Lecture 8

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1 Recap

Last week we looked at the halo model for large scale structure. The essential ingredients we discuss there were:

- In the model it is assumed that all dark matter is in virialised dark matter halos and that their density profiles are universal and only depend on the mass of the halo.
- Given this assumption, when we calculated the correlation function we found that it could be written as a sum of two terms:

$$\xi(r) = \xi^{1h}(r) + \xi^{2h}(r), \tag{1}$$

  which we called the 1-halo and 2-halo term. The one halo term contains correlations between matter *within* the halo and the 2-halo term captures large scale structure.
- On sufficiently large scales, so that the density distribution of the halo is irrelevant, we saw that the two-halo correlation function approaches the linear correlation function.

The final part of the lecture was a discussion of how you would include galaxies in this model which we will complete today. This follows the same structure as the halo model but you need to have an estimate of the expected number of galaxies per halo of mass M. One way to do this was to use the conditional luminosity function, but we closed by mentioning that to use this correctly you need to distinguish between the central galaxy of a halo and its satellites.

We also briefly discussed the virial relations for collapsing structures. We saw that the $\rho > \Delta_c \bar{\rho}$ criterion for collapse of a structure, which we derived from the model for a collapsing spherical perturbation, gave us an expression for the radius of the virialised object which was:

$$r_{\rm vir} = \left(\frac{2G}{H_0^2 \Delta_c}\right)^{1/3} M^{1/3} (1+z)^{-1} \Omega_m^{-1/3}. \tag{2}$$

Combining this with the virial theorem we found an expression for the virial temperature of a mon-atomic ideal gas

$$T_{\rm vir} = \frac{1}{5}\frac{GM\mu m_P}{r_{\rm vir}k_B} = \frac{1}{5}\frac{\mu m_P}{k_B}V_c^2, \tag{3}$$

where the pre-factor 1/5 is appropriate for a uniform sphere. The second equality expresses this in terms of the circular velocity of the virialized halo. This is

$$V_c = \left(\frac{GM}{r_{\rm vir}}\right)^{1/2} \tag{4}$$

$$= \left(\frac{G^2H_0^2}{2}\right)^{1/6} \Delta_c^{1/6}\Omega_m^{1/6}M^{1/3}(1+z)^{1/2}. \tag{5}$$

These are useful relations to have in mind throughout.

2 Putting the tools together

On the basis of this we can now go back and start calculating the properties of perturbations of various significance. To this we start with the density perturbation, δ, field. This is distributed as a Gaussian with a variance given by $\sigma(M)^2$, where

$$\sigma^2(M) = \frac{1}{2\pi^2}\int_0^\infty k^2 P(k)\hat{W}(k;R)^2 dk. \tag{6}$$

If you are given a power spectrum and a window function this is a straight-forward integral to do, although it typically needs to be done numerically.

If we now want to find peaks of various significance, we need to compare δ to $\sigma(M)$. Here in particular we are interested in *collapsed* peaks. This means that we know the δ threshold — for a collapsed object at time t we have $\delta = \delta_c(t)$, so for a $N\sigma$ peak we need to solve

$$\nu = \frac{\delta_c(t)}{\sigma(M)} = N \tag{7}$$

for M. This then give the mass of a collapsed halo with that significance. We then use this mass to estimate the virial temperature and circular velocity. The result of these calculations can be seen in Table 1.

3 Time-scales and collapse

After virialisation we are left with a dark matter halo that cannot collapse much further because dark matter is non-interactive and non-dissipative. In contrast, the gaseous (baryonic) component within this halo *can* collapse further. However the the virial temperatures

	$z = 5$			$z = 10$		
	1σ	2σ	3σ	1σ	2σ	3σ
$M_b(M_\odot)$	1.6×10^7	1.2×10^{10}	2×10^{11}	3.7×10^3	4.8×10^7	2.5×10^9
$T_{\rm vir}$ (K)	3.9×10^3	3×10^5	2×10^6	26	1.5×10^4	2×10^5
V_c (km/s)	10	93	238	0.86	20	75

Table 1: Characteristic scales at two different redshifts calculated as described in the text. M_b is the total baryonic matter in the halo calculated using the baryon density of the Universe.

shown in Table 1 are all quite high and because for an ideal gas the pressure is proportional to the temperature, this means that there will be significant pressure forces that can balance the gravitational collapse.

To collapse the gas component further we therefore need to radiate away part of the energy that keeps the gas supported against gravity. The ingredient here is how *fast* the energy can be radiated away. This is quantified using the cooling time-scale

$$t_{\rm cool} = \frac{E}{\dot{E}} \tag{8}$$

and by comparing this to other time-scales of interest. For our situation there are two key time-scales: The time-scale for the expansion of the Universe. This is roughly given by the inverse of the Hubble parameter,

$$t_H \sim H(z)^{-1}, \tag{9}$$

and the time-scale of free-fall collapse. This is also often called the dynamical time-scale

$$t_{\rm dyn} \sim (G\rho)^{-1/2}. \tag{10}$$

This is the time-scale that collapse would proceed on if there were no pressure forces operating. For a spherical mass with initial density ρ, the time it takes it to collapse to a point would be

$$t_{\rm dyn} = \left(\frac{3\pi}{32G\rho}\right)^{1/2}, \tag{11}$$

and other definitions for $t_{\rm dyn}$ only differ by factors of unity. We then have three main scenarios:

1. $\mathbf{t}_{\rm cool} > \mathbf{t}_{\mathbf{H}}$. In this case the Universe expands faster than the gas can cool and no significant collapse can take place.

2. $\mathbf{t}_{\rm dyn} < \mathbf{t}_{\rm cool} < \mathbf{t}_{\mathbf{H}}$. In this case the system evolves quasi-statically but while some contraction takes place it is not sufficient for galaxy formation.

3. $\mathbf{t}_{\rm cool} < \mathbf{t}_{\rm dyn}$. In this case energy can be removed sufficiently quickly to allow rapid gravitational collapse to take place.

Before discussing point 2 and 3 in more detail it is worth commenting on the fact that I have implicitly assumed that $t_{\mathrm{dyn}} < t_H$. Is this always true? As we will see below it is not always true but in general it is a good approximation.

If we write $\rho = \Delta_c \bar{\rho}$, which is the criterion for a collapsed structure, we can write $\bar{\rho} = \Omega_m \rho_{\mathrm{crit}}$ and hence

$$\rho \approx \Delta_c \Omega_m \frac{3H(z)^2}{8\pi G}, \tag{12}$$

so $t_{\mathrm{dyn}} \sim (\Delta_c)^(- 1/2) H(z)^{-1} \sim \frac{1}{10} t_H$, since $\sqrt{\Delta_c} \sim 10$. So at collapse, t_{dyn} is typically shorter than t_H, but this might not hold in general (as we will see an example of below).

$\mathbf{t}_{\mathrm{dyn}} < \mathbf{t}_{\mathrm{cool}} < \mathbf{t_H}$ In this case, collapse proceeds quasi-statically. It is useful in this case to introduce the Jeans length, λ_J, and Jeans mass, M_J. The Jeans length is the length scale that a pressure wave travelling at the speed of sound, c_S, can traverse in one free-fall time, thus it is the maximum scale that pressure support can withstand gravity on.

$$\lambda_J \approx \left(\frac{kT}{\mu m_P} \right)^{1/2} (G\rho)^{-1/2}, \tag{13}$$

for an ideal gas. The scaling of the Jeans mass is then given by

$$M_J \propto T^{3/2} \rho^{-1/2}. \tag{14}$$

Thus at constant Jeans mass, a drop in temperature (from cooling) implies an increase in density according to this formula. But because the time-scale of cooling is long, it is reasonable to approximate the evolution as polytropic interspersed with small drops in temperature.

