Lecture 10

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1 Recap

Last week we discussed the cooling function in some more detail — we looked at the effect of metals and of radiation. Adding metals to the cooling increases the cooling very significantly between $T = 10^5$K and 10^6K and will also increase the cooling at low temperatures because there are many more bound states available, some with much lower excitation and ionization energies than hydrogen.

When one adds a photo-ionizing background, the main effect is to remove low-energy bound states. The exact effect does depends on the intensity and spectral shape of the ionizing background, but any source that has a significant number of photons with energies in excess of the ionization energy of hydrogen will ionize hydrogen fully and remove the first peak in the cooling curve. Photo-ionization will also boost the cooling at low temperatures, and at high redshift it will also boost the efficiency of inverse Compton cooling.

Inverse Compton cooling is the cooling of electrons by transfering their energy to the cosmic microwave background. We found that the cooling time for inverse Compton cooling is

$$t_{\rm cool,Compton} \approx 2.3 \times 10^{12}(1+z)^{-4}\ \text{years}, \tag{1}$$

which is independent of density and temperature (as long as there are free electrons around), but it is clearly too long at low redshift. Given the strong dependence on redshift it becomes much more important at high redshift and we found that at $z > 10$, inverse Compton cooling can be an efficient cooling pathway.

We then discussed of cooling flows in clusters where we showed that the change in energy in a volume, V, could be written as

$$\frac{dH}{dt} = L(<r) = \frac{5}{2}\frac{kT(r)}{\mu m_p}\dot{M}. \tag{2}$$

This would predict an extremely high luminosity in the center of the clusters if $\dot{M}$ is constant, but this is not seen observationally.

The accretion rate, $\dot{M}$, is observationally found to be $\dot{M} \sim 100 - 1000 M_\odot/\text{yr}$ in the central regions of clusters, we would naively have expected that this inflow of gas would lead to star formation at a similar rate. This is however not seen, in fact the star formation rate in the central part of the galaxy clusters is 1–2 orders of magnitude lower than the mass inflow rate. This is one example of the *cooling flow* problem.

Resolving this problem requires some additional heating source, but what this is is not clear. A number of suggestions exist in the literature and it appears likely that a combination of AGN feedback, cosmic ray heating and supernova feedback will resolve this conundrum.

2 Getting gas into halos

This has been quite extensively discussed recently. Useful starting points might be Dekel et al (2009, Nature, 457, 451) and Neistein et al (2006, MNRAS, 372, 933) which I base myself on here.

Typical galaxies at $z \sim 2$ form stars at a rate of $\sim 10^2 M_\odot/\mathrm{yr}$. In order to connect them to dark matter halos, it is useful to look at their number density. Assuming that there is one of these galaxies per halo, the typical space density of $\sim$ few $\times 10^{-4}$ Mpc^{-3} means that they have to reside in halos with halo masses below $\sim$ few $\times 10^{12} M_\odot$.

It is clearly interesting to ask whether gas accreted onto these dark matter halos is what keeps star formation going in these high redshift galaxies. To answer that we first need to determine what the growth rate of halos are with time, and then move from there to determine the growth of the baryonic content of the halos.

The growth of halos was recently discussed by Neistein et al (op cit) and Neistein & Dekel (2008, MNRAS, 388, 1792) building on the extended Press-Schechter formalism. We will not need to go through the derivation so we just note that they find that the growth in mass of a halo with mass M_h can be written

$$\frac{d \ln M_h}{d\omega} = -\left(\frac{2}{\pi}\right)^{1/2} \left[\sigma^2(M_h/q) - \sigma^2(M_h)\right]^{-1/2}, \tag{3}$$

with $\omega = \delta_c/D(t) = \delta_c(t)$ as a time variable, $\sigma^2(M)$ the variance of the density field smoothed on scale M, and $q = 2.2 \pm 0.1$.This has been checked against simulations and has been found to give good estimates.

