Lecture 13

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1 Equilibrium models of galaxy evolution

This has been discussed in a number of papers recently, see for instance, Schaye et al (2010, MNRAS 402, 1536) , Davé et al (2012), Lilly et al (2013, ApJ 772:2, 119)). The discussion below follows Davé et al in some detail.

In the previous lectures we have built up a set of tools that can be used to investigate the evolution of galaxies. It is now natural to take one step further and try to use these tools to explain some observed scaling relations for galaxies.

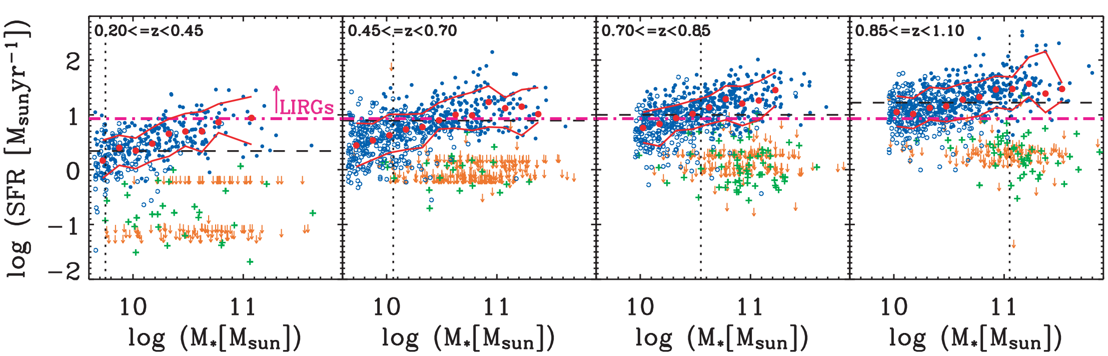

Figure 1: The relationship between the stellar mass of a galaxy (on the x-axis) and the star formation rate of the galaxy (y-axis) at four different times in the Universe. The figure is taken from Noeske et al (2007, ApJL, 660:1, L43) and the blue points are the relevant ones — these show the locations of galaxies that are actively star forming. This is often called the "blue sequence". Note that there is a positive correlation between mass and SFR and also that the relationship moves up as you go back in time.

One key observation is that the star formation rate (SFR) of actively star-forming galaxies is well correlated with their stellar mass. In other words, $SFR/M_* \propto M_*^{\alpha}$, where α appears to be slightly negative, Lilly et al (2013, ApJ 772:2, 119) argues that it should be $\alpha \sim -0.1$ at least in relatively massive galaxies, and we will adopt that here too. The time evolution is slightly less well characterised — Lilly et al argues for $\propto (1+z)^3$ at $z < 2$

and $\propto (1+z)^1.7$ at higher redshift and for reference we will use that too but be aware that this might change with new data in the near future.

From what we covered in previous lectures it is clear that matter flows into halos, some might flow out again, some will be hot and take a long time to cool down, yet other will flow into the galaxy forming in the central region of the halo. Here the gas, when sufficiently cooled, will form stars. From the galaxy's point of view we then have inflow, outflow and star formation as crucial ingredients. Since these all involve gas as a crucial ingredient it is natural to expect that an equilibrium will be set up: as more gas comes in, the star formation rate goes up, as the star formation increases more supernovae will take place and hence more outflows will be set up. It is then reasonable to expect that this will reduce the inflow of gas (and this appears to be the case in simulations) which again will reduce the star formation rate, Thus we have a self-regulating system which ought to reach an equilibrium solution.