During polytropic evolution $P \propto \rho^\gamma$ and $T \propto \rho^{\gamma-1}$. For a monatomic ideal gas $\gamma = 5/3$ and $\rho \propto T^{3/2}$. For a constant Jeans mass we can see from equation (14) that $\rho \propto T^3$.

Thus if you have a system with $M = M_J$, then drop the temperature slightly it will move to a lower temperature and then adiabatic collapse will take place (because the cooling time is long) and this will move the system back to the constant Jeans mass again. This is a quasi-static contraction and in effect the system evolves along tracks of constant Jeans mass.

$\mathbf{t}_{\mathrm{cool}} < \mathbf{t}_{\mathrm{dyn}}$ This is the interesting case. Here the loss of pressure support due to cooling is rapid and as a result the Jeans mass drops quickly without giving the system a chance to re-adjust its density. This drop in Jeans mass can lead to smaller structures being able to collapse gravitationally and hence to fragmentation. The drop in pressure support also leads to rapid collapse.

To quantify this criterion, we need to quantify the cooling time. The total energy of a mon-atomic gas is $E = 3nkT/2$, where n is the total number density. As for $\dot{E}$, it is common to write equation (8) as

$$\dot{E} = n_H^2 \Lambda(T), \tag{15}$$

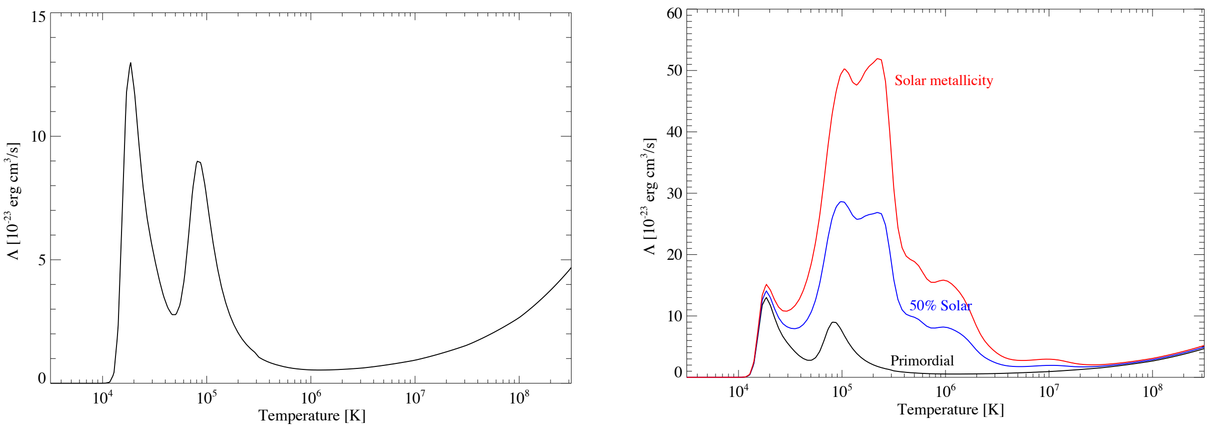


Figure 1: Left: The cooling curve for primordial gas. Right: The cooling curves for primordial gas compared to gas with varying degrees of heavier elements. The values for Λ is taken from the data provided by Wiersma et al (2010, MNRAS, 409, 132).

where Λ is the cooling function, and while I have indicated the dependence on temperature here, it also depends on metallicity.

Does it depend on the density? The main cooling processes are, as we will see later, collisional in nature, so their rates depend on the density squared. This is why the density is taken out of the equation. Note that there are different definitions in the literature and the pre-factor can be n_H^2 as here or $n_e n_H$ or even n_e^2. For a fully ionized gas it is easy to convert between these using the expressions in the lecture notes for lecture 6 and the associated problem class, but it is of course important to be aware of this issue.

As we will see later Λ has a typical scale of $10^{-23}\,\mathrm{erg\,cm^3\,s^{-1}}$, so inserting typical values into the expression (8) we have

$$
\begin{aligned}
t_{\mathrm{cool}} &= \frac{E}{n_H^2 \Lambda} = \frac{\frac{3}{2} nkT}{\left(\frac{4}{9}\right)^2 n^2 \Lambda} && (16)\\
&\approx 3.3 \times 10^9 \left(\frac{T}{10^6\ \mathrm{K}}\right) \left(\frac{n}{10^{-3}\ \mathrm{cm}^{-3}}\right)^{-1} \left(\frac{\Lambda}{10^{-23}\ \mathrm{erg\ cm^3\ s^{-1}}}\right)^{-1} \text{years} && (17)\\
&\equiv 3.3 \times 10^9\, T_6\, n_{-3}^{-1}\, \Lambda_{-23}^{-1}\ \text{years}. && (18)
\end{aligned}
$$

Note that the last equivalence, which defines T_6, n_{-3} and Λ_{-23}, suppresses all unit information and while this is a convenient notation it is prone to silly mistakes at times. It does however save space so I will make use of it at times, as long as the constituents have been defined.

Armed with this expression we can now compare t_{cool} to t_{dyn}, where the latter is defined in equation (11). If we also define $f_{\mathrm{gas}} = \rho_{\mathrm{gas}}/\rho$ to be the gas fraction of the halo with total density ρ, and note that $\rho_{\rm gas} = n\mu m_P$, we get

$$\begin{aligned} t_{\rm cool} &= t_{\rm dyn} \\ n_{-3} &= 245\, T_6^2\, f_{\rm gas}^{-1}\, \Lambda_{-23}^{-2}, \end{aligned} \tag{19}$$

which defines a locus in n–T space above which collapse can happen rapidly, and below which collapse is slowed down.

To properly quantify this we need to know what Λ looks like in detail. In the next lecture we will look at the physical processes that go into the calculation of Λ, but the result for a gas of primordial composition, $X = 0.75$, $Y = 0.25$, is shown in the left plot in Figure 1. The rise in cooling towards very high temperatures is due to Bremsstrahlung. The peak closest to $T = 10^4$ is due to collisional excitation of H followed by the emission of a Ly-α photon. The second, lower, peak is due to the same process in singly ionized He.

In any gas with some heavier elements, the cooling is significantly altered as can be seen in the right plot in Figure 1. Here we compare three different metallicities. The curves are all collisional ionization equilibrium calculations from Wiersma et al (2010, MNRAS, 409, 132). We will look at the details next time, but here we should note that cooling, especially in $T \sim 10^5$K halos is significantly increased when you add metals.

If we now move to the density–temperature diagram, we can use equation (25) to delinate the different regions of parameter space. This is shown in Figure 2. The left panel shows the trend for primordial abundances and the right panel that for solar metallicity. The black line shows the locus for $t_{\rm cool} = t_{\rm dyn}$ and the red line $t_{\rm cool} = t_H$, drawn for $z = 3$. Any object lying above both lines will be able to cool efficiently and hence collapse. The right-side y-axis shows the redshift of collapse for a given density. I have estimated this from

$$n_{\rm gas} \approx 1.9 \times 10^{-5}\, \Omega_m h^2 f_{\rm gas} (1 + z_{\rm coll})^3 (1 + \delta)\, {\rm cm}^{-3}, \tag{20}$$

which you will prove in a problem set.

