{\Lambda_h - \Lambda_c}{u} - 3(\gamma - 1)H(t)$$

Heat loss due to the expansion of the Universe

# Thermal balance

$$dE = dQ - pdV$$

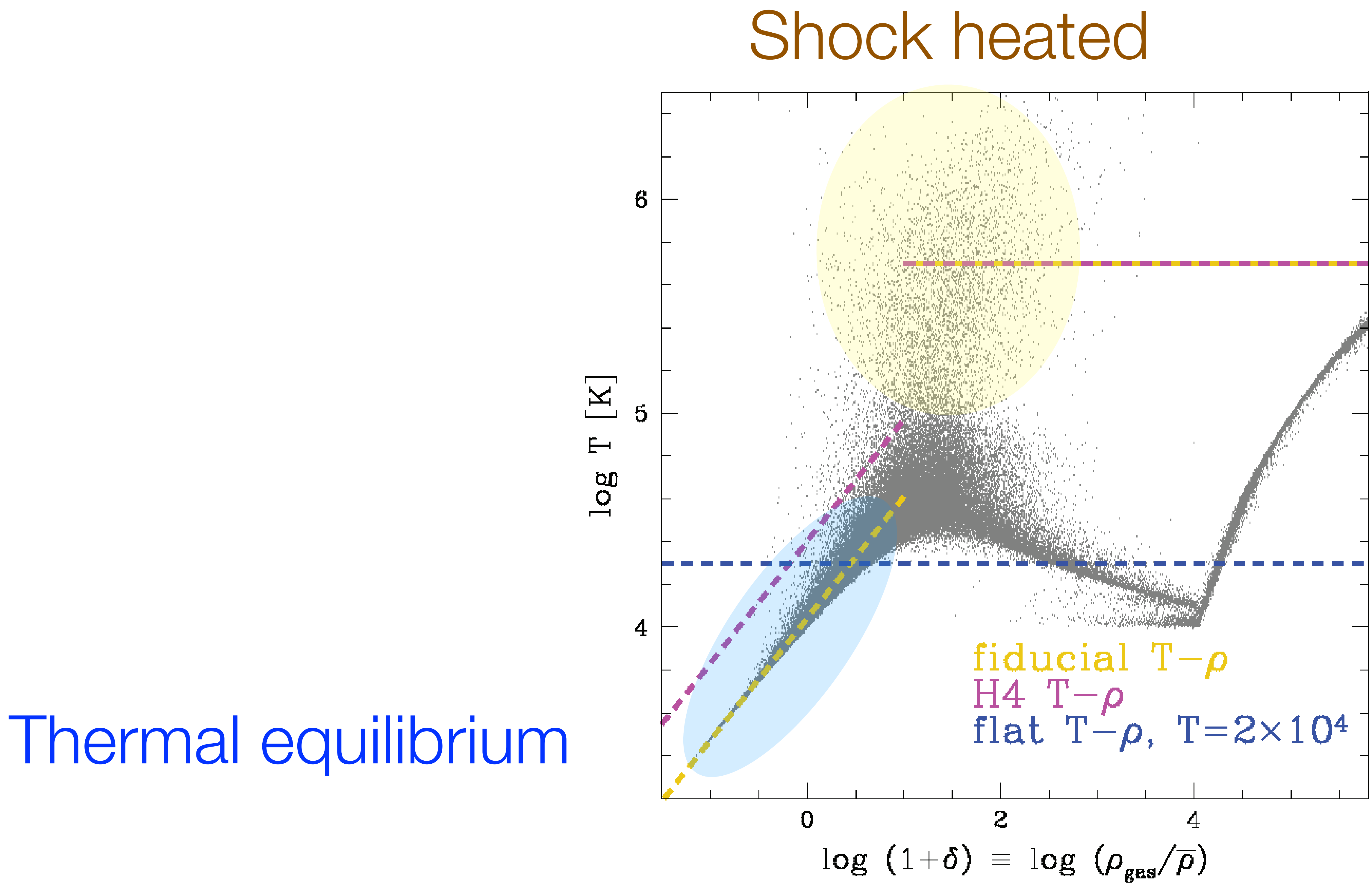
$$\frac{1}{T}\frac{dT}{dt} = \frac{1}{\mu}\frac{d\mu}{dt} + \frac{\Lambda_h - \Lambda_c}{u} - 3(\gamma - 1)H(t)$$

And we found that in thermodynamical equilibrium we have (by requiring that all terms are independent of density):

$$T \propto \rho^{0.57}$$

This is seen in simulations as well

# Thermal balance



# Strömgren spheres

$$r_S = \left( \frac{3\dot{N}_\gamma}{4\pi \alpha_r n^2} \right)^{1/3}$$

Non-expanding Universe

This is the volume (of hydrogen) that a source outputting $\dot{N}_\gamma$ photons per second can keep ionized in face of recombinations

Moving to an expanding Universe we need the ionization equation

$$\frac{\partial n_e}{\partial t} = \nabla \cdot \frac{\dot{N}_\gamma}{4\pi r^2} \hat{\vec{r}} - \alpha_r n_e n_p$$

# Strömgren spheres in an expanding Universe

$$\frac{\partial n_e}{\partial t} = \nabla \cdot \frac{\dot{N}_\gamma}{4\pi r^2}\hat{\vec{r}} - \alpha_r n_e n_p$$

Integrates to:

$$\langle x \rangle \stackrel{\text{def}}{=} \frac{1}{V}\int x dV$$

$$\frac{dV}{dt} - 3VH(t) = \frac{\dot{N}_\gamma}{\langle n_e \rangle} - \alpha_r \frac{\langle n_e n_p \rangle}{\langle n_e \rangle}$$

Allowing for expansion and clumping

$C = \langle n_e n_p \rangle / \langle n_e \rangle$ Clumping factor

$$\bar{t}_{\text{rec}} = \frac{1}{C} t_{\text{rec}}$$

$$\frac{dV}{dt} = \frac{\dot{N}_\gamma}{\langle n_H \rangle} - V\left[\frac{1}{\bar{t}_{\text{rec}}} - 3H(t)\right]$$

Gives us the volume of ionized spheres during reionization.

# Re-ionization

Different regimes:

$$t \ll \frac{t_{\rm rec}}{C} \quad \Rightarrow \quad V(t) \approx \frac{\dot{N}_\gamma}{\bar{n}_H} t$$ Inefficient recombinations

$$Q = \frac{V}{V_{\rm tot}}$$ The filling factor of ionized regions

$$\dot{n}_\gamma t = \langle n_H \rangle$$ For Q=1 - this is appropriate at early times

# Keeping the Universe re-ionized

$$Q = \frac{V}{V_{\rm tot}}$$

The filling factor of ionized regions

$$t \gg \frac{t_{\rm rec}}{C} \quad \Rightarrow \quad Q(t) = \frac{\dot{n}_\gamma}{\langle n_H \rangle} \frac{t_{\rm rec}}{C}$$

which holds at late times. To ensure that Q ~ 1 (keeping the Universe ionized) we get a lower limit to the amount of ionizing photons:

$$\dot{n}_\gamma = \frac{\langle n_H \rangle}{t_{\rm rec}/C}$$

Which can be converted to a critical SFR density

$$\dot{\rho}_*(t) \approx 0.002 M_\odot/{\rm yr}/{\rm Mpc}^3 \left(\frac{C}{6}\right) f_{\rm esc}^{-1} \left(\frac{1+z}{6}\right).$$