Lecture 15

Jarle Brinchmann

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1 Recap

Last lecture we explored the effect of neutral hydrogen on the transparency of the Universe. We saw that only a tiny amount of neutral hydrogen was sufficient to create a large optical depth to distant objects. This effect is generally known as the Gunn-Peterson effect and we found that

$$\tau_{\rm GP} \sim 1.7 \times 10^5 \left(\frac{n_{\rm HI}}{n_{\rm H}}\right) \left(\frac{n_{\rm H}}{\bar{n}_{\rm H}}\right) \left(\frac{\Omega_m h^2}{0.147}\right)^{-1/2} \left(\frac{\Omega_b h^2}{0.023}\right) \left(\frac{1+z}{4}\right)^{3/2} \left(\frac{X}{0.75}\right), \qquad (1)$$

which makes it clear that $\tau_{\rm GP}$ can actually be used to prove very small over-densities as well if other quantities can be estimated or taken to be approximately constant.

Tracing the optical depth to hydrogen absorption back in time, it is possible to conclude that the Universe was essentially completely ionized by $z \sim 6$, so re-ionization must have taken place before that time. However the large optical depth in Ly-α precludes its use for probing back to sizeable neutral fractions.

Alternative methods such as the 21 cm absorption in hydrogen and polarization of the Cosmic Microwave Background were also touched upon briefly. From observations of the CMB we can also conclude that reionization must have taken place at $z < 13$ (at least for simple reionization models).

We also looked at the characteristics of photo-ionization equilibrium. This is a situation where

$$\left(\frac{n_{\rm HI}}{n_{\rm H}}\right) = \left(\frac{t_{\rm ion}}{t_{\rm rec}}\right), \qquad (2)$$

where

$$t_{\rm rec} = \frac{1}{n_e \alpha_r}, \qquad (3)$$

and $\alpha_r \approx 4 \times 10^{13} T_4^{-0.7}$ cm^3/s is the recombination coefficient of hydrogen. Combining the assumption of photoionization equilibrium with observations we also concluded that $\tau_{\rm GP} \approx 1$ at $z \sim 3$.

2 Reionization

How then does re-ionization proceed? To start with, a simple scenario is to consider a single source of hydrogen ionizing photons (with energy $E > 13.6\mathrm{eV}$). This can be a quasar or a star-forming galaxy for instance. While QSOs are efficient at ionizing their surroundings, it is believed there were too few of them at really high redshifts to explain re-ionization. Thus currently the favoured sources for re-ionizaton are star-forming galaxies.

We assume that the source emits $\dot{N}_\gamma$ photons per second. If we consider the change in the electron density at a distance r from the source, and we assume spherical symmetry, we have that

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \vec{v}) = \nabla \cdot \frac{\dot{N}_\gamma}{4\pi r^2}\hat{\vec{r}} - \alpha_r n_e n_p, \tag{4}$$

where the left hand side gives the creation/destruction of electrons as well as the flow into/out of the region. The first term of the right-hand side gives the flow of ionizing photons into the region, so effectively the number of ionizations, while the second term quantifies the number of recombinations.

2.1 Strömgren sphere

If we consider an equilibrium solution to equation (4), ie. one where the left hand side disappears, we have

$$\nabla \cdot \frac{\dot{N}_\gamma}{4\pi r^2}\hat{\vec{r}} = \alpha_r n_e n_p. \tag{5}$$

To make progress we now assume that we have a spherical region which is fully ionized within a radius r_S, and fully netural outside. This is generally a very good approximation to the structure expected in a real ionized region. We can then integrate equation (5) over a volume V and using the divergence theorem to convert the integral of the gradient to a surface integral we have

$$\int \frac{\dot{N}_\gamma}{4\pi r^2} dS = \frac{4\pi}{3} r_S^3 n_e n_p \alpha_r. \tag{6}$$

Since the left-hand side integrates to $\dot{N}_\gamma$, we have an expression for r_S which is:

$$r_S = \left(\frac{3\dot{N}_\gamma}{4\pi\alpha_r n_e n_p}\right)^{1/3} \approx \left(\frac{3\dot{N}_\gamma}{4\pi\alpha_r n^2}\right)^{1/3}, \tag{7}$$

where in the last equality we have made use of the fract that for an ionized gas $n_e \approx n_p \approx n$, which is true for a fully ionized gas of hydrogen and is the normal expression for the radius of a Strömgren sphere. Note that n here is the number density of particles in the region before it is ionized.