The dotted sloping lines show lines of constant gas mass,

$$M_{\rm gas} = f_{\rm gas}^{3/2} \left(\frac{5k}{G\mu m_P}\right)^{3/2} (\mu m_P)^{-1/2} n^{-1/2} T^{3/2} \left(\frac{6}{8\pi}\right)^{1/2}, \tag{21}$$

which you also will derive in a problem set.

Some key points to note from Figure 1 and 2 is that cooling is very efficient at $T = 10^4$–10^5K. Thus systems with virial temperatures in this region are expected to cool very efficienctly (in the absence of heating).

We can also see that cooling is more efficient at high redshift and is relatively smaller halos. Since CDM has a lot of small-scale structure and we saw earlier that the small scale structure collapses first, we must conclude that a very large fraction of gas could cool and potentially form stars at high redshift. If this were to happen the mass density in stars would approach that of baryons in total, but in reality $\Omega_* \approx \Omega_b/10$. To resolve this minor conundrum we have either to postulate a source of feedback that will blow gas out of their halos, or introduce heating sources. The latter will of coruse always be present when stars

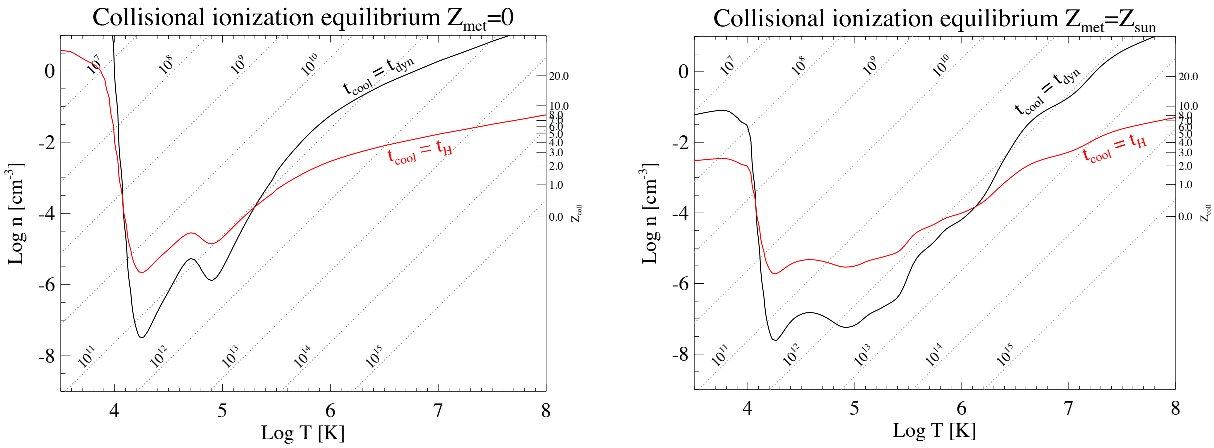
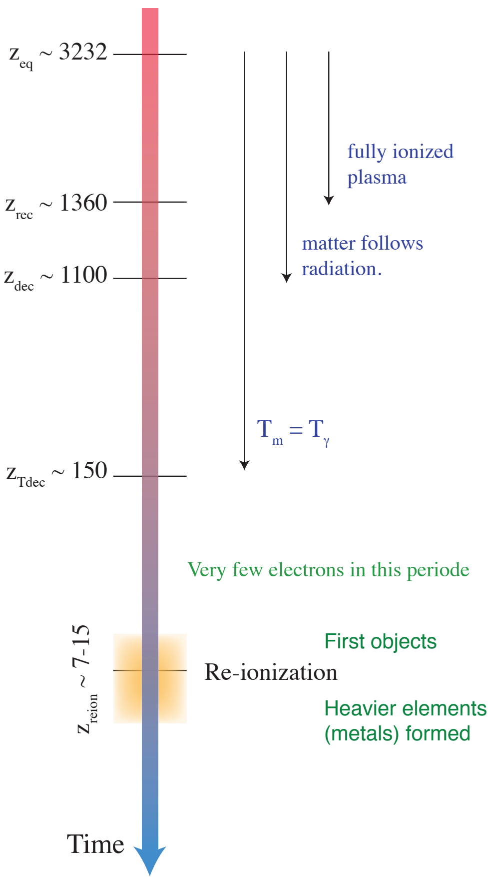


Figure 2: The density-temperature diagram. On the left we show the diagram for primordial gas and on the right for solar metallicity. In both cases the black line shows the locus for $t_{\rm cool} = t_{\rm dyn}$ and the red line $t_{\rm cool} = t_H$, drawn for $z = 3$. Where relevant I have set $\Omega_m = 0.3$ and $f_{\rm gas} = 0.15$. Objects that lie above both the red and black lines are able to cool efficiently. The right y-axis shows the redhshift at which the density of a collapsed object is the value of n given on the main y-axis. The dashed lines show lines of constant $M_{\rm gas}$.

and/or AGNs are present and they are crucial ingredients for understanding the behaviour of gas at high redshift.

The final thing to note from Figure 2 is that massive clusters today, with gas masses of perhaps $10^{13} M_\odot$, can cool only inefficiently. This is one reason why galaxy clusters today contain a lot of hot gas.

4 Radiative Processes



We will now turn to a more in-depth discussion of the radiative processes that cool gas. To set the context it is useful to quickly remind ourselves of the radiative evolution of the Universe. The key fact for us, which we will return to later below, is that all cooling processes require some free electrons. Thus we would like to have a feel for when there are free electrons around.

The figure on the left illustrates the evolution schematically. The Universe is almost fully ionized until the time of recombination. At that point the ionization fraction of the Universe dropped rapidly and some time later it became too low to sustain the close link between photons and baryons. Thus after the redshift of decoupling, $z_{\rm dec}$, photons and baryons were no longer strongly coupled.

However, there was a minor amount of ionization left. This tiny amount of ionization was sufficient to ensure that the photon and baryon *temperature* was essentially the same until $z_{\rm Tdec} \sim 150$. After that the matter temperature declined $\propto (1+z)^2$ and the photon temperature as $\propto (1+z)$.

From the time of recombination, the number of free electrons was in general very low in the Universe. It would have been possible to create some free electrons from virial heating in very rare peaks at high redshift, but the epoch of re-ionization was when the Universe became mostly ionized again. This is the state until now — while many regions of neutral matter exists today, the low density intergalactic medium is generally ionized and there are plenty of free electrons around.

5 Radiative (cooling) processes

The main radiative processes that lead to cooling of ionized plasmas are listed in Table 2. These are all two-body processes, so the likelihood of the process occuring is $\propto n_1 n_2 \sim \rho^2$. This is the root cause why we separate out a $n_{\rm H}^2$ in our expression for the cooling function, Λ, writing $\dot{E} = n_H^2 \Lambda$. Note also that all the reactions require the presence of electrons. The need for electrons means that these processes are inefficient until the Universe re-ionizes.

