Lecture 9

Jarle Brinchmann

1/04/2014

1 Recap

Last lecture we finished off our look at the halo model with a brief discussion of how one can incorporate galaxies in this formalism and stressed the difference between central galaxies and satellites. We then compared the cooling time-scale with the expansion and collapse time-scales to identify in what part of the temperature-density diagram gas structures could collapse.

We also looked briefly at the physical origin of different cooling mechanisms. Of the various radiative cooling processes, the main cooling source at high temperature is Bremsstrahlung, or free-free emission. This dominates above temperatures of $T \sim 10^7$K and the cooling is then $\propto n_e^2 T^{1/2}$. The low temperature cut-off in the cooling curve is set by the ionization cut-off of H. By considering the detailed equilibrium between collisional ionization and recombination it is possible, but cumbersome, at least analyctically, to find that if the ionization energy is ΔE, the fractional ionization is about 50% for $E \gtrsim \Delta E/10$. For hydrogen this is $\sim 10^4$K and a similar value cuts off collisional excitation as the excitation energy $n = 1 \to n = 2$ in hydrogen is ≈ 10.6eV.

Finally we discussed that the peaks in the cooling function were caused by collisional excitation, followed by radiative deexcitation.

Today we will complete our discussion of the main cooling mechanisms with a discussion of Compton cooling and the effect of photoionization heating before moving on to the structure of gas in halos.

2 Compton cooling

If you have radiation passing through a plasma with free electrons, the radiation and matter will interact through Compton scattering. It turns out (e.g. B1.3.6 in Mo, van den Bosch & White), that the change in the energy density of the radiation, u_γ, can be written

$$\frac{du_\gamma}{dt} = \frac{4k_B}{m_e c}\sigma_T n_e u_\gamma (T_e - T_\gamma), \tag{1}$$

where $\sigma_T = (8\pi/3)(q^2/m_e c^2)^2 \approx 6.65 \times 10^{-25}\,\mathrm{cm}^{-2}$ is the Thomson scattering cross section, n_e is the electron number density, T_e is the electron temperature and T_γ is the temperature of the radiation.

In our case the photons come from the cosmic microwave background (CMB) and we have $T_\gamma \ll T_e$. Thus we have a net gain of energy in the photons and hence a net loss in the electrons. This scattering process is known as inverse Compton scattering and the cooling as Compton cooling.

In that case we have that the cooling, $\dot{E}$ is

$$\dot{E} \propto n_e u_\gamma T_e \propto n_e T_\gamma^4 T_e \propto n_e T_e (1+z)^4, \tag{2}$$

since the energy density in radiation is $u_\gamma = a T_\gamma^4$, and the temperature of the CMB declines as $1+z$.

This cooling is therefore more efficient at high redshift, but from the discussion in the previous lecture we know that it would not work at arbitrarily high redshifts because there would be essentially no free electrons around to interact with.

It can work before the time of re-ionisation if you form a large enough dark matter halo that the virial temperature is above $\sim 10^4$K, in which case the shock heating of the gas is sufficient to ionise that halo. However very few such halos are expected to exist at $z > 10$ (c.f. Table 1 in last weeks notes) so this is not a wide-spread phenomenon.

We can use the expressions above to calculate the cooling time due to Compton scattering. The energy density in the plasma is $E \propto n_e k T_e$, thus

$$t_{\mathrm{cool}} = \frac{E}{\dot{E}} \propto \frac{k T_e n_e}{n_e T_e (1+z)^4} \propto (1+z)^{-4}. \tag{3}$$

It is therefore independent of T and of the electron density n_e. It is independent of temperature only insofar as there are a lot of electrons around — should the temperature so that the matter is mostly neutral, the Compton cooling will be inefficient. Quantitatively we have

$$t_{\mathrm{cool,Compton}} \approx 2.3 \times 10^{12} (1+z)^{-4}\ \mathrm{years}. \tag{4}$$

However the independence of density and temperature means that Compton cooling can act on reasonable time-scales in regions where the other cooling processes are inefficient.