I will return to some more accurate calculations of this quantity below, but it is enlightening to look at this using a more approximate approach. So let us put

$$\sigma(M) = \sigma_8 \left(\frac{M}{M_8}\right)^{-(n+3)/6} \tag{4}$$

and assume that n is constant from M to M/q (this is not correct in detail of course!). Putting that in we can immediately see that $dM_h/d\omega \propto \sigma_8$ and $\propto M^{(3-n)/6}$, but in most cases we are more likely to want the derivative with time which we can have by observing that for an Einstein-de Sitter Universe

$$\frac{d}{d\omega} = -\frac{1}{\delta_c}\frac{1}{H(z)(1+z)}\frac{d}{dt}. \tag{5}$$

Inserting this we have

$$\frac{dM_h}{dt} \approx \delta_c H_0 (1+z)^{5/2} \left(\frac{2}{\pi}\right)^{1/2} \kappa \sigma_8^{-1} M_h \left(\frac{M_h}{M_8}\right)^{(n+3)/6}, \tag{6}$$

with $\kappa = \sqrt{q^{(n+3)/3} - 1}$. One point in particular to note here is that we predict the merging rate to increase with redshift as $(1+z)^{2.5}$.

Quantitatively we have (for $n = -2.35$, $\Omega_m = 0.258$)

$$\frac{dM_h}{dt} \approx 230 \left(\frac{M_h}{10^{12} h^{-1} M_\odot}\right)^{1.1} \left(\frac{\sigma_8}{0.8}\right)^{-1} (1+z)^{5/2}\ M_\odot/\mathrm{yr}, \tag{7}$$

and the baryonic rate would be $f_{\rm gas}$ times this. In general these rates are too high however.

A more careful analysis as in Neistein et al (2006) for instance, gives for a cosmology similar to the real Universe

$$\frac{d\omega}{dt} \approx -0.047 \left(1 + z + 0.1(1+z)^{-1.25}\right)^{2.5} \frac{h}{0.73}\ \mathrm{Gyr}^{-1} \tag{8}$$

and

$$\frac{dM_b}{dt} \approx 6.6 \left(\frac{M_h}{10^{12} M_\odot}\right)^{1.15} (1+z)^{2.25} \left(\frac{f_{\rm gas}}{0.165}\right) M_\odot/\mathrm{yr}. \tag{9}$$

We see that these differ quantitatively from the relations derived above, but there are significant qualitative similarities.

Using these more accurate rates, one finds that at $z = 2.2$ the accretion rate of baryons is $dM_b/dt \approx 200 M_\odot/\mathrm{yr}$ for a $M_h = 2 \times 10^{12} M_\odot$. This is close to the actual star formation rate of $z \sim 2$ galaxies, so it is tempting to look for a causal connection. However we have to ask then whether the gas will reach the galaxy in the center in a cold state or would need to cool significantly.

3 The cooling radius

Heavily based on Mo, van den Bosch & White, section 8.4

We will now move to study how gas cools within a dark matter halo. We will focus on cooling so the dark matter component is not important here (although it does set up the dark matter potential). The main goal of this section is to get a feel for how the cooling time evolves with radius and time.

The cooling time, $t_{\rm cool} = \dot{E}/E$, can be written for any point in a cloud as well as you can write it for the whole system. In this case, for a spherically symmetric cloud you can write

$$t_{\rm cool} = \frac{\frac{3}{2} n(r)\, k_B T(r)}{n_H^2 \Lambda(T)}, \tag{10}$$

where the electron density, n_e, and the hydrogen density, n_H are tied through the ionization balance at each point. For our use here we will assume that their ratio is constant which is a reasonable approxiation.

If we have set up density and pressure profiles before heat loss is significant, so adiabatic profiles, we can assume that they will take the form

$$\rho(r) = \rho_0 \left(\frac{r}{r_0}\right)^{-\alpha} \tag{11}$$

$$P(r) = P_0 \left(\frac{r}{r_0}\right)^{-\beta}, \tag{12}$$

and for an ideal gas we will then have

$$T = \frac{P_0}{\rho_0}\mu m_p \left(\frac{r}{r_0}\right)^{\alpha-\beta} = T_0 \left(\frac{r}{r_0}\right)^{\alpha-\beta}, \tag{13}$$

from which it is obvious that the isothermal sphere has $\alpha = \beta$ and since $\rho \propto r^{-2}$ for the singular isothermal sphere (SIS), we see that in this case $\alpha = 2$.

The cooling function can be approximated as

$$\Lambda(T) \approx \Lambda_0 \left(\frac{T}{T_0}\right)^{\nu} \tag{14}$$

in a range in temperature. We will however see that for the SIS this will cancel out.

