Lecture 11

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1 Recap

Last week we looked at how cooling proceeds in a gas cloud with a particular density and pressure profile. We found that as time progresses, the radius where the cooling time is the same as the age of the system typically increases with time. In other words there is a cooling front that propagates outwards in the gas cloud.

We also discussed that as gas falls onto a virialised halo, it is likely to be shocked and indeed numerical simulations find that a shock will develop typically close to the virial radius. The gas that passes through this shock is heated to a temperature near the virial temperature of the halo.

We then asked when this cooling radius equals the virial radius and found that this implied a critical cooling level, or given a cooling level, a critical mass. This critical mass divides halos into those that have a cooling radius *outside* the virial radius (the lower mass ones) and those that have the cooling radius *inside* the virial radius. In the first case the gas might be shock heated, but it will cool again almost immediately and in fact in simulations it is found that only a very weak shock develops. Thus the gas will accrete onto the halo "cold". The second case is one where the gas first is shock heated to near the virial temperature and this gas will typically settle in a near hydro-static equilibrium "atmosphere" and cool out of this only slowly.

This assumes spherical symmetry and I then moved to explain that in reality we expect that a lot of the accretion is non-spherical and this led to the concept of cold streams of gas accreting onto galaxy clusters and into central galaxies.

2 Stellar and AGN feedback

The scalings we have derived give us an idea of the physical properties of dark matter halos. In particular it helps us understand the dynamics of gas within the halos.

When we derived the Press-Schechter mass function we noted that it predicted an excess of low- and high-mass halos relative to the galaxy luminosity functions. One possible resolution of this mismatch is to propose a mechanism that suppresses star formation by blowing the initial gas clouds apart.

That there are some such mechanisms comes from the simple observation that a) the most massive stars explode as supernovae which release large amounts of energy and b) many galaxies have active galactic nuclei (AGN) in their centers, again these put out very large amounts of energy and so are likely to influence their surroundings.

To get a feel for whether these energies are relevant for galaxies, it is useful to look at the typical binding energy of a gas cloud with mass $M_{\rm gas}$:

$$E_b = f_S \frac{GMM_{\rm gas}}{r_{\rm vir}}. \tag{1}$$

If we now assume that the gas to dark matter mass is equal to the overall baryon fraction of the Universe, Ω_b, we can write this as

$$E_b = f_S \frac{GM^2}{r_{\rm vir}} \frac{\Omega_b}{\Omega_m} = f_S V_c^2 M \frac{\Omega_b}{\Omega_m}. \tag{2}$$

If we use equation (**??**) to replace $r_{\rm vir}$ we get

$$E_b = f_S \left(\frac{G^2 H_0^2}{2} \right)^{1/3} \Delta_c^{1/3} (1+z) \Omega_b \Omega_m^{-2/3} M^{5/3} {\rm erg} \tag{3}$$

For our two scenarios this corresponds to a binding energy of $\sim 10^{58} h^{-3}$erg for the $10^{12} M_\odot$ halo at $z = 0$ and $\sim 10^{52} h^{-3}$erg for the $10^8 M_\odot$ halo at $z = 10$. Since the typical energy in a supernova (SN) explosition is $\sim 10^{51}$erg we see that even a single SN would input an interesting amount of energy into small halos.

By equalling the preceding equation of the binding energy to the energy of one SN we find that for a halo mass of $\sim 10^7 M_\odot$ the energies are comparable. However stars do not form alone, what we really want to know is how much supernova energy you get if you form a galaxy with stellar mass M_*.

To find this it is convenient to write the energy output in supernovae as:

$$E_{\rm SN} = \epsilon_{\rm SN} M_* c^2. \tag{4}$$

M_* is the mass of stars that have formed, c is the light speed and $\epsilon_{\rm SN}$ quantifies the efficiency of converting the rest-frame energy of a galaxy to supernova energy.

To estimate $\epsilon_{\rm SN}$ we need to know the distribution of stellar masses when forming a particular mass of stars. This is given by the Initial Mass Function (IMF). Adopting a Kroupa IMF we find that if we adopt a minimum mass for a star exploding as a supernova to be $M = 8M_\odot$ we produce 0.01 supernovae per solar mass of stars; for a Salpeter IMF the corresponding number is 0.007 if we integrate down to $0.1M_\odot$. Thus we need to form $\sim 100M_\odot$ of stars to create on supernova.