2.2 Strömgren spheres in pressure equilibrium

In the preceding sub section we made the assumption that there is no net expansion of the Strömgren sphere into the ambient medium. Whether this is a good approximation or not depends on the time-scales in the problem, but it is generally a reasonable approximation.

If you do ionise a sphere quickly then it would become over-pressurised relative to the surrounding medium and pressure balance will have to kick in. To study the resulting evolution we need an estimate of n_e and n_p. We know that for a fully ionized gas that we have $n_e = \frac{14}{27} n_{\rm tot}$ and that $n_p = \frac{4}{9} n_{\rm tot}$. Inserting above we then get

$$r_s = \left(\frac{3\dot{N}_\gamma}{4\pi\alpha_r n_a^2} \right)^{1/3} \left(\frac{n_a^2}{n_p n_e} \right)^{1/3}. \tag{8}$$

The last parenthesis can be rewritten as

$$\frac{n_a^2}{n_p n_e} = \frac{n_a^2}{\frac{14}{27} n_{\rm H\,II} \frac{4}{9} n_{\rm H\,II}} \approx \left(\frac{27}{13} \frac{T_{\rm H\,II}}{T_a} \right)^2, \tag{9}$$

This gives us our final expression

$$r_{\rm final} = r_S \left(\frac{\frac{27}{13} T_{\rm H\,II}}{T_a} \right)^{2/3}. \tag{10}$$

While the resulting expansion of the sphere can be quite substantial if the ambient medium is low temperature, it does however take quite some time.

3 Strömgren spheres in an expanding Universe

This is covered in many references, this exposition follows fairly closely that of Madau, Haardt & Rees (1999, ApJ, 514, 648). Another good reference is Bolton & Haenhelt (2007, MNRAS, 382, 325).

Reionization requires that the entire volume of space is reionized. It therefore makes sense to focus on the volume of the Strömgren spheres. We therefore start with this and then move on to how many of these you need to fill space.

We will ignore peculiar velocities in equation (4). This leads to

$$\frac{\partial n_e}{\partial t} = \nabla \cdot \frac{\dot{N}_\gamma}{4\pi r^2} \hat{\vec{r}} - \alpha_r n_e n_p. \tag{11}$$

Note that in writing down this equation we have of course made the same assumptions as in the original equation, namely that we have ignored the peculiar velocity of the ionizing source, and we have assumed that the surrounding medium is uniform. Relaxing these assumptions typically requires a numerical simulation to explore the evolution of the expanding ionized region. That is unnecessary for our needs here.

If we integrate equation (11) over volume we have

$$\frac{\partial}{\partial t}\int n_e dV = \int \nabla \cdot \frac{\dot{N}_\gamma}{4\pi r^2}\hat{\vec{r}} dV - \int \alpha_r n_e n_p dV. \tag{12}$$

The first term on the right hand side can be converted to a surface integral and if we assume that the temperature is roughly constant within the ionized region we can also take α_r (which is temperature dependent) outside the second integral on the right. If in addition we introduce the notation for volume average quantities:

$$\langle x \rangle \stackrel{\text{def}}{=} \frac{1}{V}\int x dV, \tag{13}$$

we can re-write equation (12) as

$$\langle n_e \rangle \frac{dV}{dt} + V\frac{d\langle n_e \rangle}{dt} = \dot{N}_\gamma - \alpha_r \langle n_e n_p \rangle V. \tag{14}$$

Since the Universe is expanding, we have that $n_e \propto a^{-3}$ and hence

$$\frac{1}{\langle n_e \rangle}\frac{d\langle n_e \rangle}{dt} = \frac{d\ln\langle n_e \rangle}{dt} = -3\frac{\dot{a}}{a}, \tag{15}$$

so we get

$$\frac{dV}{dt} - 3VH(t) = \frac{\dot{N}_\gamma}{\langle n_e \rangle} - \alpha_r \frac{\langle n_e n_p \rangle}{\langle n_e \rangle} V. \tag{16}$$

If we now introduce the recombination time-scale, and observe that for a fully ionized gas, $n_e \approx n_p \approx n_H$, we have

$$t_{\text{rec}} = \frac{1}{\alpha_r \langle n_e \rangle} \tag{17}$$

and an effective recombination time-scale

$$\bar{t}_{\text{rec}} = \frac{\langle n_e \rangle}{\alpha_r \langle n_e n_p \rangle} = \frac{1}{C} t_{\text{rec}}, \tag{18}$$

where $C = \langle n_e n_p \rangle / \langle n_e \rangle^2$ is called the *clumping factor*. C plays a key role in reionization because if matter is clumpy (as it is likely to be), recombinations are much more efficient there (because they depend on the density squared) and hence you need more photons to reionize the Universe.