The density dependence on the energy loos, means that we can write

$$\dot{E} \propto \rho^2 \Rightarrow t_{\rm cool} \sim \frac{\rho}{\dot{E}} \sim \rho^{-1}. \tag{22}$$

Type	Reaction	Name
Free-Free	$e^- + X^+ \longrightarrow \gamma + e^- + X^+$	Bremsstrahlung
Free-Bound	$e^- + X^+ \longrightarrow X + \gamma$	Recombination
Bound-Free	$e^- + X \longrightarrow 2\,e^- + X^+$	Collisional ionization
Bound-Bound	$e^- + X \longrightarrow e^- + X\cdot$	Collisional excitation

Table 2: Various radiative transitions of importance in forming galaxies

Thus we see that cooling is more efficient in more dense gas.

But note that here, and in most of the following, I am assuming that the radiation produced in these reactions actually can escape the cooling cloud. This is not necessarily the case, but for now we make this assumption of optically thin gas.

I am also ignoring any sort of photoionization, so collisions will dominate. This setup is referred to as collisional ionization equilibrium (CIE).

6 The features of the cooling curve

Figure 3 shows the cooling curve of a gas with primordial abundances, assuming CIE. This has three characteristic features: A sharp cut-off around $T = 10^4$K, two peaks, and a slow rise towards high temperatures. The goal of this section is to understand the nature of these three features.

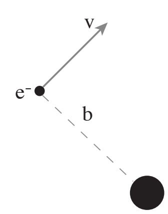
6.1 Bremsstrahlung

Let us start with the turn-up at high temperatures. This is caused by free-free emission, or Bremsstrahlung. Thus it is useful to briefly review the nature of this process. We will do this in a very approximate way.

Bremsstrahlung takes place because when electrons interact with ions, their trajectory is slightly bent and this leads to an acceleration/deceleration of the electron and as is well known, an accelerated charge radiates.

To get an order of magnitude estimate of the effect it is sufficient to consider that essentially all the interaction takes place at the closest approach between the electron and the ion. In that case we have a setup like shown just below. Newton's second law then gives

$$m_e a = \frac{q^2}{r^2}, \tag{23}$$



in CGS units. At closest approach the electron is a distance of b away. This separation is essentially the mean separation of the ions, $b \approx n_{\rm ion}^{-1/3}$. Thus we can write the acceleration of the electron as

$$a = \frac{q^2}{m_e b^2}. \tag{24}$$

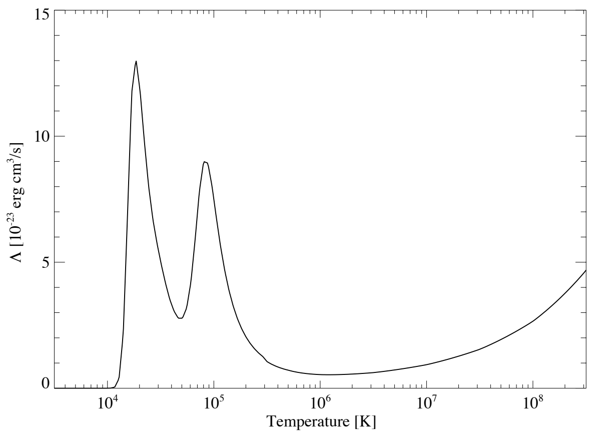


Figure 3: The cooling curve of primordial gas from Wiersma et al (2009).

Armed with Larmor's formula for the radiation of an accelerated particle with charge q we have:

$$\frac{d\mathcal{E}}{dt} = \frac{2q^2}{3c^3}|\vec{a}|^2 = \frac{2}{3}\frac{q^6}{c^3 m_e^2 b^4}. \tag{25}$$

To get an estimate of the energy radiated per interaction, we need to know how long it lasts. This Δt is given approximately by $\Delta t \sim b/v$. A more careful treatment would integrate over the path traced by the electron. Put together this gives that

$$\Delta\mathcal{E} = \Delta t \frac{d\mathcal{E}}{dt} = \frac{2}{3}\frac{q^6 n_{\rm ion}}{m_e^2 c^3 v}, \tag{26}$$

where I also inserted $b = n_{\rm ion}^{-1/3}$. This is the energy per electron, so the total energy radiated from a volume is

$$\Delta E = n_e \Delta\mathcal{E}. \tag{27}$$

In thermal equilibrium we have that

$$k_B T \approx \frac{1}{2}v^2 \quad \Rightarrow \quad v = \sqrt{\frac{k_B T}{m_e}}. \tag{28}$$

What about the frequency dependence? This is set by the time of interaction. Since the interaction has a duration $\sim b/v$, there will be little radiation emitted at frequencies longer than $\omega \sim v/b$, where ω is the circular frequency, while below this frequency you have essentially constant emission[1]. Thus the emission per volume and per time and per frequency is given by by

$$j_\omega = \frac{dE}{d\omega dt dV} \approx \Delta E \approx \frac{q^6}{m_e^2 c^3}\left(\frac{m_e}{k_B T}\right)^{1/2} n_e n_{\rm ion}. \tag{29}$$

[1]This is because you have a finite period of emission of radiation Δt. And when you Fourier transform this you get a sinc function with width $1/\Delta t \sim v/b$.

We then need to integrate this up to a maximum frequency. This can be determined by noting that an electron with energy $k_B T$ cannot emit radiation with energy above this, so we can take $h\nu = k_B T$ as the upper frequency cut-off.

This then gives an expression for the total energy radiated that is

$$j = \int_0^{k_B T/h} \frac{q^6}{m_e^2 c^3} \left(\frac{m_e}{k_B T}\right)^{1/2} n_e n_{\rm ion} = \frac{q^6}{m_e^2 c^3} \left(\frac{m_e k_B T}{h}\right)^{1/2} n_e n_{\rm ion}, \tag{30}$$

where the essential ingredient for us is that the energy radiated is $\propto n_e n_{\rm ion} T^{1/2} \sim n^2 T^{1/2}$. This is exactly the form of the up-turn seen at high temperatures in the cooling function in Figure 3.

At high temperatures, Bremsstrahlung is the dominant cooling mechanism and to get an accurate expression for this cooling one needs a more careful analysis, but this would follow mostly the steps sketched out above. The final result for a fully ionized gas with primordial abundance can be written as

$$j = C_B \approx 1.6 \times 10^{-23} \left(\frac{T}{10^8\,\mathrm{K}}\right)^{1/2} n_e^2 \mathrm{erg/scm}^3 \tag{31}$$

6.2 The low temperature cut-off

Why is there a sharp drop in the cooling curve at low temperature? This is due to the fact that there are no free electrons around in the gas. In a primordial gas, which is what we focus on at the moment, the main source of electrons at low temperature is hydrogen, with an ionization energy of 13.6 eV[2].

To find the temperature for ionization of hydrogen, we need to be able to estimate the temperature at which hydrogen can be collisionally ionized. To do this for CIE you need to equate the rate of collisional ionization to that of recombination. I do not go through the derivation here, but a good rule of thumb is that the temperature is given by

$$T_{\rm ionization,H} \approx \frac{1}{10} \frac{13.6\ \mathrm{eV}}{k_B} \approx 1.6 \times 10^4\ \mathrm{K}, \tag{32}$$

or in other words about 1/10 of the temperature implied by the ionization energy. Thus below $T \sim 10^4$K there will be very few free electrons and the cooling reactions will be inefficient.