This corresponds to a $\epsilon_{\rm SN}$ of

$$\epsilon_{\rm SN} \approx \frac{10^{51}{\rm erg}}{100 M_\odot c^2} \approx 5.5 \times 10^{-6}. \tag{5}$$

Thus star formation is not very efficient at turning rest mass into kinetic energy from supernovae. If we compare this to the binding energy in equation (2) we get

$$\frac{E_{\rm SN}}{E_b} = \frac{5c^2\epsilon_{\rm SN}M_*}{3M_{\rm gas}V_c^2}, \tag{6}$$

where we have assumed a uniform gas cloud ($f_S = 3/5$) and used that $M_{\rm gas} = M\Omega_b/\Omega_m$. Inserting values we find

$$\frac{E_{\rm SN}}{E_b} \approx 10 \left(\frac{M_*}{M_{\rm gas}}\right)\left(\frac{V_c}{300{\rm km/s}}\right)^{-2}. \tag{7}$$

Thus the energy produced by SN can be a significant fraction of the binding energy if a significant fraction of the original gas mass is turned into stars.

However to blow gas out of the galaxy or halo, it is also necessary that the energy couples closely to the baryons and whether it does this or not is less well understood.

Now we could also use this mechanism at high masses, but in that case we would have to have a mechanism that efficient at low masses, then gets progressively less efficient and then gets more efficient again. This is not easy to do and no natural model exist for this. Instead we think that at high masses feedback associated with active galactic nucleic activity is important.

We can carry out a similar analysis for the energy output from AGNs. In this case we write the energy output from and AGN using the standard equation for energy output from accretion onto a black hole:

$$E_{\rm AGN} = \epsilon_{\rm AGN}M_{\rm BH}c^2, \tag{8}$$

where the efficiency of converting the rest-mass energy of infalling matter into radiation is typically taken to be of the order of $\epsilon_{\rm AGN} \approx 0.1$.

To make the link to the preceding we need to be able to link the black hole mass to the properties of the galaxy. It is not obvious how to do this a priori, but it turns out from observations that there is a close connection between the spheroidal part of galaxies (often referred to as the bulge) and the mass of the black hole at their center. From Magorrian et al (1998) we have $M_{\rm BH} \approx 0.002M_{\rm bulge}$ where the pre-factor might be uncertain by a factor of a few.

If we then calculate the equivalent of equation (7), we find that

$$\frac{E_{\rm AGN}}{E_b} \approx 200 \left(\frac{M_{\rm bulge}}{M_{\rm gas}}\right)\left(\frac{V_c}{300{\rm km/s}}\right)^{-2}\left(\frac{\epsilon_{\rm AGN}}{0.1}\right). \tag{9}$$

So *if* the energy output from AGN couples tightly to the baryonic content of a halo, it can provide a very efficient way to eject matter out of galaxies. Whether this happens, and if so how, is an area of considerable current research.

2.1 Is feedback a universal fix?

This section is based on Peñarrubia et al (2012, ApJL, 759, 42), see that paper for further information.

So does these ideas suffice? It is possible that they do, but it is not certain. There are two observational results that make things a bit more tricky. The first is that dark matter simulations predict a very large number of small halos orbiting a galaxy like the Milky Way, but very few faint satellites to the Milky Way are seen. This is sometimes called the missing satellites problem and a way out of this is to postulate that star formation is very inefficient in small halos, in addition to being stopped by feedback.

However in contrast to this are the observations of dwarf galaxies around the Milky Way that find that these galaxies have cored profiles (see Figure 1. In this case their density distributions can be approximately written

$$\rho_{\rm obs}(r) \approx \frac{\rho_0 r_s^3}{(r_c + r)(r_s + r)^2}, \tag{10}$$

where r_c is the core radius and r_s is a scale radius — the form of this is of course chosen to be similar to the NFW form.