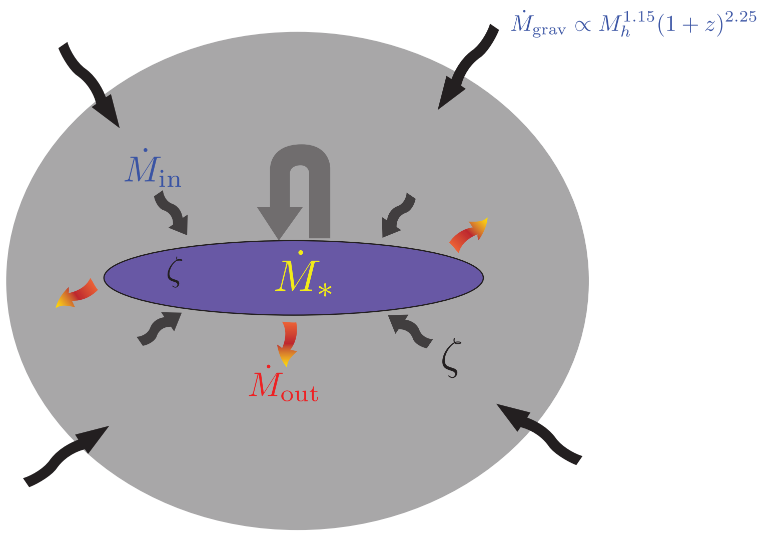

Figure 2: An illustration of the simple system considered here. Matter flows into dark matter halos, as indicated by the large arrows with one labelled according to the relationship we discussed in lecture 10. Some of that infalling mass will reach the galaxy in the centre with a rate $\dot{M}_{\rm in}$, and this will to some extent be turned into stars with a rate $\dot{M}_*$. Some of this mass will in turn be released to the ISM through stellar winds and supernova explosions, leading to $\dot{M}_{\rm out}$ which may or may not escape the galaxy. In the latter case you would expect it to return on a relatively short time-scale.

Armed with this, and our previous discussions we can create a simple illustration of the galaxy in a dark matter halo. Figure 2 shows a simple version of this, indicating the balance between the different flows of gas. I have here used a different notation for the gas flowing into galaxies, $\dot{M}_{\rm in}$, and that flowing into dark matter halos, $\dot{M}_{\rm grav}$. The latter is well-predicted by the extended Press-Schechter formalism we discussed in Lecture 10 and confirmed by simulations, but the former is much harder to predict accurately and we will see below that this matter.

The figure and the discussion above can be encapsulated in a simple equation

$$\dot{M}_{\rm in} = \dot{M}_{\rm out} + \dot{M}_*, \tag{1}$$

or in words that the amount of infalling matter is balanced by the sum of the outflowing matter and the rate of star formation. This neglects the role of a gas reservoir, so we could generalise this by including the gas reservoir

$$\dot{M}_{\rm gas} = \dot{M}_{\rm in} - \dot{M}_{\rm out} - \dot{M}_* + R\dot{M}_*, \tag{2}$$

where R is the fraction of mass turned into stars that is eventually returned to the gas reservoir. This depends on stellar evolution and the IMF assumed and will take a value of 0.3–0.5 for different IMFs.

For simplicity I will however ignore the gas reservoir here (see the Lilly et al reference for a more complete discussion). In that case we can define a mass-loading parameter, $\eta = \dot{M}_{\rm out}/\dot{M}_*$, ie. how much of the mass in stars is blown out of the galaxy (the return fraction can obviously be encapsulated into this so we will ignore R below). The value of η is difficult to estimate *a priori* and must be calibrated with simulations. Given this we find that

$$\dot{M}_* = {\rm SFR} = \frac{\dot{M}_{\rm in}}{1+\eta}. \tag{3}$$

So we see that if we can relate $\dot{M}_{\rm in}$ to the mass of the galaxy, we might be able to infer a relation between stellar mass and SFR.

So let us turn to $\dot{M}_{\rm in}$. In Lecture 10 we looked at the rate that mass is accreted onto dark matter halos and if we multiply this by a baryon fraction, f_b, we can derive a rate of mass growth in baryons in the halo

$$\frac{\dot{M}_{\rm grav}}{\dot{M}_{\rm halo}} \approx 0.47 f_b \left(\frac{M_{\rm halo}}{10^{12}\ M_\odot}\right)^{0.15} \left(\frac{1+z}{3}\right)^2 .25\ {\rm Gyr}^{-1}. \tag{4}$$

If we take $\dot{M}_{\rm in} = \dot{M}_{\rm grav}$ we then infer that

$${\rm SFR} \approx 2.1 \frac{M_\odot}{\rm yr} \left(\frac{f_p}{0.16}\right) \left(\frac{M_{\rm halo}}{10^{12}\ M_\odot}\right)^{1.15} (1+z)^{2.25}. \tag{5}$$