One might think that perhaps some cooling could take place from excited levels of hydrogen where lower temperatures are sufficient for the excitation. However the life times of the hydrogen levels is 10^{-4} to 10^{-8} seconds, with the slowest transition being the $2S \to 1S$ transition which has a life time of 0.12s. Thus for most practical purposes we can consider tha any excited level in hydrogen will cascade instantly to the ground level. Thus any excitation in hydrogen will have to overcome the energy difference between the

[2]The ionization energy of helium is 24.6 eV for the first electron and 54.4 eV for the second.

$n = 1$ and $n = 2$ level, which is 10.2 eV. Using the same factor of 1/10, this translates to a temperature requirement of

$$T_{\text{excitation,H}} \approx 1.2 \times 10^4 \text{ K}. \tag{33}$$

Thus below 10^4K neither excitation nor ionization of hydrogen is efficient and the cooling function drops quickly. If there are metals around things are easier and although there is a drop it is not as complete as for a primordial gas. To cool a primordial gas below 10^4K, a requisite for galaxy formation!, we need molecular cooling but that is for a later lecture.

6.3 The cooling peaks

It is perhaps not obvious why the cooling function should have clear peaks. The majority of the processes listed in Table 2 have smooth behaviour with temperature. We saw this with Bremsstrahlung above, but this is also true for recombination where the emitted photon radiates away the energy from the electron in excess of 13.6eV (or lower if recombination was to a higher level in hydrogen). This is a smooth function of temperature; and this is also true for collisional ionization.

However for collisional excitation we have a limited range of energies. In collisional excitation a collision between an electron and an atom/ion will excite this, in the case of the first peak this is Hydrogen. The hydrogen will then deexcite and radiate the energy away. However this process can only operate while there are hydrogen atoms around — when the temperature gets to be too high, the hydrogen is all ionized and there are no bound transitions left.

This then is the reason for the peaks in the cooling curve. It turns out that when one solves the equilibrium equations, that the process of collisional excitation is the dominant cooling process at low energies and it operates efficiently, as explained above, only in particular ranges of temperature.

If we add metals, this argument is still true. Figure 4 shows the cooling function for solar metallicity. The left panel is what concerns us here and it shows the cooling function for CIE. The solid black line shows the overall cooling curve and the coloured curves show the contributions from individual elements as indicated.

Again it is clear that for most elements there is a broad peak and its existence is again for the same reasons as above. If we take oxygen as an example, it will at low temperatures radiate through excitation of neutral O. Then, as the temperature increases, O will be mostly ionized and cooling occurs through exictation of O^+. As the temperature increases, this pattern continues and you get a broad-ish peak in the cooling function.

In the case of oxygen it is also clear that it has two peaks. This is because there is a big jump between the ionization energy of O_5^+, which is is $\approx$ 140eV, and that of the excitation energy for the lowest level of O_6^+, which is $\approx$ 560eV[3]. Thus in the temperature range few$\times 10^5$K to few$\times 10^6$K oxygen is mostly in the form of O_6^+ but there is not enough energy to excite this ion significantly, thus its contribution to the cooling drops significantly.

[3]If you need such numbers, the NIST data base at http://www.nist.gov/pml/data/asd.cfm is a useful resource.