We can then use equation (10) for the cooling time at a given time in this dark matter halo and find that

$$t_{\rm cool}(r) = \frac{3}{2}\frac{n(r)}{n_H(r)^2} k_B T_0 \left(\frac{r}{r_0}\right)^{\alpha-\beta} \Lambda_0^{-1} \left(\frac{T}{T_0}\right)^{-\nu}, \tag{15}$$

which by using that $n(r) = \rho(r)/\mu m_p$ we can write as

$$t_{\rm cool}(r) = t_0 \left(\frac{r}{r_0}\right)^{1/\tau}, \tag{16}$$

with $1/\tau = \alpha + (1-\nu)(\alpha - \beta)$ which is $\tau = 1/2$ for the SIS. Thus for the SIS we can also write

$$r_{\rm cool} = r_0 \left(\frac{t}{t_0}\right)^{1/2}. \tag{17}$$

Thus in this case the cooling propagates from the inside out and it is referred to as a cooling wave.

It is also possible to add up the mass that is cooling out at $r_{\rm cool}$ at a given time. This is

$$\begin{aligned} \dot{M} &= 4\pi r_{\rm cool}^2\, \rho(r_{\rm cool})\, dr_{\rm cool} \\ &= 3M_0 \left(\frac{\tau}{t_0}\right) \left(\frac{t}{t_0}\right)^{(3-\alpha)\tau-1}, \end{aligned} \tag{18}$$

where $M_0 = 4\pi\rho_0 r_0^3/3$ and which can be integrated up to give the mass as

$$M(t) = 3M_0 \left(\frac{t}{t_0}\right)^{1/2} \tag{19}$$

for the SIS. Quantitatively for the SIS we have

$$t_0 \approx 1.48 \times 10^{11} \left(\frac{r_0}{1\ \mathrm{Mpc}}\right)^2 \left(\frac{\Lambda}{10^{-23}\ \mathrm{erg\ cm^3/s}}\right)^{-1} \mathrm{yrs} \tag{20}$$

$$r_{\mathrm{cool}} \approx 0.26 \left(\frac{t}{10^{10}\ \mathrm{yrs}}\right)^{1/2} \left(\frac{\Lambda}{10^{-23}\ \mathrm{erg\ cm^3/s}}\right)^{1/2} \mathrm{Mpc} \tag{21}$$

$$M_0 \approx 2.72 \times 10^{13} \left(\frac{T}{10^6\ \mathrm{K}}\right) \left(\frac{r_0}{1\ \mathrm{Mpc}}\right) M_\odot \tag{22}$$

$$\dot{M} \approx 276 \left(\frac{T}{10^6\ \mathrm{K}}\right) \left(\frac{r_0}{1\ \mathrm{Mpc}}\right)^{-1} \left(\frac{t}{t_0}\right)^{1/2} \left(\frac{\Lambda}{10^{-23}\ \mathrm{erg\ cm^3/s}}\right) M_\odot/\mathrm{yr} \tag{23}$$

4 Cold and hot accretion — how do halos get their baryons

We will now move on to a discussion of how gas is accreted onto galaxy haloes. The virial radius defines an extent of a dark matter halo and in the absence of cooling, baryonic matter within this radius will be heated to a temperature close to the virial temperature. It is therefore of considerable interest to try to compare this to the cooling radius derived in the previous section. We will not discuss all the details here, see Mo, van den Bosch & White, section 8.4 for more details and a detailed discussion is in Birnboim & Dekel (2003, MNRAS, 345, 349). However the results we derive are broadly similar to those from the more detailed analysis.

We first note that, as we have shown earlier, that the virial radius can be written

$$r_{\mathrm{vir}} = 250 \left(\frac{M}{10^{12}\ h^{-1}\ M_\odot}\right)^{1/3} (1+z)^{-1} h^{-1}\ \mathrm{kpc}, \tag{24}$$

for $\Omega_m = 0.3$. This is remarkably similar to equation (21) in numerical value. If we set $r_{\mathrm{cool}} = r_{\mathrm{vir}}$, and use that the age of the Universe is given by $t = 2/3(1+z)^{-3/2} 10^{10} \Omega_m^{-1/2} h^{-1}\mathrm{yr}$ at high redshift, we get

$$\frac{\Lambda}{10^{-23}\ \mathrm{erg\ cm^3/s}} \approx 2.47 \left(\frac{M}{10^{12} h^{-1}\ M_\odot}\right)^{2/3} (1+z)^{-1/2} \tag{25}$$

$$\approx 10.8 \left(\frac{T_{\mathrm{vir}}}{10^6\ \mathrm{K}}\right) \left(\frac{\mu}{0.59}\right)^{-1} (1+z)^{-3/2}\, h^{-1} \tag{26}$$