The mass dependence is not quite the same as seen in the observations, while the redshift evolution is intermediate between the high and low redshift trends reported above. However, the assumption that $\dot{M}_{\rm in} = \dot{M}_{\rm grav}$ is a questionable one. Gas falling into massive halos is expected to be heated up and some of the gas might be stopped from accreting onto the galaxy by other physical processes, such as galactic winds and photoionisation. It is possible to capture this into a prevention factor ζ so that

$$\dot{M}_{\rm in} = \zeta \dot{M}_{\rm grav}. \tag{6}$$

It turns out that in intermediate mass halos a key process reducing mass inflow is heating through virial shocks and a simple approximation for this was presented by Faucher-Giguere et al. (2011) who advocated a function

$$\zeta_{\text{shocks}} \approx 0.47 \left(\frac{1+z}{4}\right)^{0.38} \left(\frac{M_{\text{halo}}}{10^{12}\ M_{\odot}}\right)^{-0.25}, \tag{7}$$

which eventually gives us

$$\text{SFR} \propto M_{\text{halo}}^{-0.1}(1+z)^{2.63}, \tag{8}$$

which is encouragingly similar to the observational results summarised above. However this gives the star formation rate of the galaxy in terms of the halo mass and it is not obvious that the mass of a galaxy is a constant fraction of the halo mass, indeed we know that this is incorrect when we consider a wide range of halo masses. However over a narrow range in halo mass this is more likely to be an acceptable assumption and this might be related to the observational results. But the discussion is more important for the light it throws on which factors might be important in determining the scaling relation than for the detailed predictions it might make.

2 The intergalactic medium

Mo, van den Bosch & White chapter 16; Padmanabhan chapter 9; a good review of the physics of the IGM is provided by Meiskin (2009, Rev. Mod. Phys., 81, 1405). The discussion of the Ly-α-forest is based on Schaye (2001, ApJ, 559, 507).

The majority of baryons in the Universe reside in the intergalactic medium (IGM). Simply for that reason it is a crucial ingredient of the Universe to understand. It is the reservoir of gas that is accreted into halos and it contains the gas that is ejected from galaxies.

Despite this crucial importance, it is hard to observe as it is generally diffuse and hot. But the simple fact that the IGM today is ionized has important implications — for the Universe was mostly neutral after the epoch of re-combination, at $z \approx 1100$. Thus at some point the Universe must have become re-ionized. This process, how it occurred, when it occurred and what the sources were that caused it, is a very active topic of research, both from the theoretical and the observational perspective.

Here, we will first see how absorption by the netural medium can be used a probe of the diffuse IGM, and thereafter we will explore the thermal balance of the IGM and see how this changes with redshift and will lead us into the epoch of re-ionization.

Re-ionization is truly one of the key processes in the Universe, and one which is not very well understood. Our focus on studying the IGM will therefore be tilted towards methods that are useful to constrain the process of re-ionization in particular. But we will start with Ly-α-absorbers.

3 The Ly-α forest

In the spectra of distant quasars there are numerous weak absorption lines — more frequent the higher redshift you go to (Figure 3). These are predominantly Ly-α absorption lines from the intervening IGM. Their ubiquitousness makes it natural to ask what the absorbing objects are, what masses, what sizes etc. Here we will build a simple model of this — the goal is to get a handle on the masses, densities and sizes of these systems and how this depends on redshift and other parameters. The full equations are left for the problem class, I will focus on scaling relations predominantly. The focus here are on systems that are optically thin and have column densities $N_{\rm HI} < 10^{17}$ cm^{-2} with some comments on the more optically thick systems further below.

3.1 The Jeans length

The structures are coherent and obviously the gas within them is affected by gravity, but since they gaseous structures, pressure forces will also be important. Thus there are two important time-scales. Firstly the dynamical time

$$t_{\rm dyn} = \frac{1}{\sqrt{G\rho}} \approx 3.2 \times 10^7 \text{ yrs} \left(\frac{n_{\rm H}}{1 \text{ cm}^{-3}}\right)^{-1/2} \left(\frac{1-Y}{0.76}\right)^{1/2} \left(\frac{f_g}{0.16}\right)^{1/2}, \tag{9}$$

where Y is the mass fraction of helium which is set to 0.24 unless explicitly given, and f_g is the fraction of mass in gas, excluding stars and molecules. For the Ly-α-forest a reasonable assumption is that this is $f_g \approx \Omega_b/\Omega_m$ because as we will see, this is a low-density system where the gas is mostly ionised. The value of this ratio is a bit uncertain now, WMAP7 told us it was 0.20 while Planck prefers a value of 0.154, thus I have settled on using 0.16 whenever not explicitly given.