Clearly the time-scale for Compton cooling given in equation (17) is too long to be useful at $z = 0$. To get a feel for where it is of importance, we can set $t_H = t_{\mathrm{cool}}$ and we have that Compton cooling is efficient for $z > 6.2$, at lower redshift the time-scale is longer than the age of the Universe. However to be dynamically important t_{cool} must also be shorter than the dynamical time, t_{dyn}. This translates into

$$\left(\frac{3\pi f_{\mathrm{gas}}}{32 G \mu m_p n}\right)^{1/2} = 2.3 \times 10^{12} (1+z)^{-4}, \tag{5}$$

which clearly becomes a function of density and gas fraction. This translates into

$$1 + z \approx 13.6 \left(\frac{n}{1\ \mathrm{cm}^{-3}}\right)^{1/8} f_{\mathrm{gas}}^{-1/8}. \tag{6}$$

which gives a typical redshift of $z \sim 10$, depending on the assumptions made for n — an n associated to an over density a factor of 10 above the mean will have efficient Compton cooling (relative to $t_{\rm dyn}$) until $z \approx 7$, while at a density contrast of 200 above mean, Compton cooling is only efficient at $z > 13.5$. Thus Compton cooling is clearly important in the early Universe, but only at the highest redshifts.

3 Heating

In a real situation there is not only cooling of course — there will also be an important heating contribution. An important heating source in the cosmological context is photoionization heating — it is not the only one of course but it is the one we will discuss here. In Figure 1 the effect of adding a UV ionization field to the cooling calculations is shown in the right-hand panel. Comparing this to the left-hand panel which shows the cooling function for a collisional ionization equilibrium, we note that there is less cooling at $T \sim 10^4$K and the first peak due to hydrogen is mostly gone.

The reason for these changes can be summarised as follows

- The photoionization background leads to a higher ionization of the elements, thus the neutral fraction of hydrogen drops significantly so that there are very few bound electrons left for collisional excitation in hydrogen to be efficient. Thus the hydrogen peak disappears.

- Because of the added photoionization, there will be more free electrons left even at lower temperatures. Since the cooling processes we considered last time require free electrons, there will now be a significantly increased cooling at temperatures a bit below 10^4K. Although the photoionization will also dissociate H_2 which will reduce the cooling at lower temperature.

- The additional electrons will also increase the efficiency of Compton cooling, allowing it to operate also in halos with low virial temperature.

More formally, the rate of photoionization is naturally related to the flux of ionization photons. This is generally given by

$$\Gamma_{\rm pi} = \int_{\nu_{\rm pi}}^{\infty} \frac{4\pi J(\nu)\sigma_{\rm pi}}{h\nu} d\nu, \tag{7}$$

where the term in the integrand gives the number of photons per second per Hertz. $\nu_{\rm pi}$ is the frequency corresponding to the ionization energy $\nu_{\rm LyC} = 13.6{\rm eV}/h$ for hydrogen; $\sigma_{\rm pi}$ is the photoionization cross section, and for hydrogen this is given by

$$\sigma_{\rm LyC} \approx 6.3 \times 10^{-18} \left(\frac{\nu}{\nu_{\rm LyC}}\right)^{-3} \ {\rm cm}^2, \tag{8}$$

Wiersma, Schaye & Smith 2009, MNRAS, 393, 99

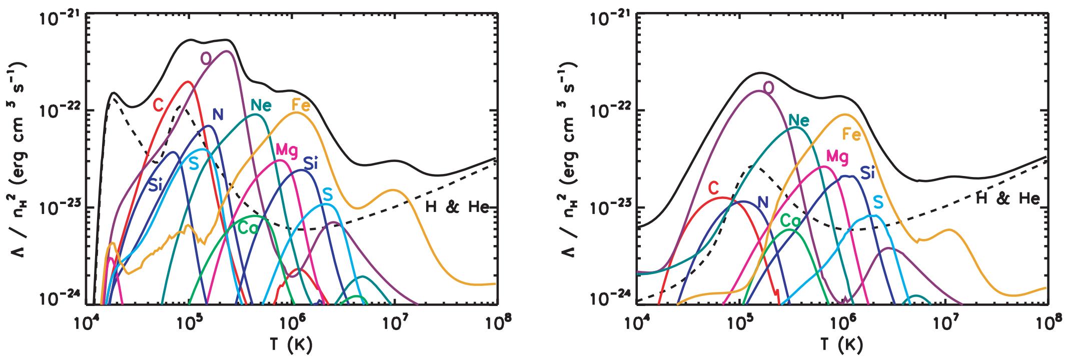

Figure 1: Cooling curves from Wiersma et al (2009) for solar abundance plasma. The left hand panel shows the cooling function for CIE and the right for photo-ionization equilibrium. The different colours show the cooling due to different elements.