Wiersma, Schaye & Smith 2009, MNRAS, 393, 99

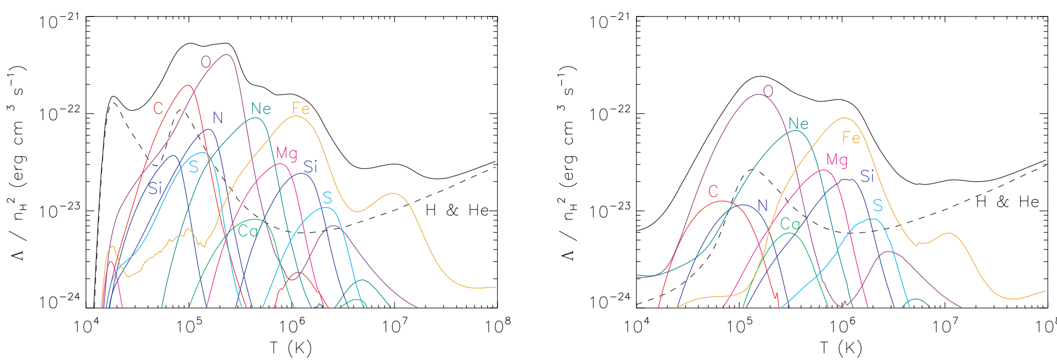
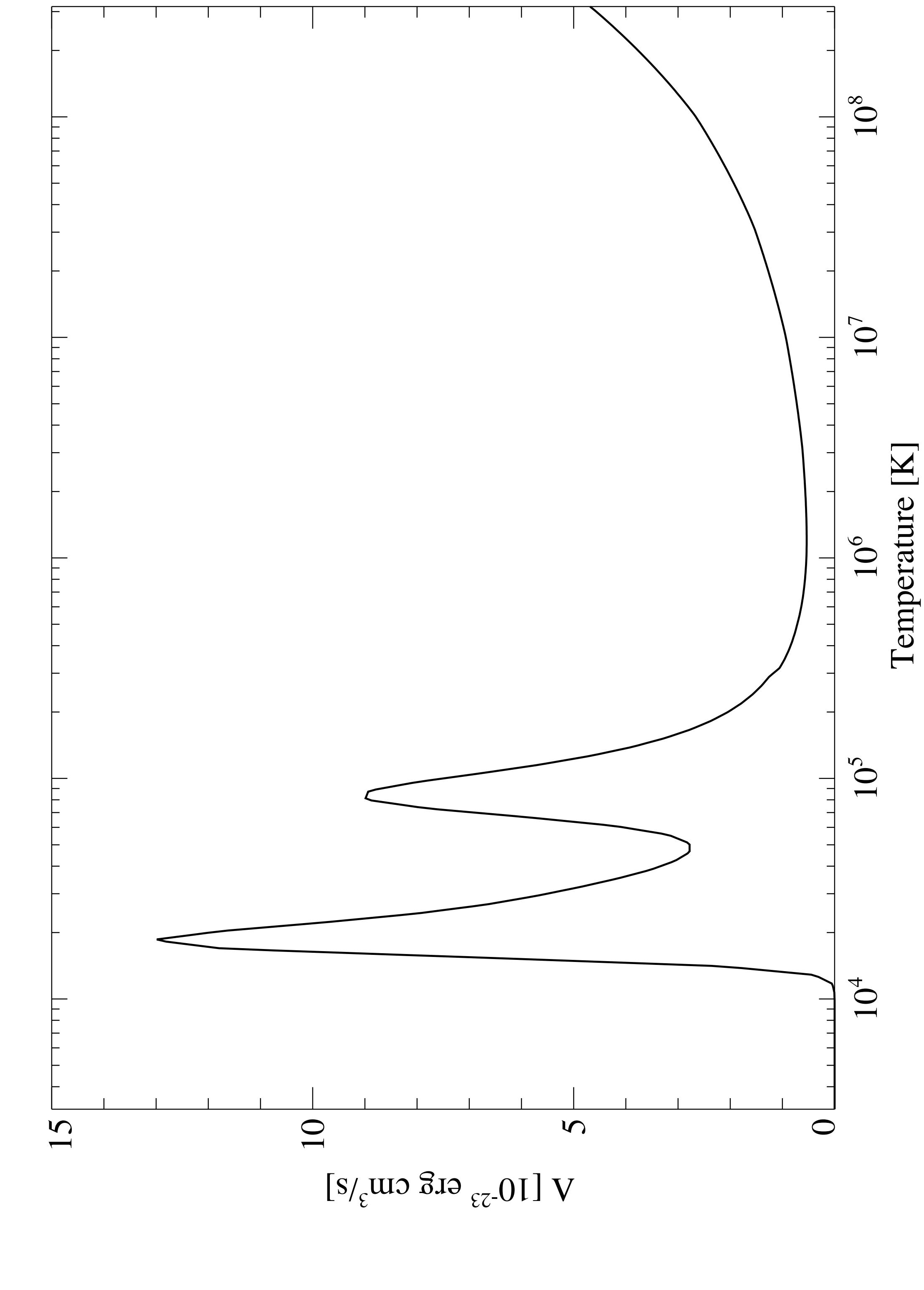
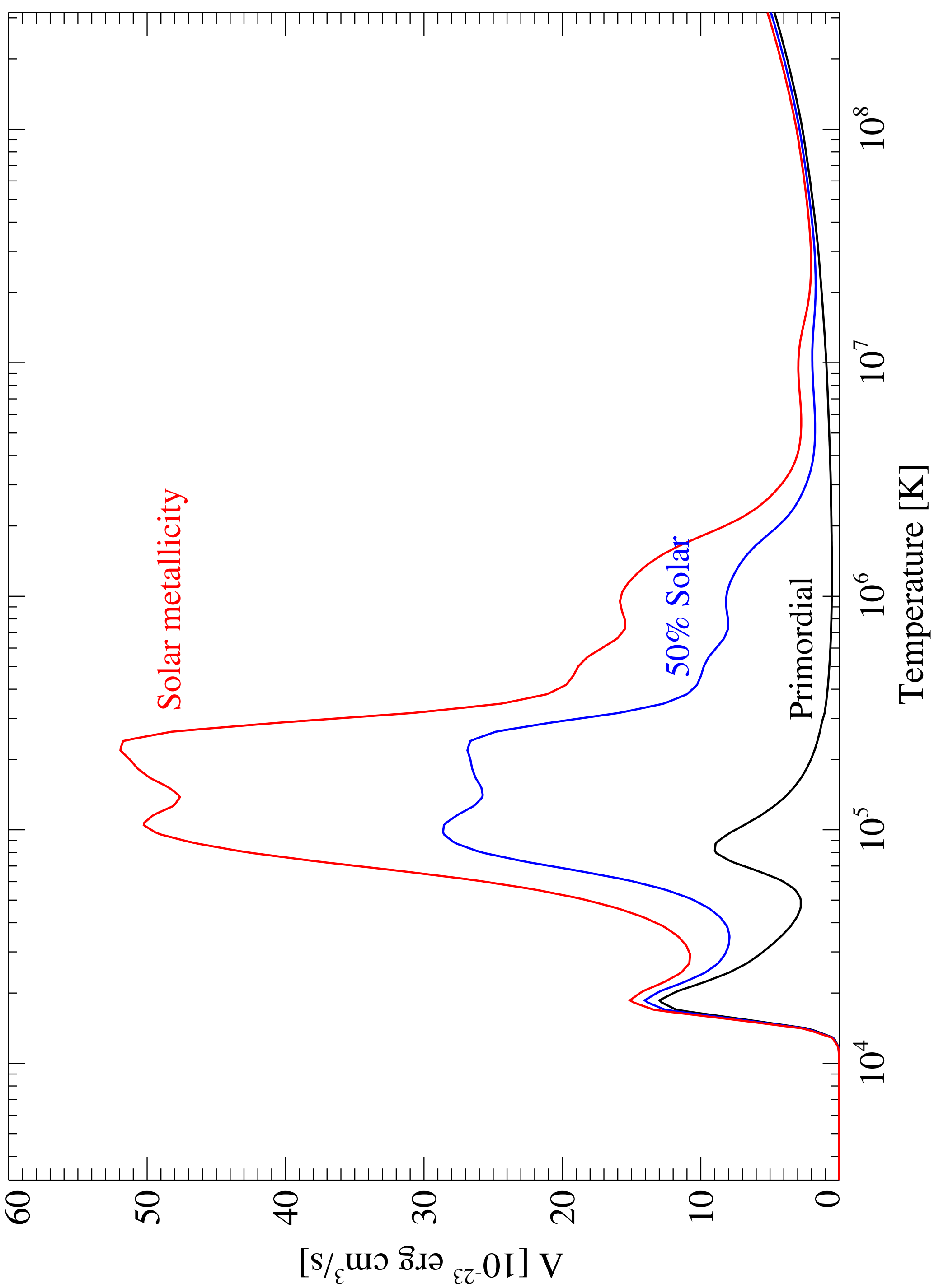
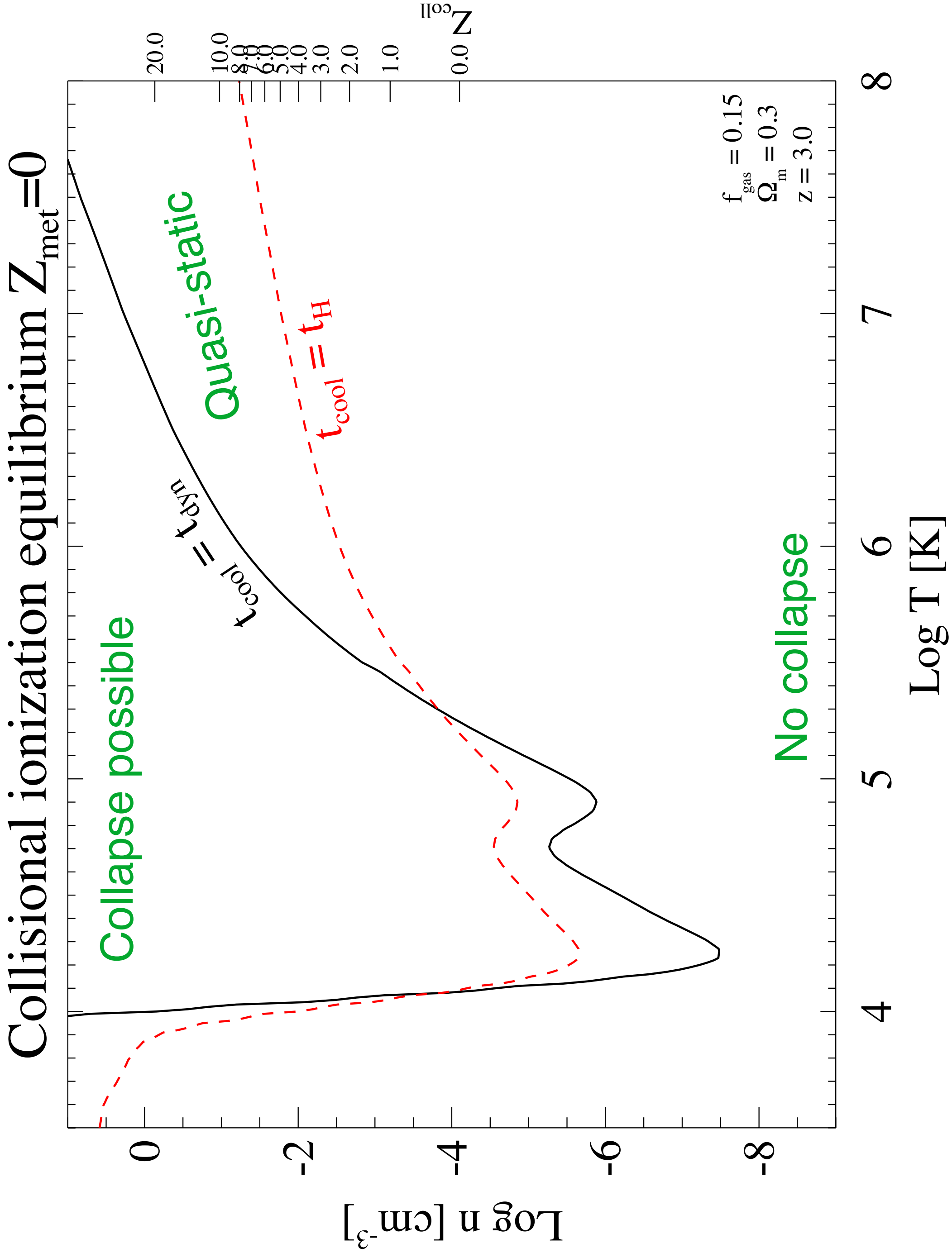