Secondly, the sound crossing time of a structure of size L which is

$$t_{\rm sc} = \frac{L}{c_s} \approx 6.3 \times 10^7 \text{ yrs} \left(\frac{L}{1\text{kpc}}\right) T_4^{-1/2} \left(\frac{\mu}{0.59}\right)^{1/2}, \tag{10}$$

where c_s is the sound speed, $T_4 = T/10^4$ K is the temperature in units of 10^4 Kelvin and μ is the mean molecular weight which in the following I will keep at the value appropriate for a fully-ionised primordial plasma ($\mu \approx 0.59$). I have assumed that the ratio of specific heats, $\gamma = 5/3$. These time-scales will both be important for the evolution of the system. We will for the moment assume that neither cooling nor Hubble expansion are important. The density and sound-speed are both taken to be "characteristic" quantities — below we will argue that these are the quantities appropriate for the denser parts of the clouds.

Setting $t_{\rm dyn} = t_{\rm sc}$ defines a characteristic length-scale, this is the Jeans length

$$L_J = \frac{c_s}{\sqrt{G\rho}} \approx 0.52 \text{ kpc } n_{\rm H}^{-1/2} T_4^{1/2} \left(\frac{f_g}{0.16}\right)^{1/2}. \tag{11}$$

Gas that satisfies this criterion is expected to be in hydrostatic equilibrium with gravity and pressure forces balance each other as can be seen from the equation for hydrostatic equilibrium for instance.

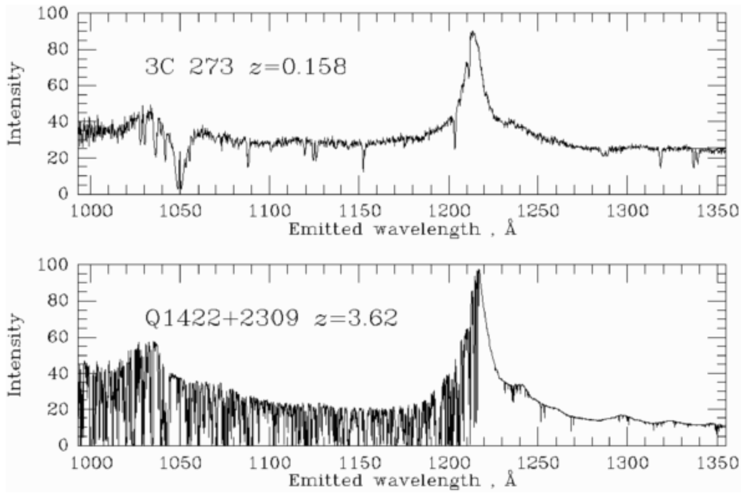

Figure 3: An illustration of the effect of the Ly-α-forest. The top panel shows a low redshift QSO with Ly-α in emission and redwards of this a number of narrow emission lines that are due to neutral hydrogen between us and the QSO. The lower panel shows the same for a $z = 3.62$ QSO and here can be seen the veritable forest of Ly-α absorption lines. The nature of the absorbers responsible for this is the topic of this lecture. The figure is adapted from Ned Wright's web page http://www.astro.ucla.edu/~wright/Lyman-alpha-forest.html.

3.2 Column density

Observationally what we are sensitive to is the column density — ie. the amount of matter that a light-ray traverses on its way towards us. The simplest is to define the column density of hydrogen, this is simply

$$N_{\rm H} = n_{\rm H} L_J \approx 1.6 \times 10^{21}\ {\rm cm}^{-2}\ n_{\rm H}^{1/2} T_4^{1/2} \left(\frac{f_g}{0.16}\right)^{1/2}, \tag{12}$$

and can be thought of as the Jeans column density. Note that by convention column densities are given in cm^{-2}. However observationally the material that we see in the Ly-α-forest is neutral hydrogen so to connect to observations we are really mostly interested in the column density of neutral hydrogen. This is given by

$$N_{\rm HI} = \frac{n_{\rm HI}}{n_{\rm H}} N_{\rm H}, \tag{13}$$

where $n_{\rm HI}/n_{\rm H}$ is the neutral fraction of the gas.

3.3 Ionisation balance

To calculate the neutral fraction we need to understand the ionisation balance of our gas. It is reasonable to assume that at least at late times, photoionisation and recombination equal each other. In that case we can write

$$\underbrace{n_{\mathrm{HI}}\Gamma}_{\text{Photoionisation}} = \underbrace{\alpha_r n_e n_{\mathrm{HII}}}_{\text{Recombinations}} , \tag{14}$$

where Γ is the photoionisation rate, we will typically write this as $\Gamma = \Gamma_{-12}\ 10^{-12}\ \mathrm{s}^{-1}$, and α_r is the recombination rate for hydrogen. Since the absorbers are optically thin, the appropriate recombination coefficient is that for Case A, which can be written

$$\alpha_r \approx 4 \times 10^{-13}\ T_4^{-0.76}\ \mathrm{cm}^3\ \mathrm{s}^{-1}. \tag{15}$$

Note that for more accurate work better fitting formulae exist.