where LyC denotes the Lyman continuum which corresponds to an energy of 13.6 eV, a wavelength of $\lambda_{\rm LyC} = 912$Å and a frequency of $\nu_{\rm LyC} \approx 3.29 \times 10^{15}$Hz.

It is useful to see how this integral behaves if the ionizing radiation is given by a power-law (e.g. Haardt & Madau 1996, ApJ, 461, 20)

$$J(\nu) = \left(\frac{\nu_{\rm LyC}}{\nu}\right)^{\beta} J_{-21}\ \text{erg cm}^{-2}\ \text{Hz}^{-1}\ \text{sr}^{-1}, \tag{9}$$

which upon insertion in equation (7), and only considering hydrogen, gives us

$$\Gamma_{\rm pi,H} \approx \frac{1.2 \times 10^{-11} J_{-21}}{\alpha + 3}\ \text{s}^{-1} \tag{10}$$

Given these generic scalings it is also common to quote $\Gamma_{\rm pi,H}$ to units of of 10^{-12} s^{-1}.

We can also easily calculate the energy input from photoionisation by writing (again specialising to hydrogen):

$$\Lambda_{h,H} = n_{\rm HI} \int_{\nu_{\rm LyC}}^{\infty} \frac{4\pi J(\nu)\sigma_{\rm LyC}(\nu)}{h\nu} h(\nu - \nu_{\rm LyC}) d\nu, \tag{11}$$

where the integrand in equation (7) has been augmented by the energy per photon. We also see that this expression is proportional to the density of hydrogen, and not to the density squared as is true for (most) cooling processes. We can also evaluate this integrand and find that

$$\Lambda_{h,H} \approx \frac{3.3 \times 10^{-21} J_{-21}}{\alpha^2 + 5\alpha + 6}\ \text{erg/s}. \tag{12}$$

Now consider a region of the Universe with sufficiently low density that we can consider Case A conditions (ie. that photons resulting from recombinations to the ground state escape the region without ionising hydrogen further). If we have an ambient UV background given by equation (9), we can see that the typical ionisation rate is quick and if we assume that recombinations also are swift we will set up photo-ionisation equilibrium, in which the number of ionisations should balance the number of recombinations, or in other words

$$n_{\rm HI}\Gamma = \alpha_r n_e n_{\rm HII}, \tag{13}$$

where α_r is the Case A recombination coefficient which can be approximated by (e.g. Draine 2011)

$$\alpha_r \approx 4\times 10^{-13} T_4^{-0.713}\ {\rm cm^3 s^{-1}} \tag{14}$$

Now, firstly we can use this to estimate a time-scale for recombinations to assess whether an equilibrium is expected. This gives us

$$t_{\rm rec} \approx 1.4\times 10^7\ {\rm yr}\left(\frac{T}{10^4\ {\rm K}}\right)^{0.713}\left(\frac{\Omega_B h^2}{0.022}\right)^{-1}\left(\frac{1+z}{11}\right)^{-3}\left(\frac{\Delta}{10}\right), \tag{15}$$

where Δ is the overdensity relative to the mean density. Since the Hubble time at $z = 10$ is of the order a Gyr, we see that it is reasonable to expect a photo-ionisation equilibrium here.

Secondly we can use this to study the ionisation state of the Universe, something we will return to this in later lectures but for now we will do this to highlight a puzzle. If we take equation 13 for photoionisation equilibrium we can calculate the fractional ionization, $x = n_e/n$, which is given by

$$\frac{1-x}{x^2} \approx 10^{-7}\,(3+\alpha)J_{-21}^{-1}\,\Omega_B h^2(1+z)^3. \tag{16}$$

From this equation it is possible to conclude that the ionization must be very large, with a neutral fraction $1-x \sim 10^{-7}$. Thus how can we form stars?