This defines a critical cooling level, above which the cooling radius is outside the virial radius. The locus of this relation is illustrated in Figure 1 where three lines corresponding

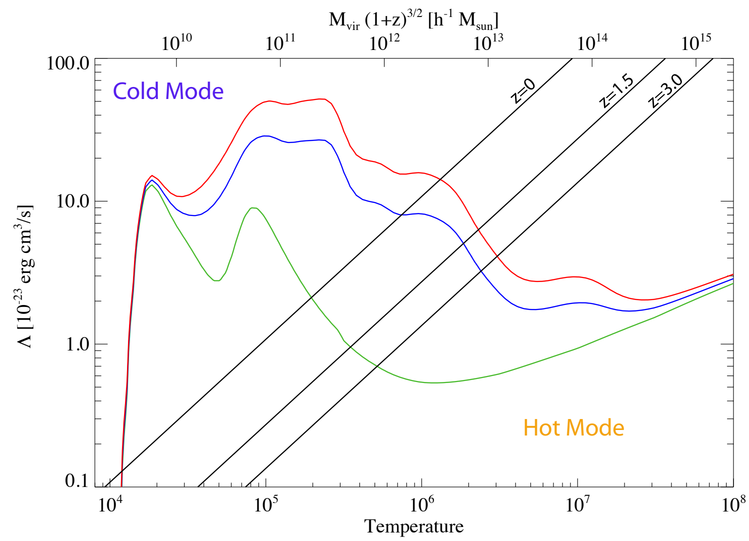

Figure 1: The cooling function of primordial gas (black), 1/2 solar metallicity gas (blue) and solar metallicity gas (red). Overplotted are the tracks for the critical Λ using equation (25).

to $z = 0$, 1.5 and 3.0 are shown on top of the cooling functions for zero metallicity, half solar and solar metallicity going from the lowest to the highest. What is clear is that at high redshift a large number of halos will have a cooling radius that is outside the virial radius in this simple picture.

In general one would expect to have a shock at the virial radius where gas from the surrounding inter-galactic medium falls into the high pressure environment of the virialised halo. This is indeed the case when the cooling radius is within the virial radius. In that case gas falling into the halo will be heated up to $\approx$ the virial temperature of the halo and then slowly cool down to form baryonic structures. This is often referred to as hot mode accretion.

In the alternative scenario there is a substantial cooling already at the virial radius and in addition, Birnboim & Dekel (2003) showed that any shock developing at the virial radius in this situation is not stable. Thus in this case the gas entering the halo will not be shock heated to high temperatures but will rather reside in what cosmologists call the cold phase, what interstellar medium researchers will refer to as the warm phase, around $T \sim 10^4$K. This will then be able to stream efficiently in to the center of the halo where the "cold" gas potentially will feed the formation of galaxies there.

4.1 Two phases?

In the preceding the terms cold and hot phase were thrown around, but is there really a distinct set of phases in the dark matter halo? The fact of the matter is that it is. In fact there are at least three: At low temperature there is a peak in the cooling curve at

$T \sim 10^2$K — when you add heating to a system around that temperature it will mostly be able to cool efficiently and stay at that temperature. This is what we would refer to as the cold phase in the ISM. But what happens if you continue to add heat? Eventually you will start to heat the system too much for the low temperature cooling to work and as the cooling curve then declines with temperature you have an unstable situation: Adding more heat means less cooling and hence increasingly higher temperature.

This process will continue until $T \sim 10^4$K at which point cooling due to collisionally excited transitions will be efficient. Around that temperature we are again in a stable situation — increasing the temperature will increase the cooling and hence regulate the temperature. This then is the warm medium (if you talk about the ISM), usually called the cold phase by cosmologists.

If you then add more heat, you will eventually ionized the atoms fully and again you are in an unstable situation where adding heat decreases cooling and hence increases the temperature. This contnues until Bremsstrahlung starts to dominate, which at primordial abundances happens around $T \sim 10^9$ and later for more metal rich systems (c.f. Figure 1). This then makes up the hot phase.

5 Spherical and non-spherical accretion

The concept of spherical accretion and accretion shocks which we looked at above, has been known since at least White & Frenk (1991, ApJ, 1991, 52). In reality, however, the Universe is not that smooth and we saw in an earlier lecture that the morpohlogy of the large-scale structure is expected to consist of sheets, filaments and halos. And it is therefore reasonable to ask whether accretion really happens spherically or along filaments and what the consequence of this would be.