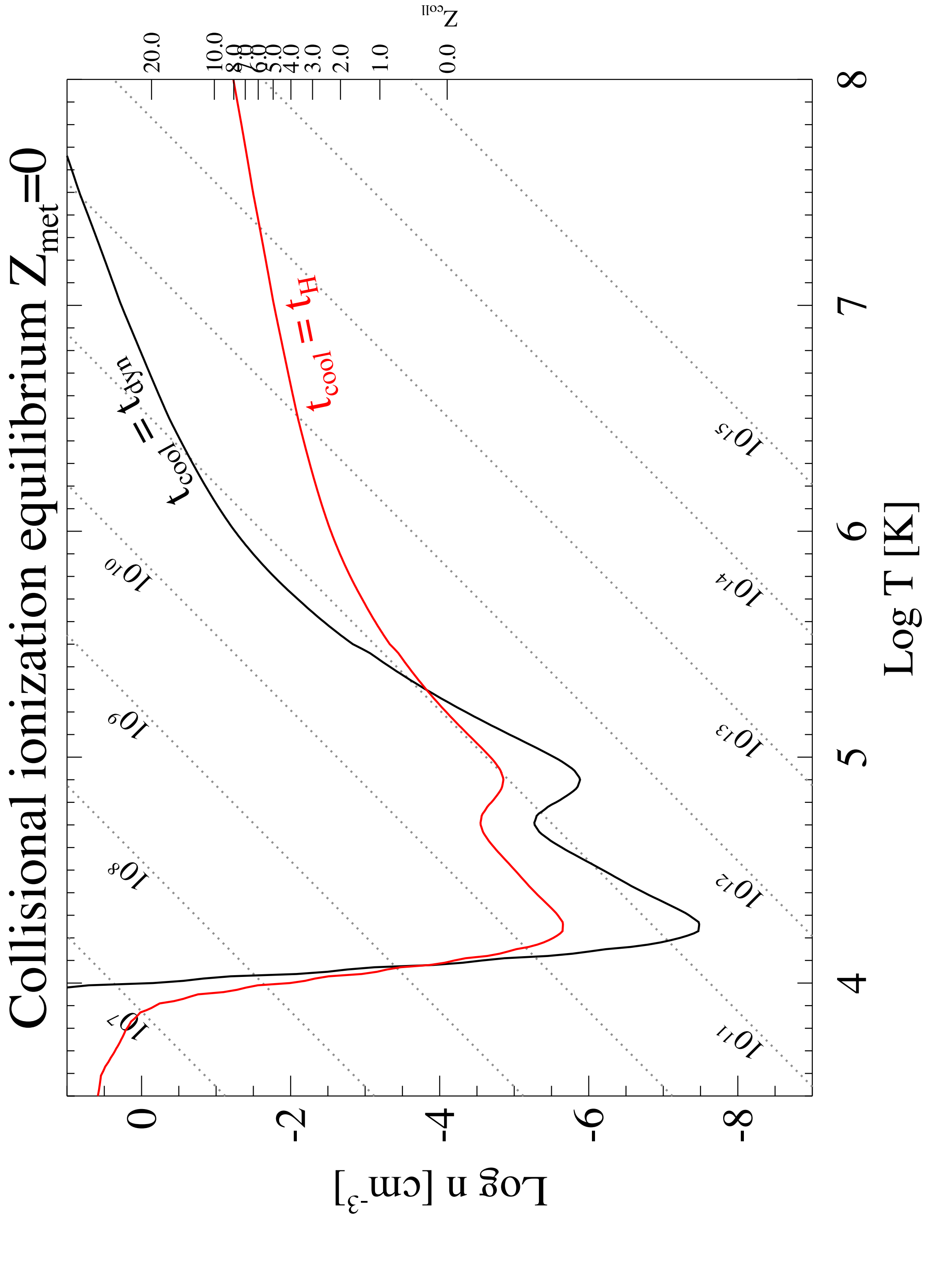
Figure 4: Cooling curves from Wiersma et al (2009) for solar abundance plasma. The left hand panel shows the cooling function for CIE and the right for photo-ionization equilibrium. The different colours show the cooling due to different elements.