The absorbers are also expected to be highly ionised because the temperature of the gas is found to be typically $\sim 10^4$K. In that case we can write

$$\frac{n_{\mathrm{HI}}}{n_{\mathrm{H}}} \approx \frac{n_{\mathrm{HI}}}{n_{\mathrm{HII}}} = \alpha_r n_e \Gamma^{-1}. \tag{16}$$

For a fully ionised gas, we can write

$$n_e = n_{\mathrm{H}} \frac{1 - Y/2}{1 - Y}. \tag{17}$$

If this looks unfamiliar, write down the derivation for yourself. Inserting this into the previous equation we get the following expression for the neutral fraction

$$\frac{n_{\mathrm{HI}}}{n_{\mathrm{H}}} \approx n_{\mathrm{H}} \frac{1 - Y/2}{1 - Y} \frac{\alpha_r}{\Gamma} \tag{18}$$

$$\approx 0.47 n_{\mathrm{H}}\ T_4^{-0.76}\ \Gamma_{-12}^{-1}. \tag{19}$$

Inserting equation 18 into equation 20 we get

$$N_{\mathrm{HI}} \approx 2.3 \times 10^{13}\ \mathrm{cm}^{-2} \left(\frac{n_{\mathrm{H}}}{10^{-5}\ \mathrm{cm}^{-3}}\right)^{3/2} T_4^{-0.26} \Gamma_{-12}^{-1} \left(\frac{f_g}{0.16}\right)^{1/2}. \tag{20}$$

3.4 Non-uniform gas

In general we expect that a forming structure would have some variation in density. If we, for simplicity, assume that the absorber is approximately spherically symmetric with a density profile $\rho \propto r^{-n}$, we can calculate more explicitly what the column density is as it is going to an integral along a line through the cloud (see Figure 4), so

$$N_{\mathrm{HI}} \propto \int_{-\infty}^{\infty} \rho\left(\sqrt{l^2 + b^2}\right) \frac{n_{\mathrm{HI}}}{n_{\mathrm{H}}}\, dl \tag{21}$$

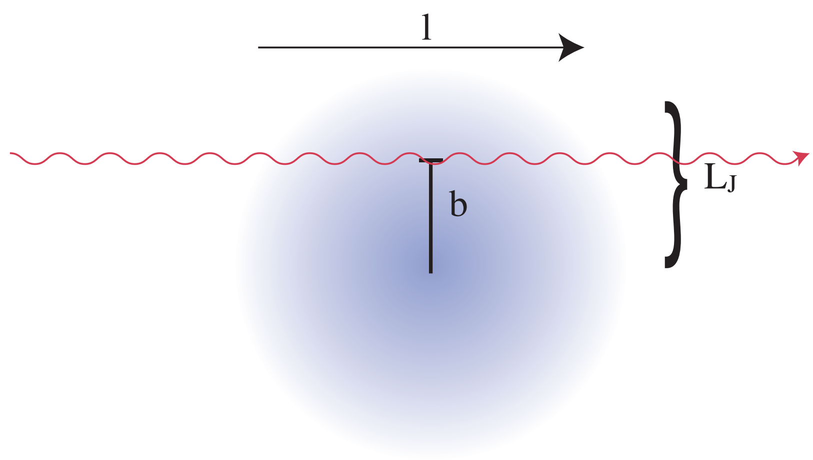

Figure 4: An illustration of the geometry for column density calculation.

and since $n_{\rm H} \propto \rho$ we can use the scaling in equation 18 to write

$$N_{\rm HI} \propto \int_{-\infty}^{\infty} \rho\left(\sqrt{l^2+b^2}\right)^2 dl \propto \int_{-\infty}^{\infty} \left(b^2+l^2\right)^{-n} l \, d\ln l. \tag{22}$$

For $n > \frac{1}{2}$ the main contribution to this integral is for $l \leq b$, thus we can approximate the integral by $N_{\rm HI} \propto \rho(b)^2 b$, indeed the integral gives $b^{1-2n}\sqrt{\pi}\Gamma(n-1/2)/\Gamma(n)$ where $\Gamma(n)$ is the gamma function. Given the equation for the neutral fraction, and that $\rho \propto n_{\rm H}$ we can then conclude that the integral scales as $N_{\rm HI} \propto n_{\rm HI}(b)b$. This means that for reasonable density profiles there is a typical density where most of the absorption takes place and this is close to the maximum density of the system.