Figure 2 shows an illustration of the temperature and density of gas around galaxy halos from van de Voort et al (2011, MNRAS, 414, 2458). What we can see here is that in low mass halos there is significant streams of cold gas penetrating the virial radius and not getting shocked. This importance of non-spherical accretion of "cold" gas along filaments was first highlighted by Katz et al (2003, ASSL conf. proc. 281) and has been developed extensively in the last 10 years.

The nomenclature that has appeared for this topic is to talk about gas that has $T \sim 10^4$K as "cold" and call gas with $T \gg 10^5$K "hot". The most relevant way of looking at this is to take this temperature to be the maximum past temperature. If a gas particle has never been heated above $T > 10^5$K it will not have been processed by an accretion shock in a massive halo.

In simulations it is found that hot mode accretion leads to fairly spherical accretion, while the cold mode accretion in halos with virial temperature $> 10^4$K typically is found to proceed along filaments. Thus the accretion geometry is fairly different in these two modes. When this is studied it is found that the importance of hot mode accretion *onto halos* increases with halo mass. The rate is in fairly good agreement with what you would expect from a straightforward application of the extended Press-Schechter formalism. For

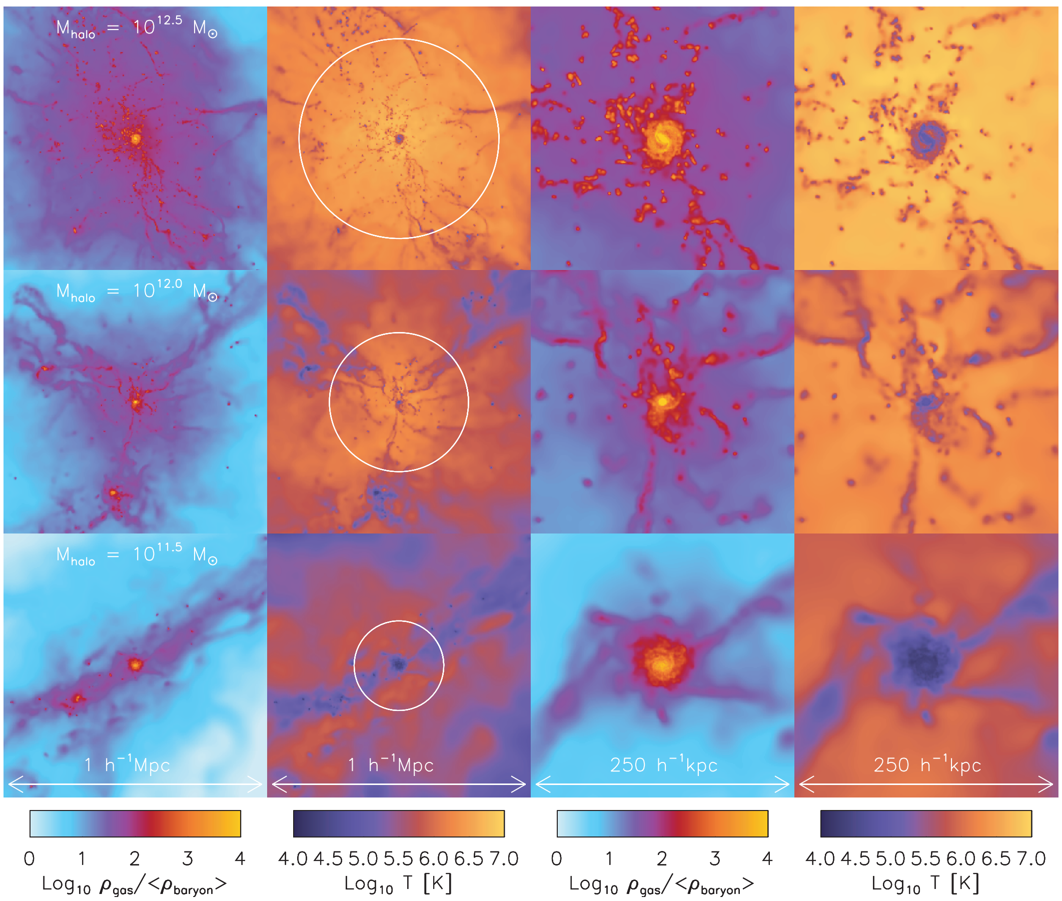

Figure 2: An illustration of the physical conditions around a galaxy halo from the OWLS simulations as presented in van de Voort et al (2011, MNRAS, 414, 2458). The two left-most panel shows a region of 1 h^{-1} comoving Mpc around halos of different mass as indicated, while the right-most columns show a zoom of the central 250 h^{-1} comoving kpc. The white circles show the virial radius and the panels show the gas overdensity and temperature as indicated. Note that in low-mass halos there are "cold" streams of matter penetrating the virial radius.