Temperature [K]
Λ [10-23 erg cm3/s]
0
5
10
15
10^4
10^5
10^6
10^7
10^8

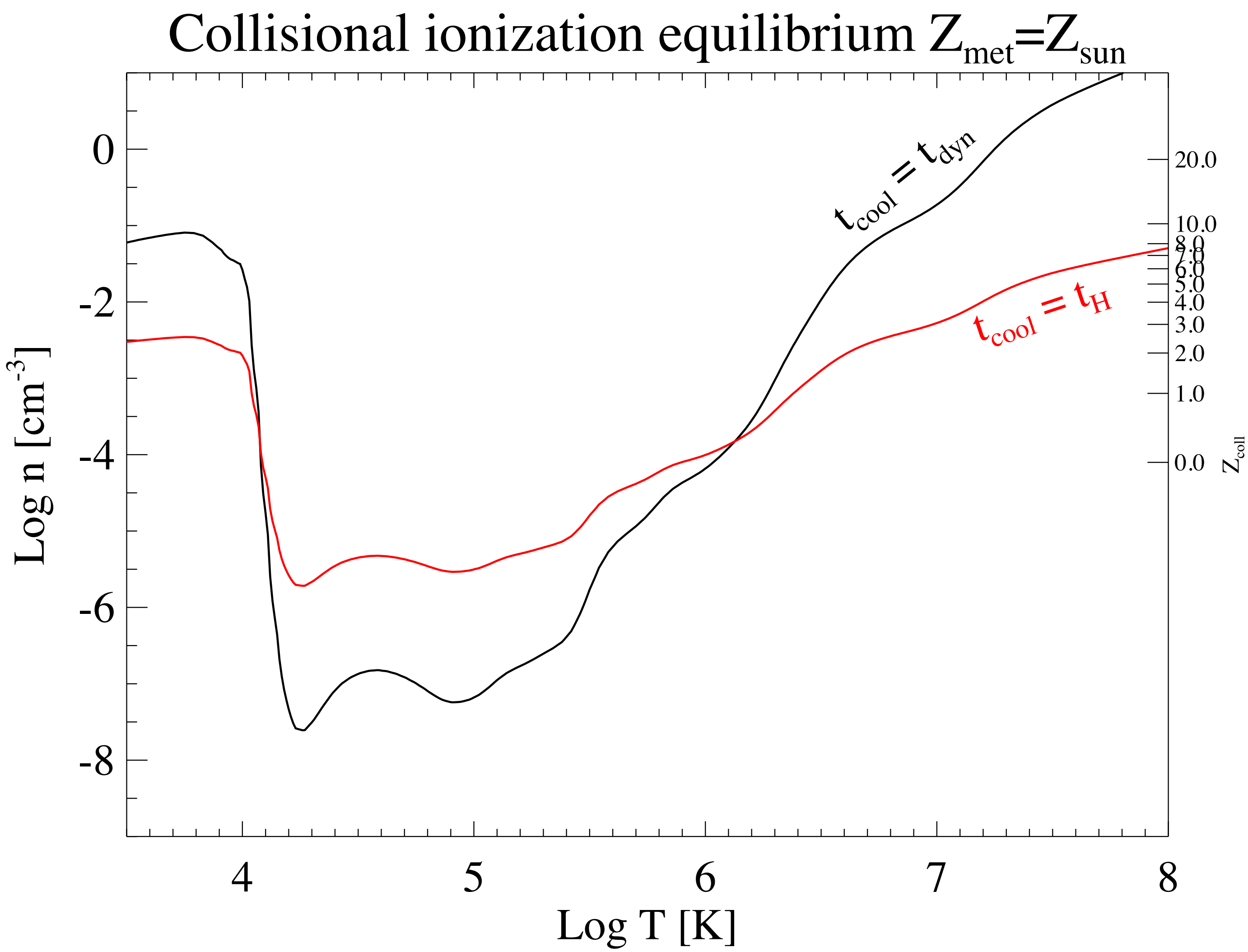


Solar metallicity
50% Solar
Primordial
Temperature [K]
Λ [10-23 erg cm3/s]
0
10
20
30
40
50
60
104
105
106
107
108

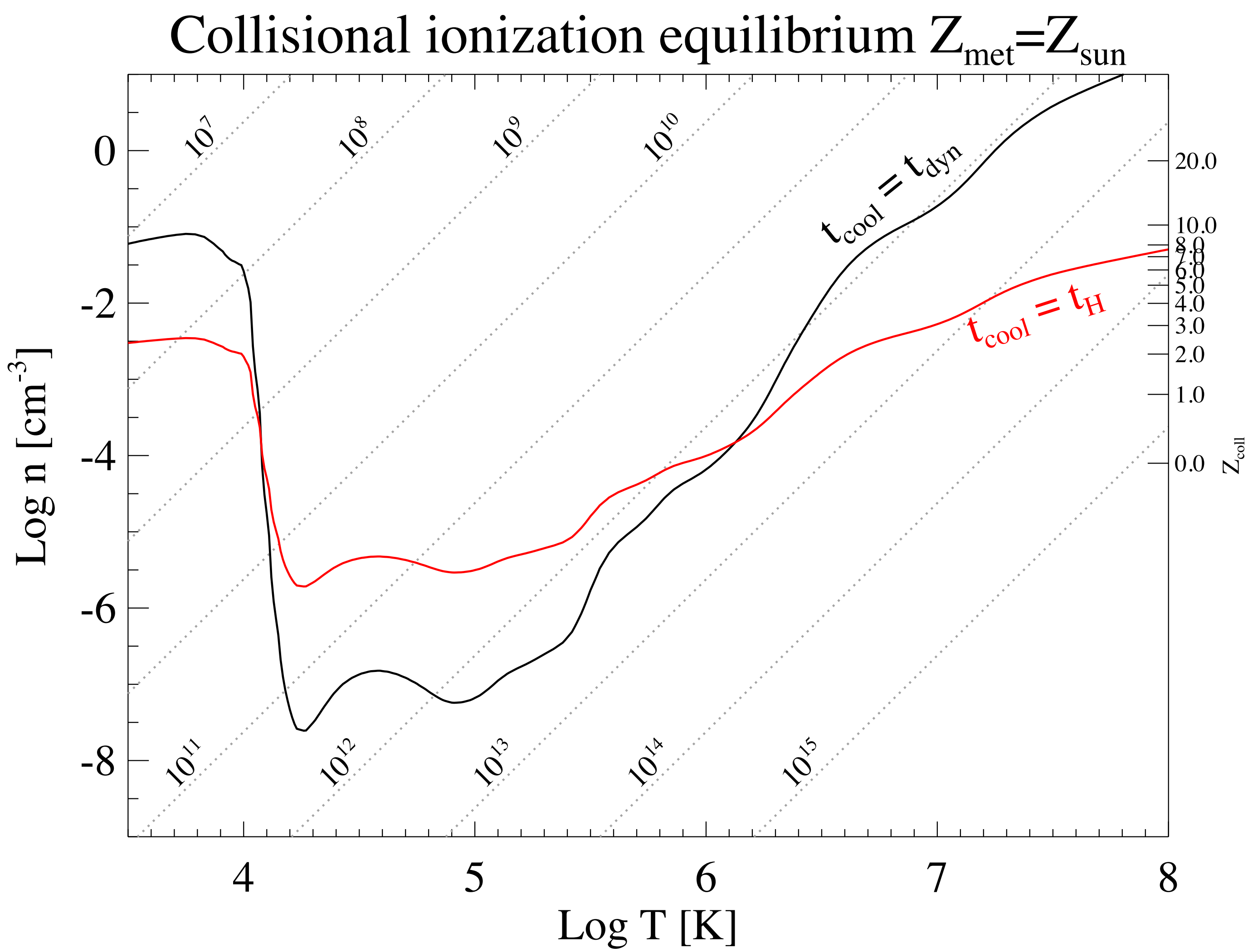

Collisional ionization equilibrium Z_{met}=0
Collapse possible
Quasi-static
No collapse
$t_{cool} = t_{dyn}$
$t_{cool} = t_H$
$f_{gas} = 0.15$
$\Omega_m = 0.3$
$z = 3.0$
Log T [K]
Log n [cm^{-3}]
Z_{coll}
20.0
10.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
4
5
6
7
8
0
-2
-4
-6
-8



Collisional ionization equilibrium Z$_{met}$=0
Log T [K]
Log n [cm^{-3}]
t$_{cool}$ = t$_{dyn}$
t$_{cool}$ = t$_{H}$
10^{7}
10^{8}
10^{9}
10^{10}
10^{11}
10^{12}
10^{13}
10^{14}
10^{15}
Z$_{coll}$
20.0
10.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0



Collisional ionization equilibrium $Z_{met}=Z_{sun}$
Log n [cm^{-3}]
0
-2
-4
-6
-8
4
5
6
7
8
Log T [K]
$t_{cool} = t_{dyn}$
$t_{cool} = t_H$
20.0
10.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
Z_{coll}



Collisional ionization equilibrium $Z_{met}=Z_{sun}$
Log n [cm^{-3}]
Log T [K]
$t_{cool} = t_{dyn}$
$t_{cool} = t_{H}$
10^7
10^8
10^9
10^{10}
10^{11}
10^{12}
10^{13}
10^{14}
10^{15}
Z_{coll}
20.0
10.0
3.0
2.0
1.0
0.0
0
-2
-4
-6
-8
4
5
6
7
8