How can these cores be formed? A common argument is that they are caused by baryonic processes, particularly those related to feedback. It is then useful to calculate what the energy required is to cause this. One can do this by calculating the required change in potential energy. To do this you first calculate the potential energy of the profile in equation 10

$$W = -4\pi G \int_0^{r_{\rm vir}} \rho(r) M(r) r \, dr, \tag{11}$$

where $r_{\rm vir}$ is the virial radius. You calculate this first for an NFW profile and then for the cored profile and take the difference in potential energy. This require an expression for r_s and for an NFW profile $r_{\rm vir}/r_s$ is called the concentration and is denoted c. It is known that c depends on virial mass of a structure and for instance Bullock et al (2001, MNRAS, 321, 559) found that $c \propto (1+z)^{-1}$ which can be combined with Macció et al (2007, MNRAS, 378, 55)'s finding that at $z = 0$ c is related to the virial mass through

$$\log c \approx 1.02 - 0.109 \log \left(\frac{M_{\rm vir}}{10^{12} h^{-1} M_\odot} \right) \tag{12}$$

to give the behaviour of $c(M_{\rm vir}, z)$.

Armed with this it is now possible to calculate

$$\Delta W = W_{\rm NFW} - W_{\rm cored} = g(M_{\rm vir}, r_c, z, \Omega_m), \tag{13}$$

something that was calculated by Peñarrubia et al and the conclusion is that if the necessary energy ingestion comes from star formation you need a quite a lot of star formation in low mass systems. Whether this can be reconciled within a cold dark matter paradigm or whether it is easier to explain the cores by postulating a slightly warm dark matter particle is not yet possible to say with current observations but there is some tension between CDM predictions and observations at the very lowest masses.

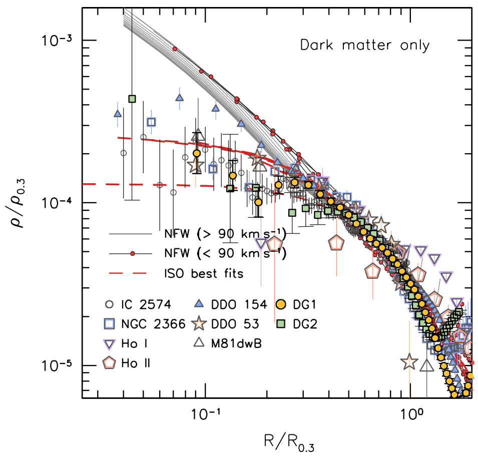

Figure 1: Large symbols: The density profiles of nearby dwarf galaxies from the THINGS sample. The small red symbols show various NFW profiles and the grey lines are from SPH simulations of Oh et al (2011, ApJ 142, 24). Note how the observed baryon densities show a core that is not seen in the simulations. Figure taken from Oh et al (2011) see that paper for more details..

3 The formation of disky structures

The formation of disk galaxies is a very natural question to pursue. If you start out with a slightly non-spherical perturbation, gravitational collapse will tend to accentuate the non-spherical nature and lead to the formation of a flattened disk-like structure, or a cigar like structure.

Now assume you have a rotating cloud, slightly flattened cloud of gas. If the temperature in the gas is $T > 10^4$K, cooling would be usually be efficient and the gas will start collapsing on the gravitational collapse time-scale. The radiation emitted by the cooling will be isotropic so if we ignore the presence of dark matter, the angular momentum, J, will be preserved during the collapse. In reality there will of course be a dark halo present but we will return to what its presence implies shortly.

3.1 Angular momentum and the spin parameter

See Peacock 17.2, Mo, van den Bosch & White 7.5.4

The angular momentum $J \sim MRV_{\rm rot}$ is generally different from what you would measure if all particles were rotating and the circular velocity, V_c. However we are often interested in knowing how dominated by rotation a system is. We therefore like to compare the angular momentum of a system to that if it is all rotating with the circular velocity, $J_{\rm circ} \sim MRV_c$. Since the circular velocity is

$$V_c^2 = \frac{GM}{R}, \quad (14)$$

we have that $J_{\rm circ}$ is given by

$$J_{\rm circ} \sim M^{3/2} R^{1/2} G^{1/2}, \tag{15}$$

and we can compare this to the observed angular momentum J, defining λ, the *spin-parameter*:

$$\lambda = \frac{J}{J_{\rm circ}} = \frac{J}{M^{3/2} R^{1/2} G^{1/2}}, \tag{16}$$

which we normally rephrase by introducing the binding energy $|E| \sim GM^2/R$ which gives the standard expression for λ viz:

$$\lambda = \frac{J|E|^{1/2}}{GM^{5/2}}. \tag{17}$$

Observationally, spiral galaxies have $\lambda \sim 0.3$–0.6, while the spin parameter for dark matter halos (in numerical simulations!) is well described by a log-normal distribution:

$$P(\lambda)\, d\lambda = \frac{1}{\sigma_{\ln\lambda}\sqrt{2\pi}} \exp\left[-\frac{\left(\ln\lambda - \ln\bar{\lambda}\right)^2}{2\sigma_{\ln\lambda}}\right] d\ln\lambda, \tag{18}$$

with $\bar{\lambda} \approx 0.035$ and $\sigma_{\ln\lambda} \approx 0.5$. This distribution is shown in Figure 3.1 and it is clear from that figure that values of the spin parameter of $\lambda > 0.3$ are extremely unlikely ($P(\lambda > 0.3) \approx 8.4 \times 10^{-6}$). Thus if baryons originally have the same spin as dark matter halos, they need to spin up significantly to reproduce the observations of spiral galaxies in the Universe.

4 Spiral galaxy formation

4.1 Collapse without dark matter

As I remarked above, it is necessary to spin up the forming spiral galaxy. It is instructive to first look at this process assuming no dark matter is present. In that case we observe that since the mass is conserved during collapse, the binding energy is

$$|E| \sim \frac{GM^2}{R} \propto R^{-1} \tag{19}$$

$$\Downarrow$$

$$\lambda \propto R^{-1/2}. \tag{20}$$

This in turn means that if we start out with a structure of radius $R_{\rm initial}$, we can predict that its final radius is given through

$$\frac{R_{\rm initial}}{R_{\rm final}} = \left(\frac{\lambda_{\rm initial}}{\lambda_{\rm final}}\right)^{-2} = \left(\frac{0.035}{0.5}\right)^{-2} \approx 200, \tag{21}$$

for an illustrative change in spin parameter. Thus we would need to shrink the size of the proto-spiral by two orders of magnitude. How long would this process take?

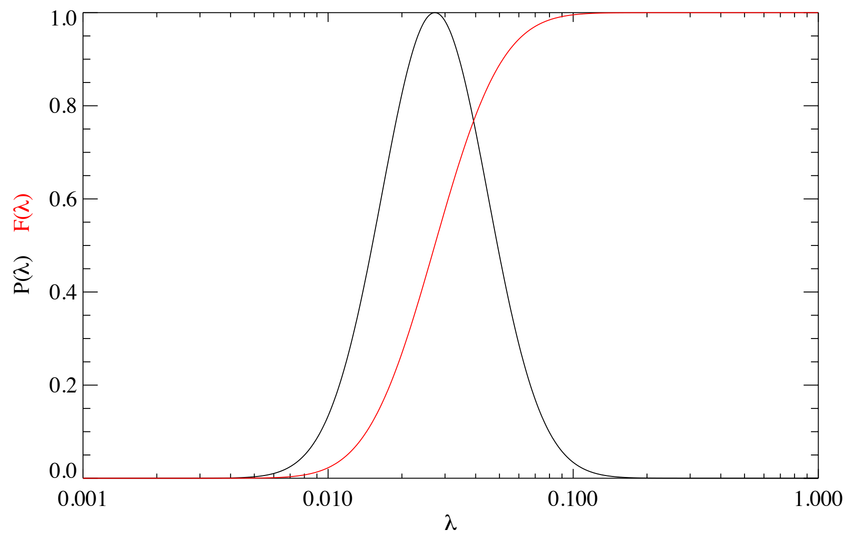

Figure 2: The black line shows the likelihood distribution of the spin parameter, λ, normalised to a peak of 1, for dark matter halos in the standard ΛCDM cosmology. The black line shows the cumulative likelihood for λ.

To fix our attention, let us focus on a galaxy like our own, with a size of $R \approx 10$kpc. From the argument above, the starting size would have to be $R_{\rm initial} \approx 2$Mpc.

How long would it take to contract to the size of 10 kpc? If we assume that cooling is efficient, and that the mass is $M = 10^{11} M_\odot$, then the time-scale of collapse is set by the gravitational collapse time:

$$t_{\rm ff} = \frac{\pi}{2} \left(\frac{R^3}{2GM} \right)^{1/2} \approx 1.5 \times 10^{11} \text{ yrs.} \tag{22}$$

This is considerably longer than the age of the Universe — thus this scenario for forming our disk is simply unfeasible. As we will see, we need to include dark matter in our considerations to make this work.