3.5 Densities

As we have done before, we can relate the hydrogen density to the mean and write

$$n_{\rm H} = (1+\delta)\Omega_b \rho_{\rm crit}(1-Y)(1+z)^3. \tag{23}$$

Inserting this into equation 20 we get

$$N_{\rm HI} \approx 2.7 \times 10^{13}\ {\rm cm}^{-2}\ (1+\delta)^{3/2} T_4^{-0.26} \Gamma_{-12}^{-1} \left(\frac{1+z}{4}\right)^{9/2} \left(\frac{\Omega b h^2}{0.02}\right)^{3/2} \left(\frac{f_g}{0.16}\right)^{1/2}. \tag{24}$$

We can now take a step back and consider this. If we consider systems that are just about virialising they have overdensities of the order of ~ 200. They will then correspond to $N_{\rm HI} > 10^{17}$ cm^{-2} systems at $z \sim 3$, which is just at the maximum column density for Ly-α-forest systems. Thus the majority of systems responsible for the formation of the Ly-α-forest have not yet virialised.

We have also given the numbers for $z \sim 3$ and there is a strong evolution with redshift explicit in the equation. However Γ also evolves fairly strongly with redshift and in fact the two effects mostly cancel out. Thus the conclusions for $z \sim 3$ absorbers will be applicable more or less to $z \sim 0$ absorbers. The latter are much less well-studied because to study the Ly-α-forest we need rest-frame UV observations and those are much easier to carry out at high redshift.

3.6 Sizes and masses

We can now evaluate the characteristic sizes and masses of the absorbers by inserting the expression for N_{HI} back into the size estimates in equation 11 and we find that

$$L \sim 100 \text{ kpc} \left(\frac{N_{\mathrm{HI}}}{10^{14} \text{ cm}^{-2}} \right)^{-1/3} T_4^{0.41} \Gamma_{-12}^{-1/3} \left(\frac{f_g}{0.16} \right)^{2/3} . \tag{25}$$

Thus the Ly-α-absorbers are large systems of the order of 100 kpc, and even the densest systems are still of order ~ 10 kpc.

To get the mass we write it $M \sim \rho L_J^3$ and this gives a total gas mass of

$$M_g \approx 8.8 \times 10^8 M_\odot \left(\frac{N_{\mathrm{HI}}}{10^{14} \text{ cm}^{-2}} \right)^{-1/3} T_4^{1.41} \Gamma_{-12}^{-1/3} \left(\frac{f_g}{0.16} \right)^{5/3} . \tag{26}$$

Thus the systems are fairly low mass at all relevant densities.

3.7 When does this break down?

What happens when $t_{\mathrm{sc}} \ll t_{\mathrm{dyn}}$? In that case there is no collapse and indeed the system might easily evaporate under influence of e.g. the Hubble expansion. In the opposite situation, when $t_{\mathrm{sc}} \gg t_{\mathrm{dyn}}$, we know that $v \sim L/t_{\mathrm{dyn}} \gg c_s$ and when velocities are much larger than the sound speed we will have shocks or fragmentation of the system. The system is out of hydrostatic equilibrium but the shocks and/or fragmentation will bring the system to an (new) equilibrium on a time-scale set by t_{dyn}. Thus at any given time we expect that locally in an absorber $t_{\mathrm{dyn}} \sim t_{\mathrm{sc}}$ which means that it is locally in hydrostatic equilibrium.

When does this picture break down? It certainly will break down when c_s fluctuates strongly because of large variations in density — shocks during virialisation will certainly lead to this effect. There could also be thermal instabilities which could also strongly affect the density and pressure balance. It turns out that these effects are most important for denser absorbers which are fewer but probably more closely correspond to the central parts of galaxies.

The other situation where these approximations break down in situations where the time-scales are comparable to the Hubble time. If a perturbation is larger than the sound horizon, pressure forces will be irrelevant and the analysis given above is unreliable. In terms of density this corresponds to densities around the mean or lower. Thus for underdense absorbers this formalism is questionable.

Next we will look at how this evolves with lookback time and how the forest eventually blends together to give a more dramatic effect.