That $C > 1$ for a clumpy medium you can easily verify for yourself. It is currently thought that $C \sim 1$–6, while a few years back a much large value $C \sim 30$ was favoured. Its value must be estimated using simulations at the moment.

Using this we can rewrite equation (16) as

$$\frac{dV}{dt} = \frac{\dot{N}_\gamma}{\langle n_H \rangle} - V\left[\frac{1}{\bar{t}_{\text{rec}}} - 3H(t)\right]. \tag{19}$$

To gain an intuitive feeling for the solutions of this equation, we can look at the two terms on the right. The first term contains the photon injection rate, and this is a property of the source. This will depend on what kind of source we are considering. The second term is source indpendent, so it is enlightening to explore the relative magnitude of $1/\bar{t}_{\rm rec}$ and $3H(t)$.

We can ignore the expansion term ($H(t)$) when

$$\frac{1}{\bar{t}_{\rm rec}} \gg 3H(t), \tag{20}$$

and in the previous lecture we derived an expression for $t_{\rm rec}/t_H \sim t_{\rm rec}H(t)$,

$$\frac{t_{\rm rec}}{t_H} \approx 3.6hT_4^{0.7}\left(\frac{n_{\rm H}}{\bar{n}_{\rm H}}\right)^{-1}\left(\frac{\Omega_m}{0.3}\right)^{1/2}\left(\frac{1+z}{4}\right)^{-3/2}\left(\frac{\Omega_b}{0.023}\right)^{-1}\left(\frac{1+X}{1.75}\right)^{-1}. \tag{21}$$

so suppressing some of the cosmology terms, and setting the temperature to $T = 10^4$K, we see that we satisfy equation (20) when

$$C\left(\frac{1+z}{10}\right)^{3/2}\left(\frac{n_H}{\bar{n}_H}\right) \gg 2.7. \tag{22}$$

This is true when $C \gg 1$ or $z \gg 9$ or the over density is $\gg 1$. At early stages of reionization this is likely to be true and we can then solve equation (19) to get

$$V(t) = \frac{\dot{N}_\gamma t_{\rm rec}}{C\langle n_H\rangle}\left[1 - e^{-t/t_{\rm rec}/C}\right] \tag{23}$$

At early times, or when you have a lot of ionising photons so that recombinations are inefficient, you have

$$t \ll \frac{t_{\rm rec}}{C} \quad \Rightarrow \quad V(t) \approx \frac{\dot{N}_\gamma}{\langle n_H\rangle}t. \tag{24}$$

In the other limiting case, we have very efficient recombinations and in that case we get

$$t \gg \frac{t_{\rm rec}}{C} \quad \Rightarrow \quad V(t) \approx \frac{\dot{N}_\gamma}{\langle n_H\rangle}\frac{t_{\rm rec}}{C}. \tag{25}$$

This latter expression is independent of time and represents of course the Strömgren sphere modified by the clumping factor C.

The actual process of re-ionization involves the gradual overlap of a number of sources. This is well illustrated by Figure 1 which is taken from Iliev et al (2006; MNRAS, 369, 1625), most other simulations give a similar picture.

The reionization scenarions we are exploring here all call for a single event, but note that since the recombination time is shorter than the Hubble time at $z > 8$, it is possible to have multiple (partial) reionization events and hence a complex history of reionization.

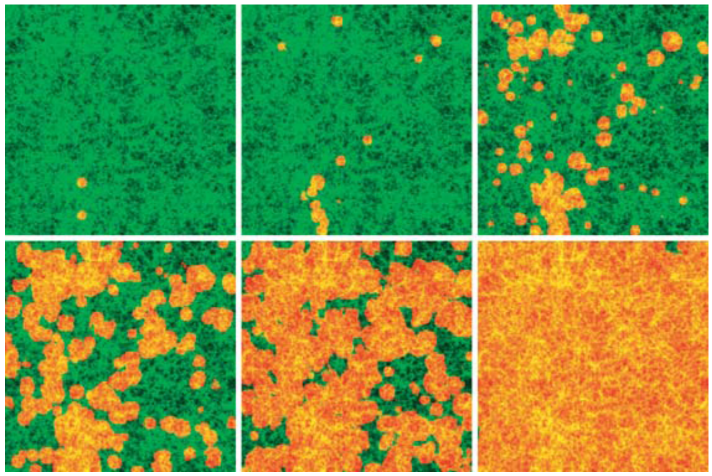

Figure 1: The evolution of ionized bubbles (shown in yellow) in a simulation of reionization by Iliev et al (2006; MNRAS, 369, 1625). The darkness reflects the density and neutral regions are shown in green. As is clear, the first ionized structures appear as separate volumes, and then as time progress (from top left to bottom right), the ionized structures start to merge and create large continuous regions with ionized material. At this stage reionization progresses rapidly to fill all space.