The answer to this conundrum lies in the fact that sufficiently dense clumps are optically thick to this ionizing radiation. With a column density of hydrogen atoms, $N_{\rm HI}$, we have that the optical depth to ionizing radiation is

$$\tau \approx N_{\rm HI}\sigma_{\rm ionization}, \tag{17}$$

for radiation with energy close to the Lyman limit. This τ is equal to unity for

$$\tau \sim 1 \quad \Rightarrow \quad N_{\rm HI} \sim 10^{17}\ {\rm cm}^2, \tag{18}$$

and for slightly larger values for higher energy photons because the cross-section declines with increasing energy. So as long as a gas cloud has sufficient column density, it will be shielded from ionising radiation and is free to cool further — at least under the approximations considered here.

4 The distribution of gas in dark matter halos

In previous lectures we have discussed the formation of dark matter halos and last lecture we looked at their general shape. Now we want to put gas into these halos and ask how this will be distributed. In general we would expect this to be different from the dark matter because baryons are subject to pressure forces/

We can work this out by considering that the system is in hydrostatic equilibrium. If that is the case, and that might not always be obvious, we have that

$$\nabla P = -\rho \nabla \phi \qquad \nabla^2 \phi = 4\pi G \rho. \tag{19}$$

In spherical symmetry and assuming that the gas is ideal this gives the radial pressure gradient as

$$\frac{dP}{dr} = \frac{d}{dr}\left(\frac{k_B T \rho}{\mu m_p}\right) = \frac{k_B}{\mu m_P}\left(\rho \frac{dT}{dr} + T\frac{d\rho}{dr}\right), \tag{20}$$

where I have also assumed that there is no ionisation gradient in the system ($d\mu/dr = 0$). This should be balanced by the radial gravity component

$$\frac{d\phi}{dr} = \frac{GM}{r^2} \tag{21}$$

which gives

$$M(<r) = -\frac{k_B T(r) r}{G \mu m_P}\left(\frac{d\ln T}{d\ln r} + \frac{d\ln \rho}{d\ln r}\right). \tag{22}$$

One point worth noting here is that it is useful to introduce logarithmic derivatives whenever you suspect the quantities might be reasonably represented by power-laws. This expression is approximate as it ignores abundance gradients and other sources of pressure, but it can be used to estimate masses of galaxy clusters although to get accurate masses a more careful modelling must be carried out.

Now in itself this formalism does not predict the run of temperature with radius. This has to be provided in another way — a particularly simple version is to consider the case of an isothermal sphere. This is also often handy as an approximation for us. In this case obviously the derivative of T is zero so we are left with

$$M(<r) = -\frac{k_B T r}{G \mu m_P}\frac{d\ln \rho}{d\ln r}. \tag{23}$$

This can be solved by combining it with the Poisson equation in spherical coordinates

$$\frac{1}{r^2}\frac{d}{dr} r^2 \frac{d\phi}{dr} = 4\pi G \rho. \tag{24}$$

The general solution is discussed in chapter 4.4 in Binney & Tremaine "Galactic Dynamics" or 8.2 in MvdBW. For us it is sufficient to focus on the singular isothermal sphere — in this case we make the *ansatz* that the density distribution is $\rho \propto r^{-b}$. By insertion into the preceding equations we find that $b = 2$, giving us:

$$\rho(r) = \frac{2k_BT}{\mu m_P} \frac{1}{4\pi G r^2} \tag{25}$$

$$M(r) = \frac{2k_BT}{\mu m_P} \frac{r}{G}, \tag{26}$$

but it is also common, as we have seen before, to see these relations phrased in terms of the circular velocity, $V_c = \sqrt{GM/r}$, which gives:

$$V_c^2 = \frac{2k_BT}{\mu m_P} \tag{27}$$

$$\rho(r) = \frac{V_c^2}{4\pi G r^2} \tag{28}$$

$$M(r) = \frac{V_c^2 r}{G}. \tag{29}$$

Another interesting case is the case of an adiabatic gas, so that $P = A\rho^\Gamma$ with Γ and A constants. In that case the equation for hydrostatic equilibrium gives us

$$\frac{dP}{dr} = -\rho \frac{d\phi}{dr} \tag{30}$$

$$\frac{A}{\rho} \frac{d\rho^\Gamma}{dr} = -\frac{d\phi}{dr} \tag{31}$$

$$A \frac{\Gamma}{\Gamma - 1} \frac{d\rho^{\Gamma-1}}{dr} = -\frac{d\phi}{dr} \tag{32}$$

which can be integrated to give

$$k_BT(r) = \mu m_p \phi(r), \tag{33}$$

so in this case the temperature follows the potential.

In reality the effect of the dark matter halo on the baryon distribution and of baryons on the dark matter is complex and we will return to this later, but this gives a good starting point for systems like galaxy clusters where a significant amount of the baryons is distributed in a hot phase throughout the halo.

5 Cooling flows

Peacock chap 17; Peterson & Fabian (2006, Physics Reports, 427, 1)

In our discussion of cooling until now, we have focused on uniform media. This is of course an over-simplifcation and density variations in a system will lead to variation in the cooling time. In particular, in a galaxy cluster, the central parts are likely to be significantly denser than the outer parts and hence they will have a shorter cooling time (which goes $\sim 1/\rho$). It is therefore possible that there will be a region in the central part of the cluster where cooling is efficient and material that enter this region will cool and lose pressure support and hence flow inwards in the cluster.

Before continuing onto a more careful assessment of the cooling in halos in general, it is therefore useful to look at the change in energy in a cluster. The appropriate quantity in this case is the enthalpy, $H = U + pV$, where U is the internal energy, p the pressure and V the volume of the system. We are interested in the *change* in enthalpy:

$$dH = dU + pdV + Vdp. \tag{34}$$

The challenging entry here is Vdp. We are interested in the time-evolution of this quantity, and if there is no significant change in the external pressure, we can approximate this to be zero. This is a decent approximation if the external pressure is set by the hot "atmosphere" of gas in the halo because this changes rather slowly. So here we will set $Vdp = 0$, what is known as isobaric cooling, but note that this can break down in real galaxy clusters, see the Peterson & Fabian (2006) review for an in-depth discussion.

The internal energy in a small volume, dV, is given by

$$dU = \frac{3}{2}\frac{\rho}{\mu m_p} kTdV, \tag{35}$$

for a mon-atomic idealised gas. The pdV term is given by

$$pdV = \frac{\rho}{\mu m_p} kTdV, \tag{36}$$

so in all we find that the change in enthalpy is

$$dH = \frac{5}{2}\frac{\rho}{\mu m_p} kTdV. \tag{37}$$

This energy loss will be radiated away, so the luminosity of the system will be equal to the change in enthalpy, so making use of the fact that $\rho dV = dM$ we can write this as

$$\frac{dH}{dt} = L(< r) = \frac{5}{2}\frac{kT(r)}{\mu m_p}\dot{M}, \tag{38}$$

where the mass accretion rate, $\dot{M}$, determines the luminosity and if the accretion continued all the way to very small scales, it would lead to very sharply peaked surface brightness there which is inconsistent with observations.

In fact observations tend to show accretion rates of $\dot{M} \sim 100 - 1000 M_\odot/\text{yr}$ which, if they proceeded to the center of the cluster, would lead to star formation rates of a similar order of magnitude. However, while the central galaxies in massive clusters certainly are more active than similar galaxies in the field, they have star formation rates much lower than this. Thus we need to stop this gas from cooling out.

Said in another way, the centers of massive clusters have a short cooling time, yet when they are observed with X-ray telescopes they show little sign of cooling. This puzzle is sometimes called the cooling flow problem. To resolve it, one needs a distributed heating source and several suggestions exist, such as AGN feedback, radio bubble heating, conduction, heating from the infall of clumpy material etc. An overview of some of these mechanisms is given in Peterson & Fabian's review cited at the beginning of the section.