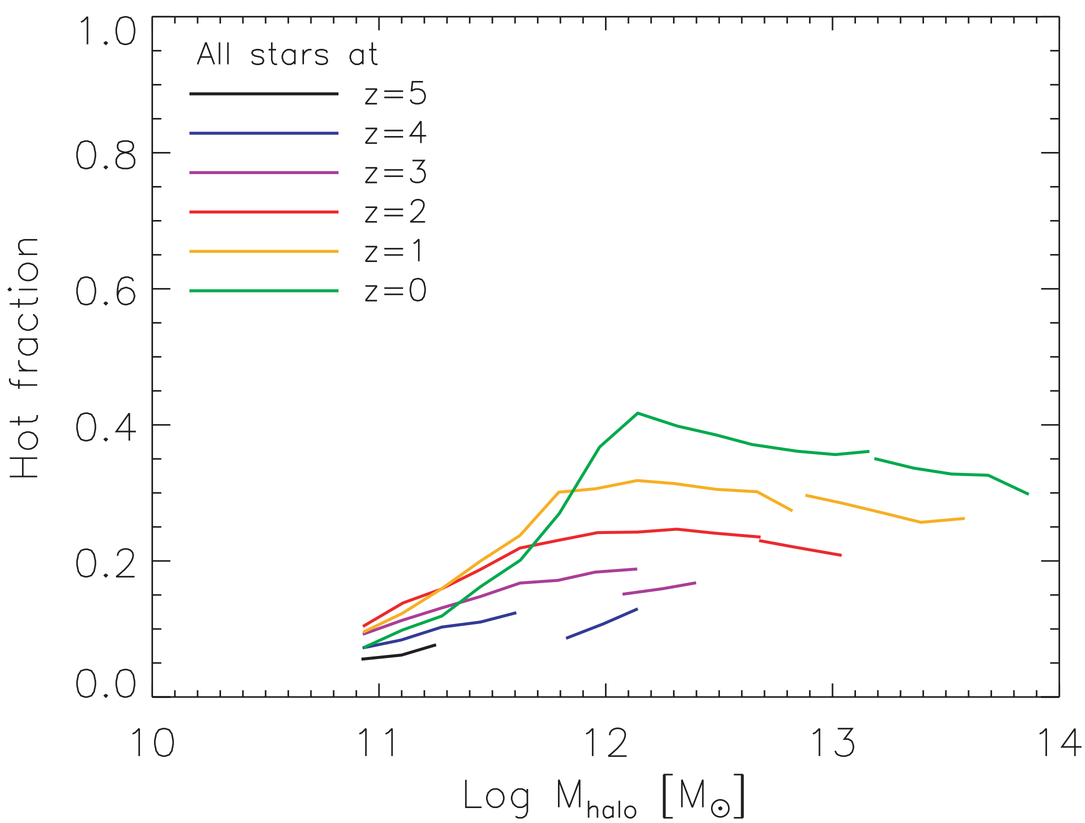

Figure 3: The fraction of stars that have formed from gas that have been heated to $T > 10^5$K at some point in the past as a function of halo mass and redshift (different colours). It is clear that hot mode accretion was unimportant at high redshift and at low redshift its importance increases with mass until it reaches an approximate threshold at high mass. Figure from van de Voort et al (2011, op cit).

a recent calculation you can refer to the van de Voort paper referenced above.

However to accrete onto galaxies inside the halos it is necessary that the gas cools down and it is not obvious that the same is true for accretion onto galaxies, and indeed it is found that the increase is less pronounced than that for accretion onto halos. It is nice to then summarise this by asking how much of all stars formed from gas that had been accreted through hot accretion. This is shown in Figure 3. In that figure the different colours correspond to the fraction of stars formed up to a given redshift from gas that has arrived as hot accretion. This is plotted against halo mass.

The most immediate conclusion from this figure is that at high redshift cold mode accretion was the main way to feed galaxies and star formation. This theoretical prediction has spawned a lot of observational studies in recent years.