4.2 Collapse with dark matter

The initial spin parameter of the halo is

$$\lambda_{\rm initial} = \frac{J|E|^{1/2}}{GM^{5/2}}, \tag{23}$$

where now E and M include both the dark matter and the gas. The final state now is one consisting of baryons only and we have

$$\lambda_d = \frac{J_d|E_d|^{1/2}}{GM_d^{5/2}}, \tag{24}$$

where subscript d indicates that the quantity is for the disk only. We are here assuming that the dark matter is not affected by the collapse of the baryons. In this case we can write down the ratio of these two spin parameters

$$\frac{\lambda_d}{\lambda_{\text{initial}}} = \left(\frac{J_d}{J}\right)\left(\frac{E_d}{E}\right)^{1/2}\left(\frac{M_d}{M}\right)^{-5/2}. \tag{25}$$

If we first focus on the ratio of energies, E_d/E, we know that we can write the gravitational binding energy of a structure as $E \approx kGM^2/R$, where k is a constant of the order of 1 which depends on the density profiles of the mass distribution. Using this we can write

$$\frac{E_d}{E} \approx \left(\frac{k_d}{k}\right)\left(\frac{M_d}{M}\right)^2\left(\frac{R_d}{R_{\text{initial}}}\right)^{-1}. \tag{26}$$

It is less clear how to relate the initial and final angular momentum. At an early stage the baryons and the dark matter should have approximately the same angular momentum per mass because they are exposed to the same tidal forces. During collapse there could in theory be transfer of angular momentum between the gas and the dark matter, but if we assume that this transfer is negligible, then the angular momentum *per mass* of the particles should be conserved, or in other words:

$$\frac{J_d}{M_d} = \frac{J}{M}. \tag{27}$$

If we adopt equation (27) and use the expression in equation (26) for the ratio of energies, we can rewrite equation (25) as

$$\frac{\lambda_d}{\lambda_{\text{initial}}} \approx \left(\frac{k_d}{k}\right)\left(\frac{R_{\text{initial}}}{R_d}\right)^{1/2}\left(\frac{M}{M_d}\right)^{1/2}, \tag{28}$$

which gives us an expression for the change in size

$$\frac{R_{\text{initial}}}{R_d} = \left(\frac{k}{k_d}\right)\left(\frac{M_d}{M}\right)\left(\frac{\lambda_d}{\lambda_{\text{initial}}}\right)^2. \tag{29}$$

If you compare this to equation (21), you see that we have gained an additional factor of M_d/M which, if the mass ratio is equal to the baryon to dark matter mass ratio, $\Omega_b/\Omega_m \sim 0.1$, we find that

$$\frac{R_{\text{initial}}}{R_d} \sim 20. \tag{30}$$

which for our 10 kpc galaxy means that the initial size was $\sim$ 200kpc. This size is of the order of the virial radius for a Milky Way galaxy and which gives us a collapse time of $t_{\text{ff}} \approx 5 \times 10^9$ years which is more consistent with observations.

But note that at $z = 4$ the age of the Universe is only $\approx 1.5 \times 10^9$ years, so it would not be possible to form a large spiral galaxy at high redshift. This is indeed also what we find.

4.3 Disks in numerical simulations

A discussion is in Mo, van den Bosch & White 11.2.6.

While this works ok, numerical simulations of this process have repeatedly had problems forming spiral disks. The problem has generally been that the disks are too small. A key reason for this is that J/M is *not* conserved during collapse.

Because collapse tend to be clumpy in the simulations, the baryonic clumps experience dynamical friction and so the baryonic structure loses angular momentum to the halo. With less angular momentum, the disks have to contract further and hence become too small.

How can this be avoided? Firstly there are numerical issues that cause loss of energy, these problems have gradually been reduced as simulations have becom larger. Another problem is that heating processes, such as feedback from star formation of active galactic nuclei, need to be included. If sufficient energy is injected, the clumpy structures are disrupted or at least made more diffuse and this reduces the amount of angular momentum lost.

Up-to-date simulations now can form realistic spirals, although it appears hard still to form bulge-less disks.