4 The filling factor of the Universe

We now want to go from the analysis of single H II-regions in the previous section, to the whole Universe. To do that, we need firstly to sum over all ionization sources, thus $\dot{N}_\gamma \to \sum \dot{N}_{\gamma,i}$. If we consider a volume $V_{\rm tot}$ of the Universe, we can then define a filling factor of ionized regions,

$$Q = \frac{V}{V_{\rm tot}} \tag{26}$$

and we will define total reionization to be $Q = 1$.

By setting $V = QV_{\rm tot}$ we can convert equation (16) to

$$\frac{dQ}{dt} = \frac{\dot{n}_\gamma}{\langle n_H \rangle} - \frac{Q}{\bar{t}_{\rm rec}}, \tag{27}$$

where $\dot{n}_\gamma = \dot{N}_\gamma / V_{\rm tot}$. If we focus our attention on the limiting cases identified above, we have at early times

$$t \ll \frac{t_{\rm rec}}{C} \quad \Rightarrow \quad Q(t) = \frac{\dot{n}_\gamma}{\langle n_H \rangle} t, \tag{28}$$

which gives as condition for reionization ($Q = 1$) that

$$\dot{n}_\gamma t = \langle n_H \rangle, \tag{29}$$

or in other words that when recombinations are inefficient, we need one ionizing photon per baryon to achieve reionization. We expect this to give a reasonable description of the earliest stages of reionization.

5 Keeping the Universe reionized

An up-to-date review of this question is provided by Robertson et al (2010, 468, 49).

The preceding result gives us an indication of how a region is ionized when recombinations are irrelevant. At later times, though, recombinations will become important and in this regime we have

$$t \gg \frac{t_{\rm rec}}{C} \quad \Rightarrow \quad Q(t) = \frac{\dot{n}_\gamma}{\langle n_H \rangle} \frac{t_{\rm rec}}{C} \tag{30}$$

the requirement to keep the Universe reionized is then

$$\dot{n}_\gamma = \frac{\langle n_H \rangle}{t_{\rm rec}/C}, \tag{31}$$

in other words that the rate of photoionizations should balance the recombination rate adjusted for clumpiness of the medium. We can convert this into a limit on $\dot{n}_\gamma$. It is most convenient to do this in comoving units as that is what observers use. In that case we have

$$\dot{n}_\gamma = \langle n_H \rangle (z=0) C \alpha_r \langle n_e(z) \rangle, \tag{32}$$

where we have cancelled $(1+z)^3$ on both sides, hence the $z = 0$ for the mean density. Using that $n/n_e = (5X+3)/(2X+2)$ and $n_H = X\mu n$ we can introduce the baryon density of the Universe and find

$$\dot{n}_\gamma \approx 3.95 \times 10^{50}\ {\rm s}^{-1}\ {\rm Mpc}^{-3} \left(\frac{\Omega_b h^2}{0.023}\right)^2 \left(\frac{C}{6}\right) \left(\frac{1+z}{6}\right)^3 T_4^{-0.7} \tag{33}$$

This is then the *minimum* rate of ionizing photons per volume to keep the Universe reionized.

To make a closer link to observational data it is convenient to convert this into a star formation rate. From stellar population synthesis models we can calculate the rate of photons per second per solar mass per year of new stars formed. If we write this as $r_Q = f_Q 10^{53}$ photons/s/$M_\odot$/yr, we find that at low metallicity $f_Q \approx 2$, so we can rewrite equation (33) to give us an expression for the minimum star formation rate to keep the Universe ionized

$$\dot{\rho}_*(t) = \dot{n}_\gamma \left(\frac{f_Q}{2}\right)^{-1} (2 \times 10^{53})^{-1} f_{\rm esc}^{-1} \tag{34}$$

$$\approx 0.002 M_\odot/{\rm yr}/{\rm Mpc}^3 \left(\frac{C}{6}\right) f_{\rm esc}^{-1} \left(\frac{\Omega_b h^2}{0.023}\right)^2 \left(\frac{f_Q}{2}\right)^{-1} \left(\frac{1+z}{6}\right) T_4^{-0.7} \tag{35}$$

This is easily achieved at $z \sim 3$ and at lower redshifts, but at high redshift it is not entirely clear whether the observed star formation rate is consistent with this limit. This is therefore an area of very active research both on the theoretical and the observational side.